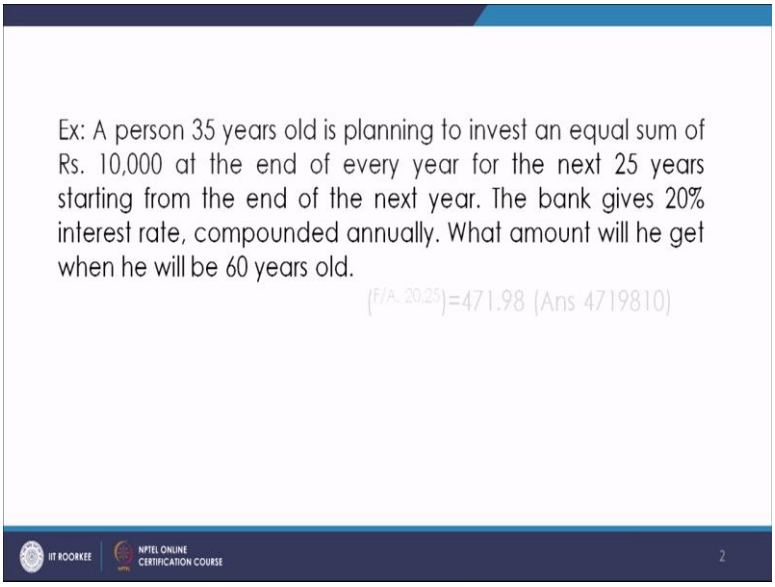
Engineering Economic Analysis
Professor Dr. Pradeep K Jha
Department of Mechanical and Industrial Engineering
Indian Institute of Technology Roorkee
Lecture 06
Problem Solving on Discrete Compounding, Discrete Payment

Welcome to the lecture on problem solving based on interest formulas. So in this lecture we will try to solve problems numerical problems based on the factors of interest what we have so far study and try to solve it. Let us see the first example.

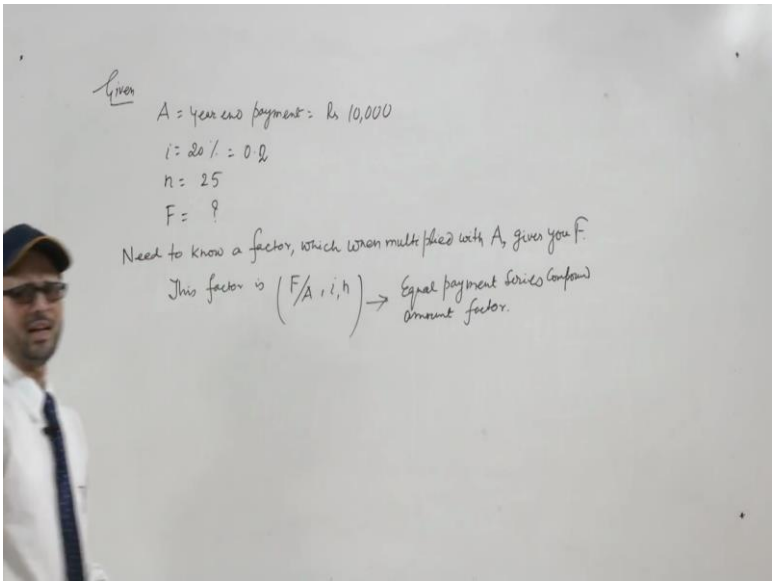
(Refer Slide Time: 1:04)



There is a problem which tells that a person 35 years old is planning to invest an equal sum of Rs. 10,000 at the end of every year for the next 25 years starting from the end of the next year. The banks gives 20% interest rate, compounded annually. What amount will he get when he will be 60 years old? So as it indicates, basically we are going to discuss all the problems based on equal payments series factors or uniform series factors.

We have already solved one problem based on single payment series present worth factor and single payment series compound amount factor. So we will discuss the problems based on equal payments series or uniform series factor. Now let us see in this problem, basically a person he is investing Rs. 10,000 equal sum at the end of every year. Year end is so it means in this problem A is given.

(Refer Slide Time: 04:31)

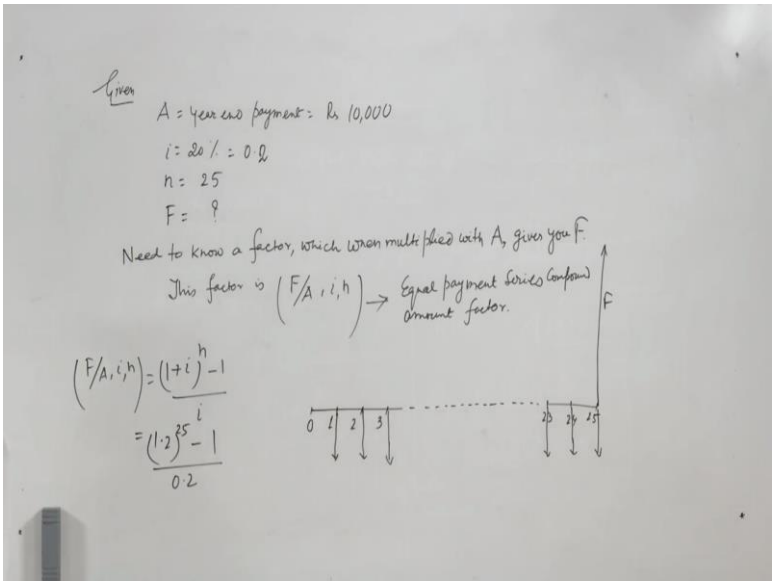


A that is yearend payment and this is given as 10,000. Then this amount he is depositing for the next 25 years, starting from the end of the first year and the bank is giving 20% interest rate. So i is 20% that is 0 point 2. Your since he is going to have the amount when he is 60 years old, so anyway he is going to deposit this amount for 25 years and that is why your interest periods are 25.

So before solving the problems based on these interest factors, it is advisable first of all to draw the cash flow diagrams. Now as you know here you have to basically find the future amount what amount he will get. So basically you are trying to get F. Now we know that you get F and you know A. So basically you need to know a factor which when multiplied with A gives you F.

And we have studied already this is a factor nothing but, this factor is F by A i n and we have also termed it as equal payment series compound amount factor. So basically first of all as we discussed that first of all we will try to draw the cash flow diagram. The cash flow diagram will look like this.

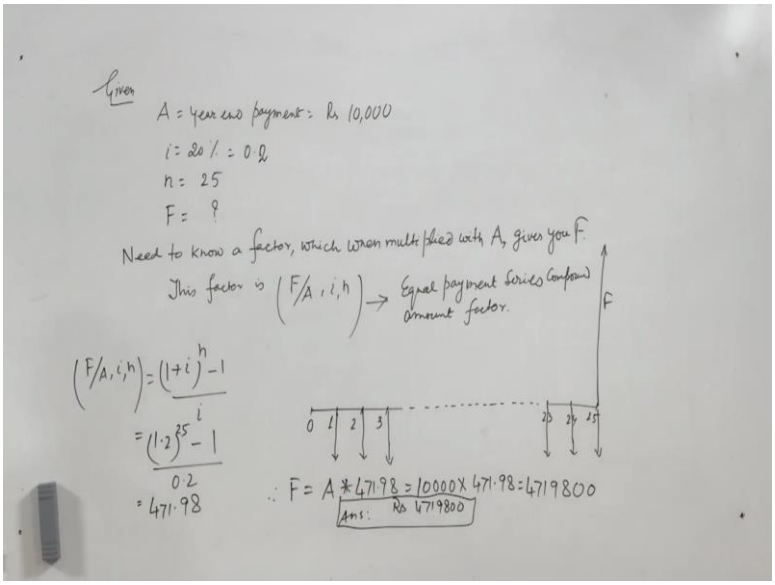
(Refer Slide Time: 06:12)



Starts from 0, 1, 2, 3 and it moves 23, 24 and 25. So every year and he is a depositing an equal amount of Rs. 10,000 and in the end he will get F. So this F for that we know that F by A i n is nothing but 1+ i n- 1 by i. And we know the values of i and n so we will put these values. 1 point 2 raised to the power 25 - 1 divided by 0 point 2. So this is basically the value of equal payment series compound amount factor.

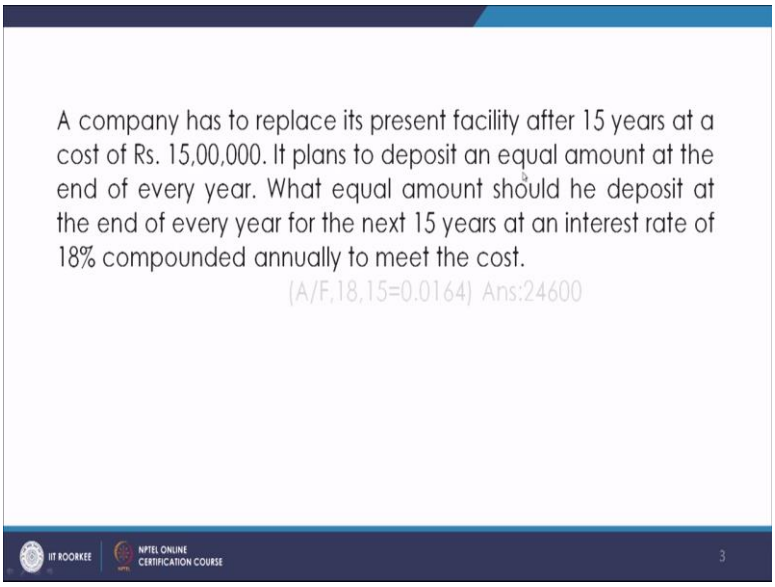
So this is basically the value of equal payment series compound amount factor and this factor comes out to be 471.98. Now this factor is to be multiplied with this amount 10,000 and it will give you the amount which will be accumulated at the end of 25 years. So your F becomes so F be is coming as A into 471.98 that is A is 10000 multiplied by 471.98 so that comes out to be 4719800.

(Refer Slide Time: 07:58)



So ultimately the answer will be the person who is depositing Rs. 10,000 at the end of every year for 25 years at a 20% interest rate which is compounded annually, he will be able to deposit 4719800 at the end of 25 years. So the answer is Rs. 4719800. So this is how we come to say we have used this equal payment series compound amount factor in this problem. Now we will move to the problem number two.

(Refer Slide Time: 08:11)

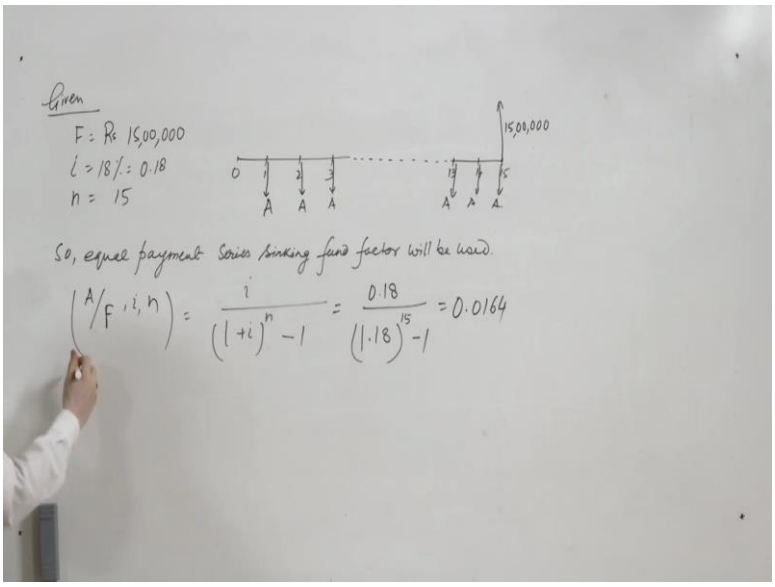


Problem number two is, a company has to replace its present facility after 15 years at a cost of Rs. 1,500,000. So the company wishes to replace the facility after 15 years and after 15 years it is expected to invest Rs. 1,500,000. It plans to deposit an equal amount at the end of

every. So for getting that amount after 15 years, it is planning to deposit certain and equal amount at the end of every year.

Now the aim is in this that what amount he should deposit every year end, what equal amount he should deposit every year end so that at the end of 15 years he is able to get Rs. 1,500,000 so that he can upgrade the facility of his industry or his company. So in this case as we know it is given that now as usual first of all it is better to draw the cash flow diagrams.

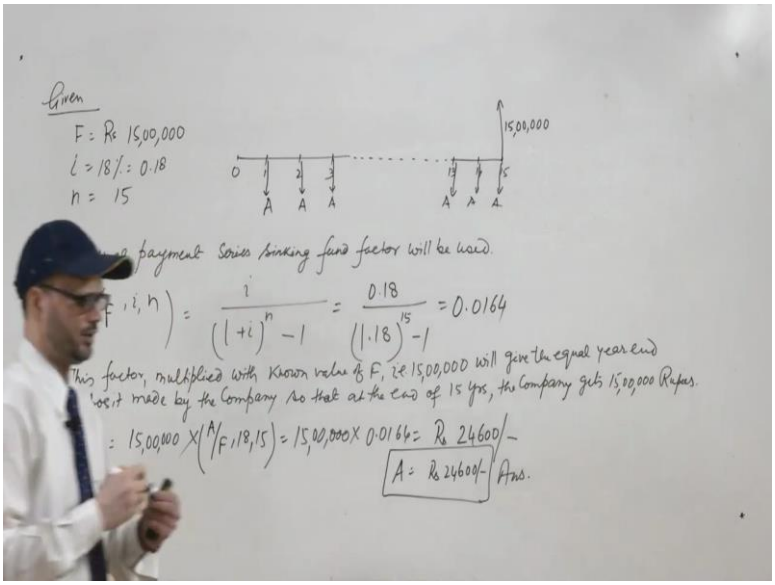
(Refer Slide Time: 12:47)



So we will draw the cash flow diagram first, at the end of 15 years the company wishes to get 15,00,000. Now for that, what amount this company should deposit now every year end. This A is to be found out. So given F as 15,00,000, i interest rate is given as 18% per annum that is 0 point 18 and n is 15.

So here this is a case equal payment series sinking fund factor where the company is setting aside a certain amount every year so that it is getting some known amount at the end of n years and in this case 15 years and that is why equal payment series sinking fund factor that is A by F. So equal payment series sinking fund factor will be used. Now as we know, equal payment series sinking fund factor is nothing but i upon 1 + i raised to the power n - 1.

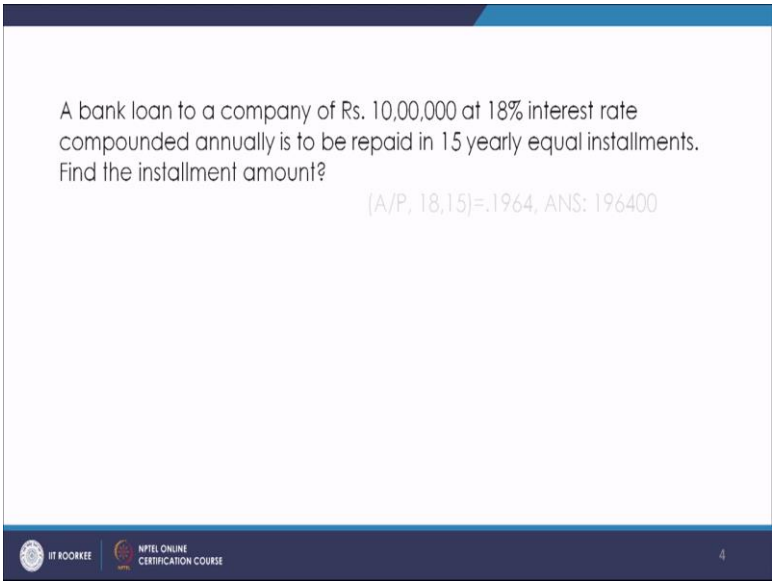
(Refer Slide Time: 14:49)



So in this case it will be 0.18 divided by 1.18 raised to the power 15 - 1 and this factor comes out to be .0164. So this factor when multiplied with F, this factor when multiplied with F will give, this factor multiplied with known value of F that is 15,00,000 will give the equal year end deposit made by the company so that at the end of 15 years the company gets Rs. 15,00,000.

So that is why A will be 15,00,000 multiplied by this factor A by F 18 15 that is 15,00,000 multiplied by .0164 and this comes out to be Rs. 24,600. This is the answer. So it means if the company deposits every year end an amount of Rs. 24,600, it is likely to get 15,00,000 of rupees at the end of 15 years. We will move to our next problem.

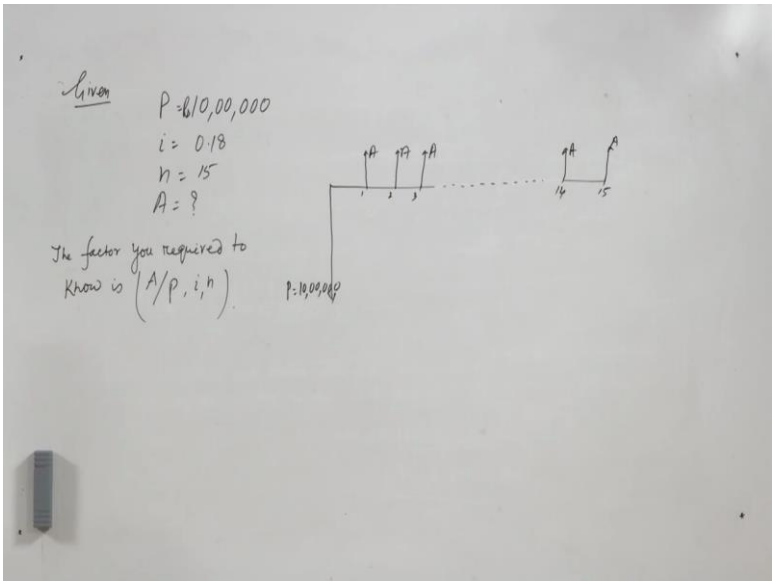
(Refer Slide Time: 15:13)



The next problem is that a bank loan to a company of Rs. 1,000,000 at 18% interest rate compounded annually is to be paid in 15 yearly equal instalments. Find the instalment amount. So basically this cases we come across in our everyday life.

We take a loan and we try to pay basically we pay month end but suppose we are supposed to pay your end so this represent such cases where the loan is given to a company of the order of 10,00,000 now. So it means P is known to you. The bank is giving the amount so P is known to us, the amount which is given now that P is known to us. Interest rate is known to us and number of interest periods are known to us that is 15.

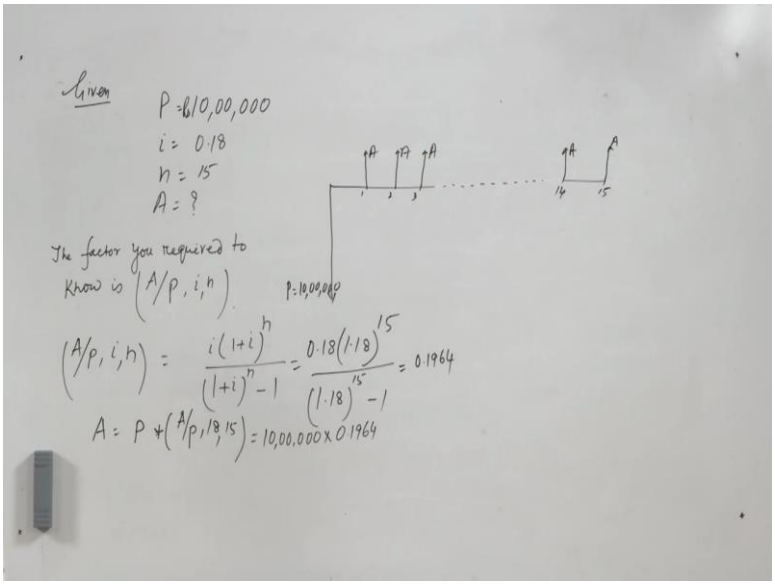
(Refer Slide Time: 18:36)



So we have to find the equal year end payment that is A. So in this case we are given P as 10,00,000. We are given interest rate as 0.18 and we are given n as 15. We have to find A. So we will again draw the cash flow diagram from the banks perspective. If we look at bank has given P today a loan of Rs. 1,000,000. Now the bank will receive every year end a certain amount from the customer. So this is A.

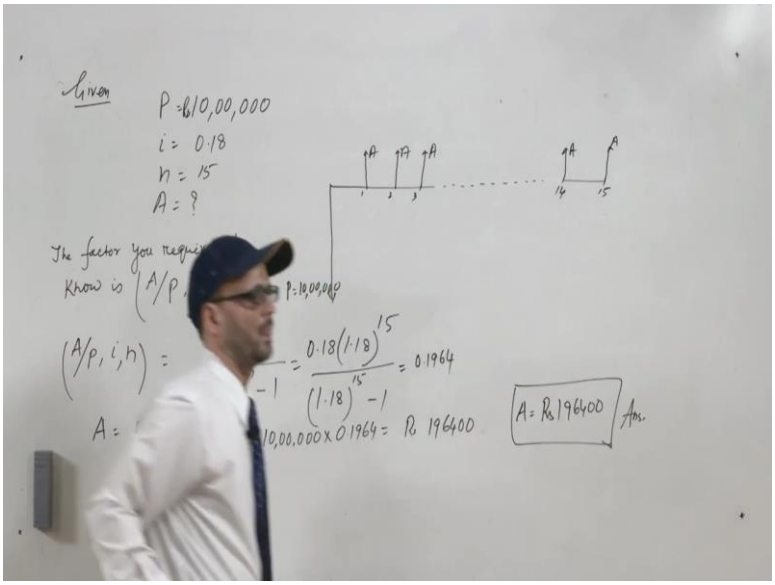
So basically it is a series where the factor required to know is the factor you require to know is A by P i n because you know P and when you will multiply this factor then this will give you a required amount of A. We know this factor as this comes out to be i into to 1 + i raised to the power n by 1 + i raised to the power n - 1. Basically the bank is recovering its capital. So equal payment series capital recovery factor will be used in this case.

(Refer Slide Time: 20:28)



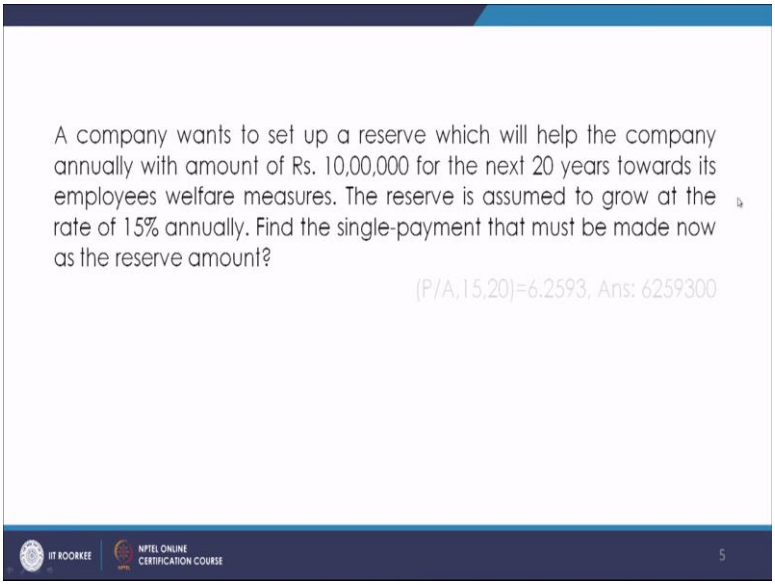
Now this is the factor and the the value of this factor is coming as. So we will calculate this, i is 0.18 into 1.18 raised to the power 15 divided by 1.18 raised to the power 15 - 1. If we calculate this factor, this comes out to be .1964. So this factor when will be multiplied with P, it will give us the equal year end payment.

(Refer Slide Time: 20:50)



So A will be given as P into A by F A by P 18 15 which we have calculated .1964 so 10,00,000 multiplied by .1964 and this will lead to Rs. 196400. So A is Rs. 196,400 is the answer. Means the bank has given the company a loan of 10,00,000 now and it will recover an amount of Rs. 196,400 every year end for the next 15 years if the interest rate is given as 18%. Now we will deal with another case another problem.

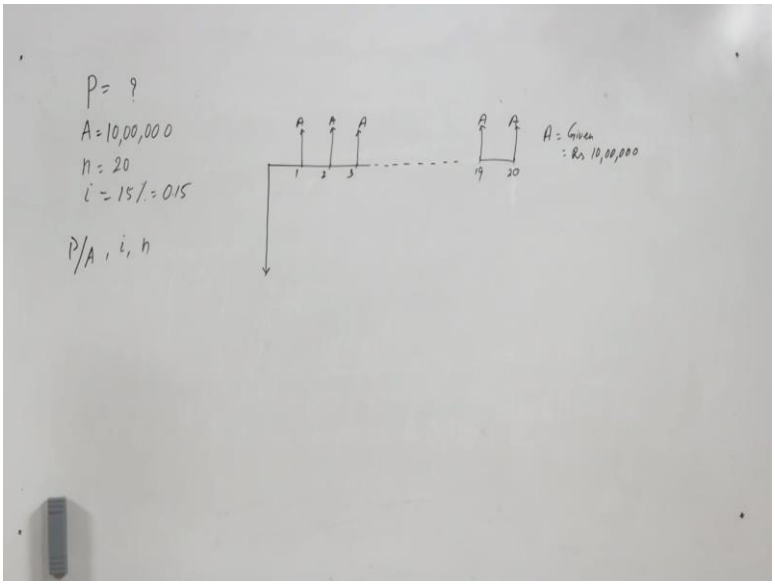
(Refer Slide Time: 21:28)



The next problem is a company wants to set up a reserve which will help the company annually with amount of Rs. 1,000,000 for the next 20 years towards its employees welfare measures. The reserve is a view to grow at the rate of 15% annually. Find the single payment that must be made now as the reserve amount. So if you look at this problem, this is a problem where the known quantity is Rs. 1,000,000 which the company is basically depositing every year end.

I mean the company has to set up a reserve, so company has to get the value of P now so that it can get annually this amount for the welfare. So basically here you have known value of A equal year end payment and that is why the factor which will be used is P by A i n. We will solve this problem now. For this problem if you look at, we have to find P because company wants to set up a reserve amount now so that from there it gets the amount.

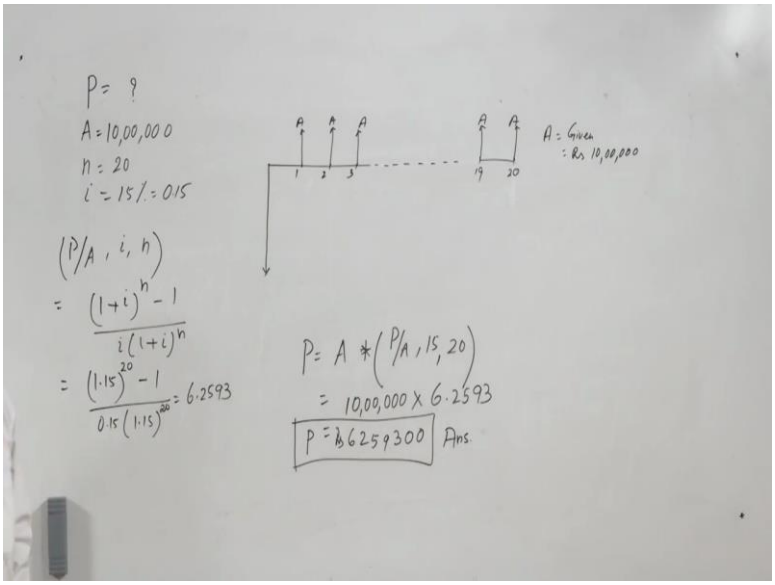
(Refer Slide Time: 25:18)



Now what is given is the equal year end payment. A is given as 10,00,000. Then number of years are given as 20 and rate of interest is given as 15% per annum so that is .15. So again as usual we will try first to see it through a cash flow diagram. So basically the company wishes to invest certain amount so for that it has to this is a sign of disbursement, so this **must** will be the negative side and this so that it gets every year end so this amount is 10,00,000.

A given that is 10,00,000. So this is a case where we have already discussed about the equal payment series present worth factor where you have to find the present worth of an investment so that it gives you an amount which is equally payable for the next n interest periods. So for that you need to know P by A i n and this is nothing but the reciprocal of the earlier factor which we have used.

(Refer Slide Time: 27:40)



So in this case P by AIM nothing but 1+ i n -1 by i into 1+ i raised to the power n. So this will be 1.15 raised to the power 20 - 1 divided by .15 into 1.15 raised to the power 20. And this factor is coming as 6.2593. Now this is a factor which when multiplied with A gives you the present which the company should invest.

So present amount can be found by A into P by A 15 20 and that we have already calculated as 6.2593 and A we know into 6.2593 which is coming as 6259300. So this is the answer. So what we see that if the company makes a reserve fund of 62,59,300 now and this investment earns 15% of interest per annum in that case the company will get every year end and amount of Rs. 1,000,000 for the next 20 years. So this is the solution of this problem.

So in this way we can solve the problems of such nature. Thank you.