Engineering Economic Analysis
Professor Dr. Pradeep K Jha
Department of Mechanical and Industrial Engineering
Indian Institute of Technology Roorkee
Lecture 05
Interest Formulas for Discrete Compounding and Discrete Payments: Equal Payment Series (CAF, CRF & PWF)

Welcome to the interest formulas on equal payment series factors. So in this lecture, we will try to find the factor calculations for equal payment series.

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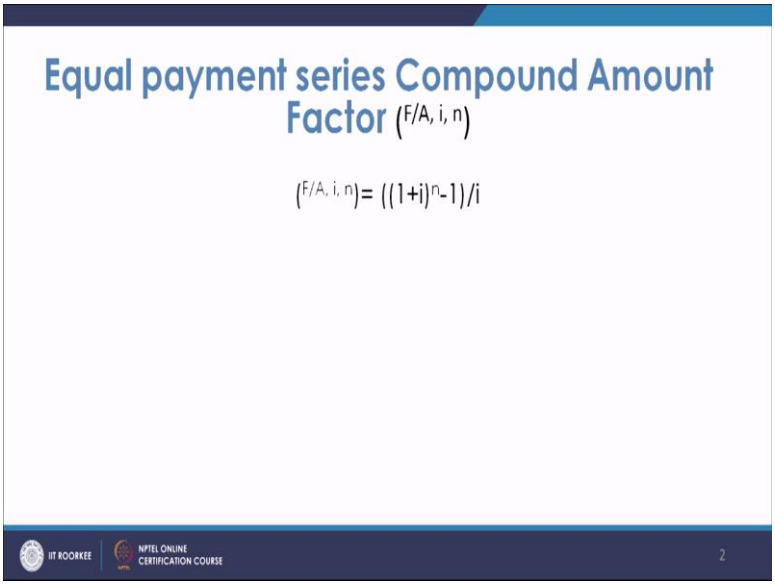

Many a times we deal with situations when we have to make equal year end payments, they are also known as annuities. Now we have equal payment series compound amount factor, this is a factor which when multiplied with A that is the equal year end payment, it will give you a future amount.

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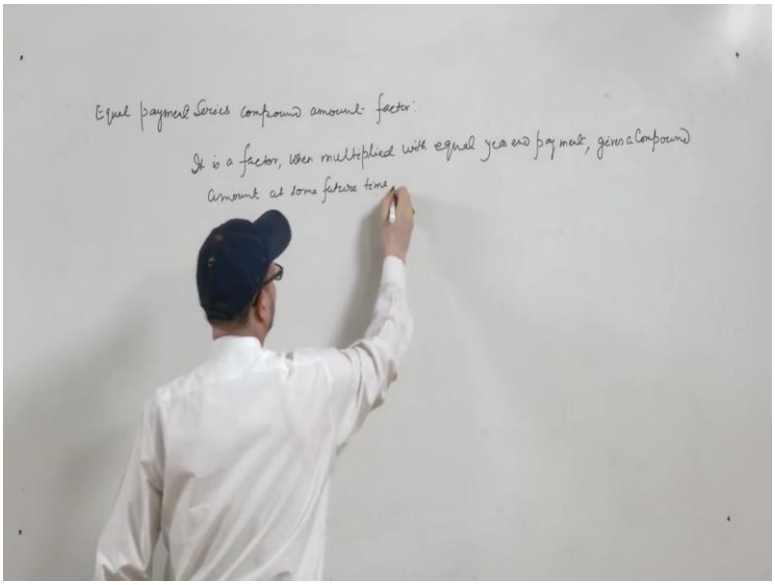

So you can define this equal payment series compound amount factor,it is a factor when multiplied with equal year end payments, gives a compound amount at some future time. So it means that you know the equal year end payments, you know the interest rate and also you know the interest period and once you know that, you wish to calculate the value of final sum that is F.

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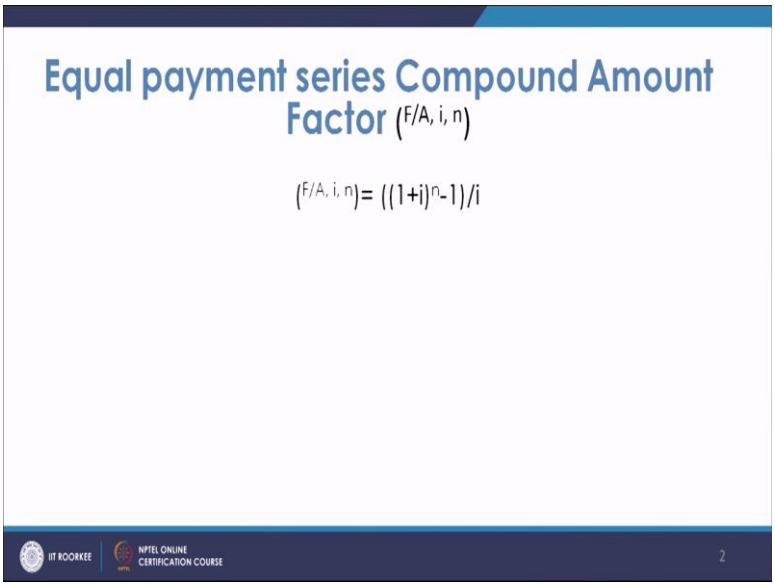

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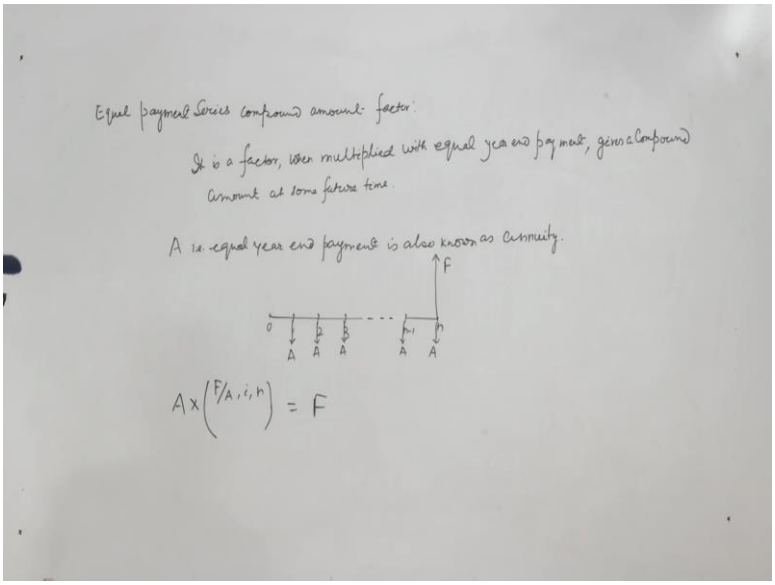

So it means the cash flow diagram will look like this. So you have n years, now every year end you are paying certain equal year end payment that is denoted by A. A is also termed as annuity. So A that is equal year end payment is also known as annuity. Now once you deposit this amount, every year end what amount of F, it will lead at the end of n years. So this factor is basically F by Ai n.

Means this factor when it is multiplied with A, it will give you F. So what we see, that this A will cut with A, you will get F. Now how to get this factor? Now if you look at this diagram, what you see is that the contribution of the A which is made at the end of nth year, it is A itself because you are drawing the amount at that time itself. So its contribution is A itself. The contribution of A which is made at the end of n - 1th year , this is nothing but A into 1 + iwhereiis the interest rate.

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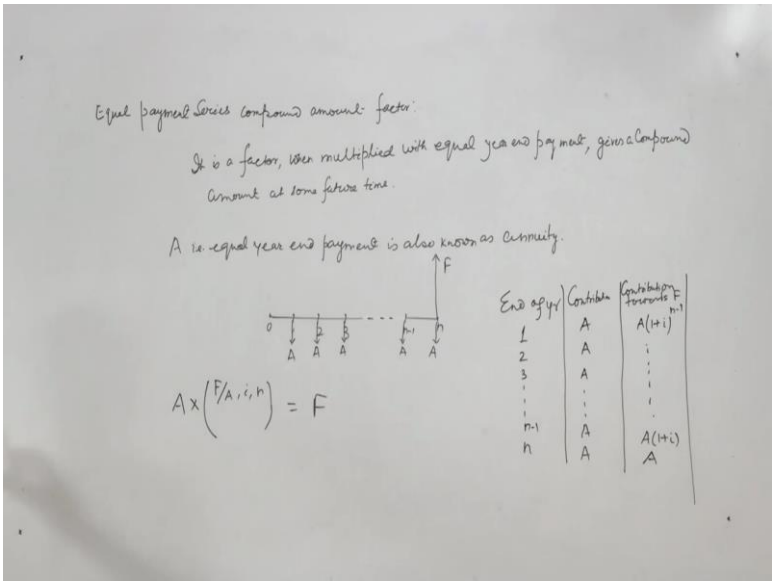

Similarly the contribution will go on changing, the next month contribution will be the equal year end payment made at the end of first year. So what you can see in a nutshell that the contribution is end of year, 1, 2, 3, it go up to n. We are contributing contribution as every time we are giving A. Contribution towards F.

So at the end of nth year the contribution of this A will be A itself. At the time n - 1, the A payment which is made, its contribution will be A into 1 +i.So this will go on and this will be A into 1 + iraised to the power n - 1. So what we see that we have deposited the equal annual amount A towards the end of the year, for continuously for n years and we wish to draw F.

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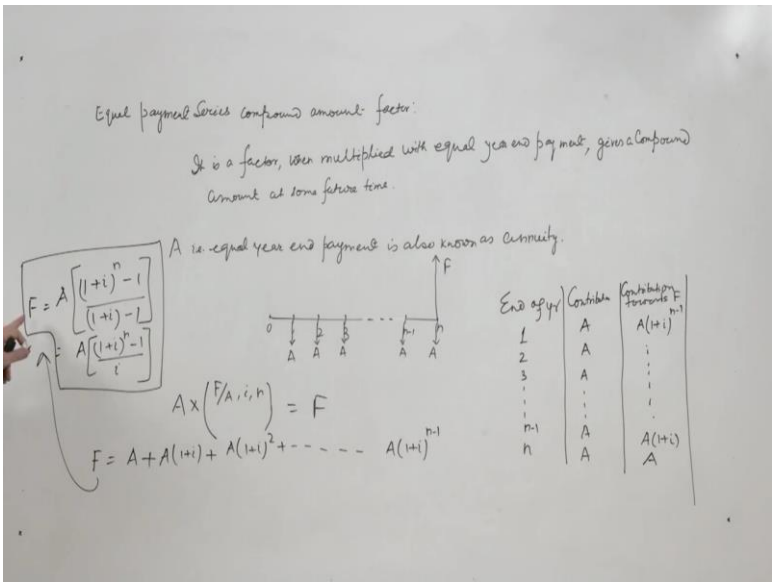

Now if we look at what is F, so we can write, this F is nothing but the sum of all these quantities. So it will be A +A into 1 + i + A into 1 + iraised to the power 2, it will go upto A 1 + iraised to the power n - 1. From this, this is basically a geometric progression series GP series whose first term is A and the common ratio is 1 +I.

So we can write, from here onwards we can write F is nothing but A into 1 + iraised to the power n - 1 upon 1 + i - 1 that is A into 1 + iraised to the power n - 1 by I. So what we have that F is A times this factor.

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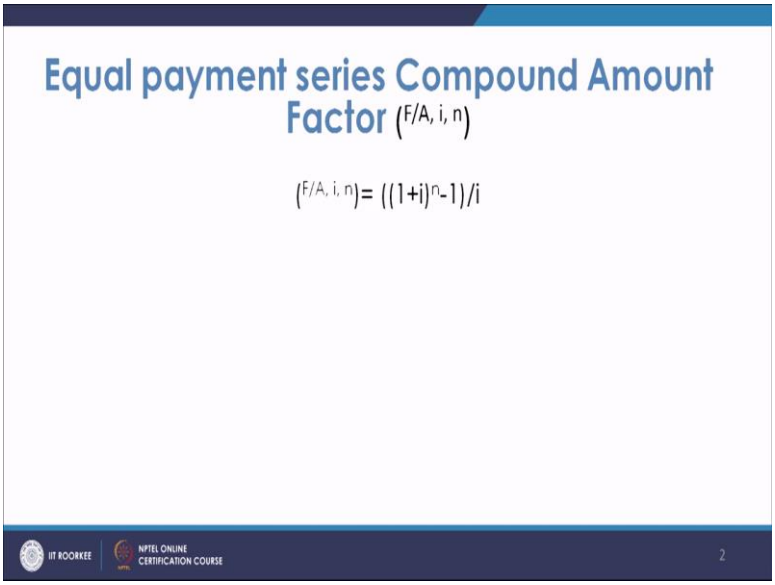

This factor when multiplied with A gives you F that is why this factor in this bracket 1 + iraised to the power n - 1 whole divided byithis is known as equal payment series compound amount factor which is shown here by 1 + iraised to the power n - 1 by i.

So this is used for the problems when we are to calculate what amount will be deposited at the end of suppose 20 years when the interest rate is suppose 10% and you are depositing every year an equal amount of suppose say 10,000. So in that case 10,000 will be multiplied by a factor F by A and interest rate and that to the interest period. So in this formula we will put theiand n and we will calculate the factor values.

This is the equal payment series compound amount factor.

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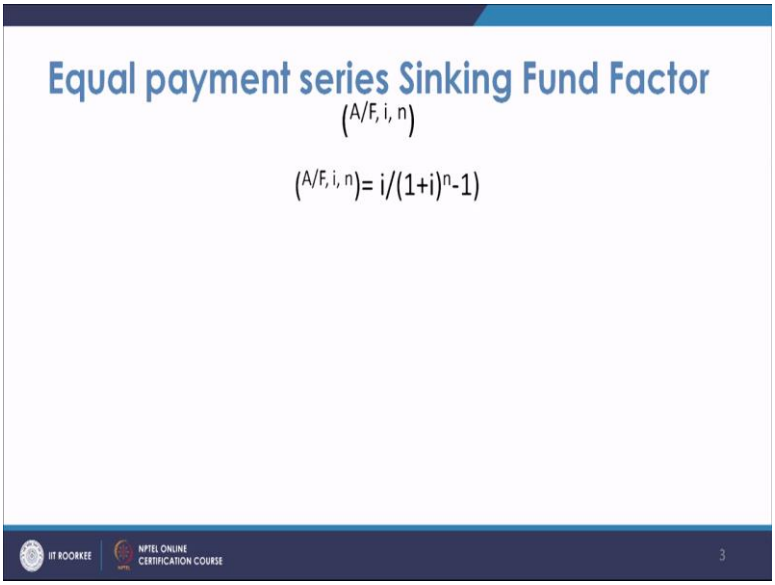

Next is equal payment series sinking fund factor. This factor is basically the reciprocal of the previous factor. In this case, this means that you are basically depositing certain amount every year end so that you get something at the end. In this case, in the earlier case, if you see in the earlier case basically you know A and you find F, so this factor will be multiplied with A and you will get F.

Whereas in this factor you know F, you have some target that you need some amount in the future, for getting that amount how much you should deposit now, now onwards every year end. So that is known as equal payment series sinking fund factor.

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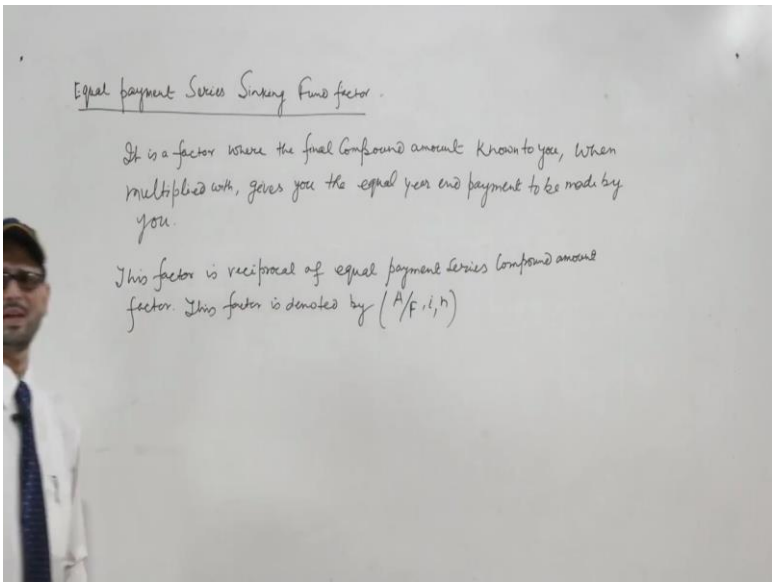

So our next factor is Equal Payment Series Sinking Fund Factor. It is a factor where basically you know the F, you know the final amount where the final compound amount known to you, when multiplied with, gives you the equal year end payment to be made by you.

So basically you are keeping aside certain equal amount every end so that you get certain amount compounded amount F at the nth year end and that is why this factor is the reciprocal of equal payment series compound amount factor. This factor is denoted by A by Fi,n. So we have got the expression for F by A in,A by F i n will be reciprocal of F by A i n.

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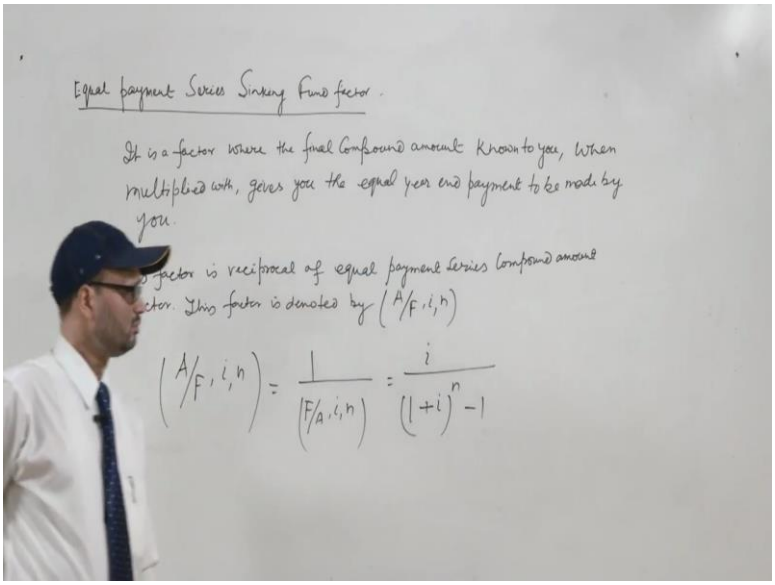

And F by A i n we have developed as 1 + iraised to the power n - 1 upon I, so this will beidivided by 1 + iraised to the power n -. So this factor is known as equal payment series sinking fund factor.

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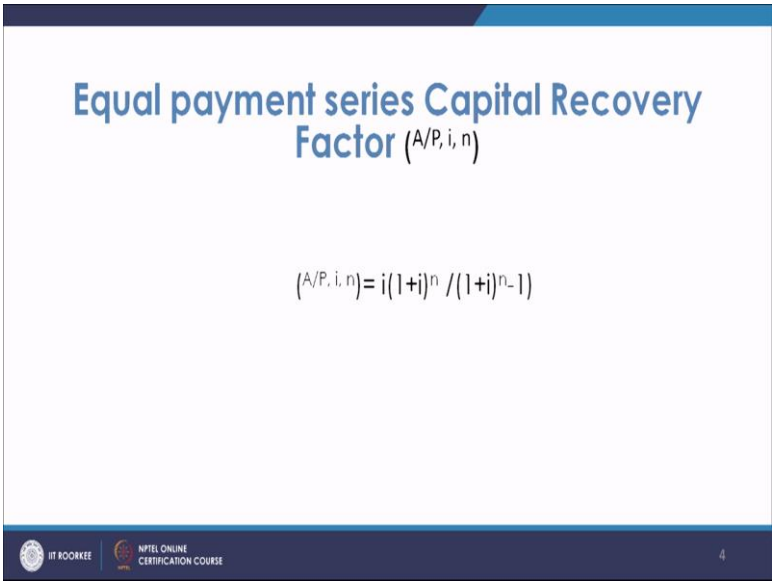

The next factor is Equal Payment Series Capital Recovery Factor. Equal payment series capital recovery factor means you know the capital which you have at present, this capital will earn interest over time. Now what should be the equal year end payment you should get in the future so that the capital which you have, the present acid which you have and also the interest which it earns, all is exhausted over n interest period.

This factor when multiplied with P that is your present investment cost, it will give you the equal year payment or equal year end income for you. So equal payment series capital recovery factor and that is why its symbol is A by P,i,n means the present worth of the capital P when multiplied with this factor gives equal year end payments for the life of the investment so that the capital amount P and interest earned on it is fully recovered.

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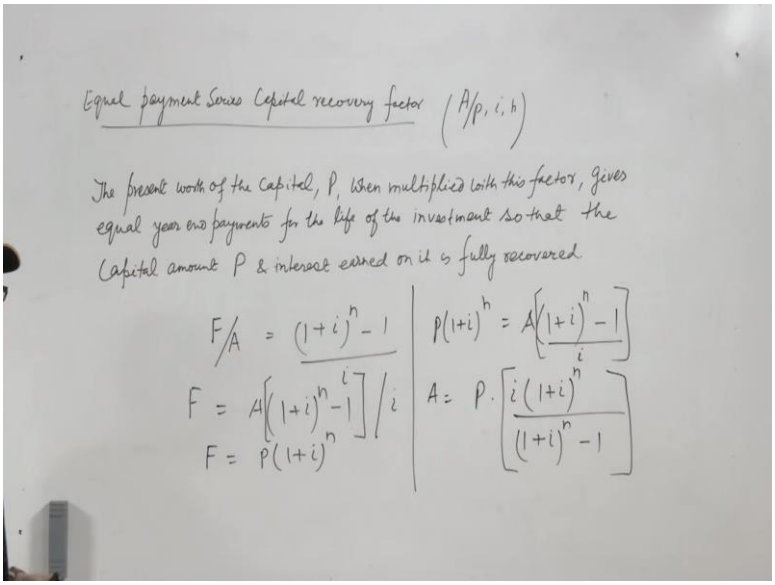

So this is the meaning of equal payment series capital recovery factor. Now we have seen that F by A is 1 + in - 1 byithat is F equal to A into 1 + 1 n - 1. Now we have to find the expression for A in terms of P. F is we know that F is P into 1 + iraised to the power n. So what we get P into 1 + iraised to the power n as A 1 + iraised to the power n - 1 by i.

Sorry here, by i. Now we have to find A by P. So P should be multiplied with this factor. So what will happen A will be equal to P intoiinto 1 + iraised to the power n divided by 1 + iraised to the power n - 1.

Means the factor which is coming here, this is when multiplied with your present investment, it gives you the equal year end payment and that is why this factor is known as equal payment series capital recovery factor.

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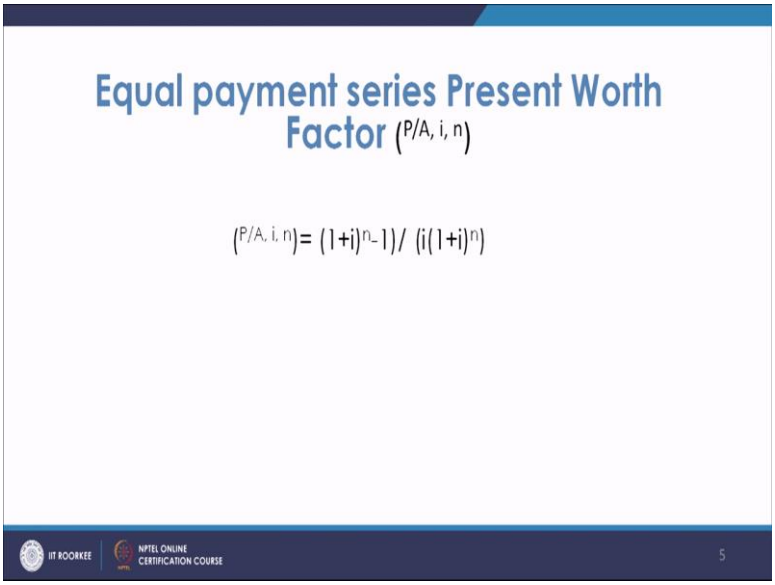

Now the next type equal payment series factor is Equal Payment Series Present Worth Factor. Now in this case, basically you try to find the present worth and you know A. So basically what investment you should do now, so that you get a known value of equal year end payment for the n years and the factor which will be multiplied with that known quantity of equal year end payment that will give you the present investment cost.

So this factor, when multiplied with known value of equal year end payment will give the present cost of the investment. That is why, this is known as equal payment series present worth factor, means you are calculating the present worth value and you know i, a at a particular interest rateiand for a particular time period n.

Now earlier we have seen that this is basically reciprocal of equal payment series, the earlier one which we have discussed, Equal Payment Series Capital Recovery Factor. In that case we knew P and we had to calculate A, in this case, we know A and we have to calculate P. So basically it is reciprocal of Equal Payment Series Capital Recovery Factor.

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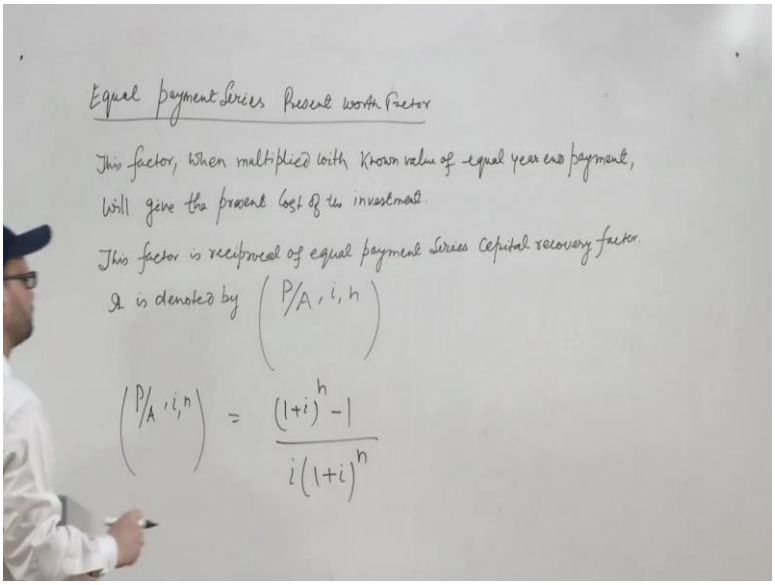

So this factor is reciprocal of Equal Payment Series Capital Recovery Factor. It is denoted by P by Ai n. We already know the value of A, we know the interest rate and we also know the number of interest periods, that is why P by A i n, it will be the reciprocal so it will be 1 + in - 1 byiinto 1 + iraised to the power n. And this is the equal payment series present worth factor.

So basically we have understood the four different factor values which are applicable whenever there is a equal payment series, it is also known as uniform series because you are paying the amount uniformly over the interest periods and that leads to a certain value of either F or P or so. So in that case we have these four types of factor values.

In the next lecture we will discuss about the problems based on these factor values. Thank you.