Engineering Economic Analysis
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Lecture 04
Interest Formulas for Discrete Compounding Indiscreet Payments: Single Payment (CAF & PWF)

Welcome to the lecture on interest formulas.

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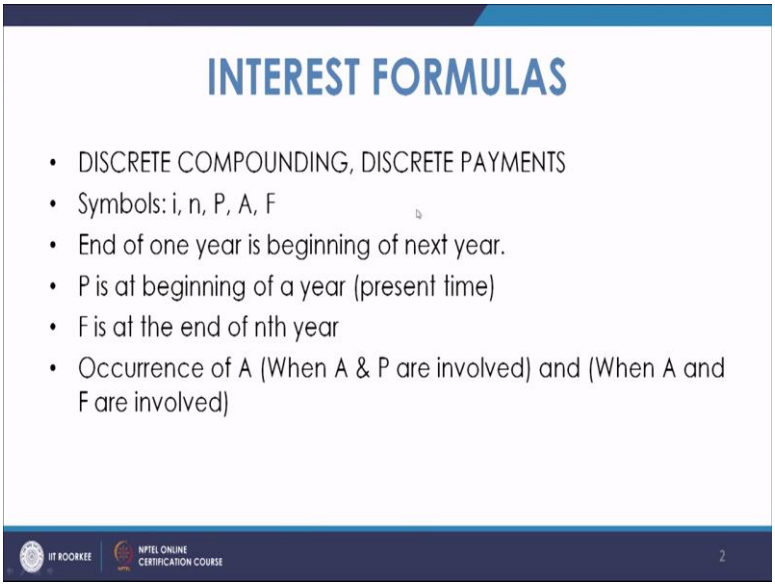

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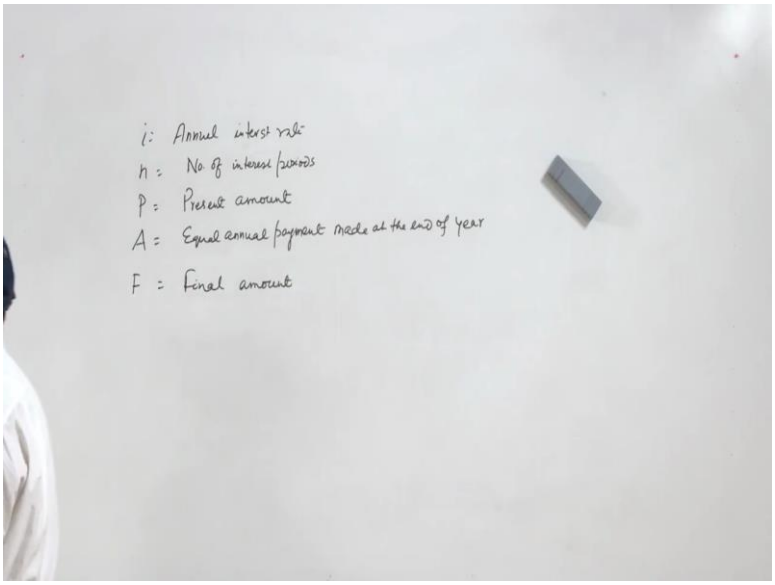

So in this lecture, we will discuss about the discrete compounding, discrete payments. In that, we will come across certain symbols. What does this symbols mean? So in these symbols you have i, n, P, A and F, where i is the annual interest rate and the n is number of interest

periods, then P is present amount at the beginning, A whenever we will discuss this will be equal annual payment made at the end of the year, F will be the final amount or compounded amount.

So we will have certain points to remember that we will be following. The concept that end of one year is beginning of the next year. So if a payment is made at the end of the year, it means the same amount will be at the beginning of the next year. P means beginning of the year that at present time at 0 time. F is at the end of nth year. Now occurrence of A, so what it mean? When A and P are involved, so when A and P are involved.

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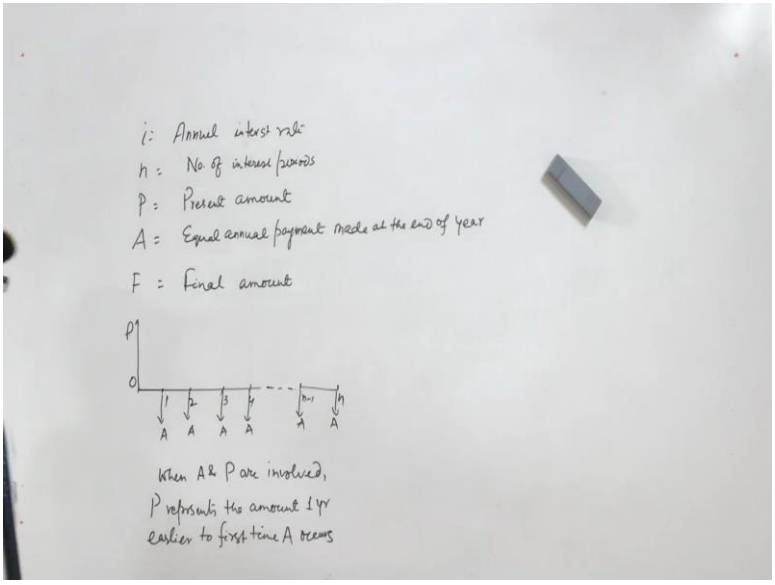

If we see at the 0 time, A starts from one year. So every year end you are paying certain amount A. When A and P is involved, P occurs at the beginning time. So this is when A and P is involved. P represents the amount one year earlier to first time A occurs.

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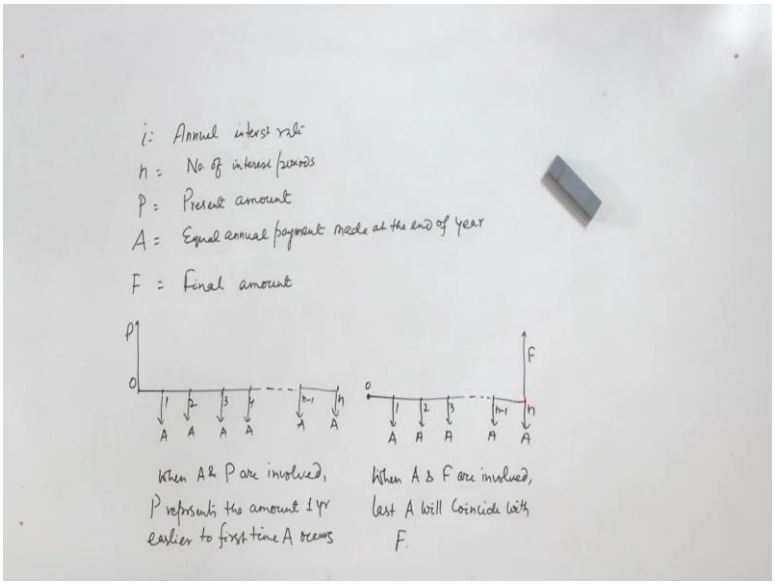

When A and F is involved, in that case A starts from the first year as usual and F will be matching with the last A. So when A and F are involved, last A will coincide with F. This represents the fact that you are starting the payment, the equal annual payment at the end of first year and it goes till the end of the nth period. P represents here at the initial point and F represents at the last time.

Now we have to discuss two types of factors, one is Single Payment Compound Amount Factor and another is Single Payment Present Worth Factor. We will discuss what are these factors.

Single Payment Compound Amount Factor: This is a factor, as the name indicates, it is a factor. This factor also is represented by F by P i n. Now what is this factor? This is a factor which when multiplied with the present amount, it will give you the future amount at some future time. When multiplied with present amount will give you compound amount and interest periods hence.

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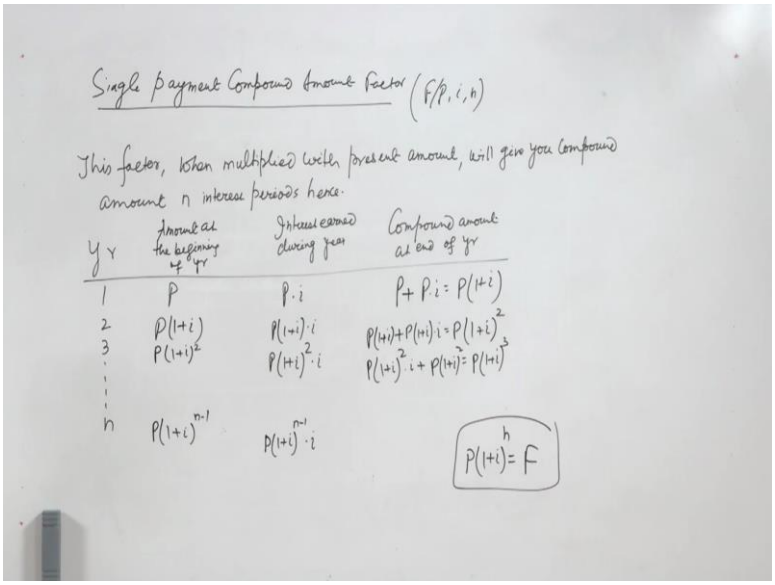

How you can find this factor, so suppose you have year 1, 2, 3, n. You have amount at the beginning of year, you have interest earned during year and you have compound amount at end of year. So if you have a present amount of P and interest rate is i, and there are n interest periods. So during the first year, the interest which will be earned will be P timesI. So your compound amount at the end of year one will be P + P into i that is P into 1 +i.

At the end of two year, in the second year, your amount is now P into 1 + i because it is a case of compound interest. So your amount which is at the beginning of year two is P into 1 +i. Now interest will be on this amount, so this will be P into 1 + i times i. So if you add these two, it will give you the compound amount at the end of year two. So if you add them, P into 1 + i + P into 1 + i into i that is p into 1 + i to the power 2.

In the beginning of the third year, you have P into 1 + i to the power 2, again you have interest accrued as P into 1 + i raised to the power 2 multiplied by i, so it will come out to be P into 1 + i raised to the power 2 into i + P into 1 + i raised to the power 2 that is P into 1 + i raised to the power 3.

So during the nth year, at the beginning, the amount which is owed is P into 1 + i to the power, raised to the power n - 1 and your interest will be P 1 + i raised to the power n - 1 into i and your final amount comes out to be P into 1 + i raised to the power n. So this is nothing but the amount which will be paid by the borrower.

Now this is a factor 1 + i raised to the power n which is multiplied with the principal amount and it gives you the final amount, the final compound amount. So that is why this factor 1 + i

raised to the power n, this is known as Single Payment Compound Amount Factor. So 1 + i raised to the power n is Single Payment Compound Amount Factor and it is represented by a symbol that we will be practicing during the course.

This is represented by a symbol F by P i n, means this factor F by P, once it is multiplied with P, this P and P will cancel and you will get F. So this is Single Payment Compound Amount Factor F by P i n.

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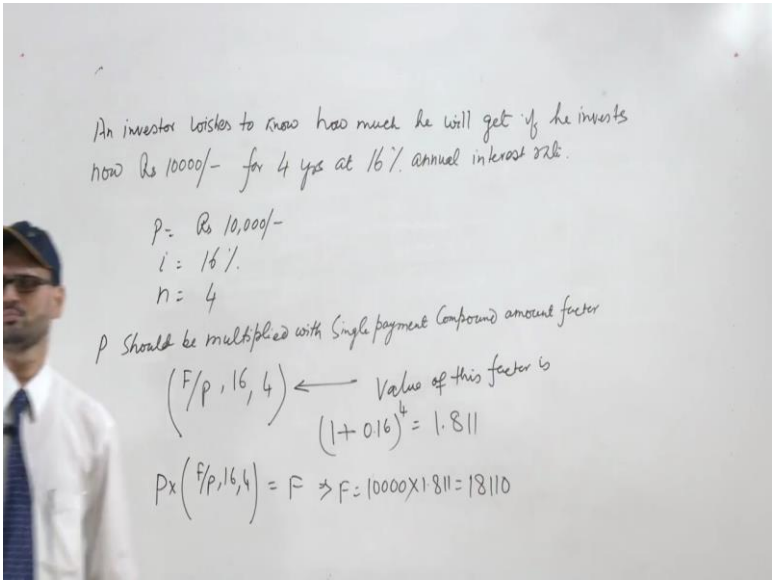

Let us see by an example, this factor will be useful for the calculations in the cases when an investor has to find that if he invests amount now, what he will be getting at the end of certain interest periods. So suppose an investor wishes to know how much we will get if he invests now Rs. 10,000 for four years at 16% annual interest rate.

So for these cases you are given that the presently the investor is investing 10,000, so P is Rs. 10000, i is 16% and n is 4. So basically P should be multiplied with Single Payment Compound Amount Factor that is F by P, rate of interest is 16 and 4. So and this factor, value of this factor is 1 + 0.16 to the power 4 that is 1.811. Means if the present amount is multiplied with this factor, it will give him the amount which you will receive at the end of four years.

So P multiplied by F by P 16 4 will be F, it means F will be 10,000 multiplied by 1.811 that comes out to be 18110. Now we are calculating this because it is a simple calculation but we have the tables supplied for this type of discrete compounding, discrete repayment for a particular interest rate.

And we can refer these values, value of these factors which is provided at the end of the book and directly you can get these factors and find the value of F. So this is how we calculate the Single Payment Compound Amount Factor. We try to calculate the value of compound amount at the end of certain interest periods.

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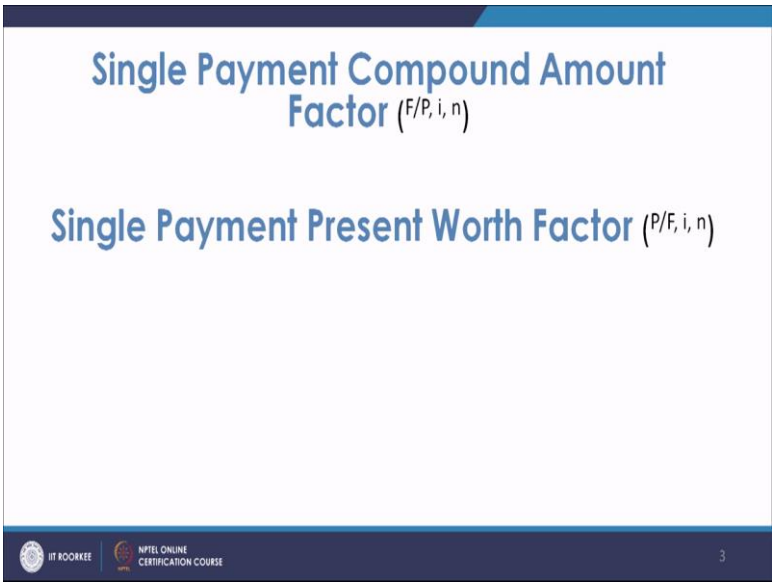

The next is Single Payment Present Worth Factor, Single Payment Present Worth Factor. Just opposite to the Single Payment Compound Amount Factor, we have Single Payment Present Worth Factor. It is a factor which when multiplied with some future amount will give us the present amount for the investment.

So as we have seen that in the earlier case, a factor when multiplied with P gives you F. In this case, it is a factor which when multiplied with F will give you P. So suppose an investor, wants to know that how much he should invest now so that he can get 18110 at the end of four years, that was the earlier case. Now they investor wants to know that he will get 18 110 at the end of four years, how much you should invest now.

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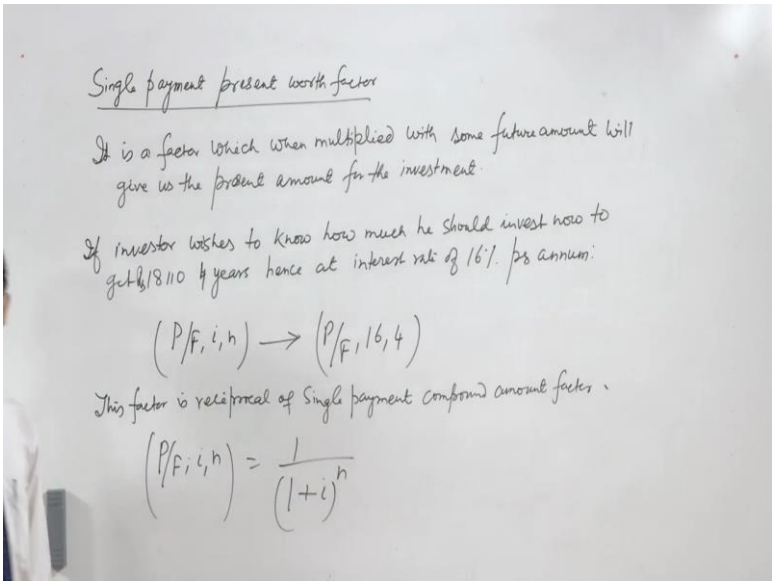

So in that case he has to, so if investor wishes to know how much he should invest now to get 18110, Rs. 18,110 four years hence at interest rate of 16% per annum. In that case a factor should be there which should be multiplied with the F and that should give him the present amount. So that factor is P by F i n. So in this case we have P by F i n that is nothing but P by F, 16, 4.

This factor is basically the reciprocal of the Single Payment Compound Amount Factor. This factor is the reciprocal of Single Payment Compound Amount Factor. So P by F i nyou can write it as 1 by 1 + i raised to the power n. And this factor is known as Single Payment Present Worth Factor because this factor when it is multiplied with F it gives you the present amount.

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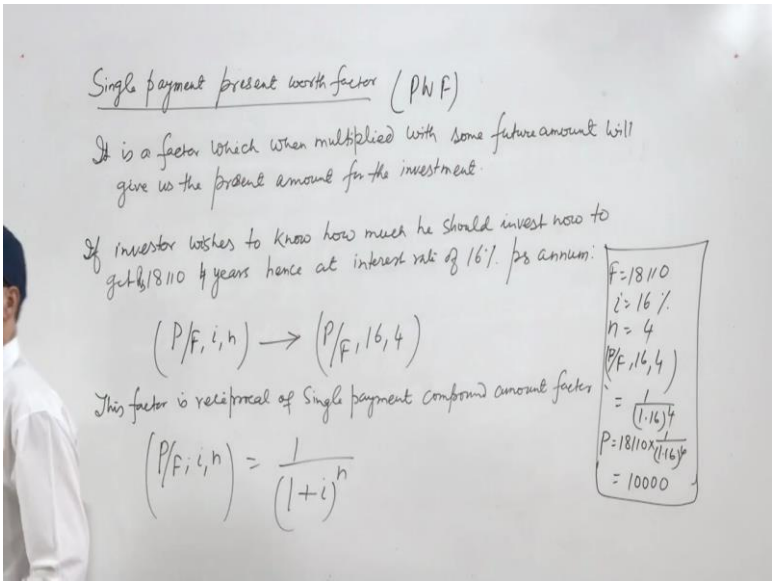

So in the present case, when your F is given as 18110 and i is 16% n is 4, so P by F i n 16,4 will be equal to 1 by 1.16 to the power 4. So you can calculate P as 18110 multiplied by 1 by 1.16 raised to the power 4 and this comes out to be 10000. So what we see that this is a factor which when multiplied with the future amount, it gives you the present amount.

So that is why it is known as Single Payment Present Worth Factor. In a nutshell, also they are called as CAF as well as PWF. So this Single Payment Present Worth Factor many a times, you will see it as PWF means present worth factor. And similarly, Single Payment Compound Amount Factor is CAF. So CAF means, Single Payment Compound Amount Factor.

So this is how once you know the factor values, you can find the values of P or F, either at the present time or at some future time. Thank you.