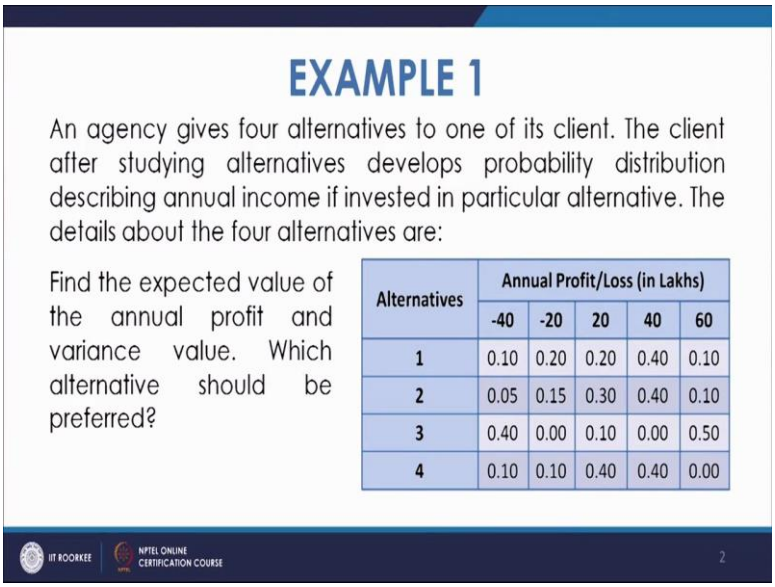
Engineering Economic Analysis
Professor Dr. Pradeep K Jha
Department of Mechanical and Industrial Engineering
Indian Institute of Technology Roorkee
Lecture 36
Problem Solving Based on Decision under Risk

Welcome to the lecture on problem solving on decision under risk and uncertainty. So in this lecture we will solve some problems based on decision under risk and uncertainty. Let us see the first question.

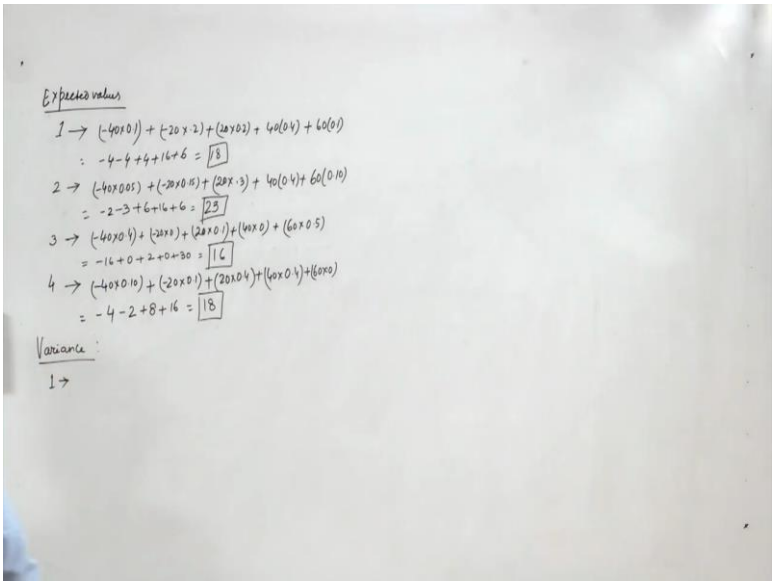
(Refer Slide Time: 00:44)



The first question is about an agency which has given four alternative to one of its client. The client after studying alternatives develops probability distribution describing annual income if invested in particular alternative. The details about the four alternatives are like this. You have alternative 1, 2, 3, 4, the probability of giving - 40 that is 40,00,000 of loss is point 1. 20,00,000 of loss is point 2.

20, 40 and 60,00,000 of profit is respectively point 2 point 4 and point 1 if alternative A1 is chosen. Similarly if the alternative is chosen as 2, 3 and 4 the different probabilities are cited. So we have to find the expected value of the annual profit and variance value. And based on that we can decide which one should be selected. So let us see if we get the expected values.

(Refer Slide Time: 06:37)

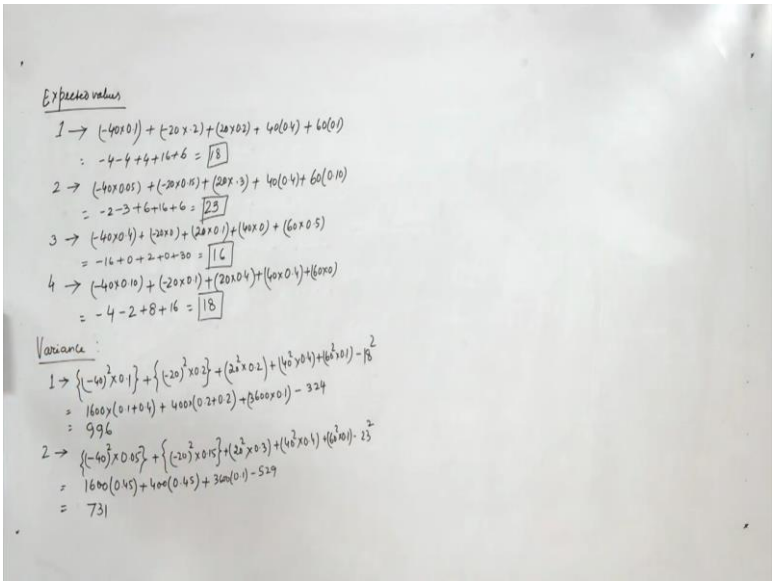


So for alternative A1, alternative one, it will be - 40 times point 1 + - 20 times point 2 + 20 time point 2 + 40 times point 4 + 60 times point 1. So this will come out to be - 4 - 4 + 4 + 16 + 6. So it will be 22 - 4 that is 18 18,00,000. Similarly for 2 if we compute the expected value will be - 40 multiplied by point 05 + - 20 multiplied by point 15 + 20 multiplied by point 3 + 40 multiplied by point 4 + 60 multiplied by point 1.

So you will get here as - 2 - 3 + 6 + 16 + 6 so 28 - 5 that is 23. Then for third alternative it will be - 40 multiplied by point 4 + - 20 multiplied by 0 + 20 multiplied by point 1 + 40 multiplied by 0 + 60 multiplied by point 5. So this comes out to be - 16 + 0 + 2 + 0 + 30, so 32 - 16 that is 16. Then for the fourth alternative we have - 40 into point 1 + - 20 into point 1 + 20 into point 4 + 40 into point 4 + 60 into 0.

So this will be equal to - 4 - 2 + 8 + 16, so 24 - 6 that is 18. So these are the values of mean or expected values when we take the probability values into account and using the four alternatives we get these expected values. Now we have to see what is the variance value for them.

(Refer Slide Time: 10:47)

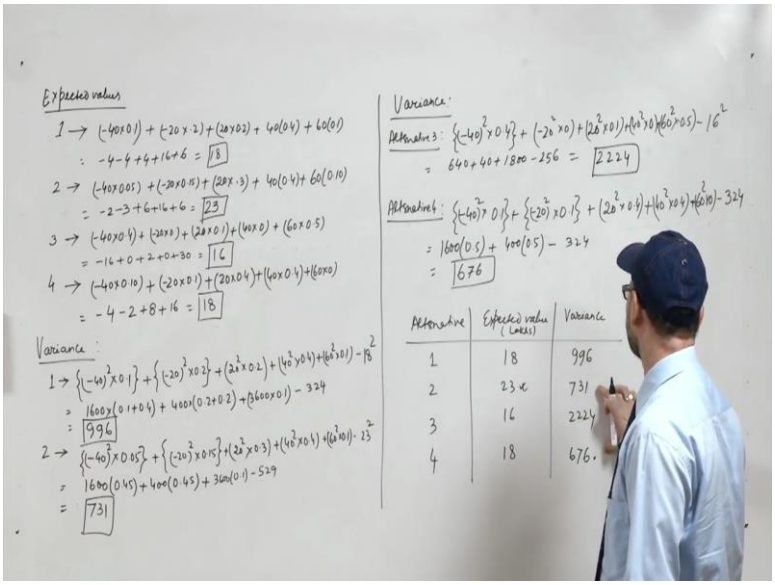


So the variance values can be computed, for the variance, again for alternative 1 we have to get to these values and we have to have the squares of these numbers multiplied by the respective probability values. So it will be - 40 square multiplied by point 1 + - 20 square multiplied by point 2 + 20 square multiplied by point 2 + 40 square multiplied by point 4 + 60 square multiplied by point 1 - the expected value square that is 18 square.

So it will be, if you look at 40 square 1600 square multiplied by point 1 + point 4 + 400 multiplied by point 2 + point 2 + 3600 multiplied by point 1 - 324, so this comes out to be 800 + 160 + 360 - 324, so this is coming out to be 996.

We go for the second alternative, for second alternative again we have - 40 square multiplied by point 05 + - 20 square multiplied by point 15 + 20 square multiplied by point 3 + 40 square multiplied by point 4 + 60 square multiplied by point 1 - the value of, the expected value is 23 so its square that is 529. So you will get 1600 multiplied by point 05 + point 4 so that a 0 point 45 + 400 multiplied by point 15 + point 3 that is point 45 + 3600 multiplied by point 1 - 529.

(Refer Slide Time: 15:46)



So if you compute this value, this comes out to be 720 + 180 + 360 - 529, so this is coming out to be 731. We will compute the variance for the next two alternatives. Variance for alternative 3, it will be - 40 square into point 4 + - 20 square multiplied by 0 + 20 square multiplied by point 1 + 40 square multiplied by 0 + 60 square multiplied by 0 point 5 - the expected value is 16 so 16 square.

So it will be 1600 into point 4 that is 640 + 400 multiplied by point 1 so 40 + 60 square into point 5 so that is 1800 - 256 and this comes out to be 640 + 40 + 1800 - 256, so it is coming out to be 2224. This is the variance of third one, now we will calculate for the fourth one.

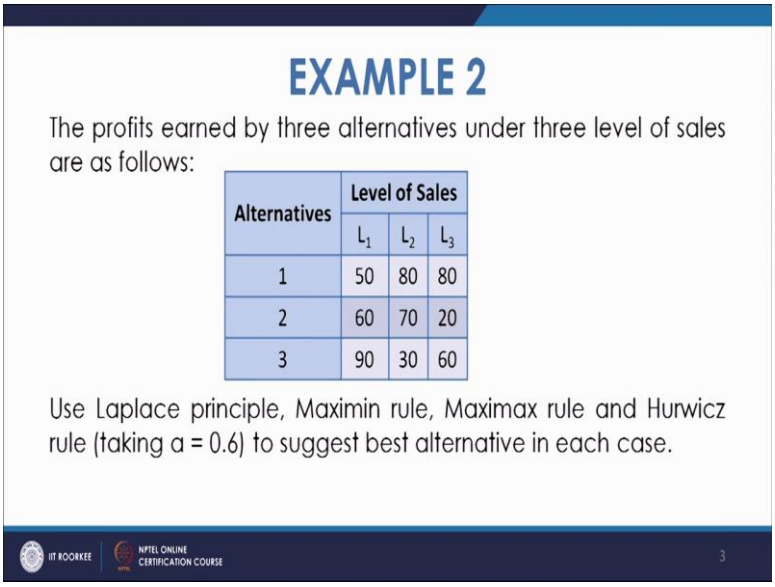
For fourth one - 40 square multiplied by point 1 + - 20 square multiplied by point 1 + 20 square multiplied by point 4 + 40 square multiplied by point 4 + 60 square multiplied by 0 - this square that is 324. So it would be 1600 multiplied by point 5 + 400 multiplied by point 5 - 3 in 24 and this will be equal to 676. So what we see is that the expected values are coming as 18, 23, 16 and 18.

So what we see alternative 1, 2, 3 and 4 expected value is coming as in terms of lakhs 18, 23, 16 and 18 and the variance is coming out to be 996, 731, 2224 and 676. So what we see is this is preferred one using the expected value and the minimum value of the variance is 676. However this option looks to be better because the variance value is quite reasonably in the case of alternative 2.

So depending upon the choice you can go for alternative 2, if the expected value is seen it is maximum here and also the variance value is closer to the minimum so alternative 2 can be

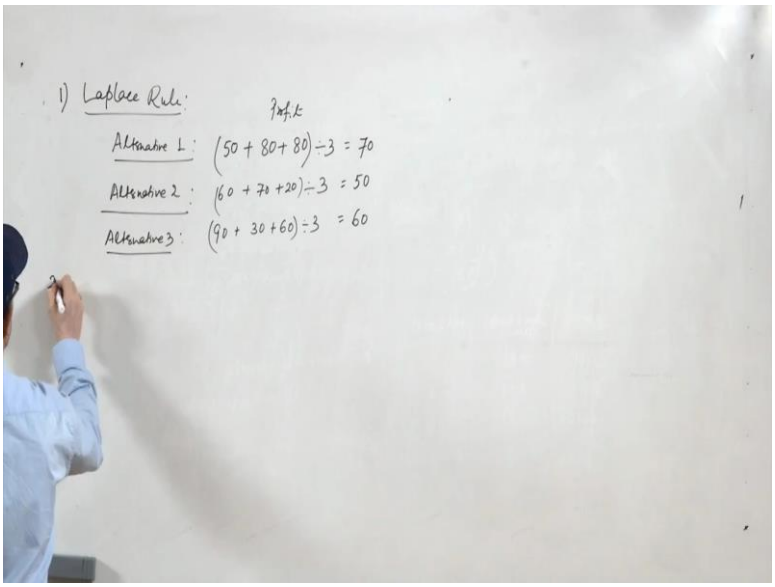
preferred. So this is how we solve the problem based on the expected value or expected variance decision making.

(Refer Slide Time: 16:24)



Next problem we will discuss about the different criterion in the case of uncertainty where the profit earned by 3 alternatives under 3 level of sales are given. The probabilities are not given in this case and you have to use the Laplace principle, Maximin, Maximax rule and Hurwicz rule and to suggest the best alternative in each cases. So we will use these different roles to come to a conclusion that which of the alternative is better in this case.

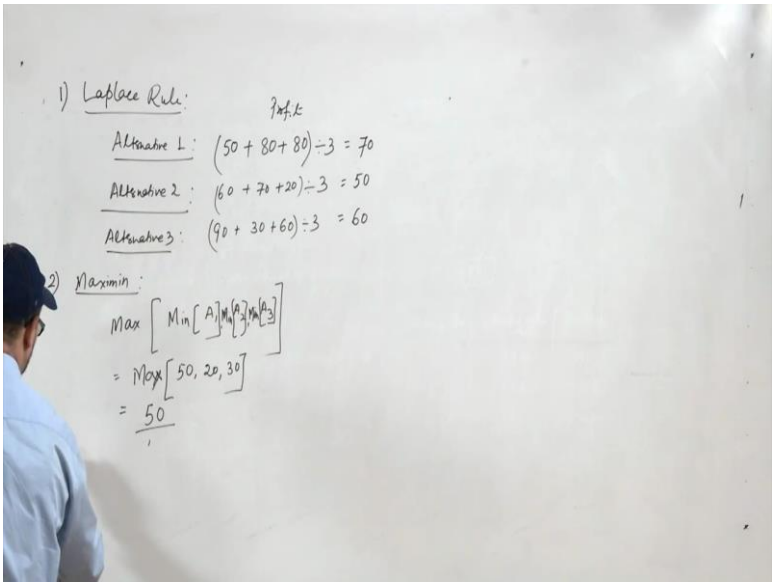
(Refer Slide Time: 19:00)



So let us start with the Laplace rules, so first Laplace rule. Now in the Laplace rule what we do is we are giving the equal probability to all the values. So for alternative 1, if you calculate the profit, profit will be 50 + 80 + 80 and equal probability is given so you have to divide by 3, so it will be 70. So it will be 70 units. Similarly for alternative 2 you are given the equal weightage that is why it will be 60 + 70 + 20 divided by 3.

So it will be 150 by 3 that is 50. You go to alternative 3 and an alternative 3 again you have 90, 30 and 60, so it will be 90 + 30 + 60 divided by 3, so it will be 180 by 3 that is 60. So using Laplace rule you see that alternative 1 should be preferred which gives you maximum of the profit out of the three alternatives.

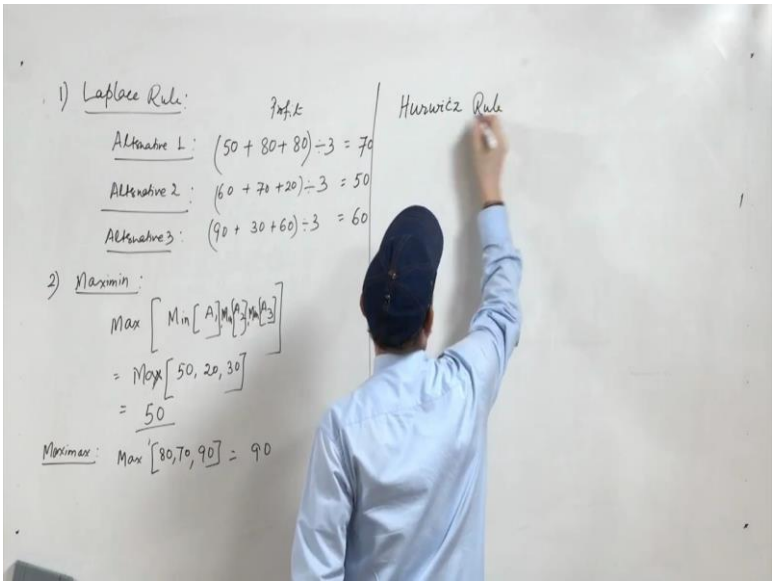
(Refer Slide Time: 20:32)



Let us go to the Maximin rule, using Maximin. Now in the Maximin what we do is, you are taking the maximum value of the minimum profit being given by each of the alternatives. So if you look at, it will be maximum of minimum by alternative A1, A2 and A3. So it will be nothing but maximum of minimum of A1 as well as minimum of A2 and minimum of A3. So basically minimum of A1 is as we have done it will be 50, it will be 20 and it will be 30.

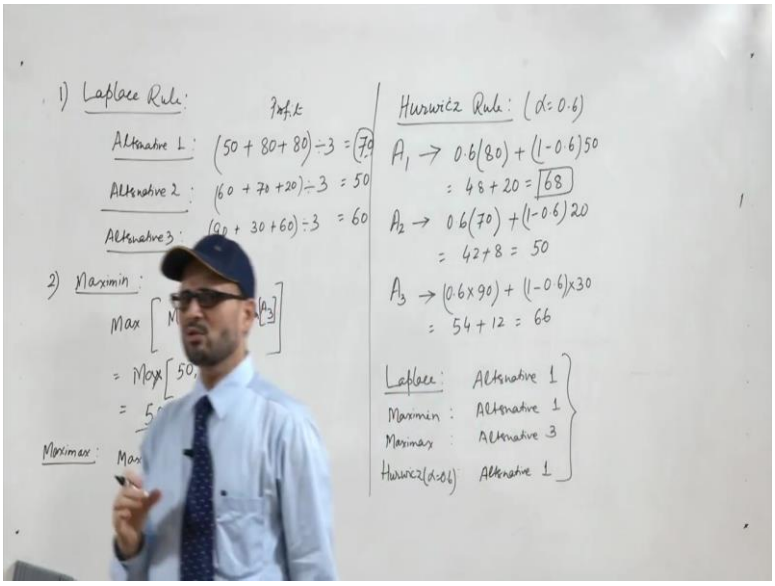
So out of this, the maximum value will be 50. So 50 we have a pessimistic view, we feel that you will get the minimum of the profit for every alternative and then we try to have the maximum of those minimum values.

(Refer Slide Time: 21:45)



If you go for Maximax, for Maximax as we know it is based on the optimistic concept. So here you are getting the maximum of the maximum values for every alternative. So that is for 1 is it is 80, for 2 it is 70 and for third it is 90. So that will be 90, so based on Maximin, you see that the alternative 1 is coming into picture whereas using Maximax the alternative 3 is under consideration.

(Refer Slide Time: 25:31)



So this is how Maximin and Minimax can be used to come to the selection of alternatives which gives you the desired objectives. Now let us discuss about the third rule that is Hurwicz rule. Now this rule talks about the parameter alpha which is the degree of optimism,

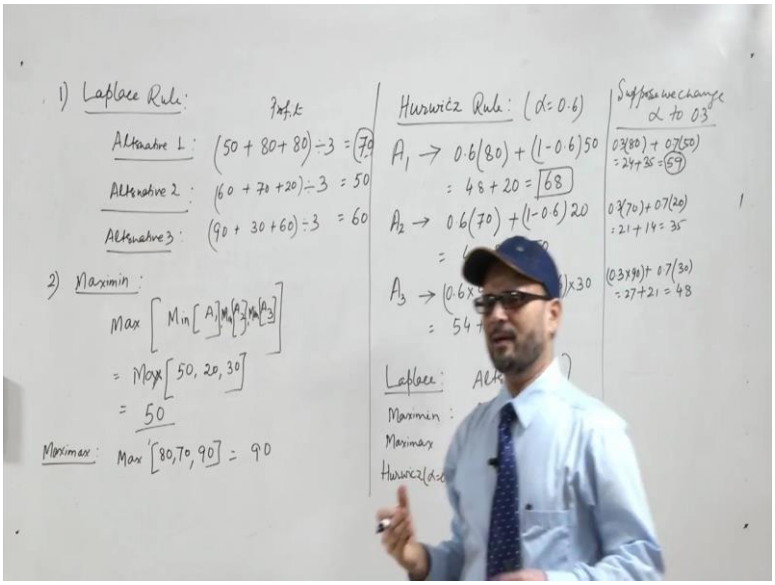
so basically in that case what you do is, you get the maximum value multiplied with this level of optimism that is point 6 and the minimum will be multiplied with one - alpha.

So basically for alternative 1, alpha is given as point 6, so basically maximum of the value is 80, so it will be point 6 multiplied by 80 + 1 - point 6 multiplied by the minimum value that is 50. So it will be 48 + point 4 into 50 that is 20, it will be 68. If you go to alternative 2, now in this case you have maximum of value is 70 and minimum is 20. So it will be 42 + point 4 into 20 that is 8, so it will be 50.

You go to alternative 3, alternative 3 maximum is 90, so using this parameter point 6 into 90 + 1 - point 6 into minimum value is 30. So it will be 54 + point 4 multiplied by 30 so 12 that is 66. So what you see is alternative A1 gives you the maximum value. So what we see is using the Laplace rule, you prefer alternative 1 which is giving you the maximum.

So using Laplace, alternative A1 one gives you the maximum values. Maximin alternative 1 again, Maximax alternative 3 and the Hurwicz where alpha is taken as point 6 the optimism parameter, for that again alternative 1. So this is how you come to the conclusion that which of the alternatives should be selected at least in the cases when the probability levels are not mentioned.

(Refer Slide Time: 27:31)



There is no certainty which is this no value of probability being given and in that cases, this is the methods these are the methods Laplace, Maximin, Minimax and Hurwicz. Hurwicz can be having different values it we change the alpha. If we change the alpha, suppose we change

the alpha, we change alpha to point 2 to point 3 suppose. Now this will basically change its value. So in that case it will be point 3 multiplied by 80 + point 7 multiplied by 50.

So in that case it will be 24 + 35 59. If you take A2 it will be point 3 times 70 + point 7 times 20, so it will be 21 + 14 35. And if we take here point 6 so this is point 3 multiplied by 90 + point 7 multiplied by 30 so it will be 27 + 21 so 48. Again using even this parameter, we see that the A1 is preferred. So for this parameter also again we see that A1 is preferred.

So this way you can have different value of the degree of optimism alpha value and based on that the alternative can be selected which should be the most preferred ones. And you can use any or either of these methods to suggest the best alternative. Thank you.