Engineering Economic Analysis
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Lecture35
Expected Variance Decision-Making under Risk

Welcome to the lecture on decision under risk and uncertainty, third lecture. We will discuss in this class about the expected variance decision-making. Basically as we know, these are the statistical parameters which are used to know how much accurate you are predicting.

So basically in the earlier lecture we had calculated the expected values by taking the probability values of every options into account and we calculate the expected values or the mean values and based on that we have to decide which of the alternative should be selected. But many times the mean values alone cannot be the decisive factor because we can come to a situation where the mean values does not differ much.

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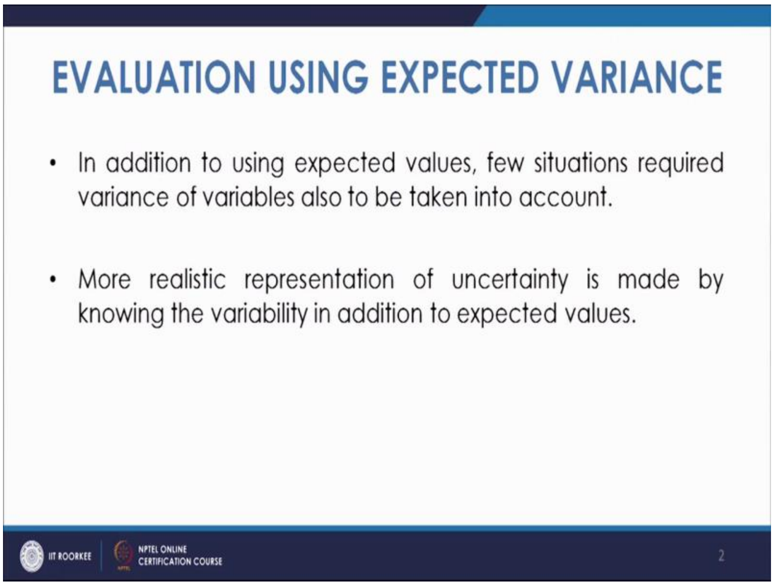

In that case the variance of the variable must be taken into account. Basically there is more realistic representation of uncertainty when the variability is taken into account. So we will see it by solving an example which will show that how variability can tell us more accurate prediction and we feel more confident in representing our results.

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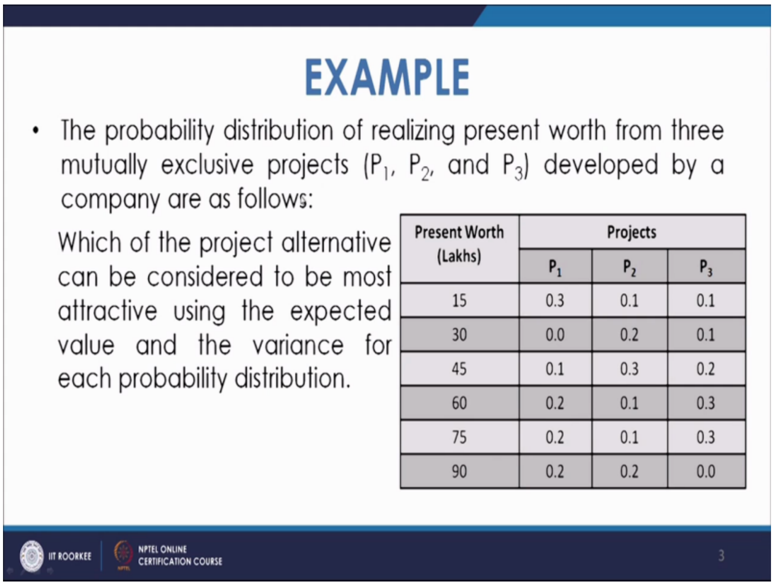

Let us see one example where it is written that the probability distribution of realising present worth from three mutually exclusive projects P1, P2 and P3 developed by a company are as follows. So basically you have the three projects and you can have these present worth values and the probability associated for getting suppose present worth of 15,00,000 is point 3. Similarly probability associated with P1 for getting 30,00,000 of present worth is 0 and so on.

So basically you have three projects and the probability distribution is shown for this which will yield this corresponding value of present worth. Now in this case as we will proceed, we will find first the expected value or the mean value but then we will also find the variance and then we will see that how you can predict that the results using expected value as well as the variance using each probability distribution. So let as solved this problem.

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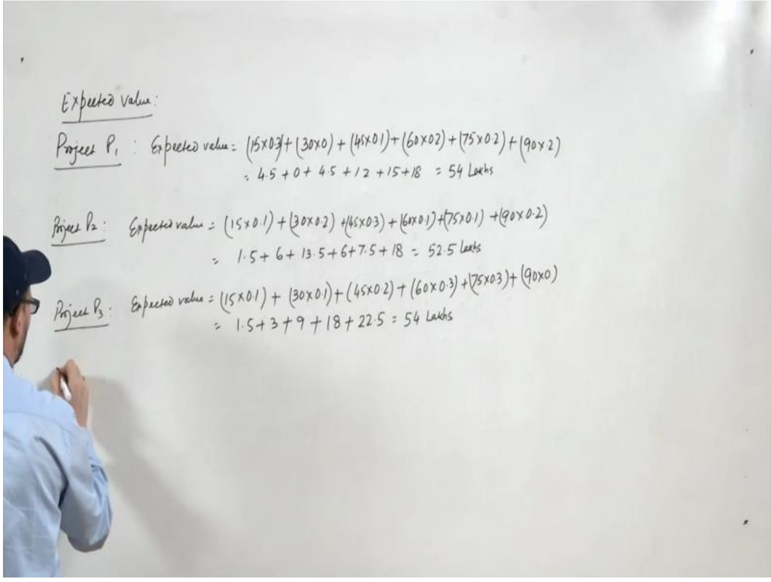

Now what we see is that if you go using expected value, what we see is, you see that you have 15,00,000 and for that your probability with first project is point 3. So if you take for project P1, expected value, it would be nothing but 15 multiplied by point 3 plus 30 multiplied by point 0 plus 45 multiplied by point 1 plus 60 multiplied by point 2 plus 75 multiplied by point 2 plus 90 multiplied by point 2.

So this comes out to be 4 point 5 plus 0 plus 4 point 5 plus 12 plus 15 plus 18. So 33 plus 45 and plus 9, so it is 54. So if we get the expected value of 54, then let us see for project P2. For project P2 in the similar fashion we can get the expected value. It will be 15 times point 1 plus 30 times point 2 plus 45 times point 3 plus 60 times point 1 plus 75 times point 1 plus 90 times point 2. So this much comes 1 point 5 plus 6 plus 13 point 5 plus 6 plus 7 point 5 plus 18.

So 31 point 5 and then 45, this is 51 and 52 point 5, lakhs of rupees. Let us go to the Project 3, so for that the expected value will be P1 so that will be 15 multiplied by point 1 plus 30 multiplied by point 1 plus 45 multiplied by point 2 plus 60 multiplied by point 3 plus 75 multiplied by point 3 plus 90 multiplied by 0. So this will be one point 5 plus 3 plus 9 plus 18 plus 22 point 5, so this is 54.

Now what we see is, using the expected value criteria, we get the expected values for the project P1, P2 and P3. Under the given probability distribution we get the expected value is 54, the 2 point 5 and 54 lakhs. Certainly we will prefer this end this but then since they are matching and you will see that it will be very close, so it is advisable to go for the checking of the variability and for that we will find the variance.

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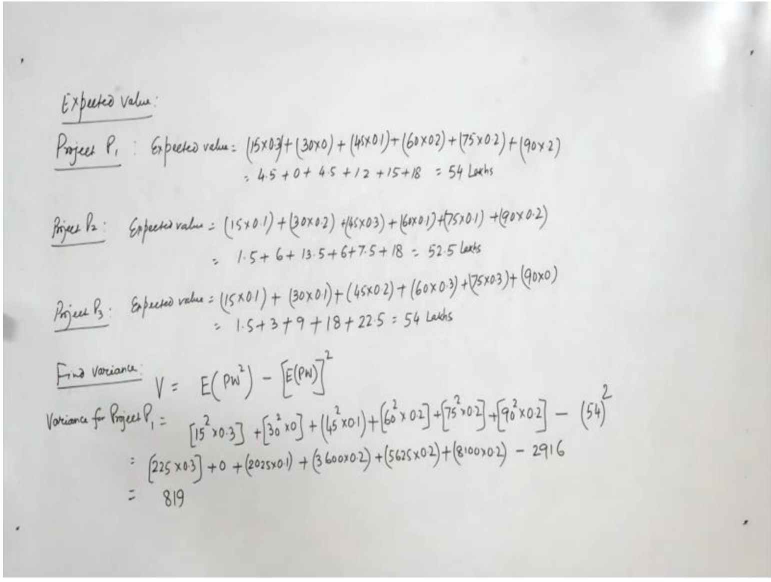

So based on these values, we will find the variance, find variance. Now for finding the variance we will use the formula variance will be equal to expected value of present worth square minus the value of present worth of and its expected value and it square. So we will find the variance for project P1. For project P1, the variance will be, we will have we have this 54,00,000 but we have the 15,00,000 individually for this probability.

So first of all we will get the expected value of the square of these values. So we will have 15 square into point 3 plus 30 square into 0 plus 45 square into point 1 plus 60 square into point 2 plus 75 square into point 2 plus 90 square into point 2. So we will have the values like this, 225 multiplied by point 3 and this will be subtracted with the value of expected present worth which is calculated as 54,00,000, so it will be subtracted from this amount.

So this is coming as plus 0 plus 2025 into point 1 plus 3600 into point 2 plus 5625 into point 2 plus 8100 into point 2 minus 2916 and this we can compute using the calculator, sorry. So this will be the 7 point 5 plus 202 point 5 plus 720 plus 1125 plus 1620 minus 2916, so this is coming out to be 819. So this is the variance for project 1. Let us see the, so in this fashion we have to calculate the variance for all the projects.

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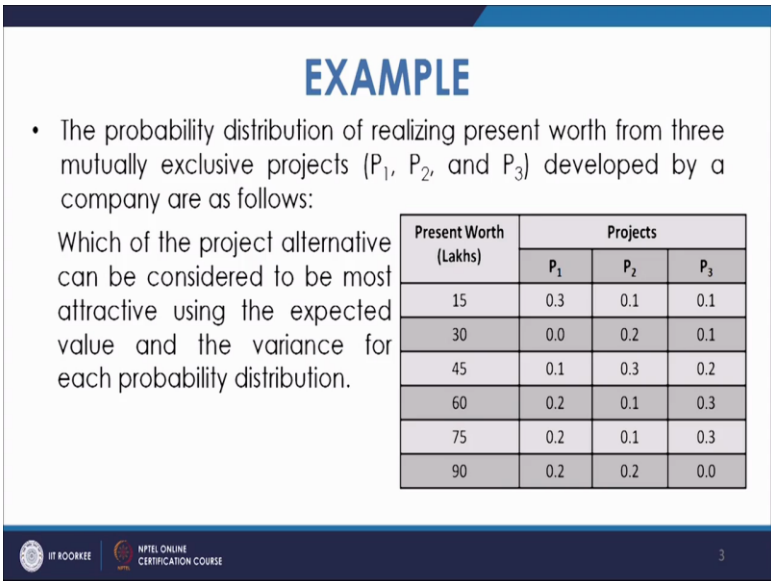

Let us calculate variance for project 2. So let us write here, we have the expected value project P1 as 54,00,000, P2 is 52,50,000 and P3 as 54,00,000. We have got expected variance, we are calculating and for project P1 it has come out to be 819, now we will calculate for P2 and P3.

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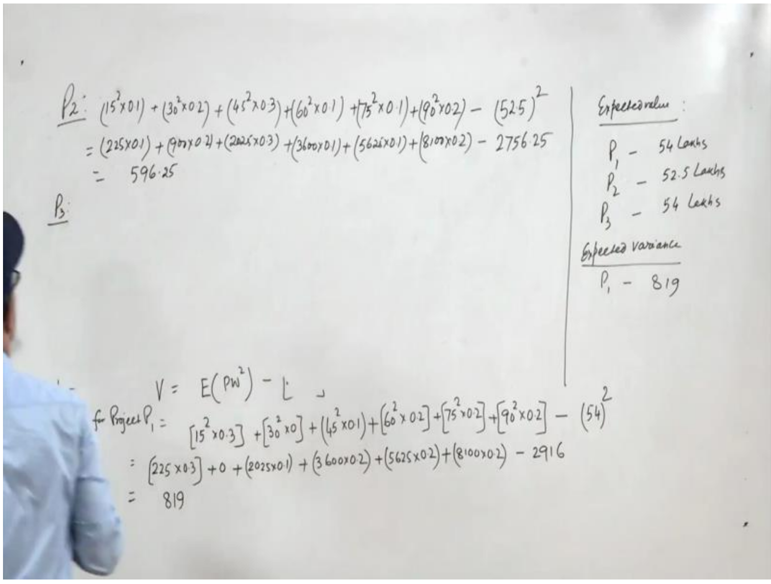

So for P2, for P2 again it will be 15 square multiplied by point 1 plus 30 square multiplied by point 2 plus 45 square multiplied by point 3 plus 60 square multiplied by point 1 plus 75 square multiplied by point 1 plus 90 square multiplied by point 2 minus we have to get the square of this expected value, that is 52 point 5 square.

So this will be equal to we have 22 5 multiplied by point 1 plus 900 multiplied by point 2 plus 2025 multiplied by point 3 plus 3600 multiplied by point 1 plus 5625 multiplied by point 1 plus 8100 multiplied by point 2 minus it will be 2756 point 25. So we can further add them, so it will be 22 point 5 plus 180 plus 607 point 5 plus 360 plus 562 point 5 plus 1620 minus 2756 point 25, so it is coming out to be 596 point 25.

What we see is that all the P1 gives 54,00,000 of expected value and P2 gives 52,50,000, the variability in terms of the parameters known as variance, 81 9 is for the project P1 and 596 point 25 is there for project P2. Let us calculate for P3.

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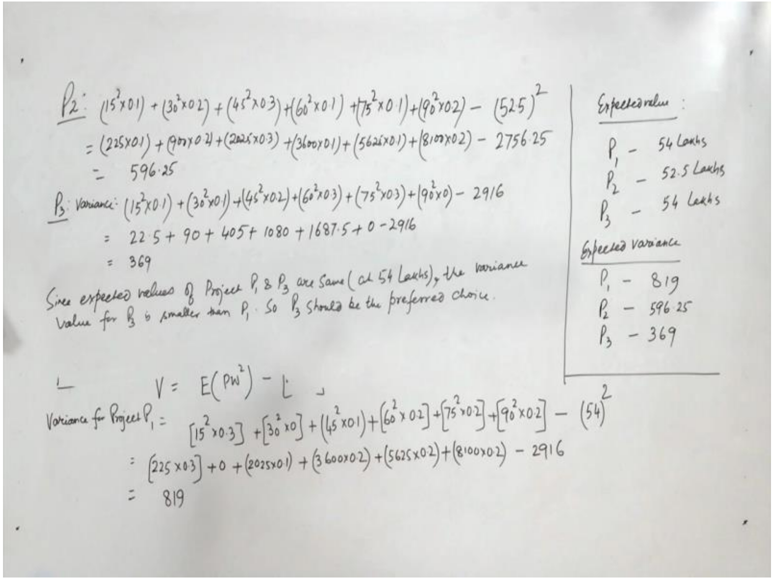

For P3 variance will be 15 square into point 1 plus 30 square into point 1 plus 45 square into point 2 plus 60 square into point 3 plus 75 square into point 3 plus 90 square into 0 minus the expected value was 54 so its square value and that is 2916. So we have 22 point 5 plus 90 plus 105 plus 1080 plus 1687 point 5 plus 0 minus 2916. We can get this value, it will be 22 point 5 plus 90 plus 405 plus 1080 plus 1687 point 5 minus 2916, it is coming out to be 369.

So what we see is, that project P1 was giving us the expected value of 54,00,000, P2 was giving 52,50,000, P3 was giving 54,00,000, so we could have preferred P1 and P3 instead of P3, however, it is difficult at this stage which one to decide whether P1 of P3. So in that case, it is better to go for the variance studies. Once we calculate the variance, what we see is for P2 it comes 596 point 25 and for P3 it comes as 369.

So it is seen that the variance value of P3 is the minimum. In that case, P3 should be preferred. So we can say that since expected values of project P1 and P3 are same at

54,00,000, the variance value for P3 is smaller than P1. Basically it is smallest of all, so P3 should be preferred, should be the preferred choice.

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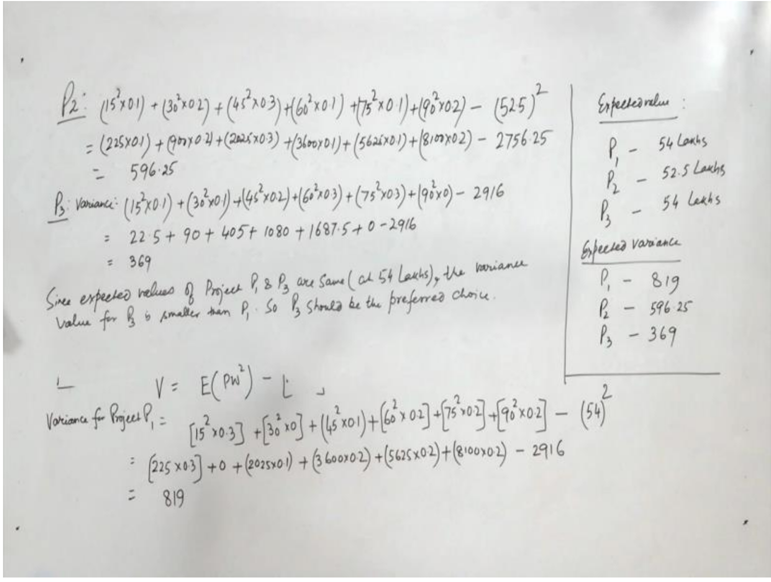

So basically this is how when you have this type of cases where the expected values are nearly equal and you have the difficulty in deciding which one should be preferred, in that case this method of getting the variance and seeing that where the variability is minimum it gives you that you are more likely to be close to this value. In this case the variability chances are less, so that is why it should be the preferred choice.

Next we will discuss about one of the technique when these type of elements are very uncertain and there are many components. In that case one of the technique which is very much known, that is known as Monte Carlo technique, that is also used. We will somewhat discuss in brief about this Monte Carlo technique, how it is used to find these values.

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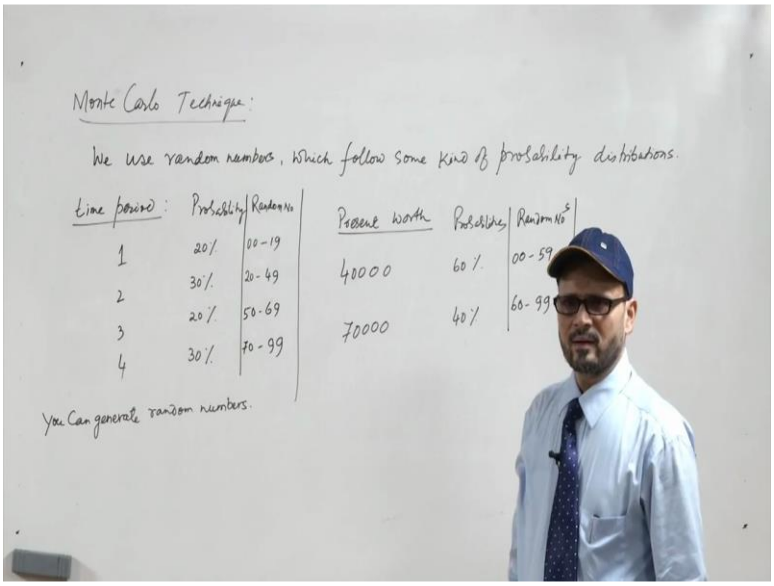

So basically the Monte Carlo technique, this technique is used under the cases when you have to decide and there are is chances of risk and uncertainty. And if there are many elements and there is a large amount of uncertainty, in that case, this Monte Carlo technique is used. Now when the Monte Carlo technique, basically we use random numbers which follow some kind of probability distributions.

So basically in the Monte Carlo analysis, you have first to generate the random numbers. Now let us say when suppose the time period is not certain, you have by and large you know time period may be 1, 2, or 3 or 4 years and the probability of having this time period is suppose 20 percent, 30 percent, 20 percent and 30 percent. Then you have, there are two possibilities of suppose present worth.

You may have the present work either as 40,000 or 70,000 and it is probability is also are given like maybe 60 percent and 40 percent. Now under these cases when you have more components involved and the probabilities are involved, in that case you have to take the effect of both and then you have to proceed. Under these cases, the Monte Carlo technique because in otherwise it will be very complex and further you have to move.

So under these cases, Monte Carlo technique is a preferred choice. So in this basically you generate the random numbers, now random numbers are generated based on this probability distribution. So you can generate uniformly distributed random numbers and you can see that you can have you can generate random numbers.

Now in this case, if you are generating a random number between 0 to 100, basically you will generate the random number between 0 to 99, there are 100 random numbers. So if this is 100 percent, you can generate the random numbers from 00 to 99. So what happens, you will generate the random number and the random numbers, since it is 20 percent, so from 00 to 19 will be the random number which will be assigned with the time period 1.

Similarly for 30, so another 30 numbers will be assigned for this, so it will be 20 to 49. Then another 20 will be for 3 years, so it will be 50 to 69 and another 30 that is 70 to 99. So this way you can assign the random numbers to these values.

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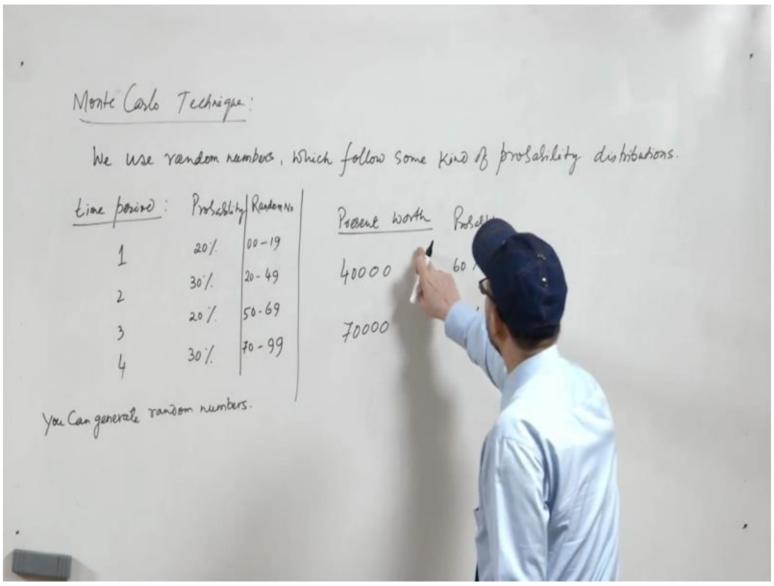

Similarly in this case also, you may assign the random numbers. So random numbers will be from 00 to 59 and 62 to 99. So this way you can assign the random numbers for these cases. Now what you can do is generate uniformly distributed random numbers, so there are many generators of the random numbers.

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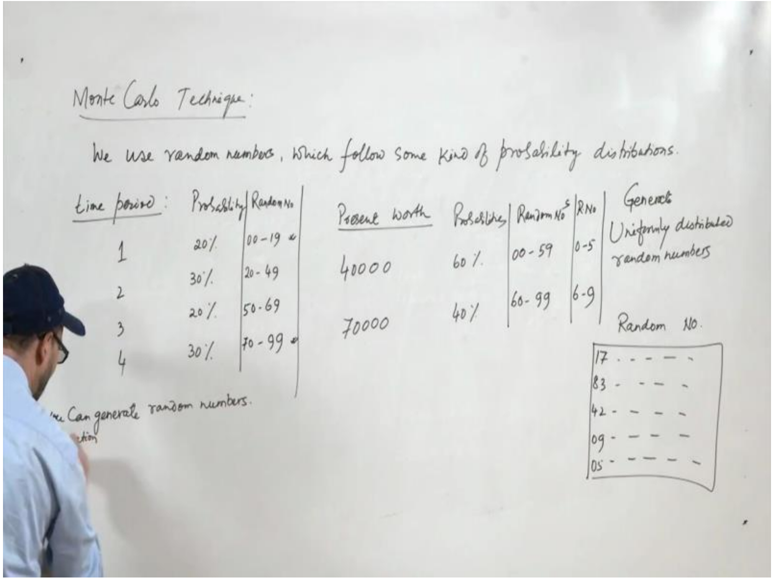

You can generate it using the Microsoft Excel or you can generate the random numbers following certain rules all following certain typical probability distribution functions by the random number generators. So basically for example when you will generate the random numbers, you will have **so** so once you generate the random numbers suppose it comes like this 17, 83, 42, 9, 5 like that it may go.

So basically you can generate the numbers and you can take the number, the first numbers of 17 and 17 will come, so for first case you will take the time period as one year. The second one as 83, at that time you will take the value as 4 year. So for every leading, you will go with similarly the random numbers if you are going for 100 random numbers. This the range will be 0 to 59 and 60 to 99. If you have a scale of 10, in that case 0 to 5 and 6 to 9.

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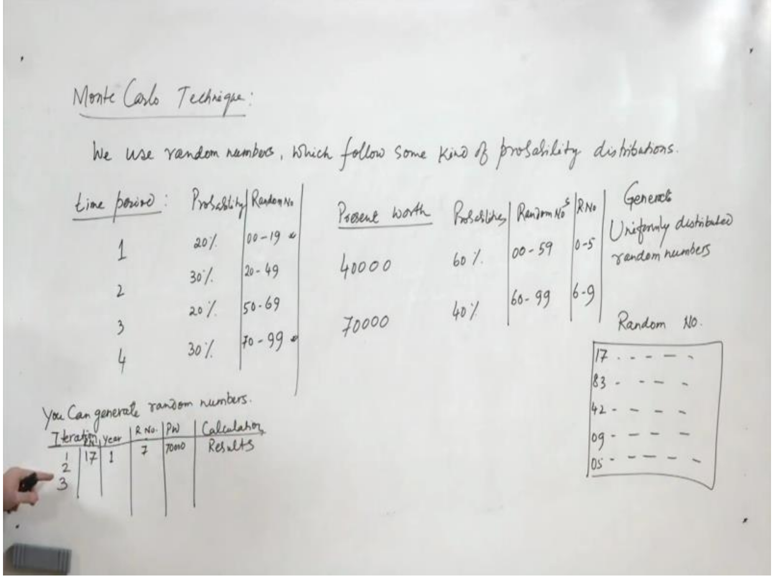

So any of the ranges can be taken and once this random number is coming according to the random number generators. So we can do the iteration and iteration wise if we go, you can take the year based on the random number. So whichever random number comes, suppose 17 comes, so you will take the year as one.

Now you will take the random number for the present worth the value and present worth value if you are taking suppose the 10 base and the first number comes 7, in that case it would be 70,000. So first (29:56) number suppose comes 7, in that case the present world will be 70,000. So this way whatever calculation you have to do, you can further do and get the results. So you can calculate the values.

Not this will basically be continued for a long time so that you come to the place where the variabilit because as long you go you take larger and larger range of random numbers and if you continue go for more distances, the chances of getting the error is minimum and in that case you can calculate whatever you have to calculate. So basically this is one of the technique which is to be used and it is quite worth to know about the technique.

It is a very useful for calculating the cases when we have many elements, the uncertainty is there in all the elements and you have to take into account the effect of uncertainty more number of elements, in that case the calculation can be done. Thank you.