Engineering Economic Analysis
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Lecture34
Expected Value Decision Making under Risk

Welcome to the lecture on decisions under risk and uncertainty. In this lecture he will be studying the case in which the expected value decision making is used.

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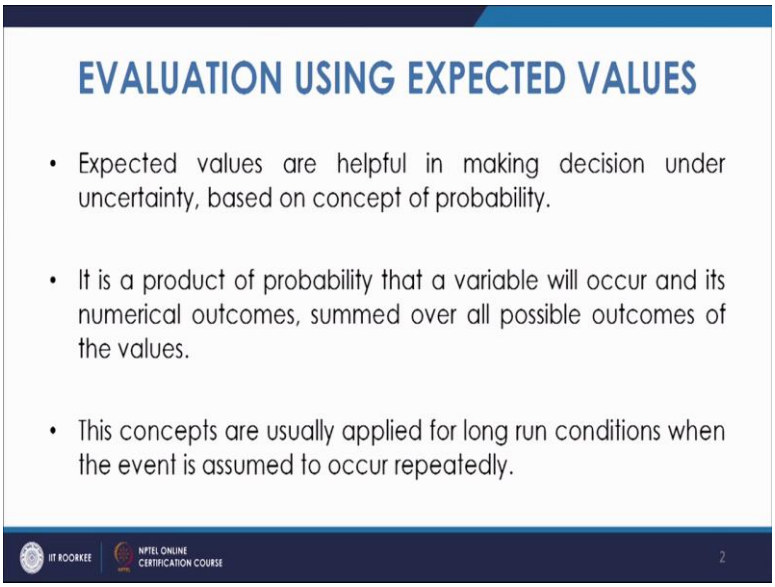

Now expected values are basically helpful in making decisions under uncertainty and basically it uses the concept of probability. We have already discussed that the probability of associated events are taken into account. The product of probability of a variable will be taken with its numerical outcomes and it will be summed over all possible outcomes.

So this way you get the expected value at these are basically used for long run conditions when the event is assumed to occur repeatedly. And sometimes also in the cases when you have initial investment and its effect is basically visualised in the future, so in those cases also, this concept is used.

Now let us take a case which has been about a flood damage in a certain area where there has been history that there is probability that flood may come and if the flood comes then it will do the damage to the crops. So there is basically a proposal that a dam should be built. Now or a levee should be built before the dam is made maybe later on.

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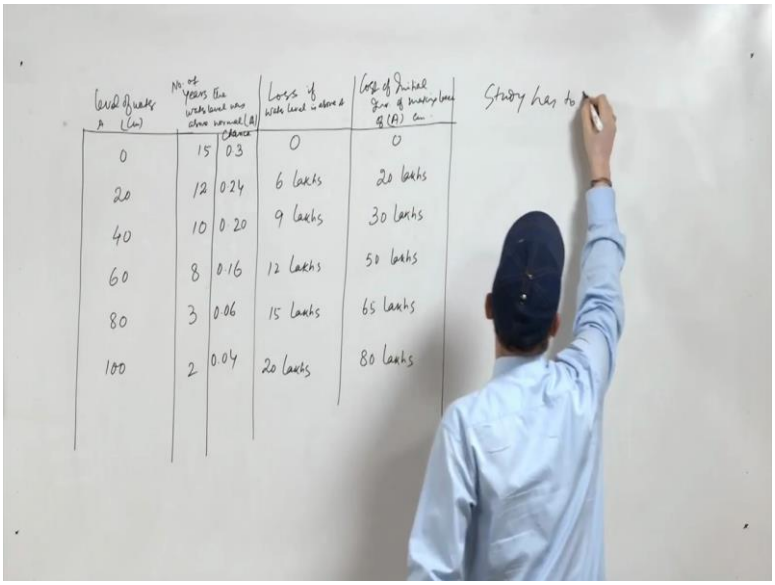

So before that basically you are trying to use it use this analysis in which there is history that how many years the level of flood water is how much. So basically the level of water is given as 5 02, 20, 40, 60, 80 and 100 centimetre. Suppose above the normal it goes. The history tells there has been occasions of 15 times during the last 50 years, the level was zero. So in that case there was no flood loss.

Now 12 times there has been the level of water 20 centimetre above normal. Then you have 10 times, 8 times, 3 times and 2 times the level of water has been as 40, 60, 80 and 100 centimetre above the zero level. And basically this these level of water causes the damage, so years, number of years, the water level was above normal. So this was the normal, so if it is A, so above A. Now third is that what will be the loss, loss is predicted if water level is above A.

So if the water level is above A, the loss is given as because there is no increase in the level, there is zero level, so loss is zero, then in this case you have 6,00,000, 9,00,000 and further you have 12,00,000, 15,00,000 and 20,00,000. This is the loss caused by the river if it, if the water level goes to 20 centimetre above the normal, it causes a loss of 6,00,000. Now further you can cost of initially investment of making levee, so of a centimetre.

So this cost is zero, this cost if you make a 20 centimetre length of levee, it will be costing maybe 20,00,000, this is 30,00,000, this is 50,00,000, this is the 5,00,000 and suppose this is a 80,00,000. Now basically depending upon this data, you have to find the probability of occurrence of the flood or the level of water in terms of the chance. And if you look at that the chance you can compute here itself, so it will be 15 by 50 into 100.

So probability will be point 3, this will be point 24, this will be point 2 this will be 1 point 6, no so this is order of 50. So you have to basically divide by 50, 15 by 50 so it will be point 3. 12 by 50 so it will be point 24, this will be point 2, this will be point 16, this will be point 06 and this will be point 04, okay. So this is the probability that the level of water will be coming to 0 that is chance is point 3, 20 as point 24, 40 as point 2, 60 as point 16, 80 as point 06.

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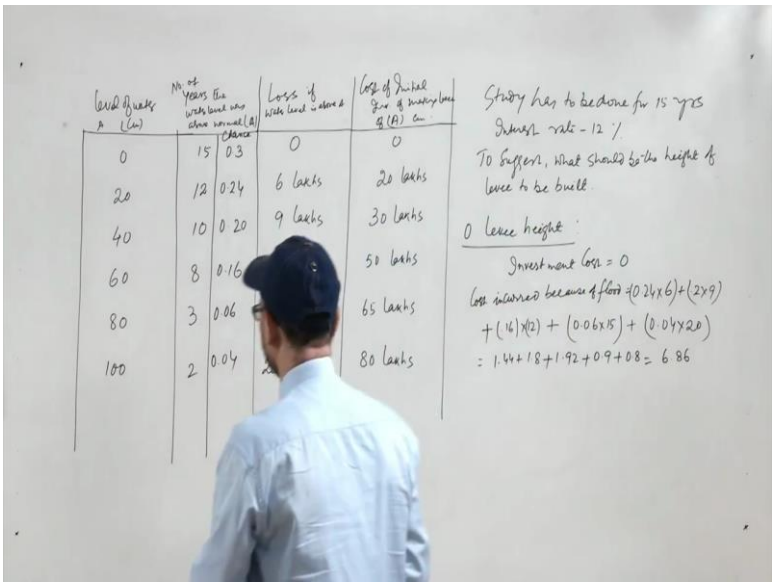

And so lesser probability is there that the river water level will exceed to this level. Now it is also seen that your study has to be done for 15 years after which it is assumed that the government will basically make a permanent arrangement by making the dam, so you have to study for 15 years. You have also interest rate as 12 percent, so using this interest rate you have to suggest what should be the height of levee to be built.

Now this is a case of expected value decision making, we can also use other methods but we will see how this expected value decision making by which you can suggest that what should be the height of the levee which should be made. Also you have to see that as you go on making larger levee heights, the initial investment is going to be higher and higher. So you have to justify what should be the basically actual height which gives you minimum of the cost.

Now let us say you are investing presently this amount, so we will talk about the annual expenditure. So if you take suppose zero levee height in that case you will have no present

cost investment, so your investment cost is 0. But you are losing all this, you are losing 6,00,000 when there is level of 20 and it is having probability of point 24.

So if you look at that, the cost incurred because of flood, it will be basically point 24 times 6 plus point 20 multiplied by 9 plus point 16 by 12 plus point 06 multiplied by 15 plus point 04 multiplied by 20. So it will be basically, if the river river reaches 20 centimetre and its chance is point 24, so that will give you loss of 6 plus point 2 times 9 plus point 16 times 12 plus point 06 multiplied by 15 plus point 04 multiplied by 20.

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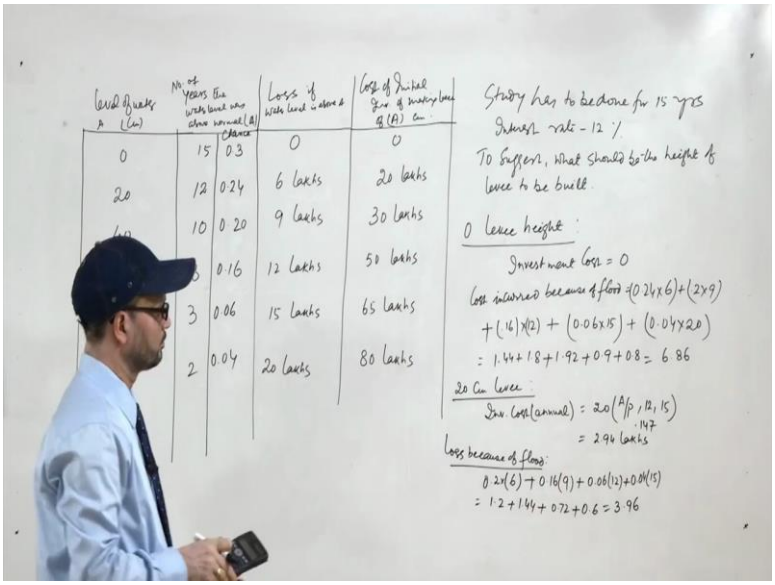

So this is when you have zero levee height and you can get it by 1 point 44 plus 1 point 8 plus 1 point 92 plus point 9 plus point 8. So this will come out to be 6 point 86. Now let us say you are going to make a 20 centimetre levee, a 20 centimetre levee, for that you are giving initial construction cost of 20,00,000 and basically this is to be used for 15 years.

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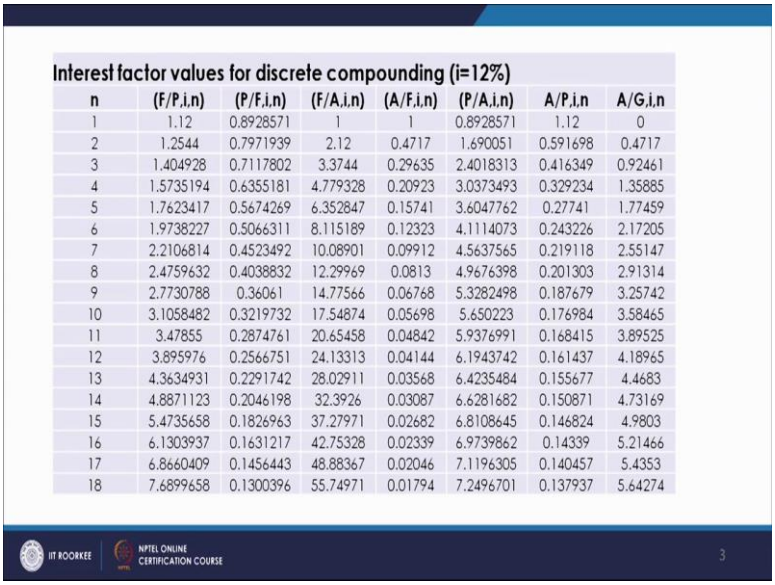

So you have to find investment cost on annual basis and so for that this 20,00,000 will be multiplied with for it is for 15 years period, so you have to multiply it with A by P I n, I is 12 n is 15. So you can see A by P I n 15, this is 12 percent for 15 years, A by P I n is point 147. So it is point 147, so it will be 2 point 94. Now let us say if the 20 centimetre levee is created, then what will be the loss because of the flood?

Now there will be no loss if there is 20 centimetre of water level but if there is 40 centimetre of water level, in that case loss will be corresponding to 20 centimetre of height and the probability of coming of 40 centimetre height water level is point 2. So point 2 multiplied by loss corresponding to 20 centimetre height, so that the 6,00,000.

Because this is levee is 20 centimetre height, so once the flood level will be 40 centimetre, then the loss will be corresponding to 20 centimetre that is 6,00,000 and the probability of having 40 centimetre height is point 2, so point 2 multiplied by 6. Similarly point 16 multiplied by 9 plus point 06 multiplied by 12 plus point 04 multiplied by 15. So this will be 1 point 2 plus 1 point 44 plus point 72 plus point 6. So this will be 3 point 96.

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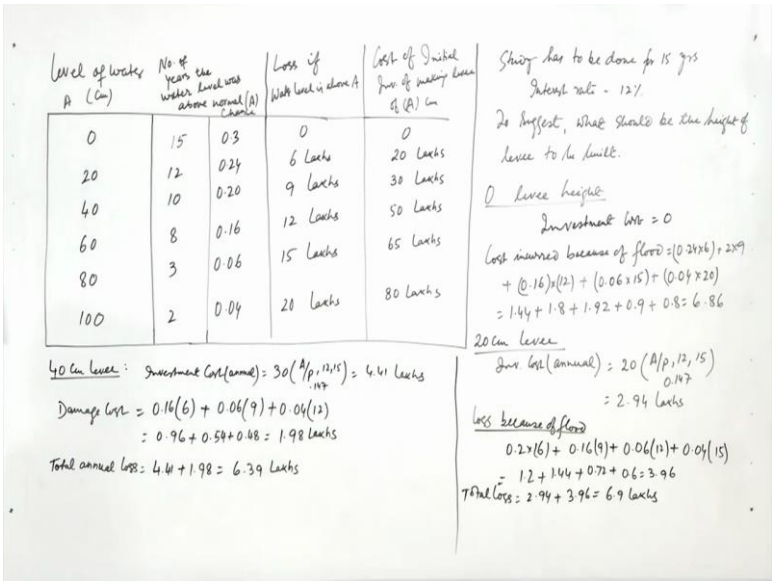

So if you look at, you will have certain total cost and total loss will be 2 point 94 plus 3 point 9 6 point 9. Now let us go to the next one we go for. Now basically we have to make a levee and basically we have to see that which levee height is giving you the minimum of loss. If you go for 10 feet levee, in that case your construction cost is, so this is 20 after that 40 centimetre levee. So for 40 centimetre levee, the cost is 30,00,000.

So again for that you are having the investment cost annual as 30,00,000 multiplied by A by P 12 15, that is point 147 4 point 41 lakhs and the damage cost will be when it is 60 centimetre, the probability is point 16 multiplied by the loss because of the 20 centimetre height, that is 6,00,000.

So it is multiplied by 6 plus again probability of 80 centimetre height, that is point 06 and it will be multiplied with the loss corresponding to 40 centimetre height of the flood water level that is 9,00,000. And then if it is 100 centimetre height, it is probability is point 04 and then the corresponding loss will be for the 60 centimetre height, that is 12,00,000. So it will be point 96 plus point 54 plus point 48 and it will be 1,98,000.

So total annual loss, it will be investment cost annual that is 4 point 41 lakhs plus the damage cost that is annual damage cost lost so it will be 1 point 98, 4 point 41 plus 1 point 98 and this will be equal to 6 point 39 lakhs.

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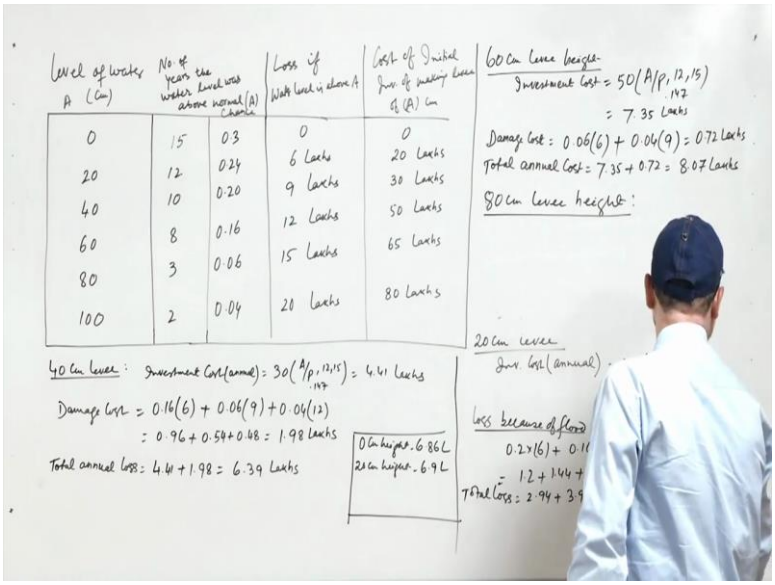

Now further if we are making a levee of 60 centimetre height let us see what is the total annual cost which you have to bear. So for 60 centimetre levee height construction you have investment cost this is equal to again for 60 centimetre height, the cost is 50,00,000. And if you want to find the annual value, it will be multiplied with the factor again A by P 12 15. Now this factor value is anyway calculated to be point 147, so it is coming out to be 7,35,000.

You further go to the loss, so damage cost, again in this case when the flood water level is 80 centimetre, its probability is point 06 and in that case the loss will be corresponding to 20 centimetre height that is 6,00,000. Then if the height of the water is 100 centimetre, its probability is point 04 and in that case the loss will be corresponding to 40 centimetre that is 9,00,000. So it will be point 36 plus point 36 and it will be point 72 lakhs.

So total annual cost comes out to be 7 point 35 plus 0 point 72 that is 8,07,000. We have to further find the loss or total annual cost for the 80 centimetre levee height and 100 centimetre levee height. We can further rub. For 0 centimetre height, it is 6,86,000 for that we have to write maybe in a box. For 0 centimetre height, 6,86,000 and for 20 centimetre height it is 6,90,000. For 40 anyway we have calculated as 6 point 39, for 60 it is coming as 8 point 07.

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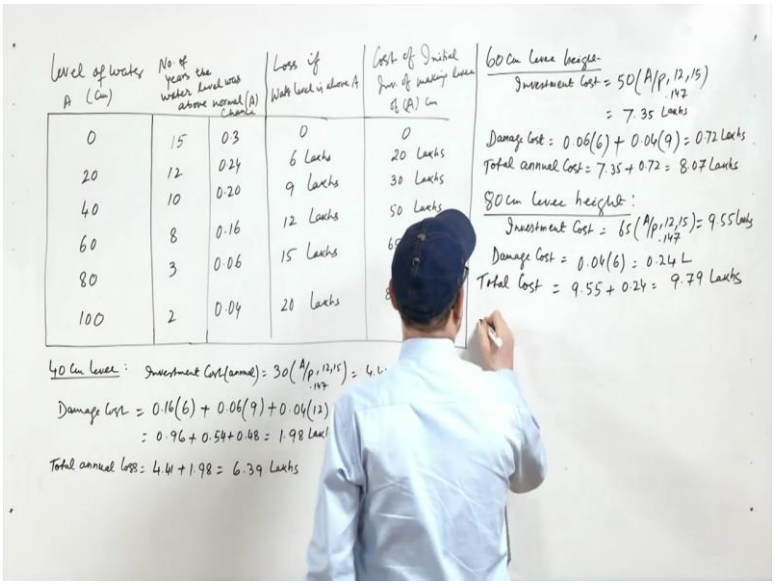

Let us calculate for the 80 centimetre levee height. For 80 centimetre levee height, the investment cost will be, this is 65,00,000, the initial cost. So 65 multiplied by A by P 12 15, that is point 147 and that is 9,55,000. Then the damage cost annually, it will be only in that case when there is 100 centimetre of flood water level and its probability is point 04.

So point 04 multiplied by the damage will be corresponding to only 20 centimetre of height and this is 6,00,000, so it is point 24 lakhs. So total cost will be 9 point 55 plus point 24 that is 9,79,000.

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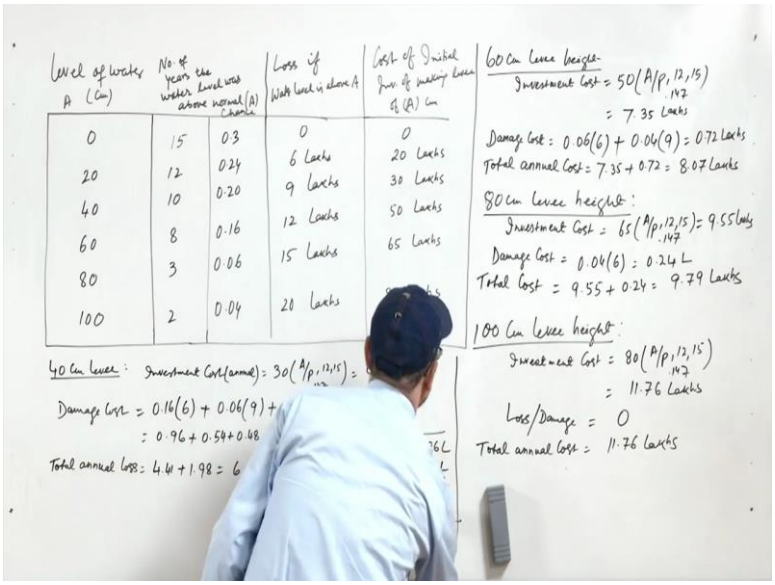

Finally if we calculate the cost for levee height of 100 centimetre. So 100 centimetre levee height, in case of 100 centimetre levee height the investment cost annually is 80 times A by P 12 15 that is point 147, so it will come out to be 11,76,000 and the loss or damage due to the flood is zero because this is the maximum height of the water which is reported. So in that case total annual cost is 11,76,000.

What we see is that on 20 centimetre height you have 6,90,000 was the annual cost. On 40 centimetre height, it was 6,39,000. On 60 centimetre height, it was 8,07,000. On 80 centimetre it was 9,79,000 and finally if you are making 100 centimetre, it is coming out to be 11,76,000.

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So what you see is the maximum loss is when you are going to make the 100 centimetre levee height and the minimum loss annually or minimum cost what you have to incur annually is 6,39,000. So it is suggested to go for making a 40 centimetre levee which should go for 15 years and in that case the annual cost is minimum. So suggested height of levee is 40 centimetre.

So this is how you solve such problems very you have been given the probability values and you have to decide what should be the height of the dam in optimum manner. Thank you.