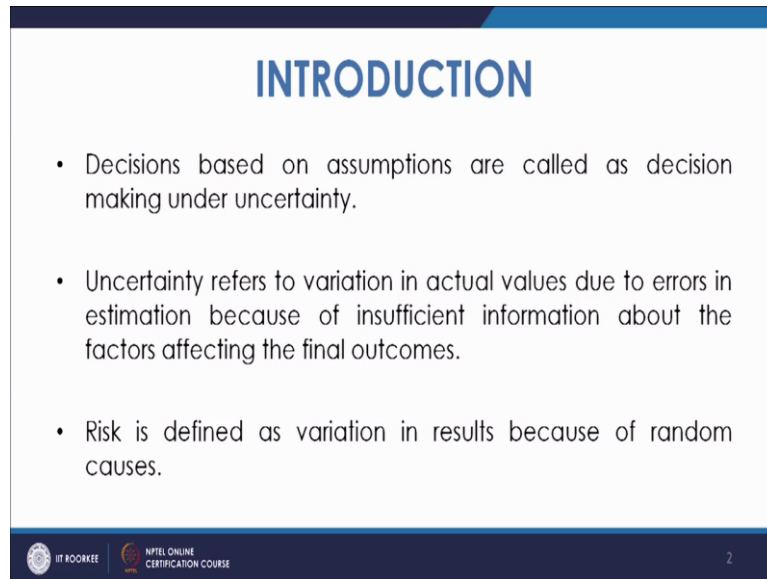


**Engineering Economic Analysis**  
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**Indian Institute of Technology Roorkee**  
**Lecture33**

**Introduction to Decision under Risk, Criteria for Decision under Risk**

Welcome to the lecture on decision under risk and uncertainty.

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**INTRODUCTION**

- Decisions based on assumptions are called as decision making under uncertainty.
- Uncertainty refers to variation in actual values due to errors in estimation because of insufficient information about the factors affecting the final outcomes.
- Risk is defined as variation in results because of random causes.

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So now decisions based on assumptions are called as decision-making under uncertainty. Basically when you know the outcomes of the decision with certainty, then it is known as decision with certainty. But in most of the cases in engineering economic analysis basically all these are predicted in future, so these outcome, you do not know what will be the outcome. So there is uncertainty involved with it, there is risk involved with it.

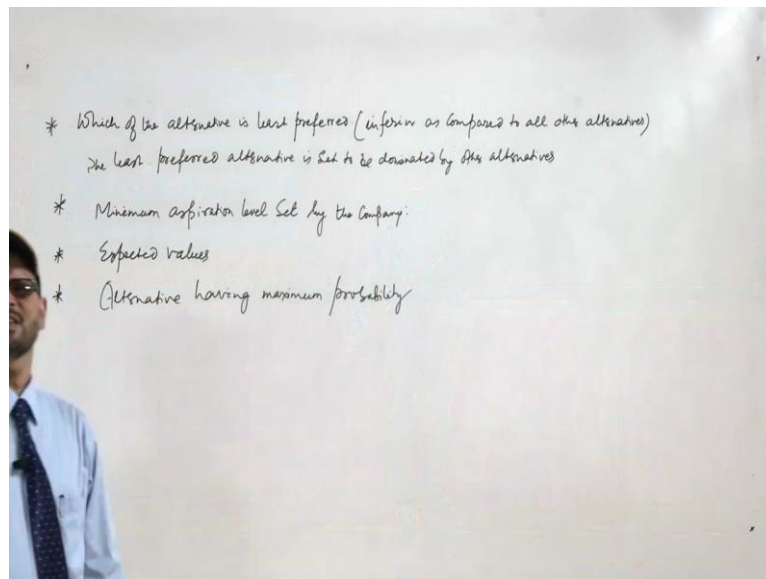
Uncertainty refers to variation in actual values due to errors in estimation because of insufficient information about the factors affecting the final outcome. So basically there are many factors and since we do not know about the behaviour of the factors, how they are going to affect, there is large amount of variation or there may be small amount of variation in what you predict and what will be the actual outcome and that is basically referred to as uncertainty.

Risk is defined as variation in result because of random causes. So basically when you do not know the causes, there may be any cause randomly, then that is defined as risk. So basically we will talk about the decisions, when there is risk and uncertainty what way the decision

should be made. Now in that case basically first of all what we see is, since the economic decisions are to be made in future always, you attach certain probability with the outcomes.

Suppose you are given certain options, you are to get certain contract and there are 3 types of contracts and the probability of getting the contracts is known to you. And every contract can be done using the alternatives and there are few alternatives and you can select the alternative. Now every alternative will give you a certain profit or loss if you are getting the individual contracts.

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So the first thing what you do is, you look at the alternatives and see that which of the alternative is least preferred, inferior as compared to all other alternatives. Now the outcomes in case of every alternative basically is also associated having certain probabilities. But still if you see that the outcomes by any of the alternatives is inferior to each one of the others, in that case you say that, it is the having the least preferred.

And basically they are said to be the least preferred alternative is said to be dominated by others alternatives. So basically when we talk about different kinds of alternatives in that case we feel that which one is dominated by others so that automatically you should leave that. Next comes the minimum aspiration level set by the company. Now what happens that as we discuss that you have different types of outcomes different outcomes in the different alternatives.

Now the company would like to go only for those alternatives where it will see that it should get at least a minimum level of aspiration. They need fix some amount of profit that is

some minimum amount of profit or maximum amount of loss so that if even if it gets any of the alternatives, it goes to any of the alternatives, it is likely to get at least some of the minimum level of profit or it can have maximum some amount of loss.

So this is known as aspiration level set by the company. Every undertaking wishes to have certain minimum profit or maximum loss, on that basis it sets its aspiration level. Then there are the expected values. Now we get the expected values by taking into account the probabilities. So basically all the values of probabilities are given the importance and based on that the mean value of outcome is basically found out.

And once you get that the alternative which gives you the maximum expected value, basically we prefer that. So basically this is known as expected value criterion of decision-making. Then there is also a criterion that is alternative having maximum probability. So basically we also can decide upon which one should be chosen and sometimes we try to go with those alternatives which has the maximum probability.

So that is known as the maximum probable criterion. So basically we go for those. Now after that you will have the comparison of the options which are basically achieved by using the different criterion and then we have to ourselves judge or the company has to judge which one should be preferred.

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\* Which of the alternative is best preferred (inferior as compared to all other alternatives)  
 The best preferred alternative is said to be dominated by other alternatives

\* Minimum aspiration level set by the company:

\* Expected values

\* Alternative having maximum probability

	prob (Loss)	Contract-A	Contract-B	Contract-C
		0.3	0.2	0.5
Alternative-1		5	5	20
Alternative-2		-10	8	25
Alternative-3		0	10	24
Alternative-4		5	15	10
Alternative-5		-15	5	8

For example let us say, you are applying for the company is applying for getting a contract and the contract company can get either as contract A contract B or contract C. Now the company getting the contract A has the probability of point 3 4 contract B it is point 2 and for

C it is point 5. Now once the contract is given, it is seen that it can be accomplished with different alternatives and there are 5 alternatives which are available. So alternative 1, 2, 3, 4, 5.

Now what we see is that if the contract A is it is getting, now if you see that, the probability of this is, if you sum this, all this is one. Now in this case, what the profit company is expected to get for all the alternatives, it is in terms of lakhs. So what we get is, here 5,00,000, 5,00,000 and 20,00,000. So basically if alternative one is selected and if contract A is got, you have a profit of 5,00,000, B for 5 and C for 20.

Similarly you have minus 10, 8 and 25. So basically you are likely to get the loss when alternative 2 is selected. Alternative 3, it is 0, 10 and 24, so in this case there is no profit no loss but you have, if you get contract B or C, you have the chances of getting 10 and 24. You have 5, 15 and 10, if you get alternative 4. And in this case you have minus 15, 5 and 8.

So now there is risk and uncertainty here, the probability associated with getting these contracts is this one. Now in this case, these are the profit amounts in lakhs, so how the company will proceed? No C, we have discussed about the different criteria, so let us see first of all if we use, which of the alternative is least preferred.

So looking at the values, we have to see which of the alternative gives you minimum amount of profit or it is giving you the inferior most results, in that case if you use this rule. The first rule, the dominance rule, it basically tells you that if you go for alternative 5, if you see these values are inferior or not better than any of the values in the respective columns. So that we alternative 5 can be removed for considerations.

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\* Which of the alternative is least preferred (inferior as compared to all other alternatives)  
The least preferred alternative is said to be dominated by other alternatives

\* Minimum aspiration level Set by the Company:

\* Expected values

\* Alternative having maximum

1<sup>st</sup> rule (dominance rule): Alternative 5 can be removed

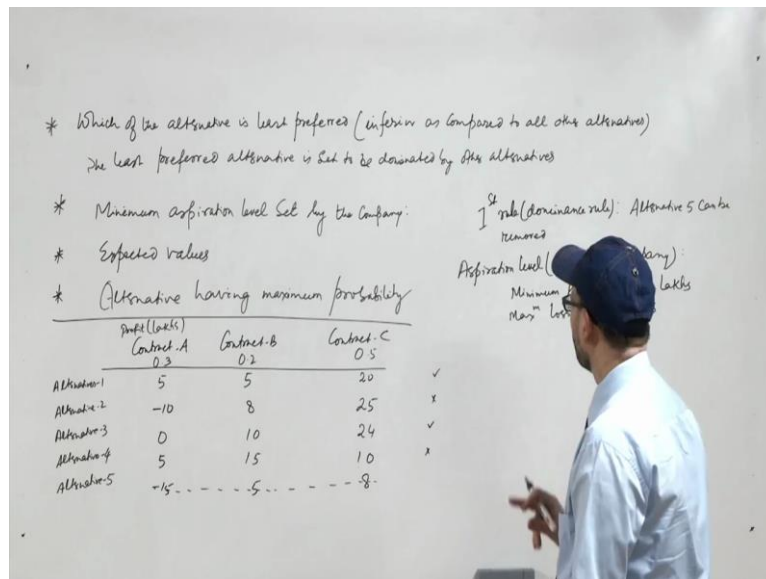
Aspiration level (Set by the Company):  
Minimum Profit of 20 lakhs  
Max<sup>m</sup> loss - 5 lakhs

	Contract-A 0.2	Contract-B 0.2
Alternative-1	5	5
Alternative-2	-10	8
Alternative-3	0	10
Alternative-4	5	
Alternative-5	-15	

Okay so what we see is that since it is giving you the inferior most values, you just remove them. Now the thing is, the next is that minimum aspiration level. Now the minimum aspiration level can be set as the by the company itself it is set as at least 20,00,000 of profit or 5,00,000 of loss.

So aspiration level set by the company, the company says that we cannot go for the alternatives which are basically likely to give us more than 5,00,000 of loss or less than 20,00,000 of profit. So the aspiration level set by the company is minimum profit of 20,00,000 and maximum loss of 5,00,000. Now suppose this is the kind of aspiration set by the company. So in that case if you look at, in this case, this is okay, this being satisfied, you have not the loss more than 5.

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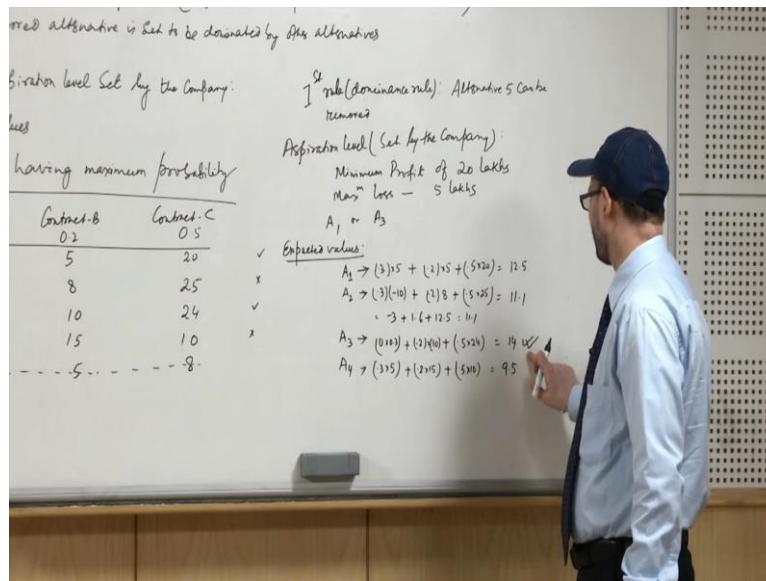


So A1 is okay, now here in fact we do not have any loss, so we have a profit. At least having 50 percent probability is there that you get a profit of 20,00,000, so this will go. Then if you look at this option, this option gives you having 30 percent probability is that if alternative 2 is chosen this will give you a loss of 10,00,000. So A2 cannot be preferred because it is giving you, it is not coming to meet aspiration level set by the company, so this will not be okay.

Then we see the alternative 3, you have loss, so there is no loss, so this way it is okay and the profit is 24, it is being met, so that is why A3 will be okay. But if you go for alternative 4, on the loss account, it is satisfying but on the profit account it is not satisfying because the minimum amount of profit is said to be 20,00,000, so this is also not satisfying. Alternative 5, we have already removed because of the dominance criterion.

So if you look at that you see that, out of these four the two is qualifying, you can go for A1 or A3. So aspiration level criteria gives you A1 or A3. So you can go for any criteria, either A1 or A3 which basically is giving you a the that criteria by which you are meeting the aspiration level.

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Then next is expected values. Now this criterion for expected values will take into account the value of these probabilities with these outcomes.

So if you take the expected values for alternative 1 it will be point 3 times 5 plus point 2 times 5 plus point 5 times 20. So it will be 2 point 5 plus 10, it will be 12,50,000. Alternative 2, it will be point 3 times minus 10 plus point 2 times 8 plus point 5 times 25, so it comes out to be minus 3 plus 1 point 6 plus 12 point 5. So it will be 14 point 1 minus 3, so it will be 11 point 1.

Then going to alternative 3, 0 times point 3 plus point 2 times 10 plus point 5 times 24, so it will be 12 plus 2 that is 14. The you go to alternative 4, here you get point 3 times 5 plus point 2 times 15 plus point 5 times 10, so it will be one point 5 plus 3 plus 5, 8 plus 1 point 5, that is 9 point 5. Now if you look at the values, what you see is, maximum is given by this option.

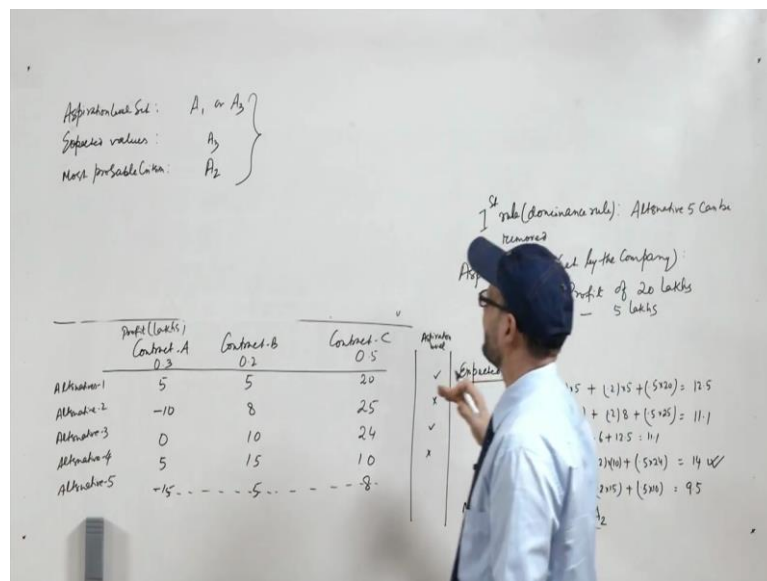
If you go for using expected value criterion, it will be preferred that its alternative A3 is selected which is likely to give you a maximum value of profit of 14,00,000. So this using expected values, you can have this A3 preferred.

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So if you look at the options which you got, in the first case anyway using the dominance, you have removed the A5 but using the aspiration level set, you go for A1 or A3. Using the expected values, you either for, you go for A3 and using the most probable criterion, you go for A2. So ultimately the organisation can take any of these criterion and it can decide that the contract can be given to whom or which alternative will be the best preferred one.

And basically based on that the estimation can be done, what will be the estimated profit and this way the estimation is done. Now we will discuss about the decisions under case of uncertainty when it is not known what should be the probabilities associated here which is mentioned, in that case there are certain criteria which we should know. So that is under the case of decision under uncertainty.

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Approximate Sol:  $A_1$ , or  $A_3$   
 Separate values:  $A_3$   
 Most probable Cost:  $A_2$

Decision under uncertainty:  
 \* Laplace Criterion:  
 $A_1 \rightarrow (5 + 5 + 20)/3 = 10$   
 $A_2 \rightarrow (-10 + 8 + 25)/3 = 7.66$   
 $A_3 \rightarrow (0 + 10 + 24)/3 = 11.33 \leftarrow$   
 $A_4 \rightarrow (5 + 15 + 10)/3 = 10$

	Contract-A 0.3	Contract-B 0.2	Contract-C 0.5
Alternative-1	5	5	
Alternative-2	-10	8	
Alternative-3	0	10	
Alternative-4	5	15	
Alternative-5	-15	-5	

So when we do not know about the probabilities, we do not have much idea about the probabilities, there are a few criterion which are basically reported. One is Laplace criterion. So using the Laplace criterion basically, equal probability is given to these outcomes. So in that case, the alternative A1 will give you, so since you have 3 values, you it will be given 5 plus 5 plus 20 upon 3.

So basically equal probability of point 33 is assigned, in that case A1 comes out to be 10. A2 comes out to be minus 10 plus 8 plus 25 upon 3, so it will be 7 point 66. A3 will come as 0 plus 10 plus 24 and upon 3 it will be 11 point 33 and for alternative 4 it will be 5 plus 15 plus 10 upon 3, so that is 10.

So in that case what you see is, alternative A3 is giving you the maximum profit where you see that the probability assigned is taken as equal because things are if they are not known, you can assign the equal probabilities.

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Hurwicz Criterion:

	Contract-A 0.2	Contract-B 0.2	Contract-C 0.5
Alternative-1	5	5	20
Alternative-2	-10	8	25
Alternative-3	0	10	24
Alternative-4	5	15	10
Alternative-5	-15	-5	-8

Decision under uncertainty:

\* Laplace Criterion:

$$A_1 \rightarrow (5 + 5 + 20) / 3 = 10$$

$$A_2 \rightarrow (-10 + 8 + 25) / 3 = 7.66$$

$$A_3 \rightarrow (0 + 10 + 24) / 3 = 11.33 \leftarrow$$

$$A_4 \rightarrow (5 + 15 + 10) / 3 = 10$$

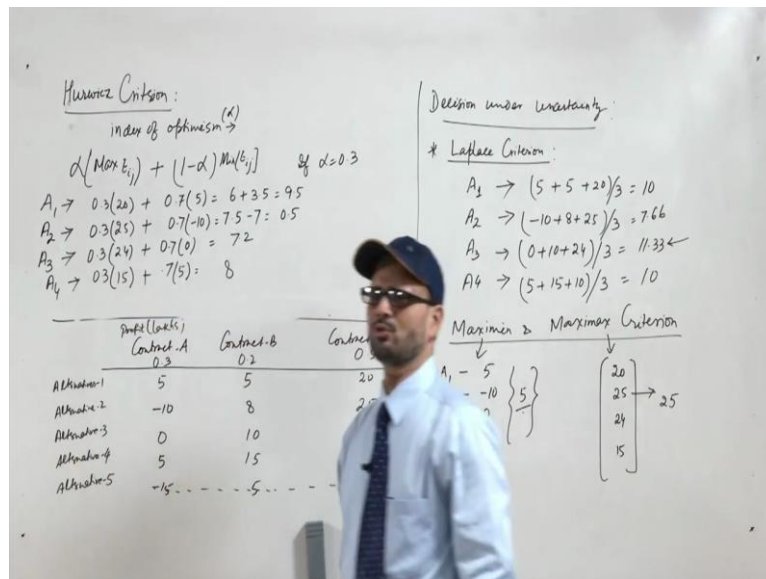
\* Maximin & Maximax Criterion

$\begin{matrix} A_1 - 5 \\ A_2 - -10 \\ A_3 - 0 \\ A_4 - 5 \end{matrix} \left\{ \begin{matrix} \\ \\ \\ \end{matrix} \right\} 5$	$\begin{matrix} 20 \\ 25 \\ 24 \\ 15 \end{matrix} \rightarrow 25$
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Now next is a criteria which is known as Maximin and Maximax criteria. Now this is known as something like a pessimistic view and this is known as an optimistic view. So what Maximin tells that basically you as you, you have a pessimistic opinion, you assume that in any case, you will get the one which has the minimum outcome or you are going with the minimum value out of all these and then you are trying to have a maximum of them.

So if you look at that, using Maximin, using Maximin if A1 you look at, in the A1 you have 5, A2 you have minus 10, A3 you have 0 and A4 you have 5. Now in that the maximum value is 5, so using Maximin you get having a pessimistic opinion you get a profit of 5,00,000. Whereas Maximax is basically maximising the maximum values, in that case you have 20, 25, 24 and 15. In that case, so in that case you have 20, 25, 54 and 15 and in that you basically go for 25.

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So these are the two criterias which are also used. Using the pessimistic and optimistic criterion you can have this as that. The next one, one more criterion is preferred is depending upon certain weightage to the optimism, we cannot have optimism to a higher value, so we give the optimism to certain value. So basically what we do is, that is known as Hurwicz criterion.

Now in this case, in the Hurwicz criteria basically, what we give is, you give an index of optimism is set. So if this index of optimum optimism is set, in that case it tells alpha that is alpha 1, so basically this will be alpha into maximum value of EIJ plus 1 minus alpha into minimum value of EIJ.

So basically you have the values and now in this case, if alpha taken as suppose point 3, in that case using the Hurwicz criteria, you have if you take A1, what you see is maximize maximum value is 20, so you will take point 3 into 20 plus point 7 into minimum value is 5. It will be 6 plus 3 point 5 that is 9 point 5. Similarly you have A2, you have point 3 as the alpha and maximum is 25 plus point 7 multiplied by minus 10.

So it will be 7 point 5 minus 7 that is point 5. A3 if you look at, if it is zero point 3 multiplied by maximum value that is 24 and minimum is point 7 0, so that is 7 point 2. And of A4, it comes point 3 as 15 point 3 multiplied by 15 plus point 7 multiplied by 5, so it will be 4 point 5 plus 3 point 5 that is it. So using the Hurwicz criteria, what you see is, your value comes out to be maximum for criterion A1 and this is based on the degree of optimism.

You can have the value either is point 3, point 2 or any value and that shows what is the degree of optimism that you are looking for. So these are the different criteria upon which you can have the estimation. You can say that you can proceed with this criteria and estimate the parameters. Apart from that, you have also a criteria that is known as minimax regret criteria.

So basically all these four criteria are used when there is uncertainty, you are not sure about the probability values and they are used for estimations. Thank you.