Engineering Economic Analysis
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Lecture 3
Interest Formulas: Simple and Compound Interest, Cash Flow Diagrams

Welcome to the lecture on interest and cash flow diagrams.

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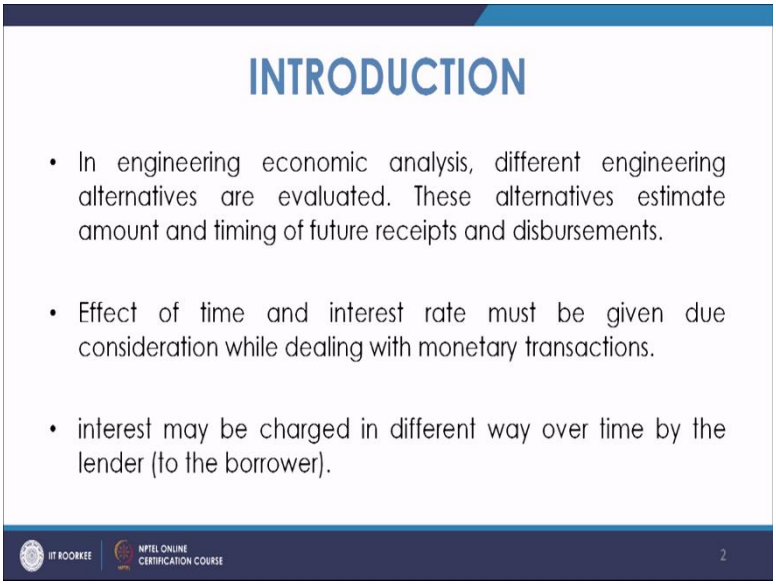

So far we have studied about interest and interest rates. In engineering economic analysis, different engineering alternatives are evaluated. These alternatives estimate amount and timing of future receipts and disbursements. So what we have seen that something a lender is investing and he is getting a return on it from the borrower. It means there are proposals and from there you have different types of receipts and disbursements.

Receipts means something somebody is getting, that this is receipt and disbursement means something somebody is paying, that this is disbursement. So in this lecture we will study about the effect of time and interest rate because it has to be given the importance. Effect of time as well as interest rate has bearing on the amount which has ultimately to be paid by the borrower.

So basically we are dealing with a monetary transactions. Some amount is taken by the borrower and he has to pay the amount itself after certain amount of time. So in that both interest rate as well as time plays an important role in ultimately what he has to pay.

Interest may be charged in different way over time by the lender. So basically the lender is charging interest in a different way to the borrower. So we will see how it is.

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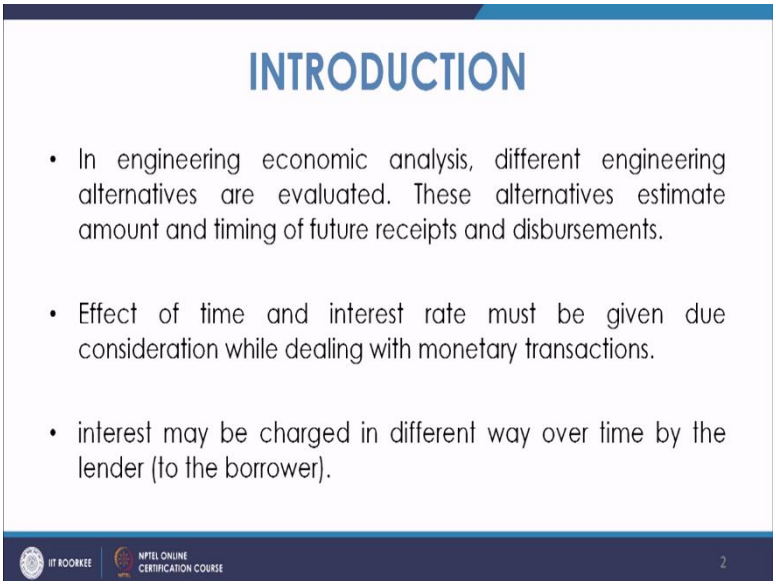

Again, the rental rate for a sum of money is usually expressed as certain percent of the sum to be paid for its use for a period of one year. We have already discussed about interest rate, it is nothing but the amount you get. If X amount is being given by the lender, then after one year he will get X + suppose Y, then Y is the interest. So Y will be certain fraction of X and that will be interest rate.

Interest rates are also quoted for periods are other than one year, known as if the speed. So normally this period can be one year or even different and usually it is taken as one year and then that way we call it as annual interest rate.

Simple and compound interest of approaches will be studied for determining the effect of time value of money. So it is lecture we will also study the simple interest and compound interest. There are two types of interests and how they affect the amount which is to be paid by the borrower that will be studied in this lecture.

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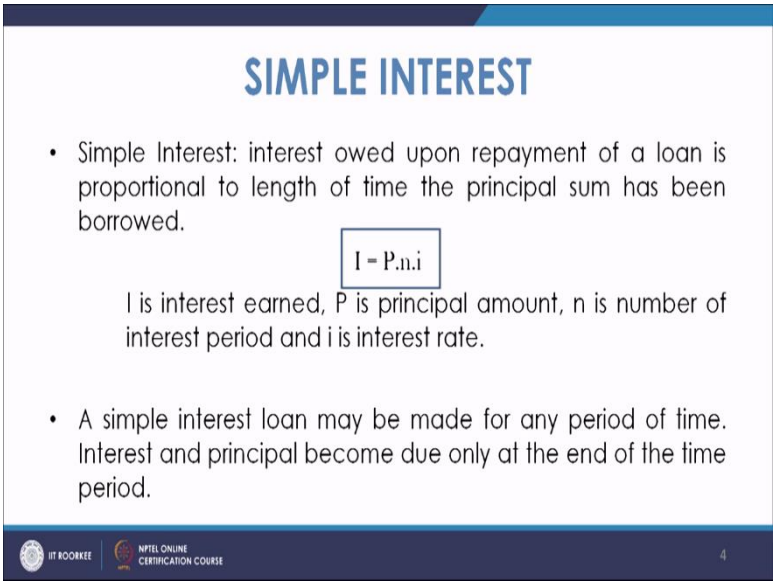

So first of all let us study what this simple interest. We all know that simple interest, it is the interest owed upon repayment of a loan and that is basically proportional to the length of time the principal sum has been borrowed.

So suppose a person is lending certain amount and if he is lending for four years, then he will get certain interest. And if he is lending the same amount for eight years, he will get the interest double of that amount. So basically that is linearly varying, in that case it is known as the simple interest.

Simple interest calculation as we know I will be P, P if the principal amount multiplied by n, n if the number of interest period and i, small i is the interest rate.

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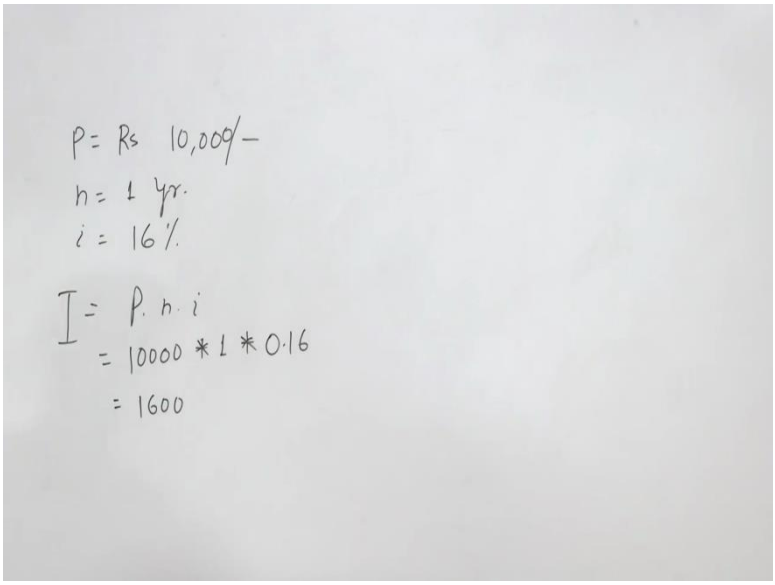

Suppose a person is lending Rs. 10,000 as P to a borrower and if he is lending it for 1 year, so n is 1 year and if the interest rate for the one year is 16%, in that case the interest amount can be calculated by this formula. So I will be equal to P into n into i. So P is 10000 multiplied by n that is one year because our interest period is also one year and then interest rate is 16%, so that a 0.16. In that case simple interest comes out to be 1600.

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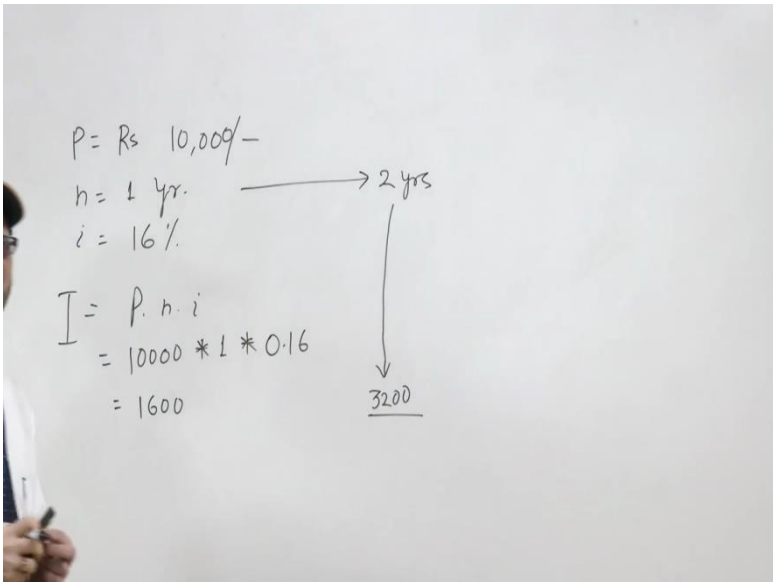

Now the thing is, that the, this one indicates that as the length of time will increase, this interest amount will go on increasing. If n becomes 2 years, this amount comes out to be 3200. So this way the simple interest amount will be varying as the time is increased.

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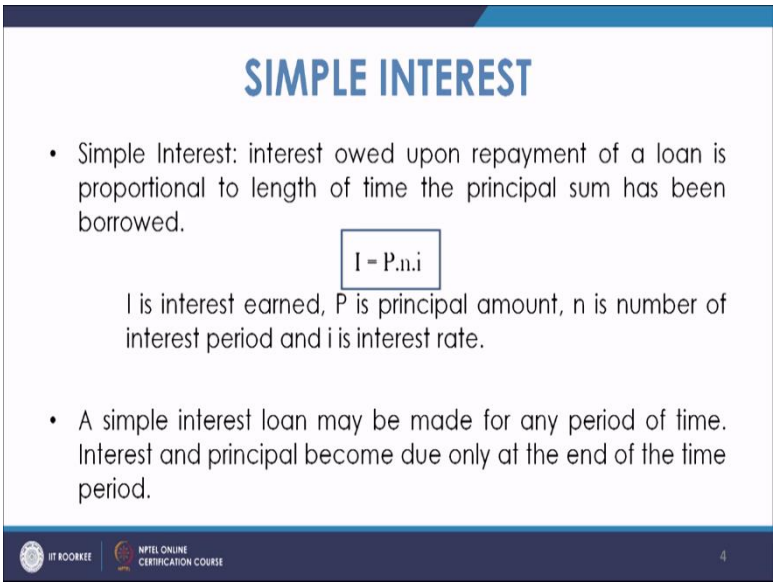

A simple interest loan may be made for any period of time. Interest and principal become due only at the end of the time period. So one of the characteristic of this simple interest is that, the person or the borrower who is taking the money from the lender, he has not to pay the interest amount in the middle period, he has to pay the accumulated interests.

Every year whatever it accumulating, at the end of the period plus the principal sum. So this is one of the characteristic of the simple interest.

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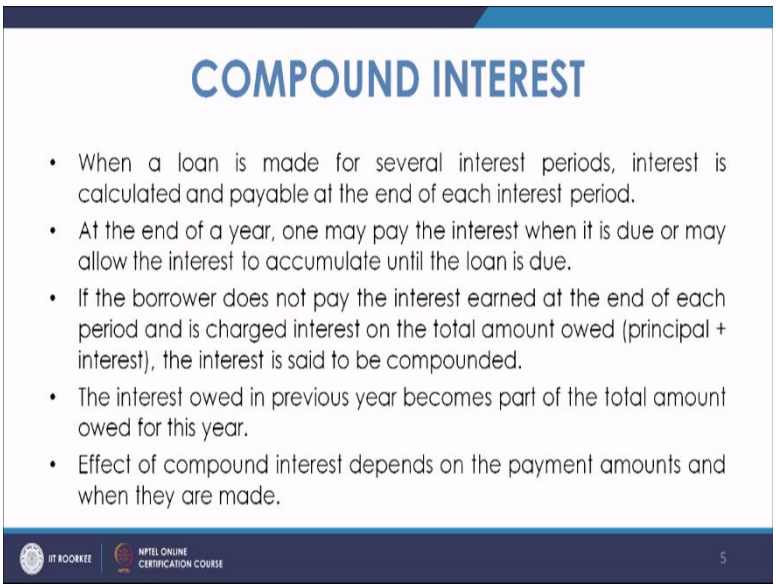

Then now we come to compound interest. What is compound interest? When a loan is made for several interest periods, interest is calculated and payaable at the end of each interest

period. Basically there may be conditioned by the lender that you should pay the interest at the end of every year otherwise the lender may say that this interest amount may be compounded or it will be added to the principal amount which is owed at the beginning of the again.

It means that the interest amount which is generated every year, it will go on increasing, so that is known as the compounding of the interest.

At the end of the year, one may pay the interest when it is due or may allow the interest accumulate until the loan is due. So there are two cases, if a person has paid certain amount or a lender has given certain amount to a borrower, he may tell the borrower to pay the interest every or he may say the borrower not to pay but the interest every year will be added to the amount owedevery year.

So every year amount owed will be increasing and correspondingly the interest accrued also will be going on increasing. If the borrower does not pay the interest earned at the end of each period and is charged interest on the total amount owed. So they basically in that case, interest is charged on the total amount owed that is principal + interest. This is known as compounding of the interest.

The interest owed in previous year becomes part of the total amount owed for this year. So basically, if you have certain amount and at the certain interest rate at the end of first year, you will have certain interest generated. This interest is added to the principal amount and this principal amount + interest generated during the first year becomes the principal amount at the beginning of the second year.

And in the second year, the interest will be a percentage of this summed amount. So now we have discussed about the two types of compounding. In one case the interest is paid annually and in another case, we don’t pay the interest annually, so interest is allowed to compound. Let us see by an example, these two cases.

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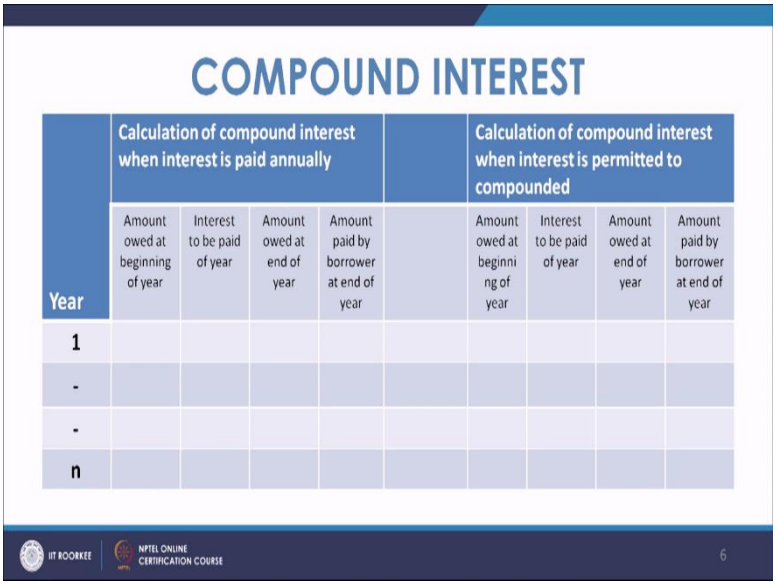

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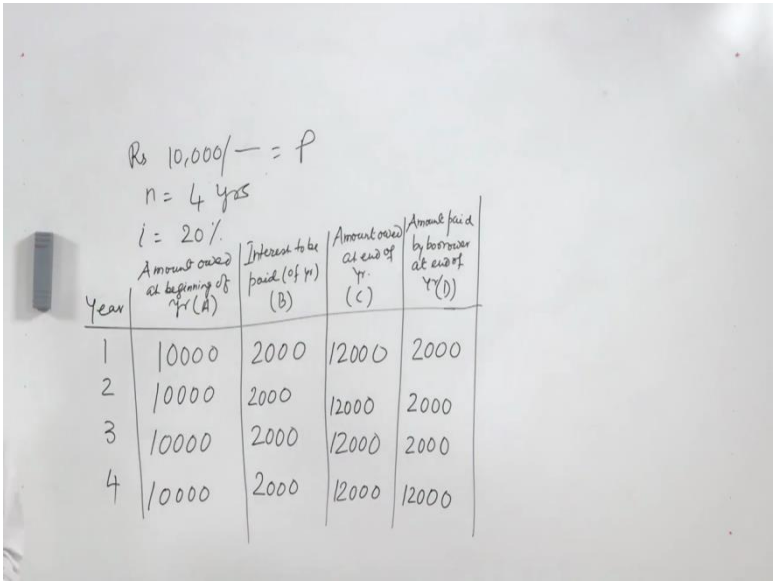

Let us assume that amount of Rs. 10,000 is borrowed by a borrower or a lender has given to a borrower. Time a n is 4 years and annual interest rate i is 20%. Now let us see how this compounding of interest is done for the two cases.

So since we have taken four years, the year will be 1, 2, 3, 4. Now amount owed at the beginning of the year that we can take as A. Similarly, interest to be paid of year that will be taken as B, amount owed at the end of year we can take it as C and amount paid by borrower at end of year that we can take as D.

Now let us see since we have taken Rs. 10,000, so in the beginning of the first year, you have Rs. 10,000 which is owed by the borrower. Now borrower is paying interest on this amount and this this interest amount will be 20% of this amount and this comes out to be 2000. So ultimately the amount which is owing at the end of first year will be 10,000+2000 and that will be 12,000.

Out of this 12,000, 10,000 as the principal sum and 2000 is the interest. The condition is that we are talking about the case when he is paying the interest annually. So basically out of this, the interest amount 2000 he has paid. So once he has paid 2000 interest, in that case now he owes an amount of 12,000-2000, so again it is left out to be 10,000.

So in the beginning of the second year he owes an amount of 10,000. Again he has to pay in the second year, interest amount of 20% of the sum. Again this comes out to be 2000. So again the total sum comes out to be 12,000 and he is paying annually the interest charged, so 2000 again he pays back.Third year again, it remains out to be 10,000, he has 2000 interest, so amount owed at the end of the third year is 12,000, he has paid 2000.

In the fourth year, he has 10,000, 2000 is the interest, so it comes out to be 12,000 and in the fourth year end he has to pay the complete amount that if the amount he owes towards end of the period, so it is 12,000. So ultimately he is paying 2000 and the end of first, second and third year and he is paying 12,000 at the end of four years.

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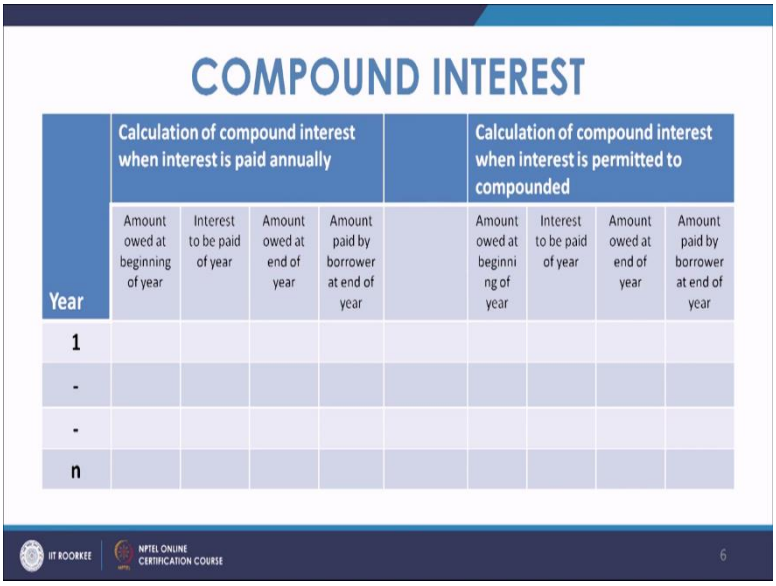

Now let us see the second case where he is not paying the interest and interest is getting compounded. So in that case if you look at, so this is a case where he is paying the interest

annually. So now if he is not paying the interest annually, not paying the interest and interest is accumulated and that is how the interest amount goes on increasing every year. In that case, A. B, C and D, these amounts will be how much, that we have to see.

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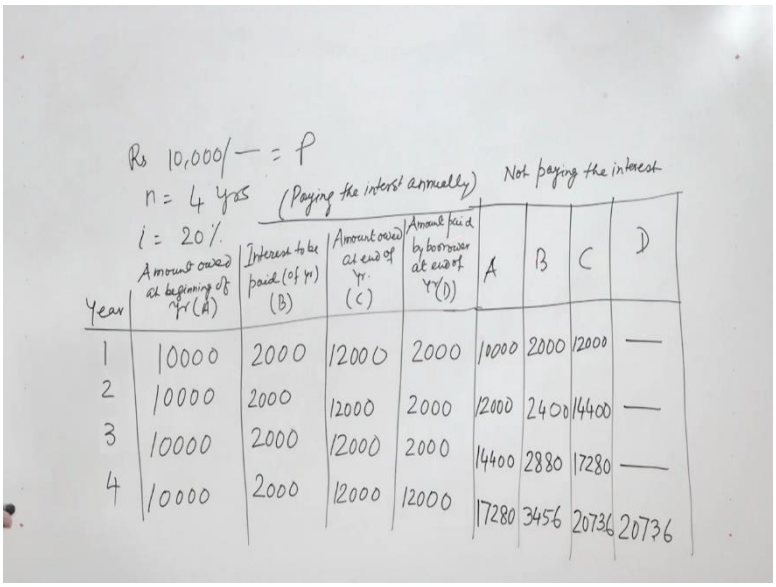

So in this first case he has Rs. 10,000, Rs. 2000 is the interest generated, total amount owed at the end of first year is 12,000 but he is not paying anything. So in the beginning of the second year, the amount which **is** he is owing is 12,000. This interest is compounded. Now this interest what he has to pay in the second year will be interest rate multiplied by this sum. So he will be 20% of 12,000 and he will be paying 2400, so this is the interest generated.

The total amount owed at the end of the second year will be 14400 and still he is not paying anything towards the end of the second period. So after the end of second year, in the beginning of the third year, now the principal sum comes out to be 14400. On 14,400, now he has to pay the interest at the rate of 20%. So that comes out to be 2880. So, it will be 17280 and still he has not paid the interest.

So ultimately in the end of the third year or at the beginning of the fourth year, the amount he owes is 17,280. On this he will be paying the 20% as the interest charge, so it will be 3456 and this comes out to be 20736. So ultimately at the end of the fourth year, he has to pay the full amount and this amount comes out to be 20736.

So there are two ways by which he can pay the amount and both are cases of compounding but in one case he has not paid. He has not paid the interest also at the end of the period. In

another case, he is paying the interest, so ultimately he is only paying 12,000, whereas in this case, he is paying 20,736.

So what we see that this case where the interest is compounded every year and it becomes a part of the total sum, this is a case of compound interest.

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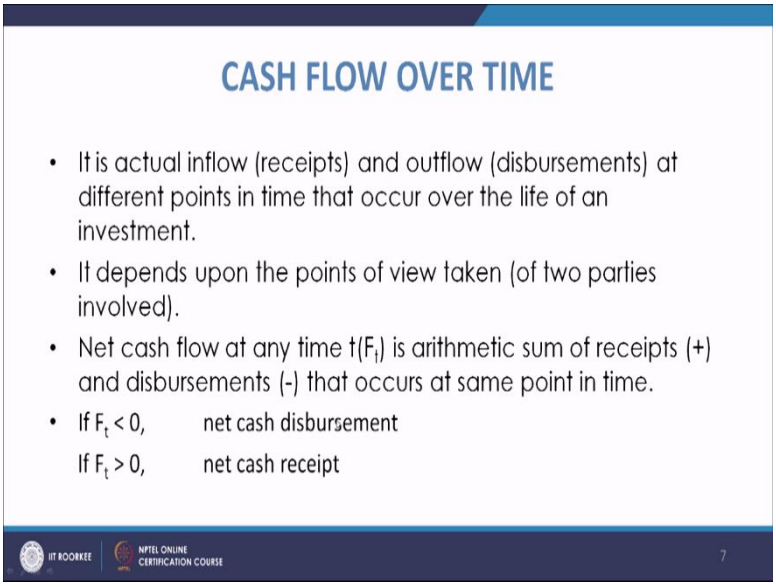

So we will discuss about the cash flow diagrams. As the name indicates, there is flow of cash or monetary transactions that is known as cash flow. It is actual inflow or receipts and outflow that is disbursement at different points in time that occur over the life of an investment. So for a particular investment, during the course of time, you have sometimes certain money either coming or sometimes the money is going out.

So when the money is coming that is inflow of money that is known as receipts. And similarly, at whichever time you are giving the amount to someone that is known as a disbursement. And this is known as cash flow, so cash is basically flowing either it is coming in or it is going out.

It depends upon the point of view taken. So you have basically two persons, one is lender and one is borrower and if we take the viewpoint of the two, it will be different. For the lender, if he is lending some amount at a particular time, he is losing at that amount of time but at a future time he will be gaining it but for the borrower, at the initial time he's gaining something and he has to pay it in the end.

So basically whatever due is taken, it means the same but certainly for one if for one case if it is positive, for another that amount same will be same but it will be negative. Net cash flow at any time Ft, at any time t that is Ft is arithmetic sum of receipts and disbursements that occurs at same point in time.

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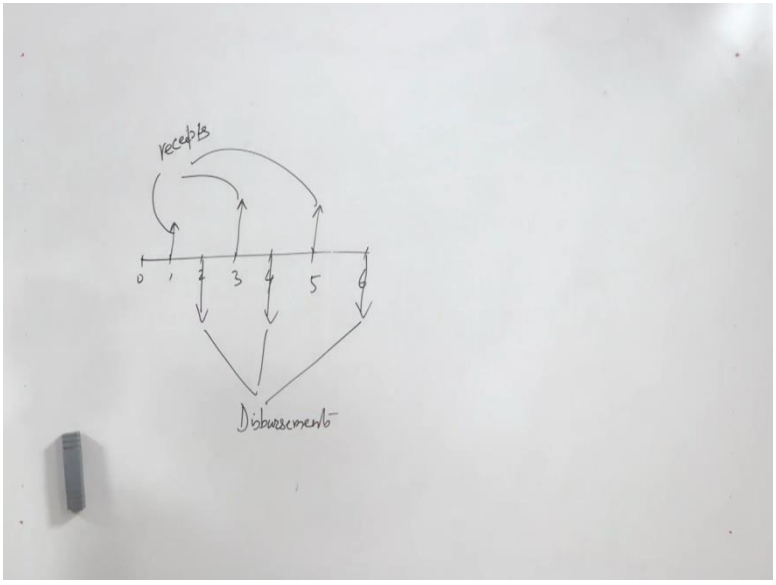

So if at any time if you have a cash flow, you have time, 1, 2, 3, 4, so you may have cash flow like this, so these are known as receipts, these are receipts and these are disbursements. So from the viewpoint of either lender or borrower, if the money is shown as positive amount it is received and if it is a negative amount, it is the disbursement.

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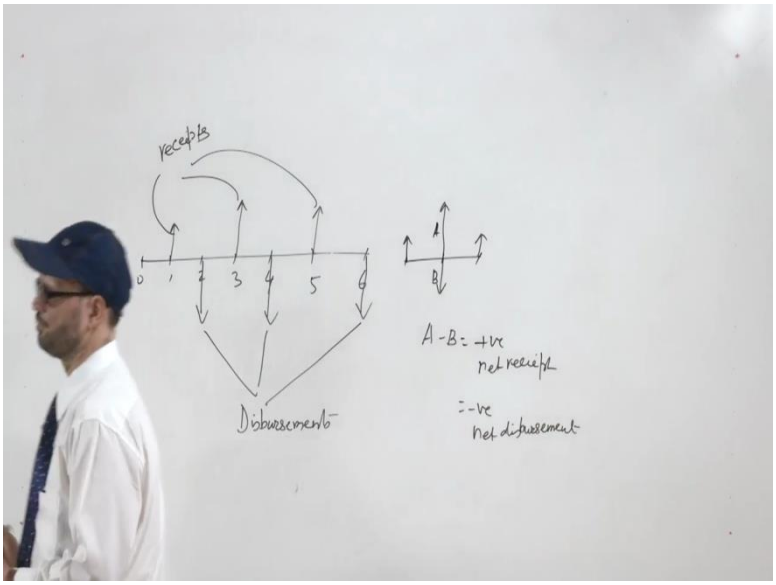

And if at any point of time, suppose you have both receipt as well as disbursement, in that case, the net amount will be A-B and this is known as net cash flow. So that is known as net cash flow and if it is positive, it is known as net receipt. And if it is negative, then it is known as net disbursement. So this way you have cash flows occurring over the span of time.

Then cash flow diagrams, cash flow diagrams are nothing but the graphical representation of the flow of cash. The diagram which we have drawn here, this is nothing but a cash flow diagram.

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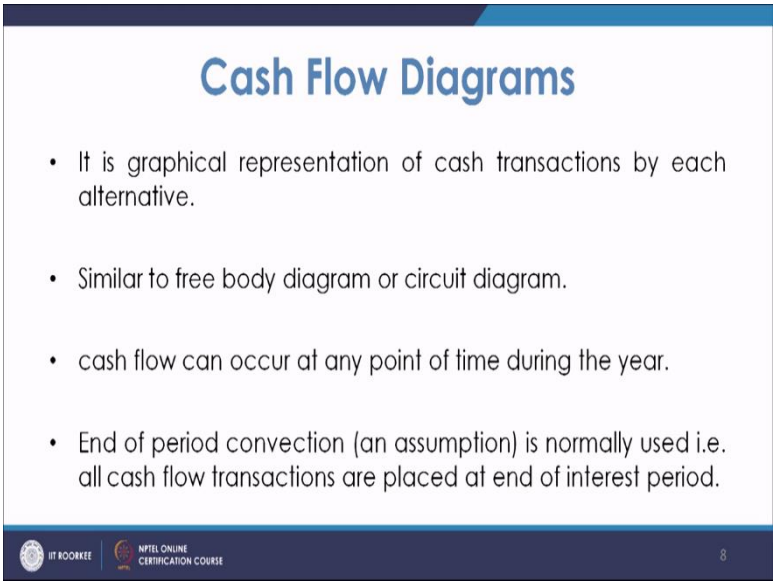

Similar to body diagram or the circuit diagram what we use. Basically it shows you at what point of time what was the flow of cash, what was the receipt and what was the disbursement, so that is a cash flow diagram.

Cash flow can occur at any point of time during the year. So basically during the year it can occur at any point of time but we will have certain adoptions that we will discuss later when we will calculate, try to calculate the amount of compound interest or so. So basically, any cash flow either it can happen sometimes during the third quarter, during the second month or during the sixth month, so to avoid any complexity.

It is assumed that the payment is made at the end of the year and that is why this end of year convention, this is normally used. So all cash to transactions are placed at and of interest period. So this is known as end of interest period that is convention.

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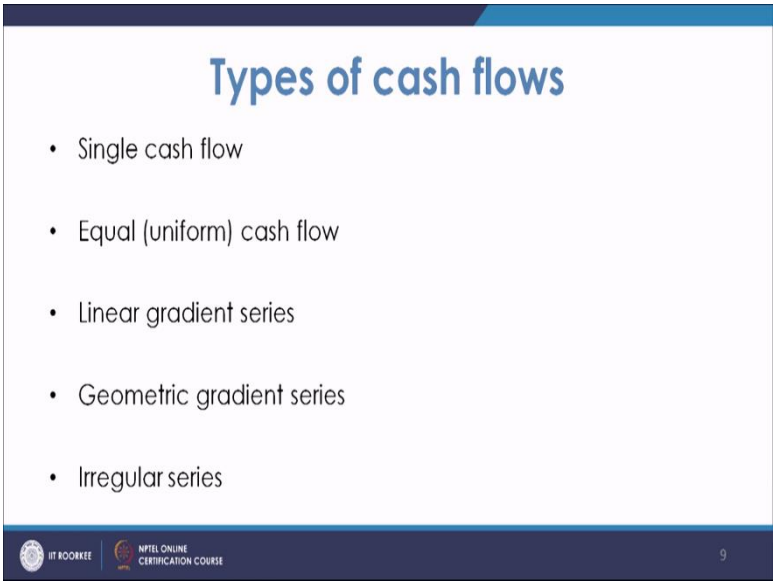

Now types of cash flows. As you will see there will be different types of cash flows. Now there are different types of cash flows and under that you have single cash flow, let us see what does it mean.

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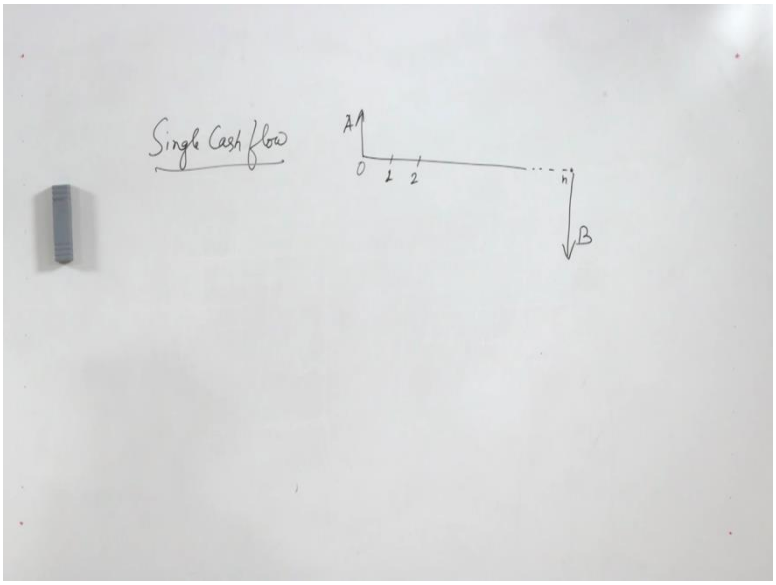

So a single cash flow means, the cash flow diagram will have only once the receipt and only once there will be disbursement. So you can have the cash flow diagrams as a receipt and certain disbursements at the end of year. So A and B. So this type of cash flows are known as single cash flow, where you have single receipt and single disbursement.

Example is that somebody in a borrower has borrowed certain amount and what he will pay at the end of certain period, that is an example of single cash flow.

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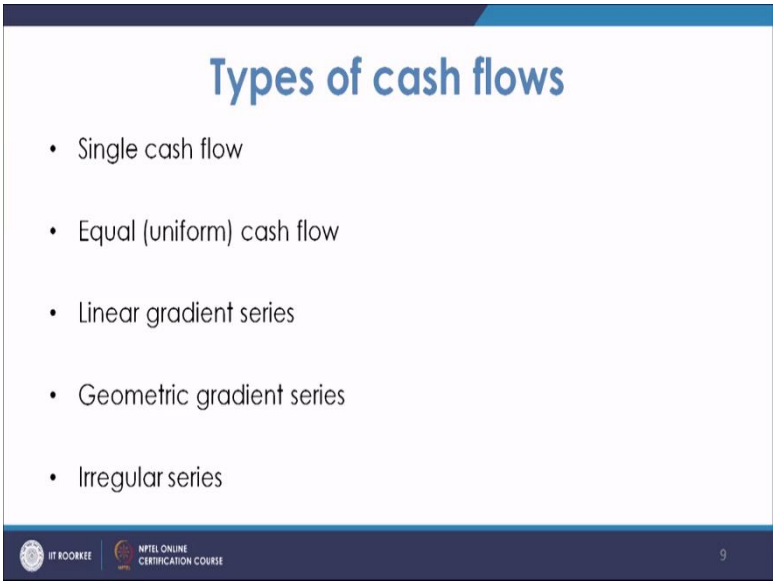

Next is equal uniform cash flow, now in this case many a times what we see is that the payment is made equally at the end of the year. So that is known as equal uniform cash flow and the cash flow diagram for such flows are like this.

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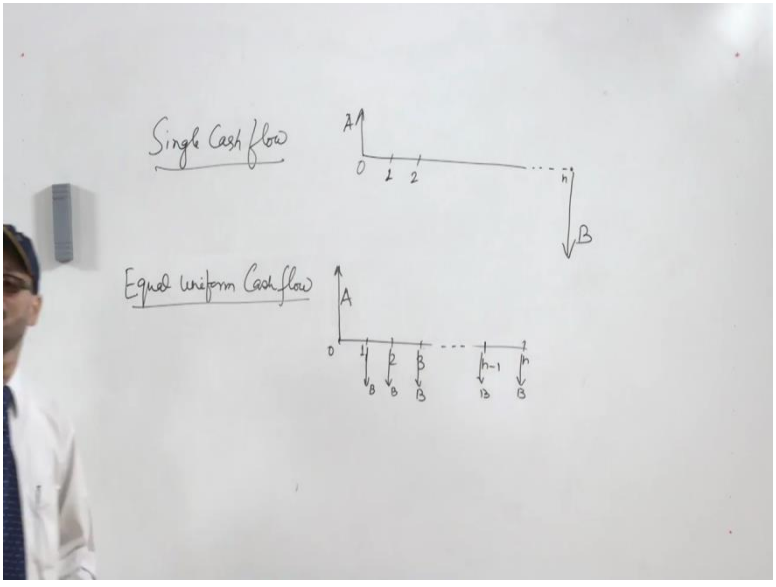

This starts from 0. So what is happening, the borrower will be paying certain amount at the end of every year. So basically he has got certain amount from the lender but he will be

paying a uniform amount to the lender for the rest of the periods. So this is an example of equal uniform cash flow.

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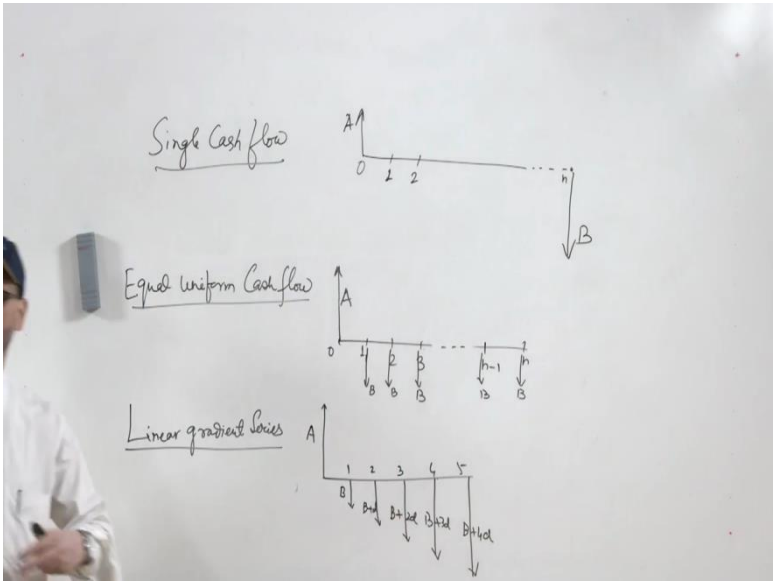

Then we have example of linear gradient series. Under this what happens that, the amount which is paid by the borrower, they have a linear gradient. It increases by a certain fixed amount. So what you can see is, if he has taken certain amount, so this is an example of B, B+D, B+2D, B+3D and B+4D.

So what we see in this cash flow diagram is that the borrower has received amount A and he is paying B amount at the end of first year and in the subsequent years, he is maintaining a gradient of D. A linear gradient of D is maintained, so every year he is paying amount of D extra, so in the second year B+D, third year B+2D like that. So this is an example of linear gradient series.

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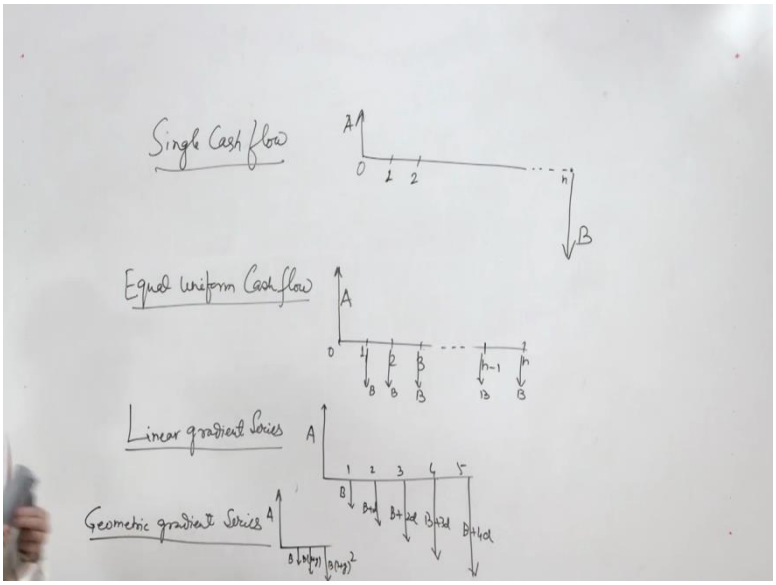

Similarly, the next is geometric gradient series. Under this series, this linear amount will be changing to a geometric factor. So basically in this case like that. So basically if you have a gradient factor, it will go on increasing in a geometric series, so this is an example of geometric gradient series factors.

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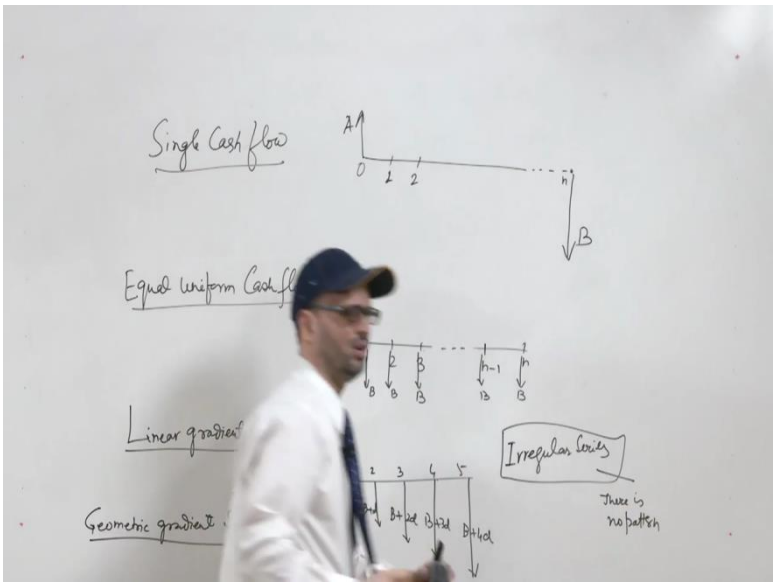

And the last is irregular series, it is nothing but there is no pattern. In irregular series, there is no pattern. So basically you can have any amount being borrowed and it is paid and that does not follow any role.So such as the cash flow diagrams represent a type of irregular series where no rules are applied. You will have to find the equivalent value by using suitable methods.

So these are the different kind of cash flows. Thank you.