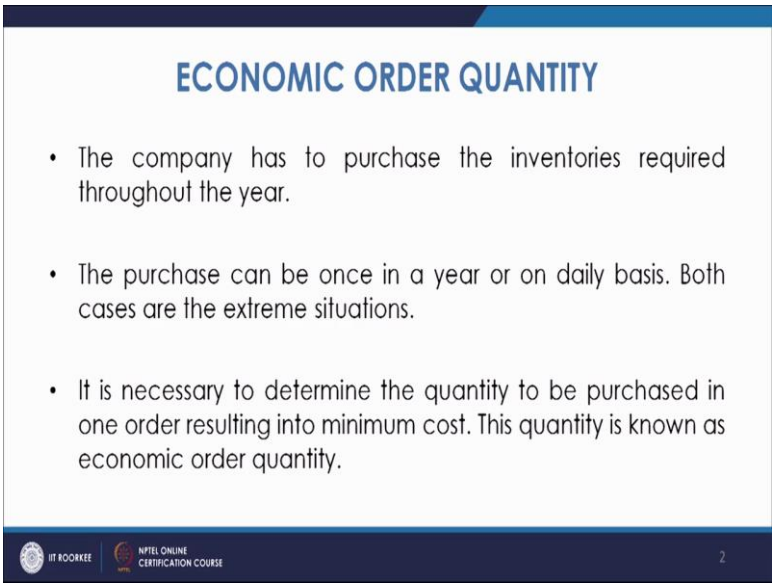
Engineering Economic Analysis
Professor Dr. Pradeep K Jha
Department of Mechanical and Industrial Engineering
Indian Institute of Technology Roorkee
Lecture 29
Economic Order Quantity

Welcome to the lecture on economic order quantity. What we see in the production operations, the company has to purchase the inventory items.

(Refer Slide Time: 00:43)



The inventory items are purchased for the demand which is required. So there is a sub demand of any item for the whole year. Now there can be two extreme cases that this purchase can be once in a year or on daily basis. Basically the cost associated with these purchases and keeping these inventories in our store, we have to keep certain things in mind. When we order for the purchase of the goods, there is some cost associated with the ordering.

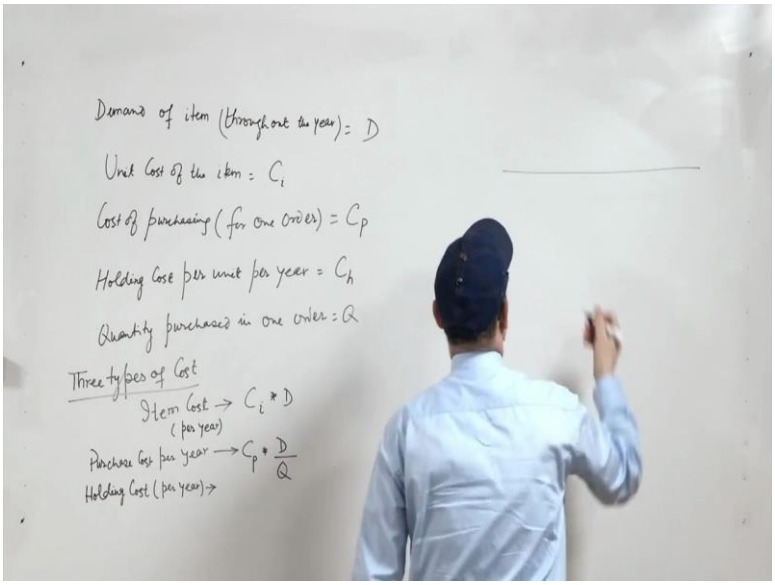
So there is some purchase cost for every order. So the number of times we give the order to purchase the quantity, the purchase cost will go on increasing. On the other hand we can take all the demand requirement at one time during the beginning of the year. Now these are the extreme cases when we keep the item in our store, it needs to be maintained. We need to maintain the cleanliness, so associated expenses are like holding cost.

Now holding cost, in that case when you purchase all the items once in a year, holding cost is quite because the items in stock is quite large in number and they are required to be kept in a good condition, so holding cost is quite high. While if we purchase on daily basis, although

the holding cost will be very small but since we are purchasing on daily basis, the ordering cost or the purchase cost will be larger.

So basically the two extremes will lead to very large cost and the total cost will be basically very very large. So basically it is necessary to the quantity to be purchased in one order, that is through one purchase order so that total cost is minimised. So the quantity which we are purchasing in one purchase order that is known as economic order quantity because it leads to the minimum total cost.

(Refer Slide Time: 08:47)



Now let us see how to get the expression for this economic order quantity. So let us say there is an item where the demand of item throughout the year is suppose D, D units are required for the whole year. Now let us say the unit cost of the item, let us say this is Ci, so this is the cost of each item. Now let us say cost of purchasing for one order, we can say we take it as Cp.

This is the cost which is required for suppose making the documents, for sending it, all these things are involved in this cost. So once you are sending the purchase order, this is the cost which is involved. Apart from that you have holding cost, holding cost will be basically specified on per unit basis per unit of the item for the whole year that is per year per unit per year. And say let as say, this is basically Ch, cost of holding the items.

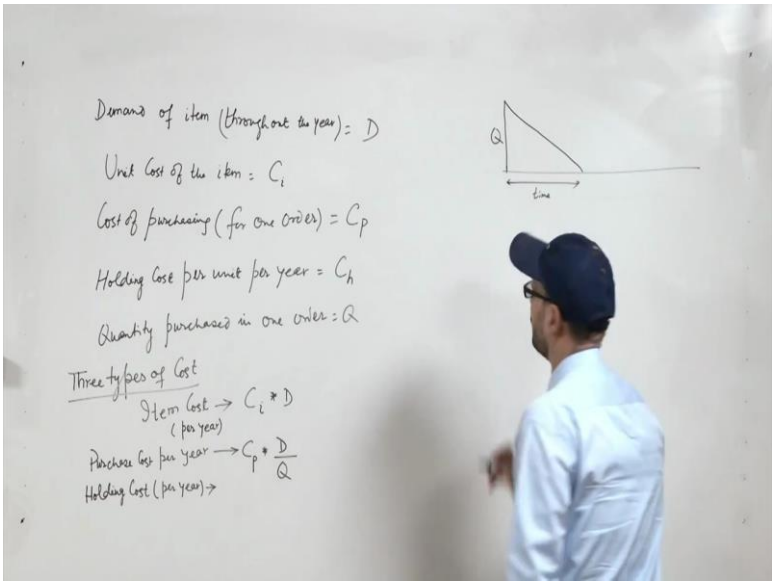
Now let us see, we take the quantity purchased in one order we will assume it to be Q. So in one order we are ordering for Q items. So basically we will be ordering for D by Q times, our ordering will be D by by Q times and that is why the cost of purchasing for every order will

be Cp times D by Q, so we will have that. So basically you have three types of cost, and cost will be the item cost.

So item cost per unit is Ci and since we have the requirement of D items throughout the year, so item cost for the year will be Ci times D, this is per year. Then comes cost of purchase per year, purchase cost per year, so what we have seen as once we give the purchase order, for every time we are incurring the cost of Cp.

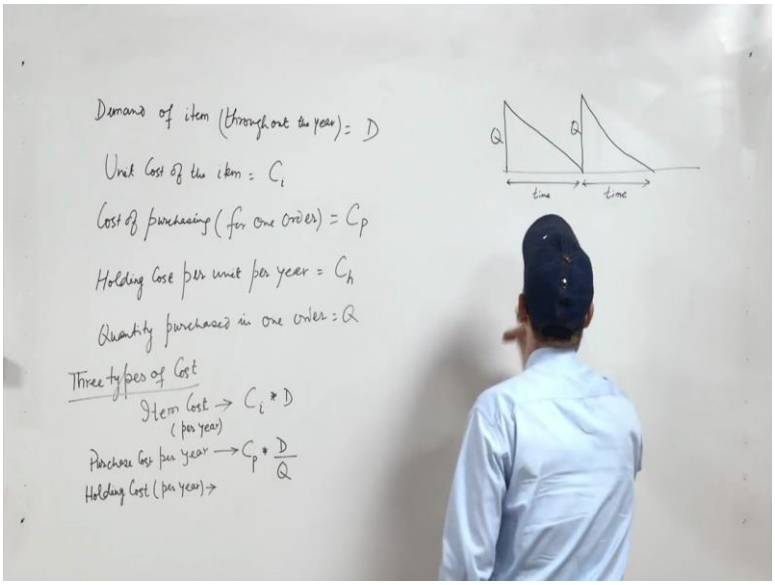
And the number of times you have to give this purchase order will be nothing but the demand requirement divided by the quantity which you are ordering once, so it will be D by Q. Then comes the holding cost, now holding cost will be basically, for this you need to know the average stock quantity at any time.

(Refer Slide Time: 09:06)



Now what we see is, in this case, the inventory flow is going like this, you are ordering the quantity Q at this time, this Q is (09:00) in this time.

(Refer Slide Time: 09:29)



So we assume that the day you order, you receive the material, so there is no time delay in receiving the material and it goes like this. So this is how the amount of inventory in the stock will be there. You have Q at this time but it will slowly be coming to 0 and at the time it is 0 the next day you are getting again Q. You do not assume any shortages neither any delay.

So average inventory will be can be assumed to be Q by 2 because it starts from Q and ends to 0, so average level of inventory you can assume it to be Q by 2. So holding cost per year will be holding cost per unit per year multiplied by the average inventory level. So Q by 2, this is the holding cost. Now let us see that we have to find for which Q this total cost will be 0. So total cost total cost will be basically item cost + purchase cost+ holding cost.

(Refer Slide Time: 14:43)

This is for the year. Now if this total cost is written as T, so T will be item cost that is Ci time D +purchase cost that is Cp times D by Q + Ch times Q by 2. So the total cost is to be minimized, so you can get for minimizing the total **sst** cost and to obtain the value of Q for which this cost is minimum, we will differentiate this T respect to Q and equate it to 0. So we will find Dt upon the DQ and this will be equated to 0.

So once we do the differentiation what we get is - of Cp times D by Q square + CH upon 2 equal to 0. This implies that Ch upon 2 will be DCp upon Q square or we can write Q as two times cost of purchase order once we send one order multiplied by the demand throughout the year upon the holding cost per unit per year and under root.

For tis Q, once we calculate this value, the total cost which we get will be minimum and that is why this Q is known as economic order quantity. So there are certain assumptions in the study, once is that demand is constant throughout the year. We have assumed that there is no time lag when the inventory is finished level is finished and the next inventory comes. So upon these assumptions, this is how you calculate economic order quantity.

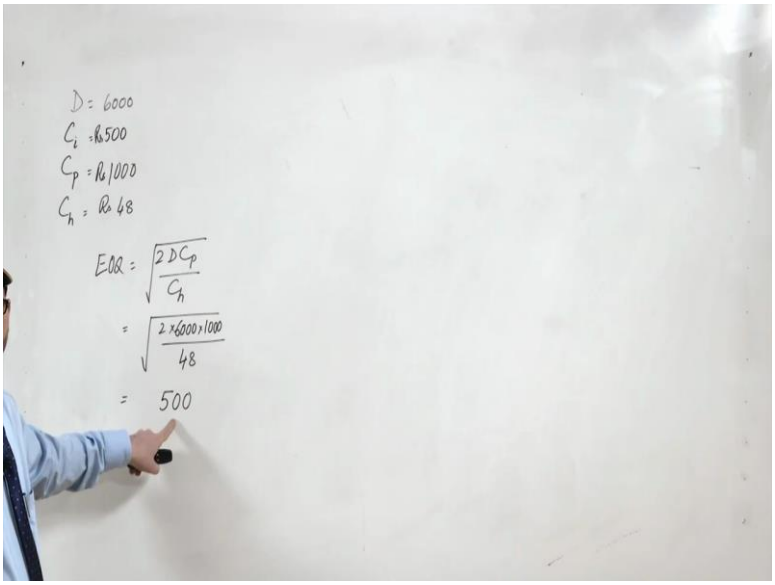
(Refer Slide Time: 15:14)

Example

- Find the economic order quantity for annual demand of an item to be 6000 units, item cost per unit as Rs 500, ordering cost per purchase order is Rs 1000 and cost of holding the inventory per unit per year is Rs 48.

Ans: 500

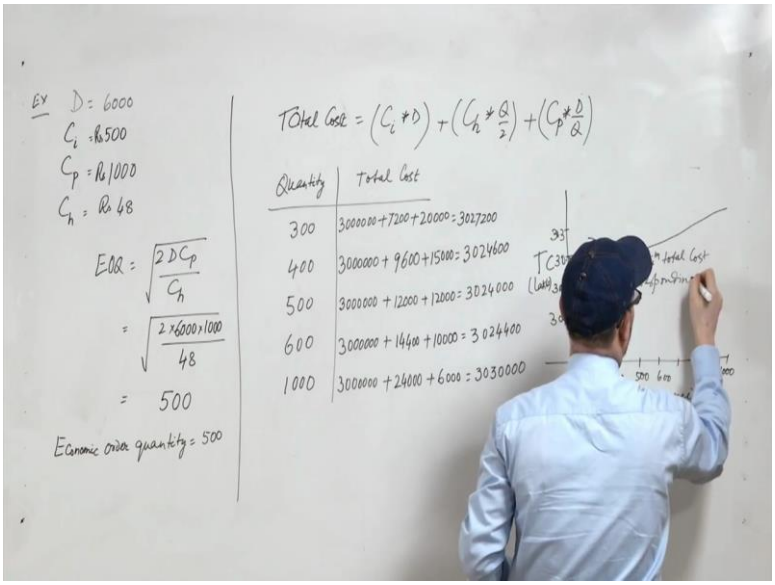
(Refer Slide Time: 17:27)



Let us see an example and try to solve it using this and see how this total cost will be minimum for this case. So to find the economic order quantity for annual demand of an item to be 6000 units. So D is 6000, then we have item cost per unit as Rs. 500, so Ci is 500. Ordering cost per purchase order is Rs. 1000, so Cp is given as Rs. 1000 and cost of holding the inventory per unit per year is given as Rs. 48 so Ch is Rs. 48.

Now in this case if we see the economic order quantity for which the total cost will be minimum, so that can be found as under root 2DCp upon Ch. So this will be under root 2 times D is 6000, Cp is 1000 upon Ch that is 48. So if we see this, so it will be 500. What we see is that if we order 500 items, the total cost will be minimum. So the economic order quantity is 500.

(Refer Slide Time: 27:36)



Let us check this by plotting the graph and looking at the values of total cost against these numbers so we can have the quantity as less than and more than this value and se what is the total cost. So what we have seen total cost expression as Ci times D + Ch times Q by 2 + Cp time D by Q. So if the quantity is plotted against total cost, what we see? If we take suppose 300 items, which is basically sent to once, so total cost will be Ci times D.

So we will have Ci is 500 times our demand this 6000, so it is 30,00,000 + Ch time Q by 2 so it will be Ch is 48 into 300 by 2 so it will be 7200 + Cp times D by Q, Cp is 1000 times D by Q, so D is 6000 and Q is 300 so it is 20 so it is 20,000. So this comes out to be 30,27,200. Now if we increase this quantity to 400, then in that case Ci times D is always constant, so it will be 30,00,000 + Ch time Q by 2, so 200 times Ch so it will be 9600.

We see that this holding cost is increased once the quantity is increased. So this will be 9600 + purchasing cost Cp times D by Q, D by Q will be 15, so 15 times Cp 15,000 and it comes out to be 30,24,600. So we can further see at 500, at 500 the item cost will be 30,00,000 + holding cost will be Ch times Q by 2 to 48 times 500 by 2 so it will be 12,000 + Cp time D by Q, D by Q will be in this case 6000 by 500, it will be 12.

So that that times Cp so 12,000. So it will be 30,44,000. Now let us see we want to increase this number, we go to 600. Item cost will be same + the holding cost Ch into Q by 2, Ch is 48 and Q by 2 is 300, so this will be 14,400 + say Cp times D by Q. Now D is 6000 and Q is 600 so D by Q is 10 ten times Cp, so it is 10,000. So it becomes 30,24,400. What we see is that cost has increased. Let us further verify by keeping it 1000.

If it is within the capacity, then using this 1000 your item cost will be coming as 30,00,000 + holding cost will be Ch times Q by 2 so 48 times 1000 by 2 it will be 24,000 + what we see is the purchase cost and that will be 1000 times 6000 by 1000 that is 6 so 6000. So it will be 30,30,000. So what we see is, that as we increase from 300 upto 500, it decreased and at 500 it was minimum.

(Refer Slide Time: 27:42)

Once we went to 600 units of order once then it increased slightly and further again there is a increasing trend as we move further. So if we plot the total cost against the quantity ordered and if we take this in terms of lakhs, if we take here 30, 30 point 5 and suppose 31, in that case and the quantity ordered is 300, 400, 500, 600 and so 7, 8, 9 and 1000.

In that case what we see is you see that it is something like this on 30,000 you have you can have the scale even smaller one. 30 point suppose say 1 and 30 point 2 and 30 point 3, so it will be in between 2 and 3 30 point 27, so it will be here. 30 point 24 so it will be here 30 point 24 agains in that case 246 and it is 24 here and then 30 point 244 and at 1000 it is 30 point 3. So what we see is, at this point you see that it is going like this.

This is the minimum point, minimum total cost of responding to Q equal to 500. So that is how we see that if your quantity ordered in one lot is 500, the total cost is minimum, this can be verified. Thank you.