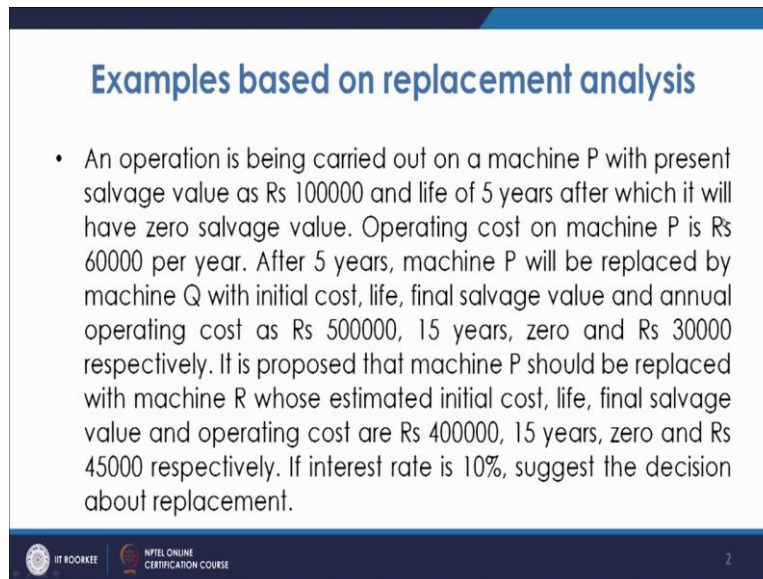


Engineering Economic Analysis
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Lecture 19
Problem Solving on Replacement Analysis

Welcome to the lecture on problem solving on replacement analysis. So in this lecture we will discuss about a problem which is based on replacement analysis and we will see how we take the different study periods and how the problem is analysed.

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Examples based on replacement analysis

- An operation is being carried out on a machine P with present salvage value as Rs 100000 and life of 5 years after which it will have zero salvage value. Operating cost on machine P is Rs 60000 per year. After 5 years, machine P will be replaced by machine Q with initial cost, life, final salvage value and annual operating cost as Rs 500000, 15 years, zero and Rs 30000 respectively. It is proposed that machine P should be replaced with machine R whose estimated initial cost, life, final salvage value and operating cost are Rs 400000, 15 years, zero and Rs 45000 respectively. If interest rate is 10%, suggest the decision about replacement.

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So let us see a problem when it is mentioned that the operation is carried out on a machine P with present salvage value as Rs. 100,000 and life of 5 years after which it will have zero salvage value. Operating cost on machine P is Rs. 60,000 per year. Now after 5 years, machine P will be replaced by machine Q with initial cost, life, final salvage value and annual operating cost as Rs. 500,000, 15 years, 0 and Rs. 30,000.

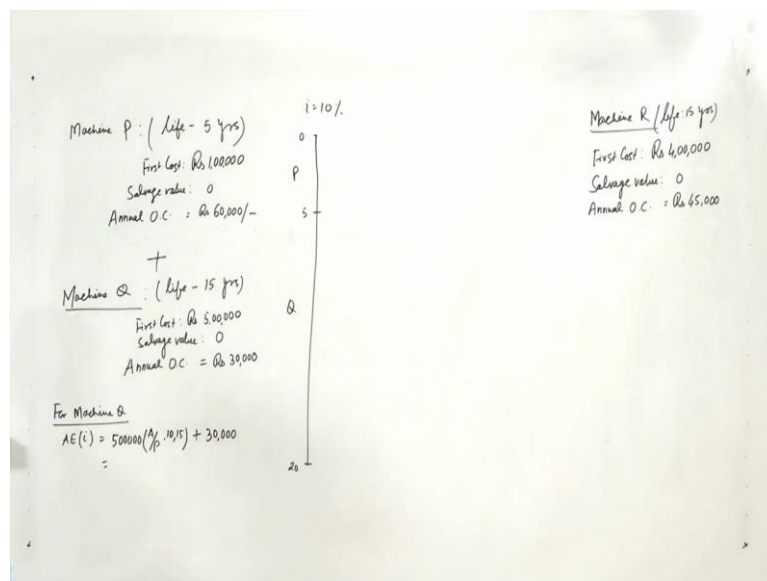
So basically for initial 5 years machine P will be used and then it will be succeeded by machine Q which has all these details. Now there is a proposal for replacement and that is for machine R. So this machine R, it has estimated initial cost of Rs. 400,000, life of 15 years, final salvage value is zero and operating cost as Rs. 45,000 per year. So basically you have a machine P which will work for 5 years and after 5 years it will be placed with machine Q.

Then another option is you have now machine R with you, so whether you should go for machine P + Q or you should go only for machine R at present. And for that there are

basically two considerations one is that if you look at machine P, it has a life of 5 years and Q has 15 years, so altogether you have 20 years.

Now in that case machine R, it has a life of 15 years, so we can have the study period of 15 years, in that case machine Q will only be used for 10 years, so its life of 5 years will be remaining. So let us see how to solve this question. So the data first of all we will write what is given.

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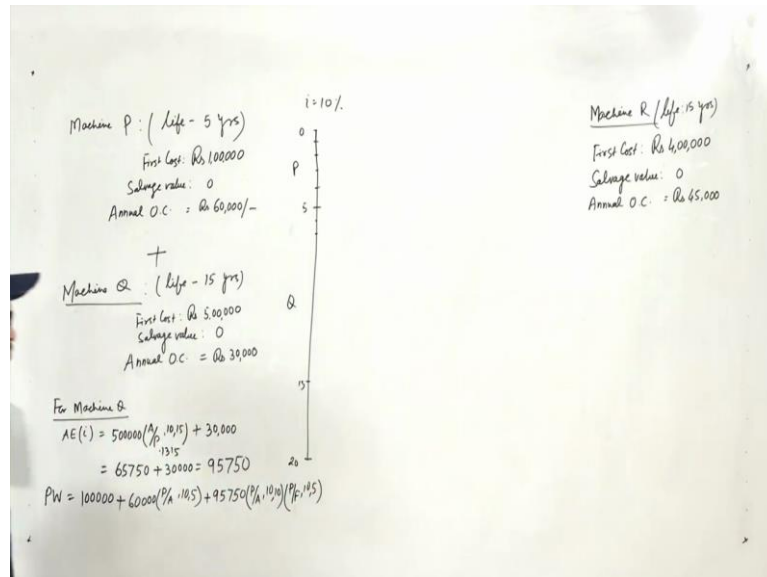
So basically you have machine P, life 5 years, first cost as Rs. 1,00,000, salvage value zero and annual operating cost is 60,000 + you have after 5 years machine Q is coming. It has life of 15 years, first cost is given as Rs. 5,00,000, salvage value again given as zero and annual operating cost is given as Rs. 30,000.

Now opposite to this you have machine R. Machine R, its life is 15 years, first cost is given as 400,000, salvage value zero and operating cost is 45,000 per year. So now the question is how to see that what should be the decision whether to go for machine P + Q where Q will be coming after 5 years or we should now go with machine R so that you can use the machine R from today itself.

Now we see that the life of these two altogether is 20 years and the life of this is 15 years. So if we take the study period is 15 years, it means 5 years of this machine will be an utilised. So in that case, we have to find the implied salvage value of this machine after 10 years.

Now since the 2 machines are having different lives and machine Q is used subsequent to machine P, the present worth basis of comparison looks better where you can find the present worth of machine P + Q at present and also present work for this because it has anyway 15 years of life and then you can also find the implied salvage value.

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So on the basis of present work you can compare whether the replacement is justified and also you can say that the machine Q when it will be not used for 5 years, what will it have the implied salvage value. So for finding the present worth, what we do is, we find the annual equivalent value for this machine. Now we have basically we have 0 to 20, so up to 5 years you have machine P and after that for up to 15 years, more 15 years would be machine Q.

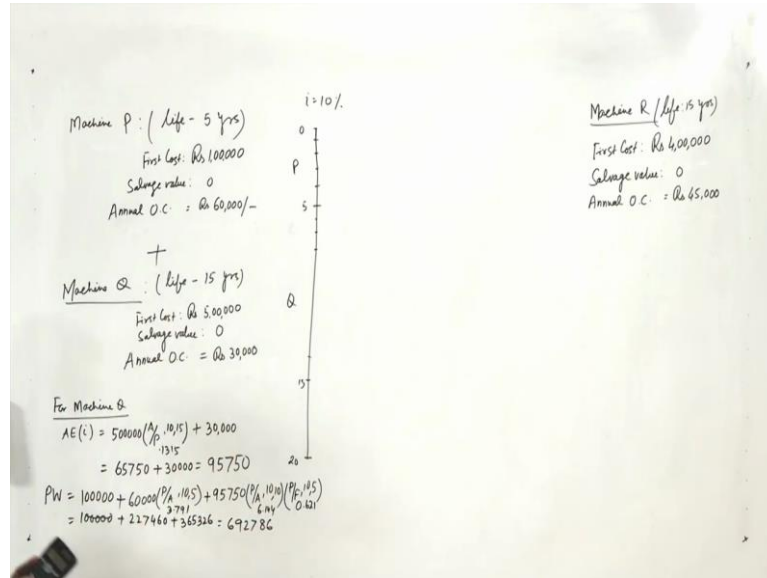
So what we do is, for this portion you find the annual equivalent value. You take the annual equivalent value of these 10 years and then you find the present worth value. So if you find the annual equivalent for machine Q, if we find the annual equivalent, the 5,00,000 multiplied by A by P I 15, so 10 and 15 + it has two components one is capital recovery with return part + you have annual operating cost, it is given as 30,000.

If you look at the factor value of A by P 10 15, so A by P 10 15 comes out to be .1315 so so it will be 95750. Now we have to find the present worth of this system P + Q. So present worth will be nothing but 1,00,000 is used here so it will be 1,00,000 + annual operating cost is 60,000 and it is used for 5 years so you have to use 60,000 for 5 years.

So 60,000 then P by A 10 5, the 60,000 which is used every year, its equivalent portion at present time that will be P by A 10 5. Now we have 10 years upto 15 we are using 30,000

operating cost and for this series we have used the annual equivalent value as 95750. So first of all this 95,750 corresponding to this every year we will get the present worth at this time and this amount would be further converted at zero time.

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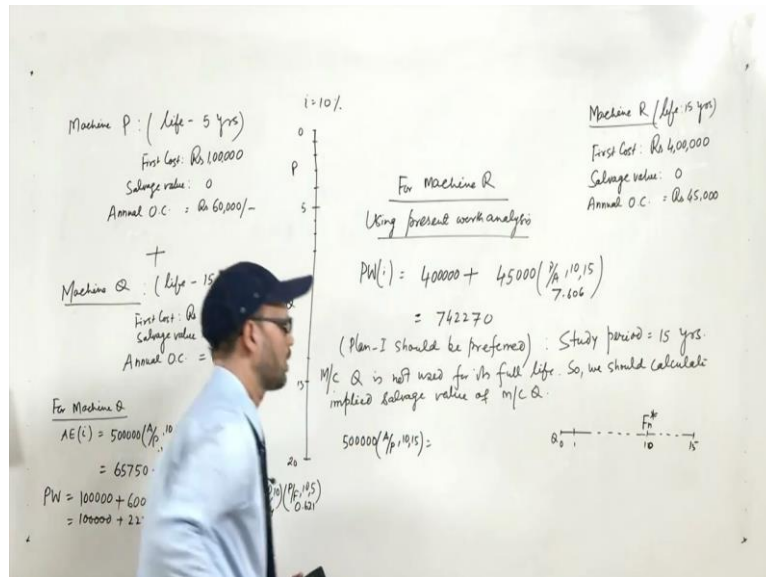
Interest factor values for discrete compounding (i=10%)

n	(F/P,i,n)	(P/F,i,n)	(F/A,i,n)	(A/F,i,n)	(P/A,i,n)	A/P,i,n	A/G,i,n
1	1.1	0.9090909	1	1	0.9090909	1.1	0
2	1.21	0.8264463	2.1	0.47619	1.7355372	0.57619	0.47619
3	1.331	0.7513148	3.31	0.30211	2.486852	0.402115	0.93656
4	1.4641	0.6830135	4.641	0.21547	3.1698654	0.315471	1.38117
5	1.61051	0.6209213	6.1051	0.1638	3.7907868	0.263797	1.81013
6	1.771561	0.5644739	7.71561	0.12961	4.3552607	0.229607	2.22356
7	1.9487171	0.5131581	9.487171	0.10541	4.8684188	0.205405	2.62162
8	2.1435888	0.4665074	11.43589	0.08744	5.3349262	0.187444	3.00448
9	2.3579477	0.4240976	13.57948	0.07364	5.7590238	0.173641	3.37235
10	2.5937425	0.3855433	15.93742	0.06275	6.1445671	0.162745	3.72546
11	2.8531167	0.3504939	18.53117	0.05396	6.495061	0.153963	4.06405
12	3.1384284	0.3186308	21.38428	0.04676	6.8136918	0.146763	4.3884
13	3.4522712	0.2896644	24.52271	0.04078	7.1033562	0.140779	4.69879
14	3.7974983	0.2633313	27.97498	0.03575	7.3666875	0.135746	4.99553
15	4.1772482	0.239392	31.77248	0.03147	7.6060795	0.131474	5.27893
16	4.594973	0.2176291	35.94973	0.02782	7.8237086	0.127817	5.54934
17	5.0544703	0.1978447	40.5447	0.02466	8.0215533	0.124664	5.8071
18	5.5599173	0.1798588	45.59917	0.02193	8.2014121	0.12193	6.05256
19	6.115909	0.163508	51.15909	0.01955	8.3649201	0.119547	6.2861
20	6.7274999	0.1486436	57.275	0.01746	8.5135637	0.11746	6.50808
21	7.4002499	0.1351306	64.0025	0.01562	8.6486943	0.115624	6.71888

So it will be nothing but 95,750, this is A amount, so we will multiply with P by A so that we get the value here. So P by A 10 and this is your 10 years so P by A 10 10, now this amount for this you have to find the equivalent amount at present time. So it will be P by F 10 5, it will be again multiplied with P by F 10 5. As it is understood that we have the annual values, for these values we are finding the equivalent here and then we are getting here.

Now let us see it will be $1,00,000 + P$ by A 10 5, so P by A 10 5 will be 3.791. Similarly P by A 10 10, P by A 10 10 is 6.144 and P by F 10 5, so P by F 10 5 is .621. Now we will multiply this 60,000 multiplied by 3.791 that is 227460 + 95750 multiplied by 6.144 multiplied by .621 so that is 365326.

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So this one added to these two quantities, so this come out to be 692786. So this quantity is the present worth for this 10 in which the machine P will be followed by machine Q. Now let us see if we go for machine R, so for machine R using present worth analysis. So for machine R, since the life is 15 years we can directly get the present worth value. The present worth it will be you have the cost 400,000 + you have annual operating cost of 45,000 used for 15 years.

So P by A 10 15, so if we took at the value P by A 10 15, it is 7.606 so 7.606. So we will add 4,00,000+ 45,000 multiplied by 7.606, so it comes out to be 742270. Now if you go for machine R, the present worth value comes out to be 742270, so you see that plan 1 requires less amount of investment initially so plan 1 should be preferred. We have taken a study period of 15 years.

Now in this case we are not using this machine Q for full of its life, so machine Q is not used for its full life. So 5 years of life is remaining, so we should calculate implied salvage value of machine Q. Now how to calculate the implied salvage value of machine Q? So machine Q, basically we are using it only for 10 years out of its total 15 years.

So its present cost is given as 5,00,000, so for finding the implied salvage value, suppose the implied salvage value at 10 year of its life is F_n^* , in that case the annual equivalent value

for 15 year analysis as well as for 10 year analysis should be equal. So in that case, you first cost being 5,00,000 so using its 15 years life when its salvage value is taking a zero, in that case the annual equivalent value would be A by P 10 15.

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Machine P: (Life - 5 yrs)
 First Cost: Rs. 1,00,000
 Salvage value: 0
 Annual O.C. = Rs. 60,000/-

Machine Q: (Life - 15 yrs)
 First Cost: Rs. 5,00,000
 Salvage value: 0
 Annual O.C. = Rs. 30,000

For Machine Q
 $AE(i) = 500000 \left(\frac{P}{F}, i, n \right) + 30000$
 $= 65750 + 30000 = 95750$

$PW = 100000 + 60000 \left(\frac{P}{A}, i, n \right) + 95750 \left(\frac{P}{A}, i, n \right)$
 $= 100000 + 217460 + 365316 = 692786$

For Machine R
 Using present worth analysis
 $PW(i) = 400000 + 45000 \left(\frac{P}{A}, i, 15 \right)$
 $= 742270$

(Plan-I should be preferred): Study period = 15 yrs
 M/C Q is not used for its full life. So, we should calculate implied salvage value of m/c Q.

$500000 \left(\frac{P}{F}, i, 15 \right) - (500000 - F_n^*) \left(\frac{P}{F}, i, 10 \right) + F_n^* (0.10)$
 $\Rightarrow F_n^* = 500000 \left(\frac{1.137 - 1.135}{1.137 - 0.1} \right) = 500000 \times 0.017 / 0.1 = 248803$ (Implied Salvage value)

Replacement is not suggested as P+Q may be better instead of M/C R.

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Interest factor values for discrete compounding (i=10%)							
n	(F/P,i,n)	(P/F,i,n)	(F/A,i,n)	(A/F,i,n)	(P/A,i,n)	A/P,i,n	A/G,i,n
1	1.1	0.9090909	1	1	0.9090909	1.1	0
2	1.21	0.8264463	2.1	0.47619	1.7355372	0.57619	0.47619
3	1.331	0.7513148	3.31	0.30211	2.486852	0.402115	0.93656
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19	6.115909	0.163508	51.15909	0.01955	8.3649201	0.119547	6.2861
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21	7.4002499	0.1351306	64.0025	0.01562	8.6486943	0.115624	6.71888

Now its F_n^* that is the implied salvage value at the end of 10 years for that, you will have 5,00,000 - F_n^* star A by P 10 10 + F_n^* star multiplied by I that is 0.10. So this is the equation to find the implied salvage value which assumes that if the machine is thrown away after 10 years, it has still certain salvage value. So now A by P 10 15 as we have calculated, .1315 and A by P 10 10 is .1627. So basically we have to solve this equation.

Then in that case F_n star can get directly, it will be to 5,00,000 into .1627 - .1315 divided by .1627 - .1. So it will be nothing but 5,00,000, this is our first equation, so it will be 5,00,000 multiplied by .0312 divided by .0627. So it will be 2,48,803. So basically what we have seen is, you have this implied salvage value which need to be mentioned although it does not affect the decision making and you can say that plan one should be preferred.

Under the plan one when we have taken the 15 years of study period, machine P + Q should be kept and machine R should not be considered. So replacement is not suggested, that is P + Q may be allowed instead of machine R. So this is how we do the analysis. Now there is other way also this question can be solved.

What we have seen is, we have taken the 15 years, among that 15 years you have full life of P as 5 years + 10 years of life of Q and 5 years of life of Q is not taken into account. Whereas you have on the other side machine R has 15 years. Now if the fate of this machine Q is not known in that case we have to only go for the 5 years life of machine P. In that case the study period has to taken as only 5 years.

So you have 5 years for machine P and you have 5 years from machine R but machine R for machine R you have 15 years of time, so you can go for comparison **aww** on the basis of annual equivalent and on the basis of that we can have the decision whether to go for replacement or not.

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Machine P: (Life - 5 yrs)
 First Cost: Rs. 1,00,000
 Salvage value: 0
 Annual O.C. : Rs. 60,000/-

Machine Q: (Life - 15 yrs)
 First Cost: Rs. 5,00,000
 Salvage value: 0
 Annual O.C. = Rs. 30,000

Machine R: (Life: 15 yrs)
 First Cost: Rs. 4,00,000
 Salvage value: 0
 Annual O.C. : Rs. 65,000

If it is not sure how machine Q will behave (or about its investment plans), it is reasonably correct to assume 5 yr study period. AE(i) will be used.

M/C P: $AE(i)_P = 1,00,000(A/P, 5, 5) + 60,000$
 $= 26,400 + 60,000$
 $= 86,400$

M/C R: $AE(i)_R = 4,00,000(A/P, 15, 15) + 65,000$
 $= 52,600 + 65,000$
 $= 1,17,600$

M/C P should be preferred. Replacement (M/C R) is not suggested.
 We can find implied salvage value of M/C R (after 5 yrs of its service)
 $4,00,000(A/P, 10, 15) = (4,00,000 - F_n^*)(A/P, 10, 10) + (F_n^* \cdot 0.10)$
 We can find F_n^* .

So if it is not sure how machine Q will behave or about its investment plans, it is reasonably correct to assume 5 year study period. So we will compare on the basis of five year and in

that case annual equivalent basis of comparison will be used. In that case also you can get the implied salvage value of machine R because its life is 15 years and we are only comparing on the basis of 5 years.

So for machine P, if we take the annual equivalent basis you will find 1,00,000 multiplied by $A_{P|5\%}^5 + 60,000$. So $A_{P|5\%}^5$ we can see $A_{P|5\%}^5$ will be .264 so it will be 26,400 + 60,000 26,400. Now if we take for machine R, annual equivalent value for machine R will be 4,00,000 $A_{P|10\%}^{15} + 45,000$. So $A_{P|10\%}^{15}$ will be .1315, so it will be 52,600 + 45,000 so it can be 97,600.

Now what we see is we have using 5 years approach 86400 is the annual equivalent value for machine P and for this machine R you have 97,600. So machine P should be preferred. Replacement machine R is not suggested. We can find in this case also the implied salvage value, we can find implied salvage value of machine R after 5 years of service. So we have already find the implied salvage value and that we can get it and we can get it by solving.

4,00,000 will be multiplied with $A_{P|10\%}^{15}$, it will be fourth 4,00,000 multiplied by $A_{P|10\%}^{15}$, so 4,00,000 $-F_n$ star $A_{P|10\%}^{10} + F_n$ star multiplied by .1. So by solving this equation we can find F_n star. So this way you analyse the problems of such nature. You have seen that under different cases you have to deal by taking the different study periods and you come at different conclusions. Thank you.