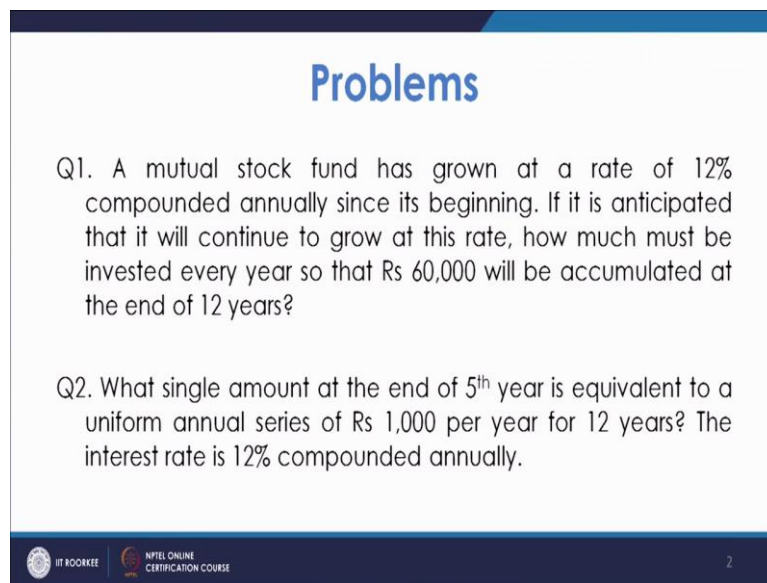


Engineering Economic Analysis
Professor Dr. Pradeep K Jha
Department of Mechanical and Industrial Engineering
Indian Institute of Technology Roorkee
Lecture 15

Problem Solving on Equivalence and Comparison of Alternatives

Welcome to the lecture on problem solving based on equivalence and methods of comparisons of alternatives. So in this lecture given to solve problems based on equivalence as well as on the different criterion we have discussed about the comparison of alternatives. Let us discuss the first question.

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Problems

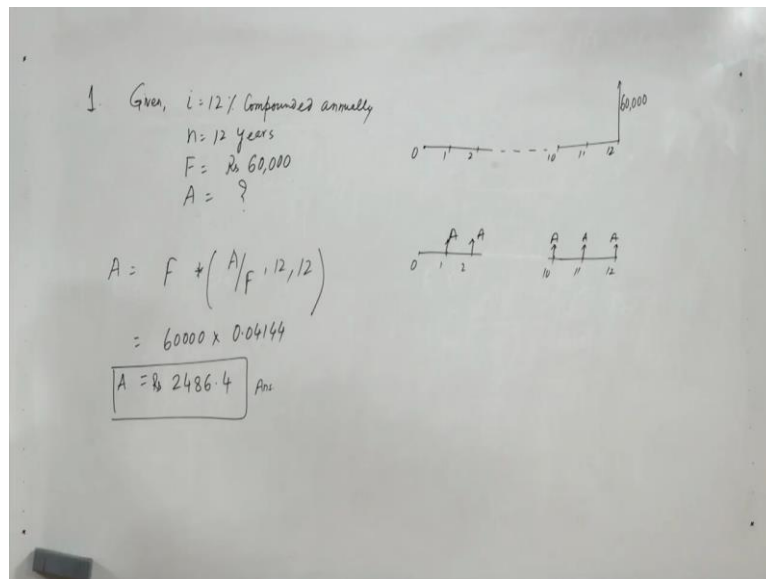
Q1. A mutual stock fund has grown at a rate of 12% compounded annually since its beginning. If it is anticipated that it will continue to grow at this rate, how much must be invested every year so that Rs 60,000 will be accumulated at the end of 12 years?

Q2. What single amount at the end of 5th year is equivalent to a uniform annual series of Rs 1,000 per year for 12 years? The interest rate is 12% compounded annually.

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The first question tells that a mutual stock fund has grown at a rate of 12% compounded annually in since its beginning. It is anticipated that it will continue to grow at this rate, How much must be invested every year so that 60,000 will be accumulated at the end of 12 years? So we can solve this problem, what is given is we will write us.

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We have been given, i is 12% compounded annually. Then it is said that Rs. 60,000 will be accumulated at the end of 12 years, so n is 12 years and F is given as Rs. 60,000. So basically you have to find what should be invested every year. So basically the cash flow diagram will be like this. So at the end of 12 years you need Rs. 60,000 should be accumulated. At the end of 12 years, the Rs. 60,000 is to be here.

Now you need an equivalent amount basically, so basically you have to find the equivalent amount A which should be deposited so that this is equivalent. So you have to find this A , which should be deposited every year end which should give you 60,000 at the end of 12 years. So you are given F , n and i , you need to have A .

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Interest factor values for discrete compounding ($i=12\%$)							
n	(F/P,i,n)	(P/F,i,n)	(F/A,i,n)	(A/F,i,n)	(P/A,i,n)	A/G,i,n	
1	1.12	0.8928571	1	1	0.8928571	1.12	0
2	1.2544	0.7971939	2.12	0.4717	1.690051	0.591698	0.4717
3	1.404928	0.7117802	3.3744	0.29635	2.4018313	0.416349	0.92461
4	1.5735194	0.6355181	4.779328	0.20923	3.0373493	0.329234	1.35885
5	1.7623417	0.5674269	6.352847	0.15741	3.6047762	0.27741	1.77459
6	1.9738227	0.5066311	8.115189	0.12323	4.1114073	0.243226	2.17205
7	2.2106814	0.4523492	10.08901	0.09912	4.5637565	0.219118	2.55147
8	2.4759632	0.4038832	12.29969	0.0813	4.9676398	0.201303	2.91314
9	2.7730788	0.36061	14.77566	0.06768	5.3282498	0.187679	3.25742
10	3.1058482	0.3219732	17.54874	0.05698	5.650223	0.176984	3.58465
11	3.47855	0.2874761	20.65458	0.04842	5.9376991	0.168415	3.89525
12	3.895976	0.2566751	24.13313	0.04144	6.1943742	0.161437	4.18965
13	4.3634931	0.2291742	28.02911	0.03568	6.4235484	0.155677	4.4683
14	4.8871123	0.2046198	32.3926	0.03087	6.6281682	0.150871	4.73169
15	5.4735658	0.1826963	37.27971	0.02682	6.8108645	0.146824	4.9803
16	6.1303937	0.1631217	42.75328	0.02339	6.9739862	0.14339	5.21466
17	6.8660409	0.1456443	48.88367	0.02046	7.1196305	0.140457	5.4353
18	7.6899658	0.1300396	55.74971	0.01794	7.2496701	0.137937	5.64274

So for finding A the equivalent value of annual deposit, what you will do is, A he will be nothing but F times you have to multiply with the factor A by F I n. So I is 12 and n is also 12 and this factor value we can refer from the table. A by F 12 12, so this is a 12% table and for 12% for 12 years, the A by F 12 12 is point 04144, so it is here. So point 04144 60,000 multiplied by point 04144 and once you multiply this it comes out to be 2486 point 4.

It means if you deposit Rs. 2486 point 4 for 12 years at 12% interest, you will get Rs. 60,000. So this 2486 point 4 is the required answer.

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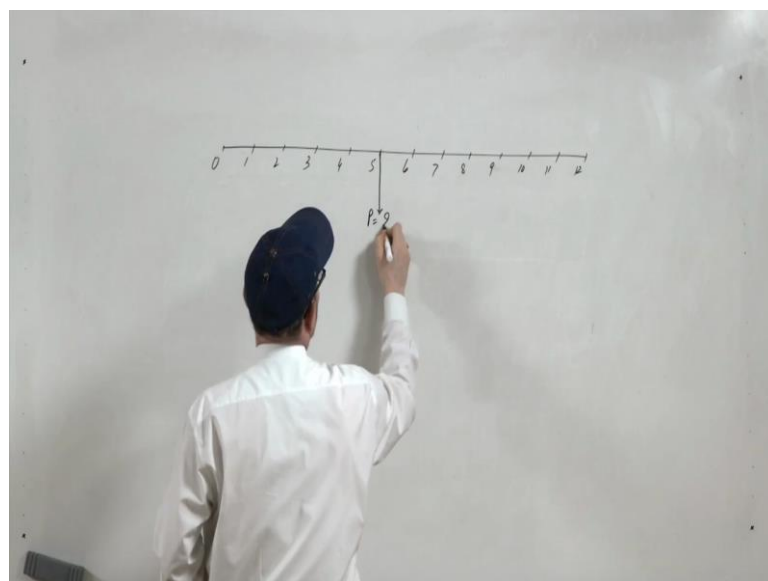
Problems

Q1. A mutual stock fund has grown at a rate of 12% compounded annually since its beginning. If it is anticipated that it will continue to grow at this rate, how much must be invested every year so that Rs 60,000 will be accumulated at the end of 12 years?

Q2. What single amount at the end of 5th year is equivalent to a uniform annual series of Rs 1,000 per year for 12 years? The interest rate is 12% compounded annually.

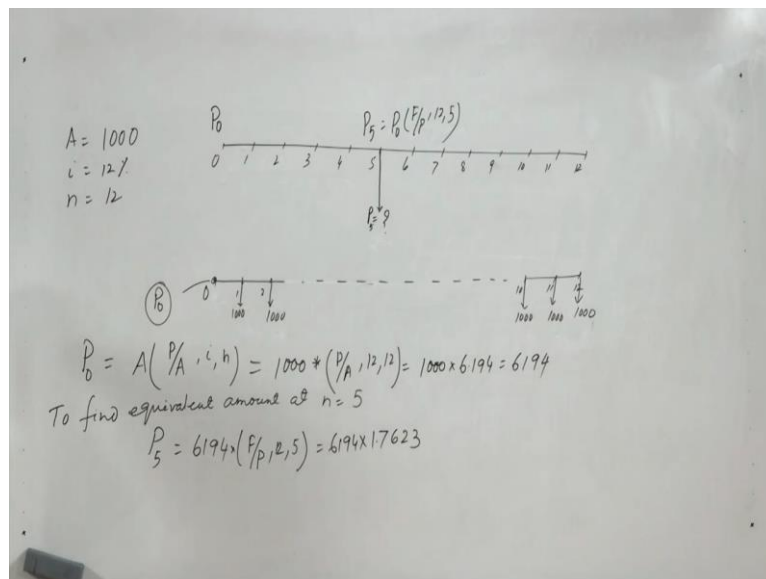
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Next question we will discuss. The next question is, what single amount at the end of fifth year is equivalent to a uniform annual series of Rs. 1000 per year for 12 years. The interest rate is 12% compounded annually. So basically in this question, you have to find the single amount at the end of fifth year. So the cash flow diagram on first. Now in this cash flow diagram you need to find the single amount at the end of fifth year. So you want to have the amount here.

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Which is equivalent to uniform annual series of 1000 per year for 12 years. Now for this the equivalent cash flow diagram is and you are given as Rs. 1000. Now for solving such problems, what you have to do is, first of all you have to find the equivalent value of this series at present time and once you get its equivalent value at present time, you can further get its equivalent at a future time.

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Interest factor values for discrete compounding (i=12%)							
n	(F/P,i,n)	(P/F,i,n)	(F/A,i,n)	(A/F,i,n)	(P/A,i,n)	A/P,i,n	A/G,i,n
1	1.12	0.8928571	1	1	0.8928571	1.12	0
2	1.2544	0.7971939	2.12	0.4717	1.690051	0.591698	0.4717
3	1.404928	0.7117802	3.3744	0.29635	2.4018313	0.416349	0.92461
4	1.5735194	0.6355181	4.779328	0.20923	3.0373493	0.329234	1.35885
5	1.7623417	0.5674269	6.352847	0.15741	3.6047762	0.27741	1.77459
6	1.9738227	0.5066311	8.115189	0.12323	4.1114073	0.243226	2.17205
7	2.2106814	0.4523492	10.08901	0.09912	4.5637565	0.219118	2.55147
8	2.4759632	0.4038832	12.29969	0.0813	4.9676398	0.201303	2.91314
9	2.7730788	0.36061	14.77566	0.06768	5.3282498	0.187679	3.25742
10	3.1058482	0.3219732	17.54874	0.05698	5.650223	0.176984	3.58465
11	3.47855	0.2874761	20.65458	0.04842	5.9376991	0.168415	3.89525
12	3.895976	0.2566751	24.13313	0.04144	6.1943742	0.161437	4.18965
13	4.3634931	0.2291742	28.02911	0.03568	6.4235484	0.155677	4.4683
14	4.8871123	0.2046198	32.3926	0.03087	6.6281682	0.150871	4.73169
15	5.4735658	0.1826963	37.27971	0.02682	6.8108645	0.146824	4.9803
16	6.1303937	0.1631217	42.75328	0.02339	6.9739862	0.14339	5.21466
17	6.8660409	0.1456443	48.88367	0.02046	7.1196305	0.140457	5.4353
18	7.6899658	0.1300396	55.74971	0.01794	7.2496701	0.137937	5.64274

So the present value for this whole equal series will be, now let us see, this we can write as P5. So we need to have first P0, so P0 will be if this is where P0 is indicated, P0 will be nothing but A times P by A I n. A is given as 1000, I is 12%, n is 12, so P0 can be found as 1000 times the factor P by A 12 12. Now P by A 12 12 you can get from the interest factor table. So P by A 12 12 you can have this P by A 12 12 is 6 point 194 6 point 194. So 1000 times 6 point 194 that is 6194.

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$A = 1000$
 $i = 12\%$
 $n = 12$

Timeline diagram showing cash flows at times 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12. P_0 is at time 0. $P_5 = P_0 (F/P, 12, 5)$ is at time 5. $P_5 = ?$ is at time 5.

$(P) \left(\frac{P}{A}, i, n \right) = 1000 * \left(\frac{P}{A}, 12, 12 \right) = 1000 * 6.194 = 6194$
 equivalent amount at $n = 5$
 $P_5 = 6194 * (F/P, 12, 5) = 6194 * 1.7623 = 10915$

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Interest factor values for discrete compounding (i=12%)							
n	(F/P,i,n)	(P/F,i,n)	(F/A,i,n)	(A/F,i,n)	(P/A,i,n)	A/P,i,n	A/G,i,n
1	1.12	0.8928571	1	1	0.8928571	1.12	0
2	1.2544	0.7971939	2.12	0.4717	1.690051	0.591698	0.4717
3	1.404928	0.7117802	3.3744	0.29635	2.4018313	0.416349	0.92461
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18	7.6899658	0.1300396	55.74971	0.01794	7.2496701	0.137937	5.64274

Now this amount is a equivalent amount at 0 times, we have to find equivalent amount at n equal to 5. So this amount should be multiplied with F by P 12 5. So this P0 and P5 will be equal to P0 times F by P 12 5. So P5 comes out to be 6194 times F by P 12 5, now F by P 12 5 again we have to see the value. F by P 12 5 is 1 point 7623, so once you multiply this number 6194 multiplied by 1 point 7623 this comes out to be 10915. So this is how we can solve such problems.

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Calculation of Internal Rate of Return

End of year	Cash Flow
0	-10000
1	-8000
2	5000
3	5000
4	5000
5	12000

$i^*=12.8\%$

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$$PW(12) = -10000 - 8000(P/F, 12, 1) + 5000(P/A, 12, 3)(P/F, 12, 1) + 12000(P/F, 12, 5)$$

$$= -10000 - 8000(0.8928) + 5000(2.402)(0.8928) + 12000(0.5674)$$

$$= -10000 - 7142.4 + 10722 + 6808.8 = -17142 + 17530.8 = 388$$

$$PW(13) =$$

Now we will discuss about the problem on internal rate of return that we had discussed that what will be the internal rate of return for which PW_i will be equal to 0. So let us see what we had received, the cash flow diagram, now in this we had the disbursement of 10,000, 8000 and three 5000 as receipts and 12,000 as the receipt.

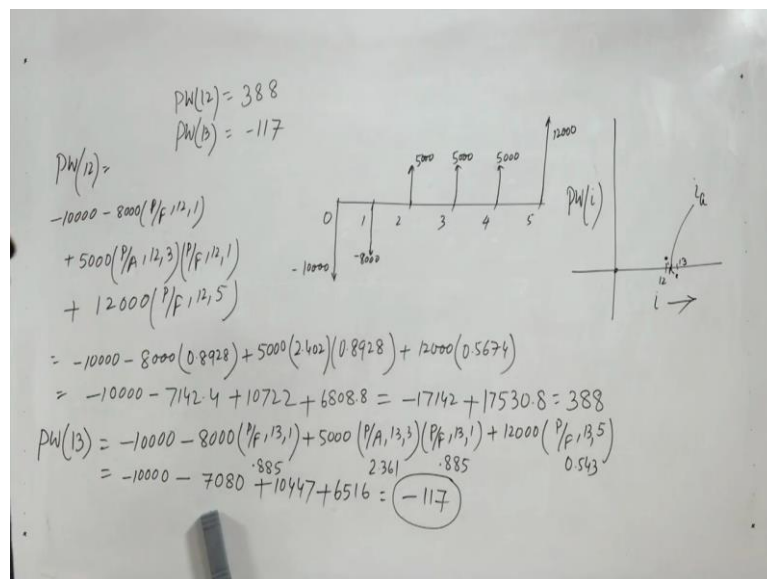
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Interest factor values for discrete compounding (i=12%)							
n	(F/P,i,n)	(P/F,i,n)	(F/A,i,n)	(A/F,i,n)	(P/A,i,n)	A/P,i,n	A/G,i,n
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9	2.7730788	0.36061	14.77566	0.06768	5.3282498	0.187679	3.25742
10	3.1058482	0.3219732	17.54874	0.05698	5.650223	0.176984	3.58465
11	3.47855	0.2874761	20.65458	0.04842	5.9376991	0.168415	3.89525
12	3.895976	0.2566751	24.13313	0.04144	6.1943742	0.161437	4.18965
13	4.3634931	0.2291742	28.02911	0.03568	6.4235484	0.155677	4.4683
14	4.8871123	0.2046198	32.3926	0.03087	6.6281682	0.150871	4.73169
15	5.4735658	0.1826963	37.27971	0.02682	6.8108645	0.146824	4.9803
16	6.1303937	0.1631217	42.75328	0.02339	6.9739862	0.14339	5.21466
17	6.8660409	0.1456443	48.88367	0.02046	7.1196305	0.140457	5.4353
18	7.6899658	0.1300396	55.74971	0.01794	7.2496701	0.137937	5.64274

So what we had seen is PW_{12} will be $-10000 - 8000 P$ by $F_{12,1} + 5000 P$ by $A_{12,3}$ that will be defined here multiplied by P by $F_{12,1} + 12000$ multiplied P by $F_{12,5}$. So this can be taken as $-10000 - 8000 P$ by $F_{12,1}$, so P by $F_{12,1}$ is point 8928 + $5000 P$ by $A_{12,3}$, so P by $A_{12,3}$ is 3 point 037 no it is 2 point 4018 so 2 point 402 multiplied by P by $F_{12,1}$ that is point 8928 + 12000 multiplied by P by $F_{12,5}$, so P by $F_{12,5}$ is point 5674.

That comes out to be - 10,000 - point 8928 multiplied by 8000 so that is 7142 + 5000 multiplied by 2 point 402 multiplied by point 8928 that is 10722 + 12,000 multiplied by point 5674 that is 6808 point 8 and that is equal to - 17142 + 17530 point 8. So 17538 - 17142 is equal to 388. So what we have seen is, we had calculated at 0% rate of interest, PWi and at 12%, the value is only a small value 388.

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Interest factor values for discrete compounding (i=13%)

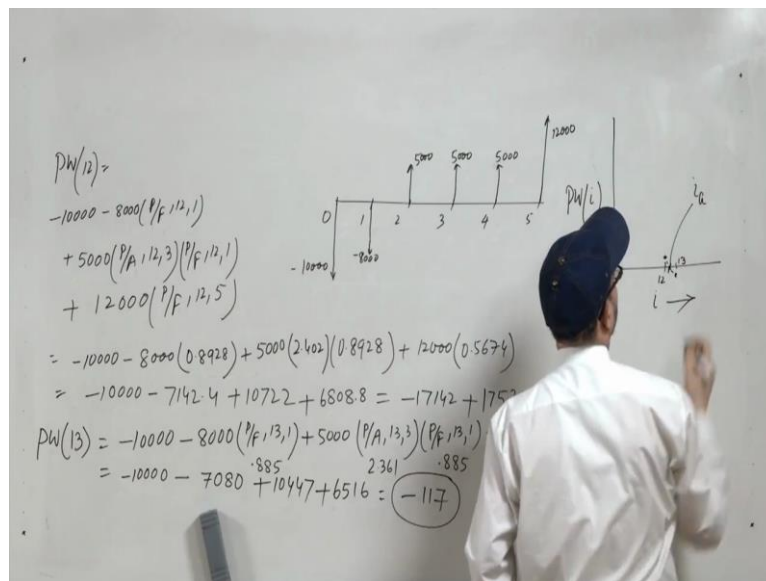
n	(F/P, i, n)	(P/F, i, n)	(F/A, i, n)	(A/F, i, n)	(P/A, i, n)	A/P, i, n	A/G, i, n
1	1.13	0.8849558	1	1	0.8849558	1.13	-7.1E-15
2	1.2769	0.7831467	2.13	0.46948	1.6681024	0.599484	0.46948
3	1.442897	0.6930502	3.4069	0.29352	2.3611526	0.423522	0.91872
4	1.6304736	0.6133187	4.849797	0.20619	2.9744713	0.336194	1.34787
5	1.8424352	0.5427599	6.480271	0.15431	3.5172313	0.284315	1.75713
6	2.0819518	0.4803185	8.322706	0.12015	3.9975498	0.250153	2.14677
7	2.3526055	0.4250606	10.40466	0.09611	4.4226104	0.226111	2.51711
8	2.6584442	0.3761599	12.75726	0.07839	4.7987703	0.208387	2.86851
9	3.0040419	0.3328848	15.41571	0.06487	5.1316551	0.194869	3.20138
10	3.3945674	0.2945883	18.41975	0.05429	5.4262435	0.18429	3.51619
11	3.8358612	0.2604977	21.81432	0.04584	5.6869411	0.175841	3.81342
12	4.3345231	0.2307059	25.65018	0.03899	5.917647	0.168986	4.09359
13	4.8980111	0.2041645	29.9847	0.03335	6.1218115	0.16335	4.35727
14	5.5347525	0.1806766	34.88271	0.02867	6.3024881	0.158667	4.60504
15	6.2542704	0.1598908	40.41746	0.02474	6.4623788	0.154742	4.83749
16	7.0673255	0.1414962	46.67173	0.02143	6.6038751	0.151426	5.05523
17	7.9860778	0.1252179	53.73906	0.01861	6.729093	0.148608	5.2589
18	9.024268	0.1108123	61.72514	0.0162	6.8399053	0.146201	5.44911
19	10.197423	0.098064	70.74941	0.01413	6.9379693	0.144134	5.62651
20	11.523088	0.0867823	80.94683	0.01235	7.0247516	0.142354	5.79172
21	13.021089	0.0767985	92.46992	0.01081	7.1015501	0.140814	5.94538

Now let us calculate the value at I equal to 3, at I equal to 13 sorry, so at I equal to 13. PW13 will be - 10,000- 8000 multiplied P by F 13 1 + 5000 P by A 13 3 P by F 13 1 + 12,000 P by F 13 5. Now again we have to see the interest table of 13%. In the 13%, P by F 13 1, P by F

13 1 is point 8849 so point 885. P by A 13 3, this is P by A 13 3, this is 2 point 361 and P by F 13 5 is point 5427 so point 543 you can take.

So if we do the multiplication further, we have - 10,000 - 7080 + 5000 multiplied by 2 point 361 multiplied by point 885 so it is 10447 + 12000 multiplied by point 543 that is 6516. So 6516 + 10447 - 10000 - 7080 that comes out to be - 117. Now see that for 13% interest rate, it has gone negative, so it has come here.

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For 13%, it has come negative, it means at some point in between this point will be the rate of return. Now this point can be found by using the interpolation method. What we see is PW12 is coming as 388 and PW13 is coming as - 117. So the interest at which it will be 0 will be, PWi star where it is 0 will be I star equal to 12 + 0 - 388 by - 117 - 388. So it is nothing but 12 + 388 by 505, so it is coming close to 12 point 8%.

So what we see is, now what we see is the I star by using interpolation method we get this value. This for this value, the present worth is coming as 0 that is why this I star of 12 point 8% is known as the internal rate of return. So you can solve the problems based on that and get the confidence more and more.

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Q3. What amount of initial deposit is required for maintaining a park forever. The park is made by the municipality and the maintenance cost is to be borne by the society. The maintenance cost every year is approximated as Rs 60000. Interest rate can be taken as 12%.

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Next question what we will discuss will be, you have to find the initial deposit amount which is required for maintaining a park forever. The park is made by the municipality and the maintenance cost is to be borne by the society. So a maintenance work is to be done forever for a park which is made by the municipality. The maintenance cost is estimated as Rs. 60,000 and interest rate is taken as 12%.

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$$CE(i) = \frac{A}{i}$$
$$A = 60,000$$
$$i = 12\% = 0.12$$

initial deposit which will provide a sum of Rs 60,000/- for ever.

$$CE(12) = \frac{60,000 \times 100}{0.12} = \boxed{Rs\ 500,000}$$

So this is a case of perpetuity type of cash flow where you need this amount for ever. So we will use the capitalised equivalent concept to find the initial deposit for such cost which have to be incurred every year for maintaining the park. So as we have discussed the capitalised

equivalent amount is nothing but A by I . So it is nothing but this amount that you have to get forever. Now in this case A is given a 60,000 and I is given as 12%.

So the initial deposit, the initial deposit which will provide a sum of Rs. 60,000 forever that is capitalised equivalent at 12% will be 60,000 divided by point 12. So this comes out to be rupees. So Rs. 5,00,000 amount if there is initial deposit, it will give you an interest rate, interest of Rs. 60,000, in other sense which will be used for maintaining the park forever. So such problems you need to solve frequently and you will get more control over solving these problems.

Thank you.