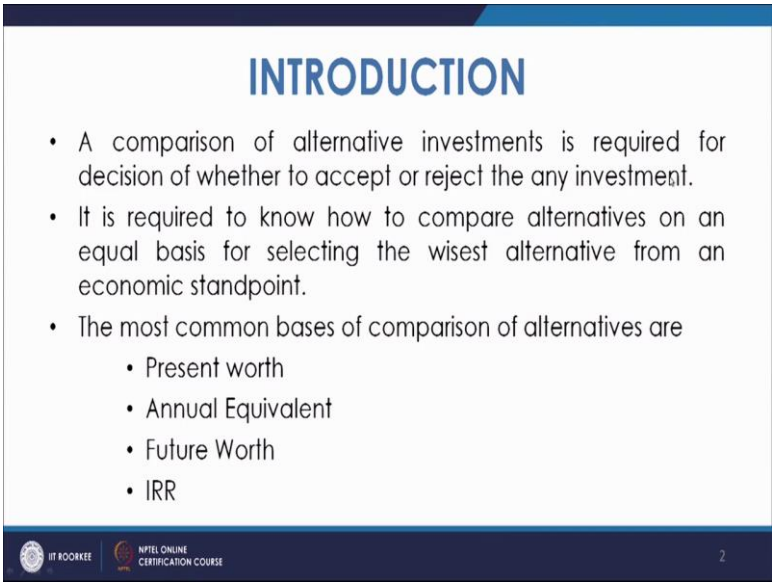
Engineering Economic Analysis
Professor Dr. Pradeep K Jha
Department of Mechanical and Industrial Engineering
Indian Institute of Technology Roorkee
Lecture 13
Methods of Comparison of Alternatives: Present Worth, Annual Equivalent, Future Worth, Internal Rate of Return

Welcome to the lecture on methods of comparison of alternatives. So basically we deal with a number of investment alternatives in engineering economic analysis and we have to compare among the alternatives which of the alternative is good or better. So basically we have to compare them and for comparing their values, there are different basis of comparison.

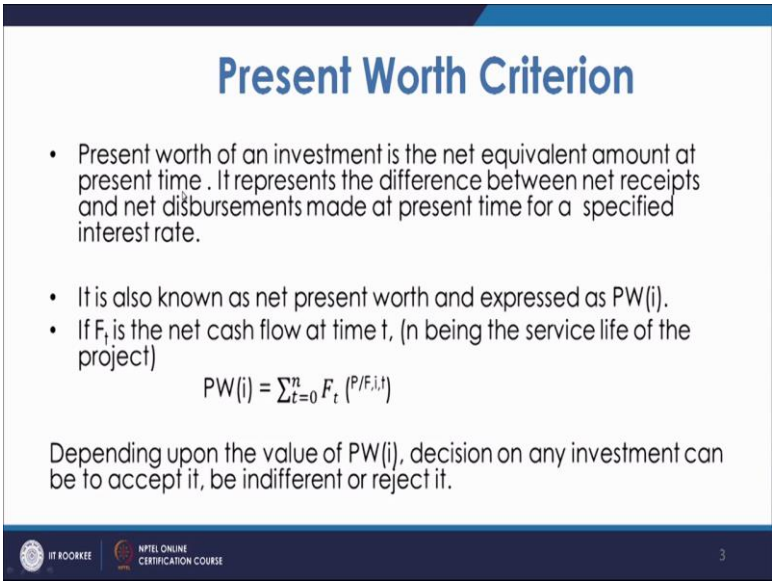
(Refer Slide Time: 01:05)



So comparison of alternative investments is required for decision of whether to accept or reject an investment mea and if you are having an investment, it has certain cash flows. You need to decide whether this cash flow will be beneficial for you for the investor whether he should go ahead or he should reject it. It is required to know how to compare alternatives on an equal basis for selecting the wisest alternative from an economic standpoint.

So basically you should know which one is the best alternative. From economic point of view you need to know about certain terminologies which is basically evaluate the alternatives values. The most common basis for companies of alternatives are present worth method, annual equivalent method, future worth method, internal rate of return. Apart from that we will also discuss about capitalized equivalent and capital recovery with return.

(Refer Slide Time: 02:25)

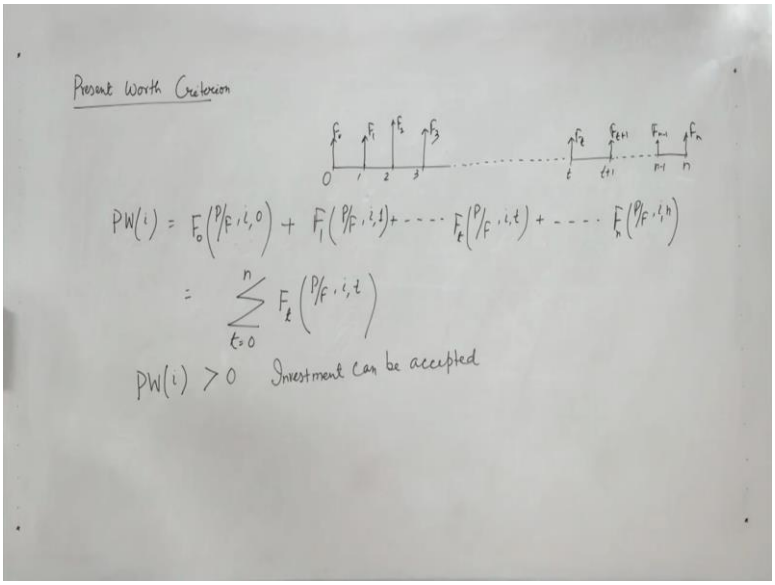


So coming first to the present worth criterion. Now present worth criterion of an investment is the net equivalent amount at present time. So basically in any cash flow, you have the receipts and disbursements occurring at different times. So basically you have to find the net value of all these receipts - that of the disbursements and basically that is to be evaluated at the present time.

So it represents the difference between net receipts and net disbursements made at present time for a specified interest rate. So given a cash flow diagram if you know the interest rate, in that case it would be the net value of the receipt and the net value of disbursement. The difference between them and its value at the present time t equal to 0 will give the present worth of that particular investment.

It is known as present worth and we express it as PWi, I shows that at particular rate of interest. So if Ft is the net cash flow at prime t, n being the service life of the project, PWi that is present worth of the investment can be written as summation t equal to 0, 2n Ft multiplied by P by F I t. So basically we will see how we can derive this formula. Let us see you have an investment as it is shown that Ft is the net cash flow at time t.

(Refer Slide Time: 08:37)



So if you have a cash flow diagram let us discuss about present worth criterion. So you deal with a cash flow diagram at different time. Now this is a cash flow diagram and the transactions are Ft that is here it is F1, here it is F0, here it may be F2, this may be F3, so this is Ft, this is Ft + 1, this is Fn - 1 and this is Fn. So these are the net cash flows at that particular time. So this is net cash flow F1 at t equal to 1.

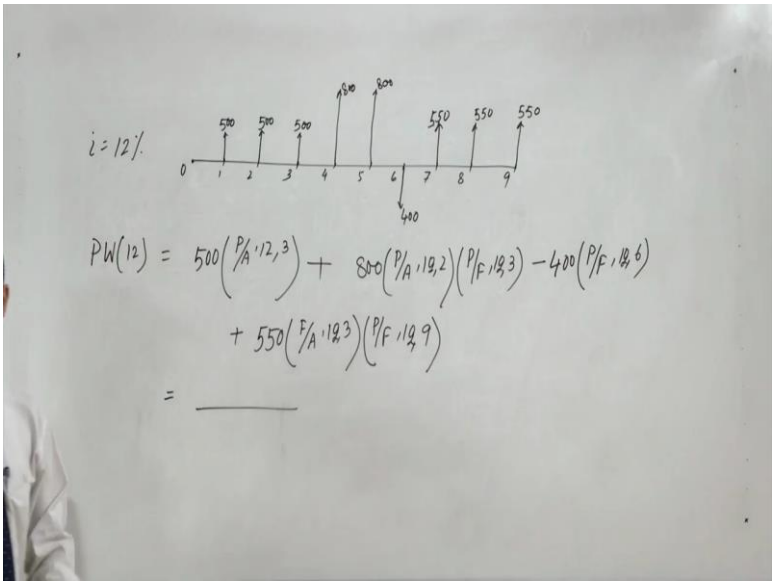
So that is why we see that Ft is the net cash flow at time t. Now present worth means, each of these components equivalent value at this time is to be found out. So present worth will be equal to that the interest rate is i. So contribution of the F0 is F0 itself and you can write this as P by F I 0 + F1 the cash flow at time t equal to 1, this will be nothing but its equivalent value at 0 time. For that you have to multiply it with the interest factor P by F I 1.

So it will be P by F I 1. Similarly it we go for P by F I t multiplied by Ft and further it will go for Fn multiplied by P by F I n. So basically it is the algebraic sum of the equivalent values of all these net cash flows at the respective times and their value is found at time t equal to 0 that is present time. So that is why it is written as summation t equal to 0 to n Ft P by F I t. So this way you can find the present worth values.

Now the present worth value should be basically greater than or equal to 0 for the investment to be acceptable or thefor the investment to run. So that is why it is written that if PWi is greater than 0 then the investment can be accepted. If PWi equal to 20 it does not meaning mean any difference because the investor is not losing neither he is gaining, so it does not make any difference.

And if PWi is less than 0 in that case the investor cannot go ahead because he is at loss, so investor won't go ahead with the project. So this criterion give us the freedom to calculate the present worth value for any cash flow diagram and based on that we will see that after we calculate the present worth criterion we can even calculate the annual equivalent or future worth. So basically they are all interlinked and we will see their implication.

(Refer Slide Time: 14:54)



Let us see a small problem which we can solve and see how we calculate the present worth value for any particular investment. So for example if we have any cash flow diagram, so suppose this is a cash flow diagram where you see certain cash flows, so suppose this is 500, this is 800, this is 400, this is 550.

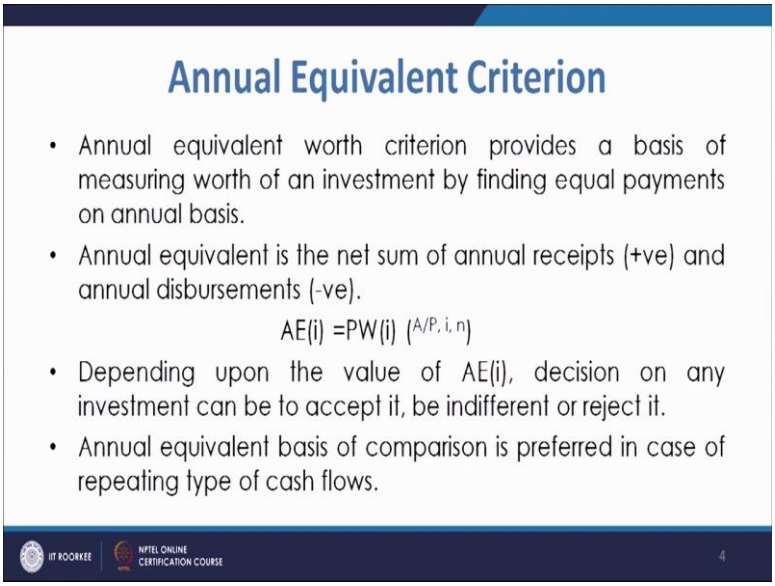
Now suppose you are given such a cash flow diagrams you need to find the present worth of such investment and for that we have already studied about the equivalence theorem. So what we can do for this and if I is given as suppose 12%, in that case what we will do is, we have to find the present worth of these 3 then this one, then this one and then this one. So this will be basically subtracted and all these in the positive side will be added.

So we can see PW12 we can write as we have so far been acquainted with the equivalence principles so directly we can see this 500 we can get the P by A theorem. So 500 multiplied by P by A 12 3, so basically these 3 will be getting the value here. Then further the two 800s, so 800 P by A 10 2, this will be defined at t equal to 3, so its value is further to be calculated at this point. So this will be further multiplied P by F 10 3.

Then this being on the negative side we will subtract it- 400 P by F 10 6 then finally we have three 550 transactions. So we can directly get its equivalent value at this point, so this 550 will be multiplied with F by A 10 3. This will its equivalent value we can get at this point and this will be multiply because the final value has to be brought in at this point. So we will further multiply with the factor P by F 10 9.

So if we refer to the table, pardon me these all these interest rates are 12%, so this all will be 12. So basically all these interest factor values we can calculate it from the interest tables and once we put all these values we can get the final P values. So all these things will be known and we can get a value. So this is how we get the present worth of any investment.

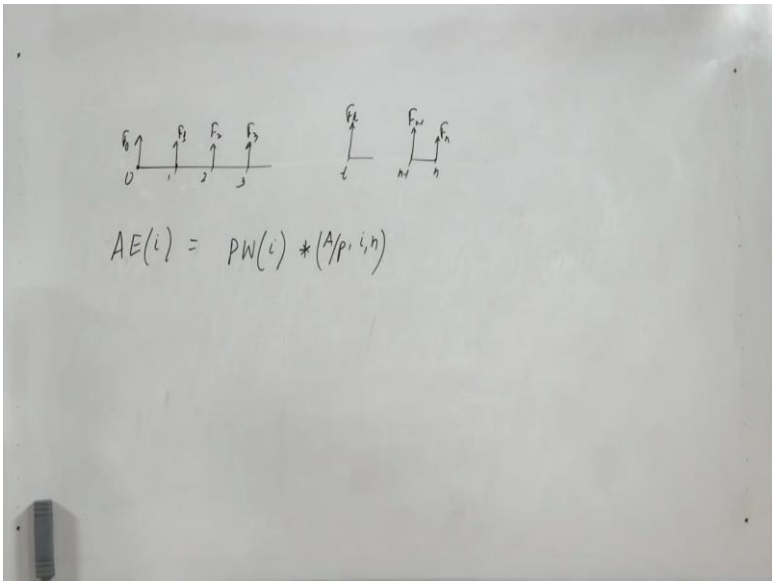
(Refer Slide Time: 15:04)



Further there is annual equivalent criteria. Annual equivalent worth criterion provides a basis of measuring worth of an investment by finding equal payments on annual basis. So basically it is giving you a basis in which any cash flow will be equivalent to certain equal payment type of series or type of cash flows. So it is the net sum of annual receipts and annual disbursements.

So basically you have receipts as well as disbursements, you find the annual equivalent value of all the receipts and annual equivalent value of the all the disbursements and then you can then find algebraic sum of it one being positive and another being negative. So what we see is, once you find the present worth value you can find the value of A by P I n and this can be multiplied, that will give us the annual equivalent value.

(Refer Slide Time: 17:57)



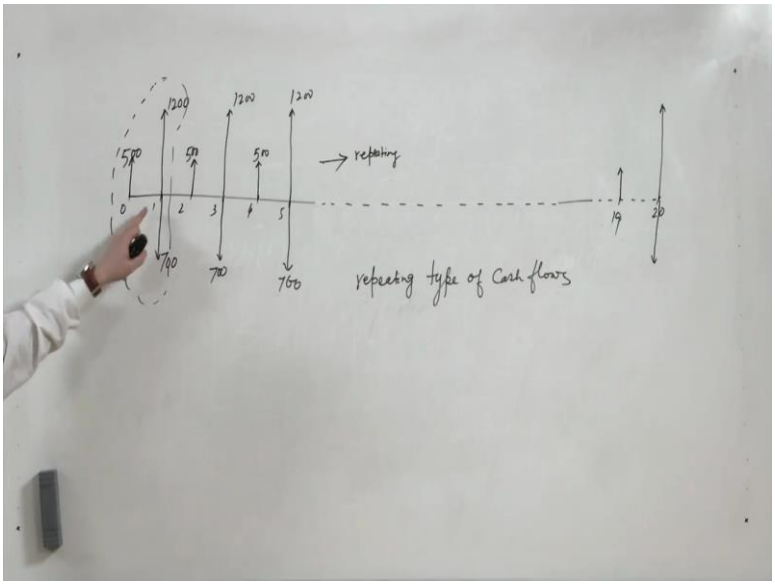
So basically if you have any cash flow diagram and once you have calculated the present worth value, so if Ft is the cash flow at time t, so in that case what you have to do is you have to find you can basically convert all these values individually and find its annual equivalent amount and then you can algebraicaically add them.

Otherwise basically we are getting the present worth component here itself and then that component can be multiplied with the factor A by P I n. So that is why you can see that this annual equivalent is nothing but once you get the present worth component, present worth component is defined as 0 time, so once this is multiplied with A by P I n, this factor, this factor once we multiply this will give us basically the annual equivalent value.

So it is nothing but PWi is written as t equal to 0 to n Ft 1 + I - t and then you can multiply it with this factor A by P I n. So A by P I n is nothing but, so if this factor is calculated, this is to be multiplied with this factor and you get the annual equivalent value.

Same as the present worth criterion, this condition holds good also here where it can be said that if it is positive, the investment can be considered as acceptable, if it is zero it is indifferent and if it is negative you cannot go ahead with the investment. Also such basis is preferred in case of repeating type of cash flows. Now let us see a particular type of cash flow which is repeating.

(Refer Slide Time: 21:34)

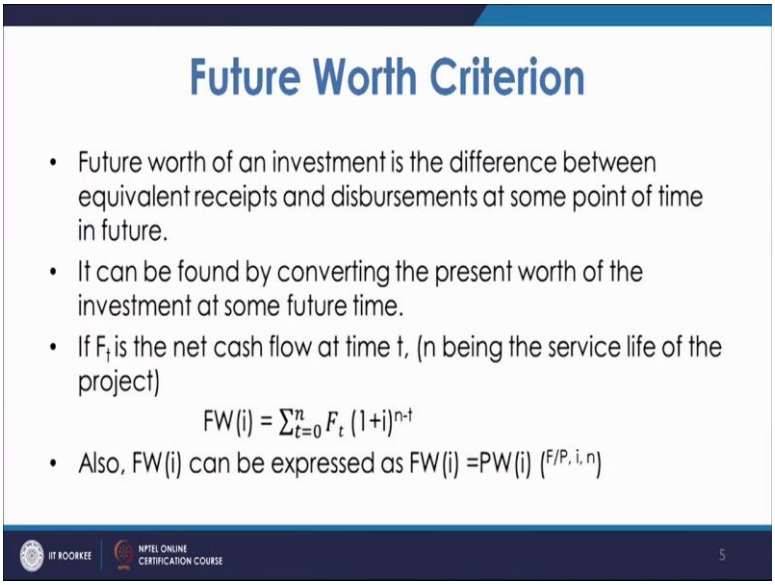


So if a cash flow is there, now let us see you have a cash flow which is repeating like if there is like this and this is repeating. So at this point also, so suppose this is 500, this is 1200 and this is 700, so basically we can think of keeping it as a continuation. So such laws are known as repeating type of cash flows where we see that this particular cash flow, this is repeating every time.

So in this time step has repeated further it has repeated here, further it has repeated here and it goes. So in such cash flows we can well use the present worth criterion but it makes cumbersome because it involves a large number of computation. In that case such annual equivalent value is basically worth because the annual equal in value for this will remain same as this one, same as this one.

So basically wide this cumbersome and to avoid large amount of calculations, for such repeating cash flows, annual equivalent basis of comparison is having more meaning. Next is future worth criterion, so future worth criterion means in this basically you find the net amount of receipts and disbursements and any future time.

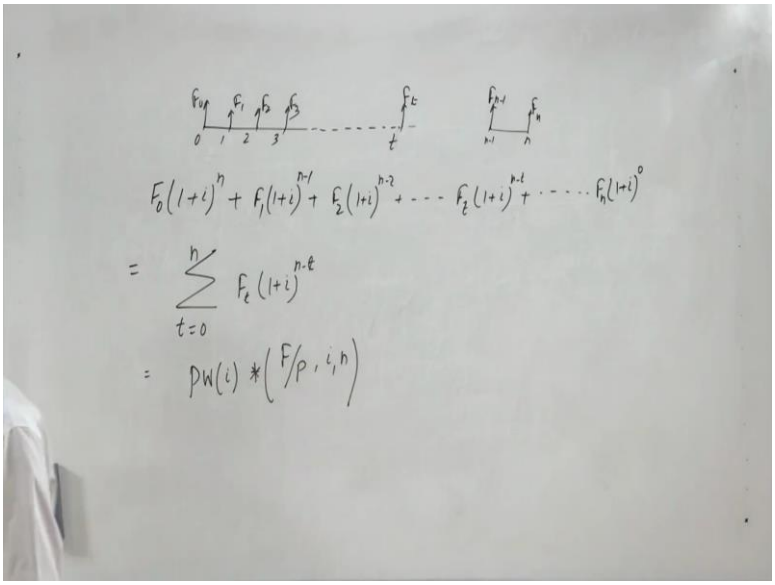
(Refer Slide Time: 22:45)



So of an investment is the difference between equivalent receipts and disbursements at some point of time future. So basically in the first case we have got the equivalent amount at present time t equal to 0. In the second case we have got the equivalent amount as the annual payment but we can also represent them in terms of an equivalent amount at the common point time in future that is t equal to n.

So it can be found by converting present worth of the investment at some future time. So basically once you have converted any cash flow and found its present worth then you can directly convert it to the future time by multiplying with F by P I n. So basically you can get the present worth value and then you can find this F by P I n.

(Refer Slide Time: 27:04)

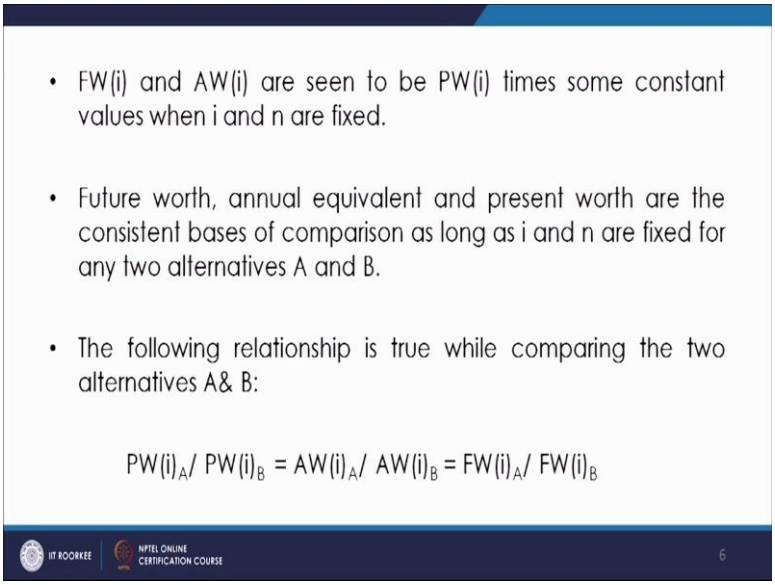


Also it can be found by this formula where FWi is t equal to 0 to n summation Ft multiplied by 1 + I raised to the power n - t. So basically this you can find like this, you have an investment, so you have F0, F1, F2, F3, Ft, Fn - 1 and Fn. So basically once you get the equivalent amount net equivalent amount at the future time at t equal to n basically this is the future worth of this particular cash flow or the investment.

So what we see is if we see the value of this particular investment basically you have to see that this F not its contribution towards the nth time that is here, it is F0 multiplied by 1 + I raised to the power n. Then F1 can be written as, its contribution at the nth time will be multiplied with so F1 has to be multiplied with 1 + I raised to the power n - 1. So basically it is earning interest for n - 1 interest periods.

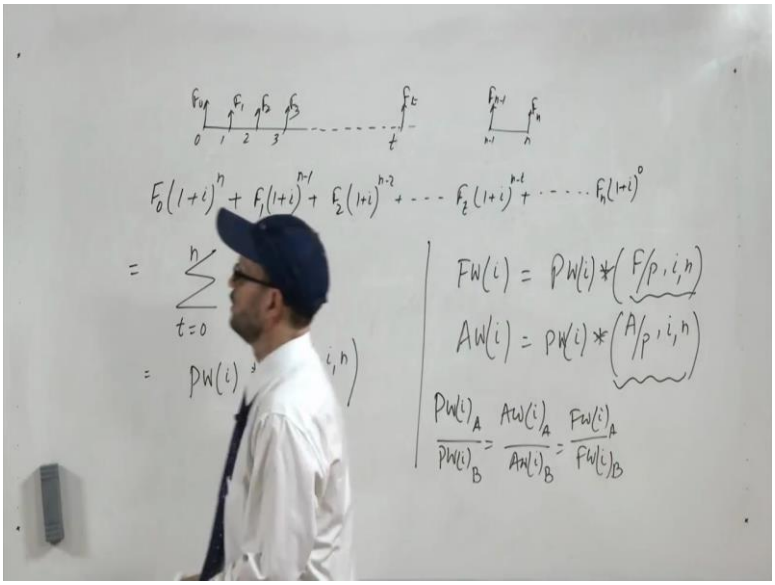
So F2 in that case will be getting F2 into 1 + I n - 2, so Ft 1 + I 1 - t and this Fn will have nothing raised to power m F Fn 1 + I raised to the power n - n so 0 so it will be only Fn, that is why it is written as Ft into 1 + I raised to the power. So it has been written as t equal to 0 to n Ft 1 + I raised to the power n - t.

(Refer Slide Time: 27:27)



We have already discussed once we get the present worth we can directly conc multiply this with this factor F by P I n and we can get the required value of future worth. So it is also equal to PWi multiplied by F by P I n. So this is how you calculate the different criterion. They are basically seen as FWi n to AWi they are basically the product of PWi times certain factors.

(Refer Slide Time: 28:28)



So what we see is FWi is PWi times F by P I n and AWi we have already discussed as PWi times A by P I n. So basically they are nothing but these are the factors which are the numbers, so basically their ratios are coming to be equal for two alternatives A and B and that is why for two alternatives A and B if you find PWi A upon PWi B it will be same as AWi A upon AWi B and that is FWi upon FWi A and B.

Next is internal rate of return, this is a rate of return which basically gives you the zero present worth value. This we will discuss in our next slide and we will see how for a certain cash flow diagram this internal rate of return basically can be generated because they are to be found by trial and error methods. Thank you.