Engineering Economic Analysis
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Lecture 12
Equivalence Calculations Involving Cash Flows

Welcome to the lecture on equivalence involving cash flows. In the last lecture we have discussed about the point where we found that receipts and disbursements can be directly added when they are calculated at the same point in time.

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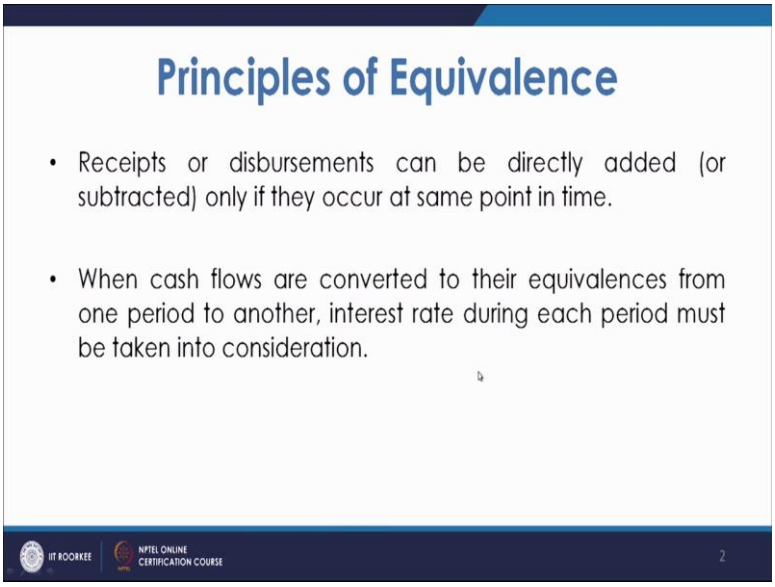

Now the next point is when cash flows are converted to their equivalences from one period to another, interest rate during each period must be taken into consideration. It means many a times this happens that the interest rates are changing with time, this is not a very unusual case.

Now when the interest rates are changing, basically you will have to find the equivalence and on that time you will have to keep in mind that the equivalent values can be converted to a particular time when the interest rate in that time period is fixed. So we can understand this by referring to a cash flow when we will try to find the present worth values.

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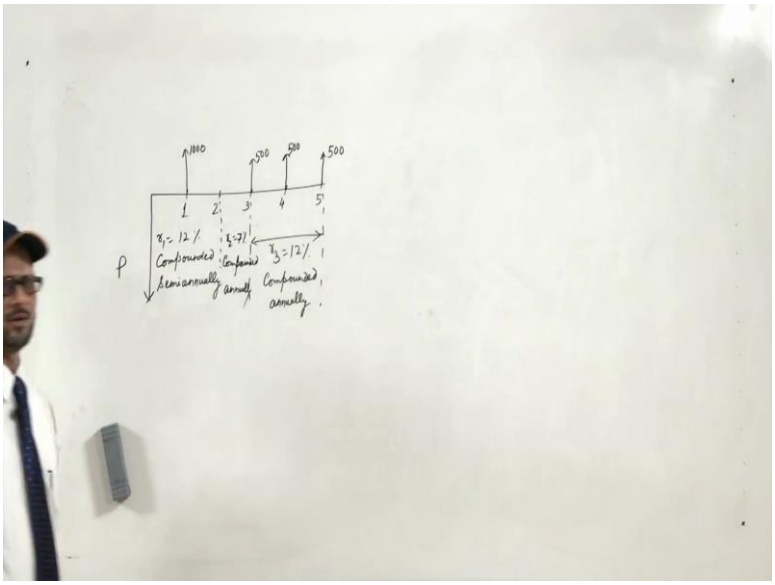

So there is a cash flow where you have to find the present worth, there are five years so n is 5. Now in this basically at this point you have Rs. 1000, 500, 500 and 500. Now the interest rates during these periods differ. Between this to this, the interest rate is varying, r1 as 12% compounded semi-annually. Then between 2 to 3 it is r2 and it is 7% compounded annually. Then between 3 to 5, in this, r3 is 12% compounded annually.

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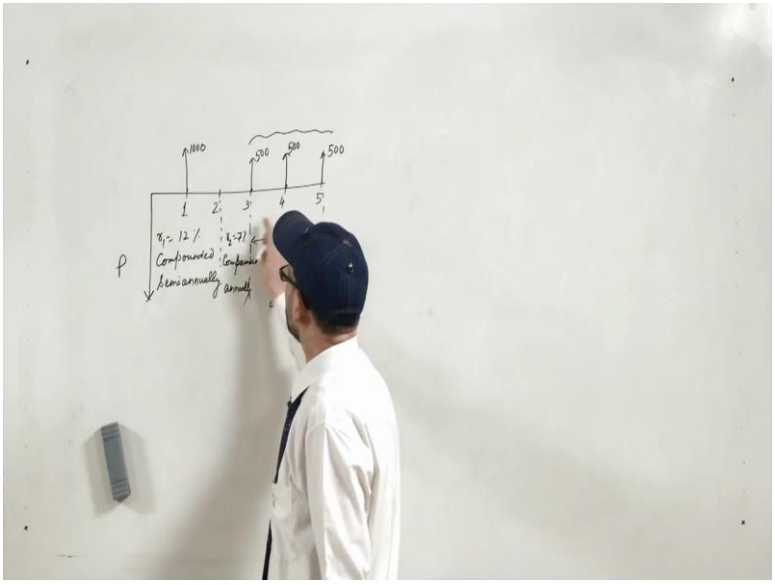

Now what we see in this question is that the interest rate during the periods vary. Now how to find the equivalent present worth value P for this cash flow diagram. So for the equivalent present amount we have to find the equivalent value of this series at this time. We cannot find

the equivalent present worth at this time because from this zone to this zone, the interest rate is varying.

So we have to only use the F by A formula so that we get the equivalent future amount at this time. Now this amount, its equivalent has to be found here, then the amount has to be converted here. This amount whatever will come, its equivalent amount has to be found here and then finally at this time. So what you have to do is, first find F for cash flows at 3, 4 and 5 years.

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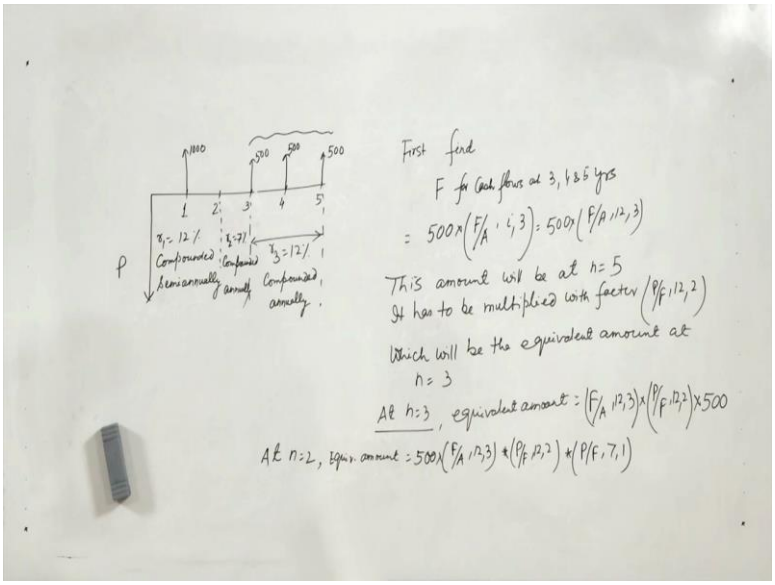

So this will be 500 multiplied by F by A i 3. So this i is 12% compounded annually, so this will be 500 multiplied by F by A 12 3. So if we look at the interest factor values, we can have the value of F by A 12 3 and this value can be used here. Further, once we get this value, this amount will be at n equal to 5. Now this amount is to be converted to this particular time.

So it has to be multiplied with factor there are two interest periods so you can multiply with P by F i n. So P by F 12 2 which will be the equivalent amount at n equal to 3. So at n equal to 3, equivalent amount is F by A 12 3 multiplied by P by F 12 2 multiplied by 500. Once you get at this particular time, now this amount has to be converted at this time. So there is one interest period and the interest rate is 7% compounded annually.

So it has further to be multiplied with a factor P by F i 1. So P by F i 1, once you have got the equivalent amount at n equal to 3, you will again multiply with P by F i 1, that will give you the equivalent amount at n equal to 2. So at n equal to 2, equivalent amount is found by 500

multiplied by F by A 12 3 multiplied by P by F 12 2 further multiplied by P by F i 1 so P by F 7 1. So this is the equivalent amount at n equal to 2.

Your ultimate aim is to get the value of P at n equal to 0. So now again during this 0 to 2 years, the interest rate is 12% compounded semi-annually. So basically this 12% compounded semi-annually, you can interpret it as 6% for every six month. So basically the effective interest rate of 6% but your interest period becomes 4 as compared to these two. Once you get the effective interest rate for six month, you have 4 interest periods.

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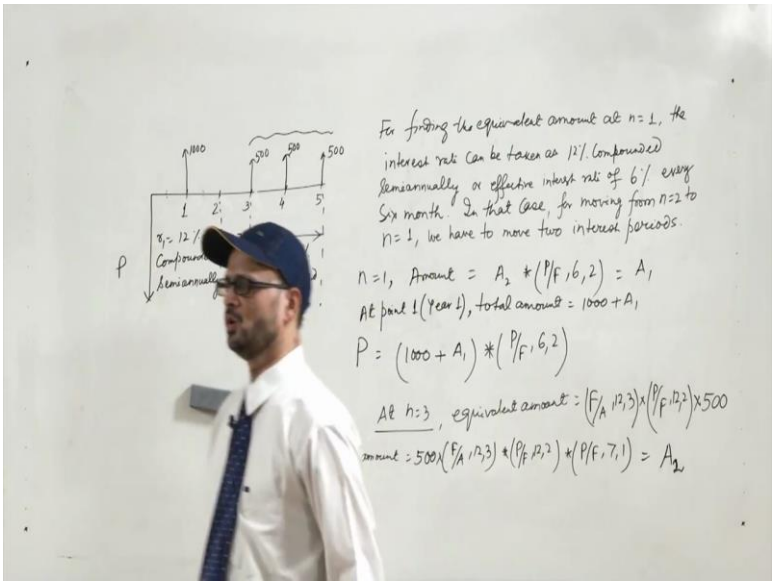

So for 6% interest rate, you will again go for two interest periods. So for finding the equivalent amount at n equal to 1, the interest rate can be taken as 12% compounded semi-annually or effective interest rate of 6% every six month, in that case for moving from n equal to 2 to n equal to 1, we have to move two interest periods because each of the interest period is of six month. So you are basically moving two interest periods.

So at n equal to 1, suppose this amount let it as A1 or A2 so now this at n equal to 1 A2, so amount will be equal to A2 multiplied P by F i n2. So basically you have 4 periods but your interest rate will be 6%, so it will be multiplied with P by F 6 and 2 and this you can take as A1 because it has moved two interest periods and the effective interest rate is 6%.

Now at this point basically A1 will be added to this 1000. So at point 1 or year 1, total amount will be 1000 + A1. And once we get this, again its equivalent amount at this time can be found by further multiplying with the factor P by F i 2 and i is 6%. So P can be found as

1000 + A1 multiplied by P by F 6 2. So what we see is that you cannot ignore these different interest rates which are applied during the different time periods.

So and you cannot add them or you cannot find the equivalent of this amount directly to this point because there is different interest rate in that domain. So that cannot be ignored, so once you get this value ultimately you will get the final P as 1000 + A1 even multiplied by P by F 6 2. So this way when we will solve some problems based on this in our coming lectures, we will come to know about it.

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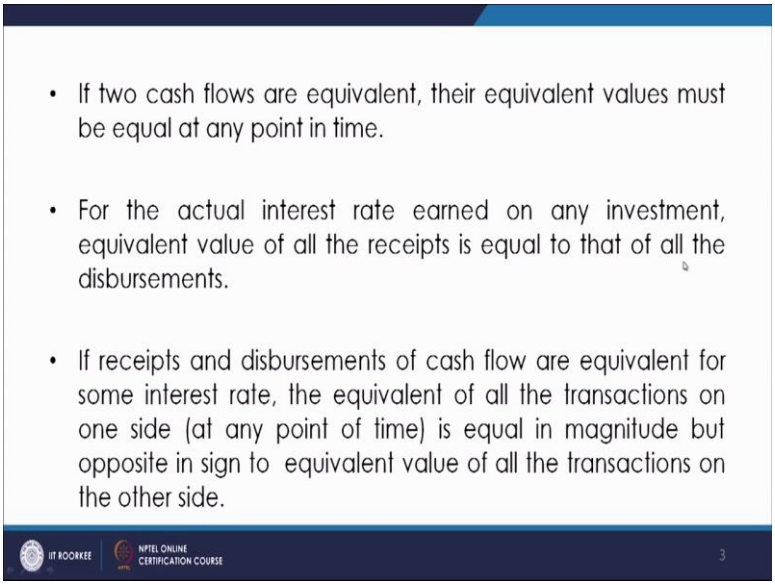

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Now there are other principles of equivalence for cash flows. If to cash flows are equivalent, their equivalent values must be equal at any point in time. This principle tells that if there are two cash flows, and the two cash flows if they are equal, so two cash flows are said to be equivalent when their equivalent values must be equal at any point in time. So for this we try to find the future value at this time, they must be equal.

And if they are equal, it means these two cash flows are equivalent. If you try to find its equivalent value for a particular interest rate at this point, if these two are equal, then you can say that these two cash flows are equal. For the actual rate of interest and on any investment equivalent value of all the receipts is equal to that of all the disbursements.

This is understood by an example, if there is any investment, now in the investment, in the cash flow, you will have receipts as well as disbursements. So basically, the interest rate which is earned on the receipt or the disbursement, equivalent value of all the receipts should be equal to equivalent value of all the disbursements. So basically receipts are taken as positive and disbursements are taken as negative.

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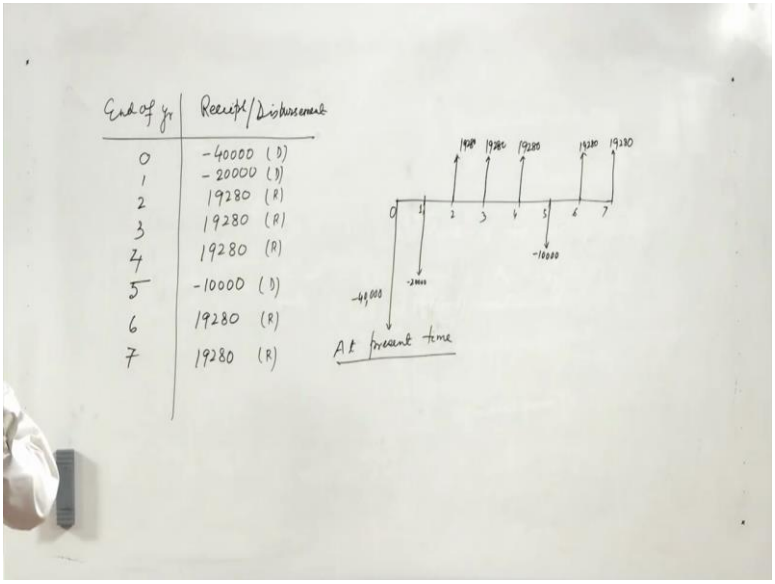

So basically they will be equal in magnitude but opposite in sign. So we can refer to a particular question and we can solve that. So we can see there is a problem which talks about receipt or disbursement at end of the year. So this is a type of cash flow which is generated in the investment. You have the negative values as disbursement and the positive values are the receipts.

The cash flow diagram can be drawn like this. Now in this you have - 40,000, - 20,000, then 2, 3, 4 you have 19,280. 5 is again -10,000, this is 19,280 and this is also 19,280. Now for this investment, the principle of this will be used where it is said that the equivalent value of all the receipts is equal to that of all the disbursements but that has to be followed at that particular time.

So if we try to find the equivalent values at a particular time, so at present time, the equivalent value of, all these disbursements must be equal to equivalent value of all these at this time and that will be basically the actual interest rate earned on this investment. So in that case you have to find the present worth value of all these receipts and the present worth value of all these disbursements.

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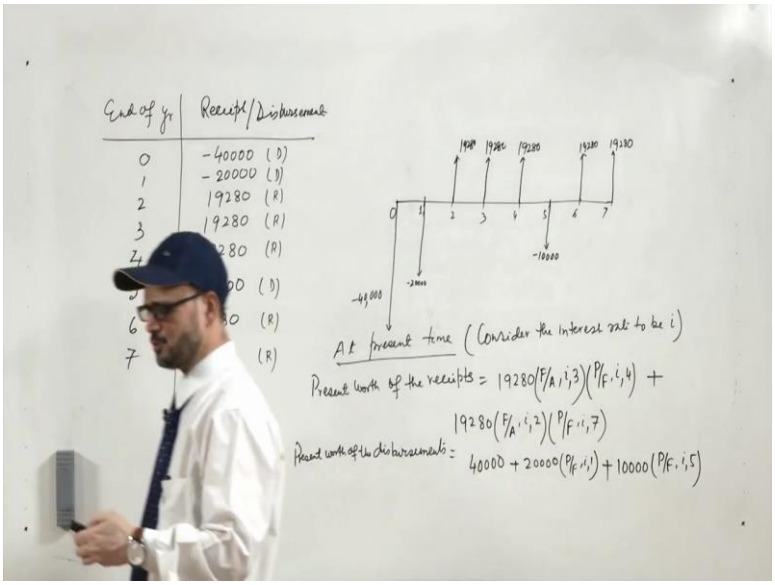

Now if we try to find the present worth of the receipts, consider the interest rate to be I. So the present worth of all the receipts will be nothing but, again you have to use the annual equivalent value, you have 3 equal annual payments, so you can find its equivalent at this particular time so 19,280. It may be multiplied with F by A i and 3.

This will be defined at number 4, so you have to bring it to time T equal to 0, so there are 4 interest period, so it will be further multiplied with P by F i 4. Then you have to add this receipt also, so you can further add 19,280 multiplied by F by A i 2, this will give you the equivalent value at this particular time. And then you have again find its present value by multiplying with P by F i 7.

Similarly the present worth value of all these disbursements, so disbursements will be, it is at this time only, so you have 40,000 + 20,000 multiplied by P by F i 1 because you are using a future time and you are converting to the present time, so you will use this factor P by F i 1 + 10,000 multiplied by 5 interest periods are there, so 10,000 P by F i 5.

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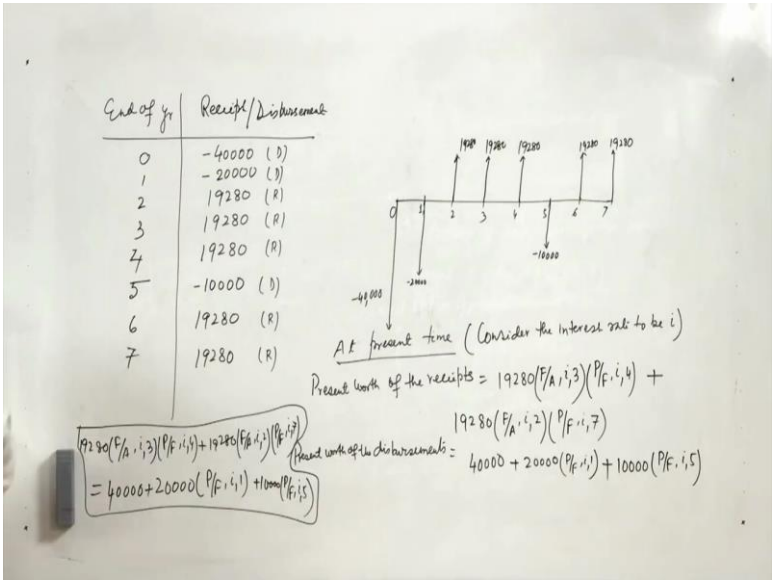

Now what this principle tells that, the actual interest rate and will be that value of i which will basically set these two equal. So what we get is, we have to equate these two, so once we equate them, you get 19,280 F by A i 3 into P by F i 4 + 19,280 F by A i 2 into P by F i 7 should be equal to 40,000 + 20,000 P by F i 1 + 10,000 P by F i 5.

So once you solve this equation, you have to solve this equation using trial and error method and once you solve it, you get i as 10%. So this is left upon you to do this exercise and try to find by trial and error and this gives you the results i equal to 10%. So basically the actual interest rate earned is 10% because it sets the equivalent value of all the receipts equal to equivalent value of all the disbursements.

In this case we have not taken the negative values because we have to anyway equate them. If you take the negative values, the summation of these two should be equal to 0. So we have taken and that is how we come to this interest rate.

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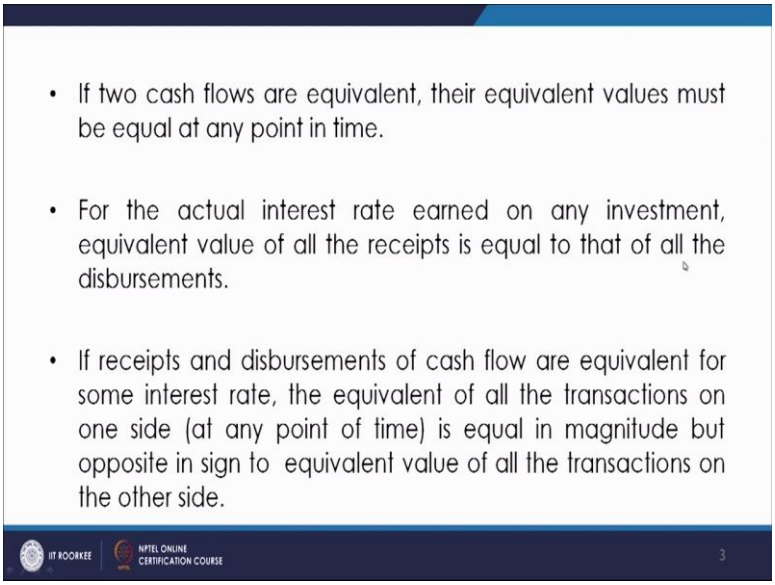

Now the third point is, that if receipts and disbursements of cash flow are equivalent for some interest rate, the equivalent of all the transactions on one side at any point of time is equal in magnitude but opposite in sign to the equivalent value of all the transactions on the other side.

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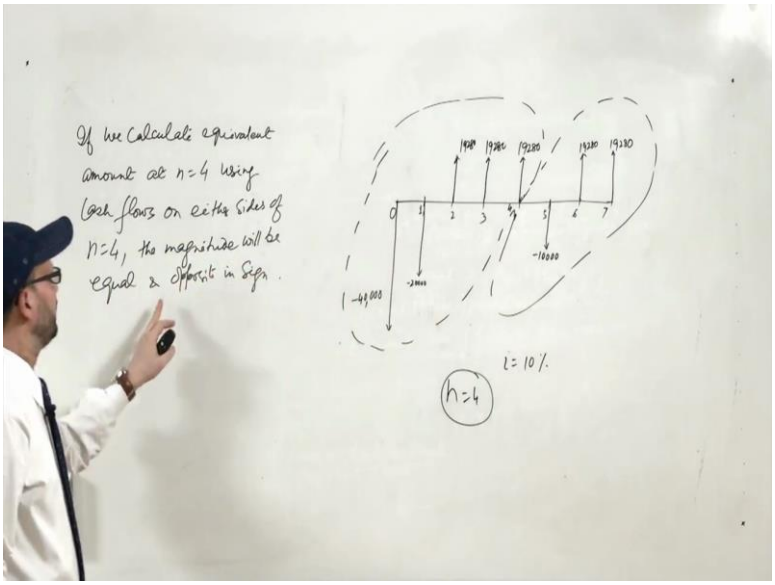

By looking at this diagram, since we have achieved the interest rate as 10%, this principle tells that if you fix any point, suppose you fix n equal to 4, in that case the equivalent value at n equal to 4 should be equivalent value at n equal to 4 using both the sections. So if we calculate equivalent amount at n equal to 4 using cash flows on either sides of n equal to 4, the magnitude will be equal and opposite in sign.

So basically if you try to find its worth at this point, it will be nothing but - 40,000 multiplied by F by P i 4 + again - 20,000 F by P i 3 + 19,280 into F by A i 3. The same amount you will get as 19,280 into F by A i 2into P by F i 3 + again - sign 10,000 P by F i 1. So basically this amount will be same and opposite in sign.

So this way if they have to be equal their values are equivalent, so this is how the equivalent values are calculated. Thank you.