Engineering Economic Analysis
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Lecture 10
Problem Solving on Frequency Compounding of Interest and Gradient Series Factors

Welcome to the lecture on problem solving based on compounding frequency and effective interest. In this lecture we will also solve problems based on gradient series. Let us see the first problem.

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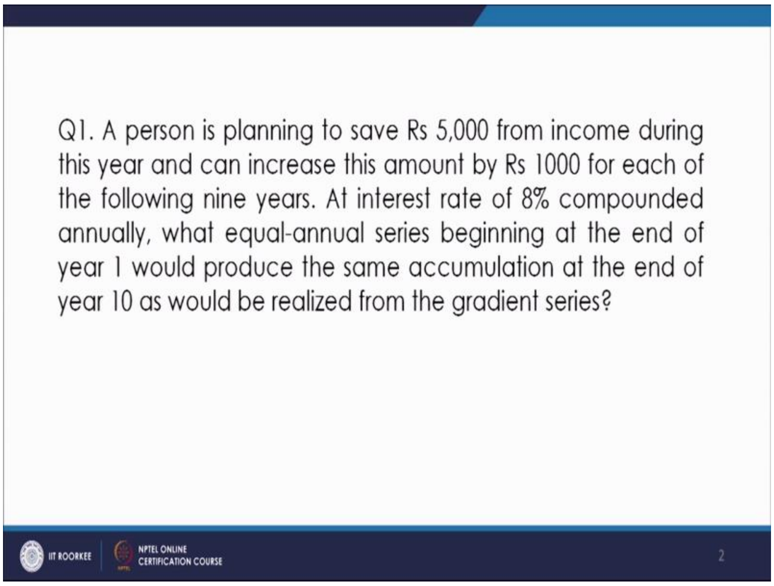

This problem is related to the gradient series. So a person is planning to save Rs. 5000 from his income during this year and can increase this amount by Rs. 1000 for each of the following 9 years. So basically the time span is 10 years, first year he is saving 5000 and from the next year onwards he is increasing this saving every year by Rs. 1000. So G is 1000, this is linear gradient amount.

Interest rate is 8% compounded annually, what equal annual series beginning at the end of your one would produce the same accumulation at the end of year 10 as would be realised from the gradient series. So basically this is a problem which talks about the cash flow series like this.

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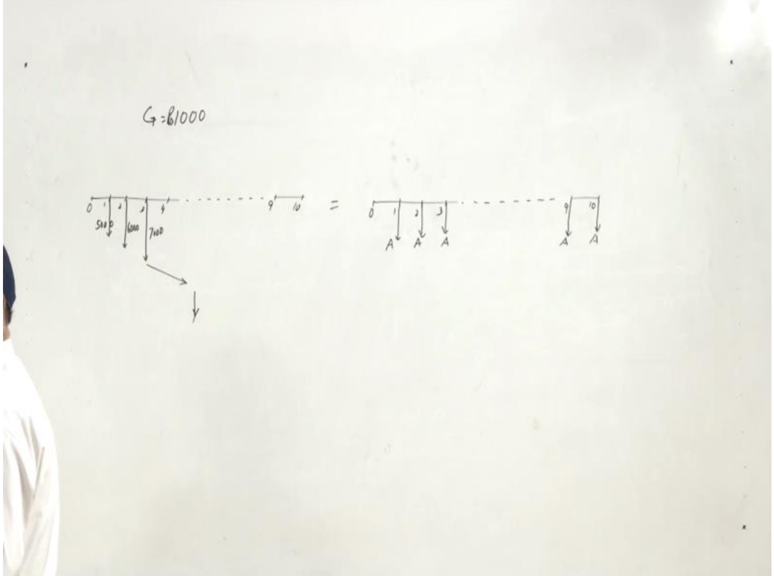

You have a 10 year time starting from 0, 1, 2, 3 and then it will go year 9 and year 10. Now the person is basically saving 5000 during this year, so in the first year he is saving 5000. Year end convention is followed. Second year he will save 1000 more so it will be 6000. Third year 1000 furthermore 7000 and this will go on like this. Accumulation can be realized by annually depositing certain amount, so this should be equal to a cash flow series which will go like this.

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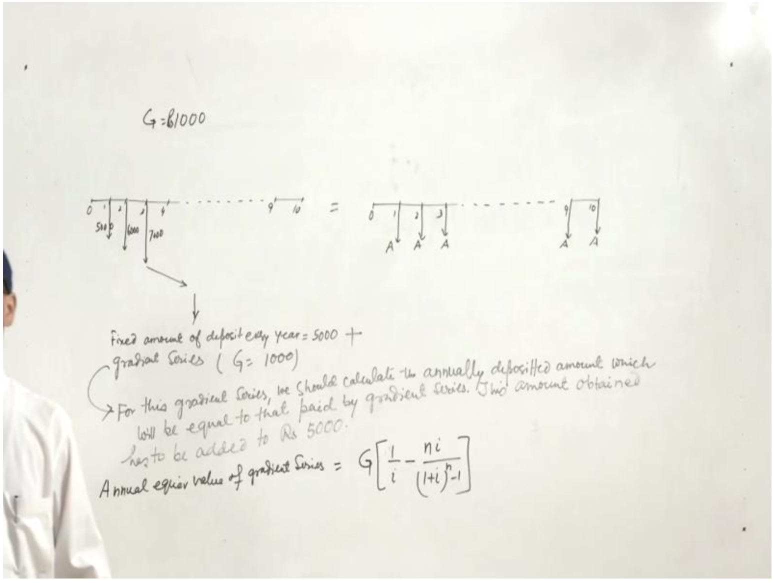

What should be basically this amount A? So in this problem what we will do is, we have to calculate now as we have already discussed, in this problem, you have two components, one is fixed component every year. So fixed amount of deposit every year is 5000 and gradient series G value of 1000.

Now first of all for this gradient series, for this gradient series, we should calculate the annual equivalent annually deposited amount which will be equal to that paid by gradient series. This amount obtained has to be added to Rs. 5000. This will give basically us the final answer.

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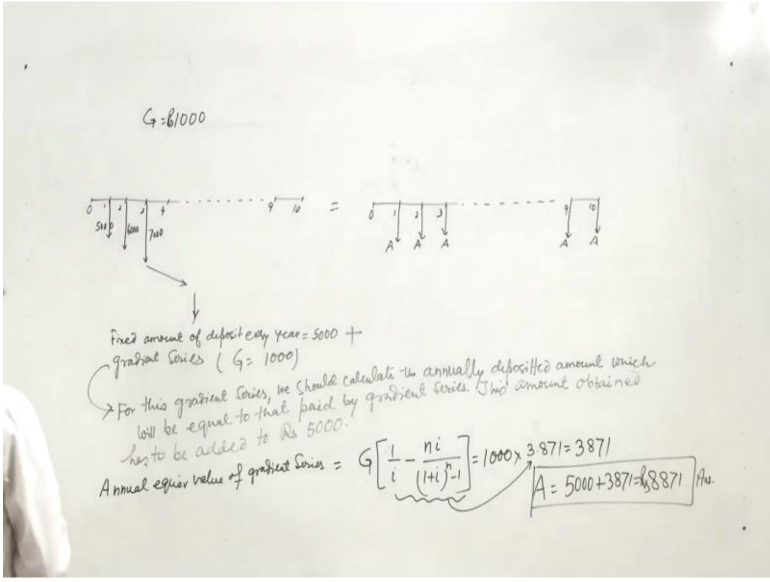

The annual equivalent value for this gradient series, gradient series we have the formula and from this we can calculate this annual equivalent value and that will be nothing but G times 1 by i - ni by 1 + 1 raised to the power n - 1. And if we calculate this value, this factor value, this comes out to be 3.8713 and G is 1000. So this value comes as 3.871. So it comes as 3871. So your final A, this A will be nothing but, the fixed component of 5000 + 3871.

So your A comes as 5000 + 3871 that is Rs. 8871. This is the answer. So what we see is, what amount you will deposit by depositing Rs. 5000 during the first year end and further increasing this amount every year by Rs. 1000. The same amount, can be depositing if you deposit every year end and amount of Rs. 8871.

So this type of problems can be solved by finding the component of the annual equivalent value using this formula first and either subtracting or adding to the fixed component. So this is the final answer. Let us go to the next problem.

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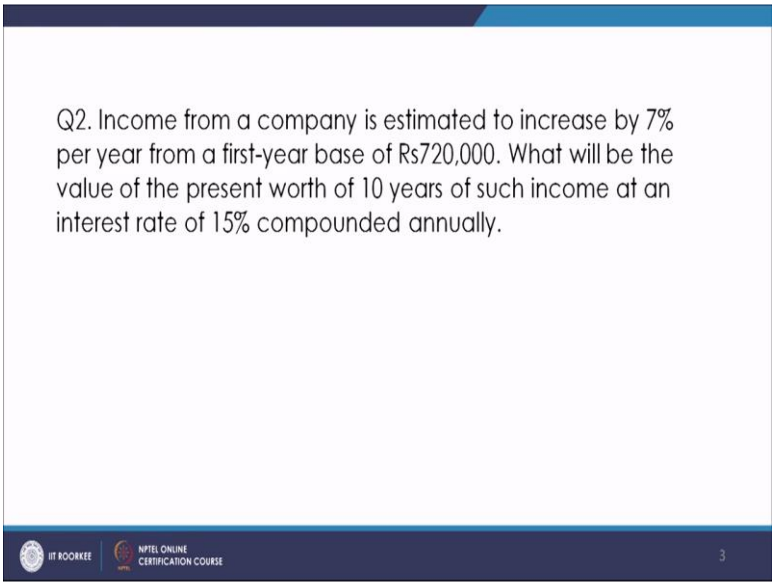

The next problem is related to the geometric gradient series factors, we have basically discussed two types of gradient series factors, one is the uniform gradient series another is geometric gradient series. In the geometric gradient series the gradient is not a constant amount but it is increasing as a percentage. What is there in this question? Income from a company is estimated to increase by 7% per year from a first year base of Rs. 720,000.

So that income is increasing every year 7% from this base, it means in this problem, it is given the G is 7% that is .07, this is the percentage increase with respect to the first deposit. F1 that is first deposit or first income it is given as Rs. 720,000. Then what will be the value of the present worth of 10 years of such income at interest rate of 15% compounded annually. So what you have been given is, annual interest rate is 15% so let us see the cash flow diagram.

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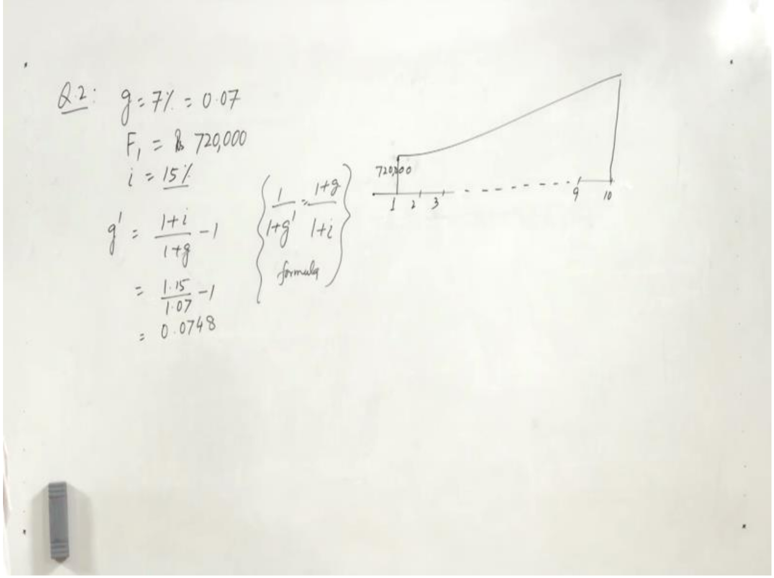

The cash flow diagram tells that in the first year A is it is 720,000 and it is going growing further upto 10 years. So basically it has to increase like this. Now we have to find basically the value of the present worth. So what is the value here which is whose value is same as what you would deposit in this type of cash flow. So we have been given I, F1 and G, So we will use the formula for gradient series factors to calculate the value of present worth.

And we know in this, first of all we will have to calculate G prime. So G prime is nothing but 1 + i upon 1 + G - 1. So what you see is, 1.15 upon 1.07 - 1 because i is 15% and G is 7%. So because we have seen the formula is 1 by 1 + G prime is 1 + G upon 1 + I, this is the formula. So G prime can be calculated as 1 + i upon 1 + G - 1 and this comes out to be .0748.

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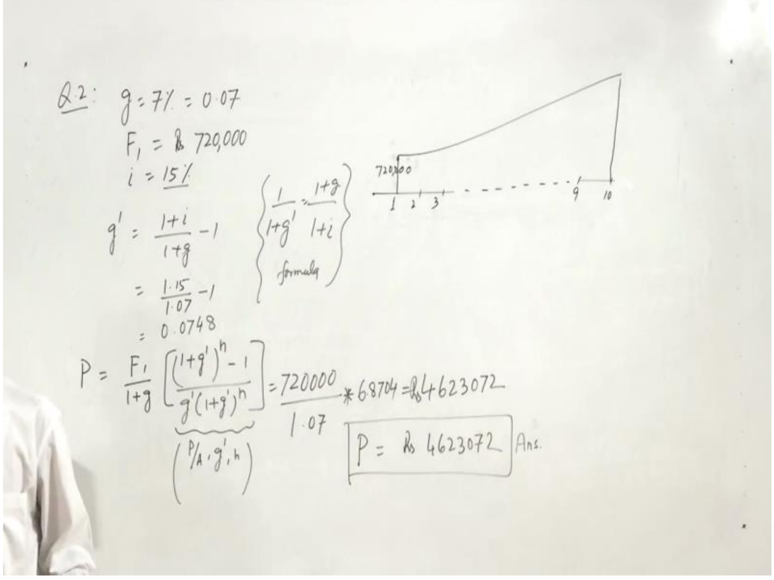

Now once we calculated the G prime, the present worth value is given by the formula F1 upon 1 + G into 1 + G prime to the power n - 1 by G prime into 1 + G prime to the power n n that is nothing but P by A G prime n. This is nothing but the factor P by A G prime n. This we have already derived while studying the geometric gradient series factor.

Now if in this, G prime we know that this .0748 and this factor comes out to be 6.8704. So what we see is this F1 is 720000 divided by1 + G , G is .07, so 1.07 multiplied by this factor comes out to be 6.8704. And that is why the value of this sum comes out to be 4623072. So this is the amount which is the present worth value of such a geometric gradient series factor. So P is 4623072, this is the required answer.

So in such cases what we have seen is, first of all, depending upon the value of i and G you have to calculate G prime. Once you get G prime you get the factorial value of P buy A G prime n. Once you calculate this, this will be multiplied with F1 upon 1 + G and that gives you basically the present worth of such geometric series.

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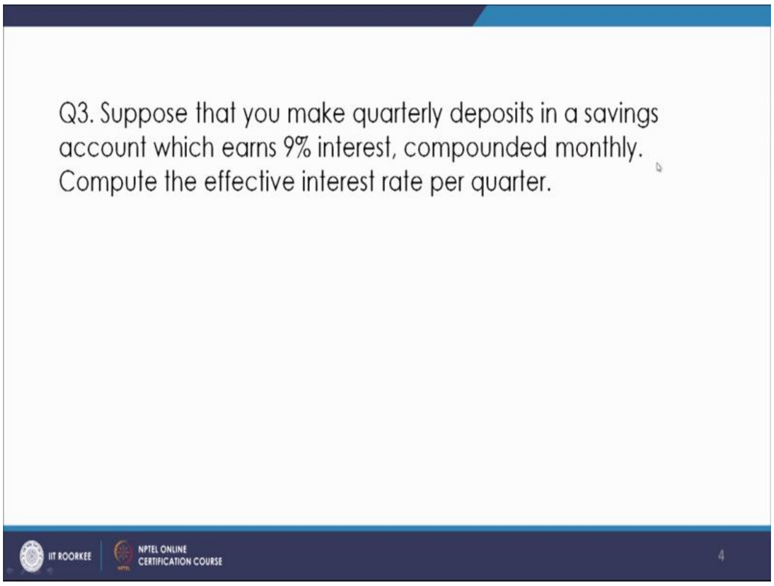

Next will be a problem based on the effective interest already we have solved few problems. Now in this case we are required to calculate the effective interest rate. So in this question suppose that you make quarterly deposits in a savings account which earns 9% interest compounded monthly. Compute the effective interest rate per quarter. Now how much will be the answer to this question.

So what you see is your r is 9%, compounded monthly and you have to find effective interest rate per quarter. It means l is in terms of years it is 1 by 4years, per quarter you have to calculate. So it means you have to calculate for three months, so l becomes 1 by 4 years. m it is compounded monthly, so compounding period in terms of years is and reciprocal to that is m. So m is reciprocal of this compounding period in years that is 1 by 12 that is 12.

So once you get that, you can use the formula of effective interest rate calculation, so effective interest rate will be 1 + R by m raised to the power l into m - 1 and that will be nothing but 1 + .09 divided by 12 raised to the power l into m that is 1 by 4 into 12 that is 3 - 1. So 1 + .0075 raised to the power 3 - 1 and that comes out to be .0227. So this is coming out to be 2.27%.

So what we see is, this is 9% interest per quarter had it been the quarterly compounding in that case per quarter the effective interest rate would have been 9 by 4 that is 2.25% but the compounding is done monthly. So basically due to that the effective interest rate for the 3 months becomes slightly more than 2.25% and what you get is 2.27%. So this how is you can calculate the effective interest rate for any time interval.

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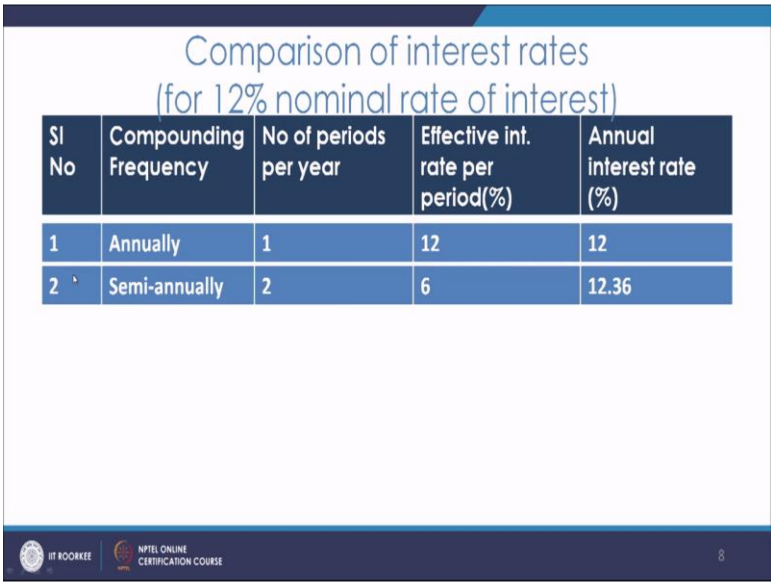

Now we will see the comparison of the interest rates when the compounding frequency is changed. So this problem talks about the annual interest rate, effective annual interest rate, how it changes when the number of periods per year or compounding frequency changes.

Now let us see when the compounding frequency annually that it is done only once, in that case, the effective interest rate per period because it is already a nominal interest rate of 12% so your effective interest rate per period is 12% and what the annual interest rate also is 12.

Now when we go for semi annual basis compounding, now in that case, as we do the semi annual compounding we are doing the compounding two times. The number of periods per year is 2 and basically the effective interest rate is 12 by 2 6% because 6% interest is levied upon every six month. So for a period of six month the effective interest rate is 6% but if we try to calculate the effective interest rate or for the year, in that case, it comes out to be 12.36.

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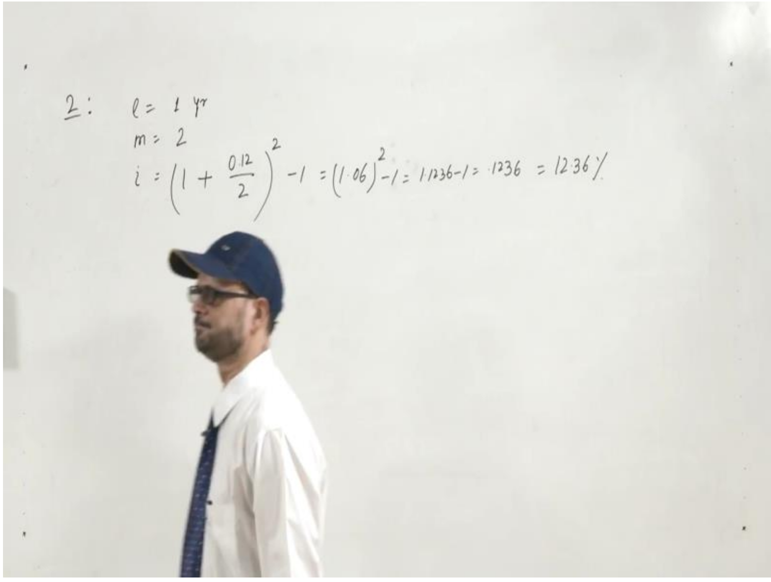

Now how it comes? So in this case as we are doing the semiannual basis, you have number of periods per year as 2 and l is basically 1. l is basically one year, m is compounding is done twice so period is 1 by 2 year and the reciprocal of 1 by 2 is 2. So effective interest rate comes out to be 1 + since R is given as 12% so .12 by 2 raised to the power l into m that is 2 - 1. So 1.06 raised to the power 2 - 1, it is 1.1236 - 1 so .1236.

So you can write it as 12.36%. So this is how we calculate when it is semi annually interest is levied upon or interest is calculated on semiannual basis, your effective interest rate for the year comes to 12.36%.

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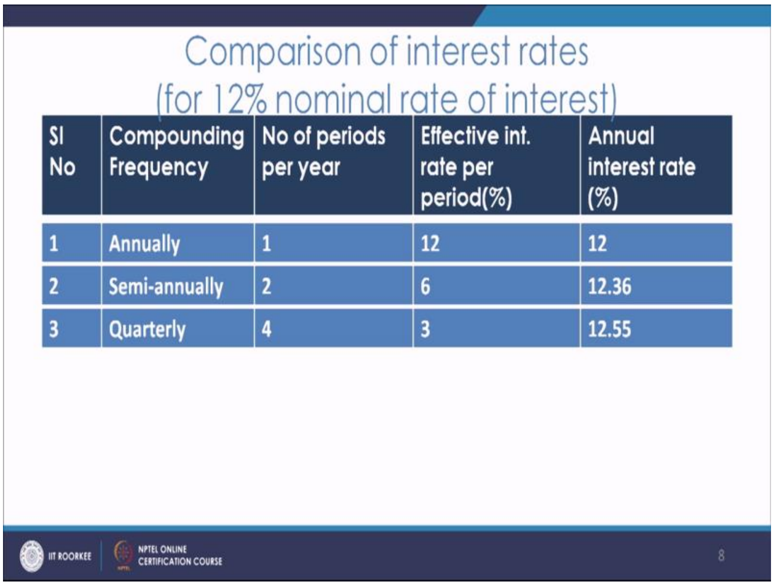

Sl No	Compounding Frequency	No of periods per year	Effective int. rate per period(%)	Annual interest rate (%)
1	Annually	1	12	12
2	Semi-annually	2	6	12.36
3	Quarterly	4	3	12.55

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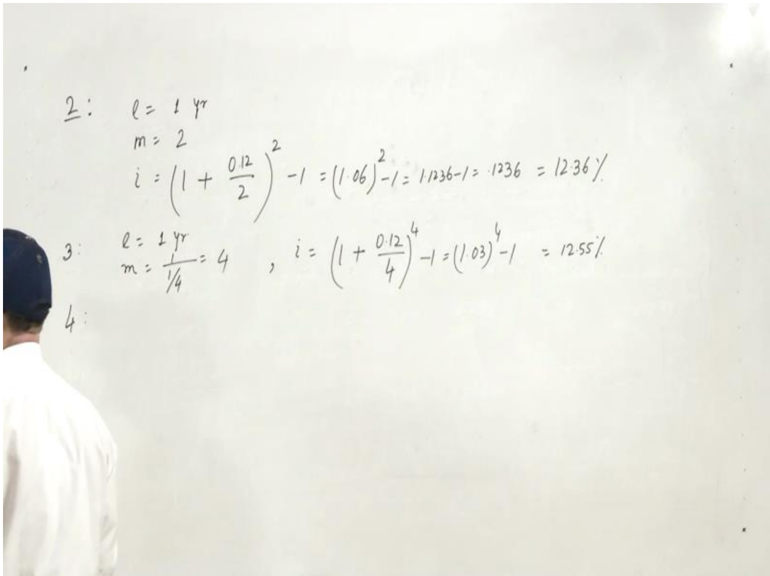

You go further, if you go for quarterly, in that case l anyway is 1 year. Once you go for quarterly compounding, in that case, m is reciprocal of 1 by 4 years that is for 3 months so 1 by 4 years that comes as 4. So in this case i will be calculated as 1 + .12 divided by M, m is 4 raised to the power l into m that is 1 into 4 is 4 - 1. So 1.03 raised to the power 4 - 1 and gives you the result of 12.55%.

So effective interest rate basically for the year changes from 12 to 12.36 when it is done semi annually and to 12.55% when it is done quarterly.

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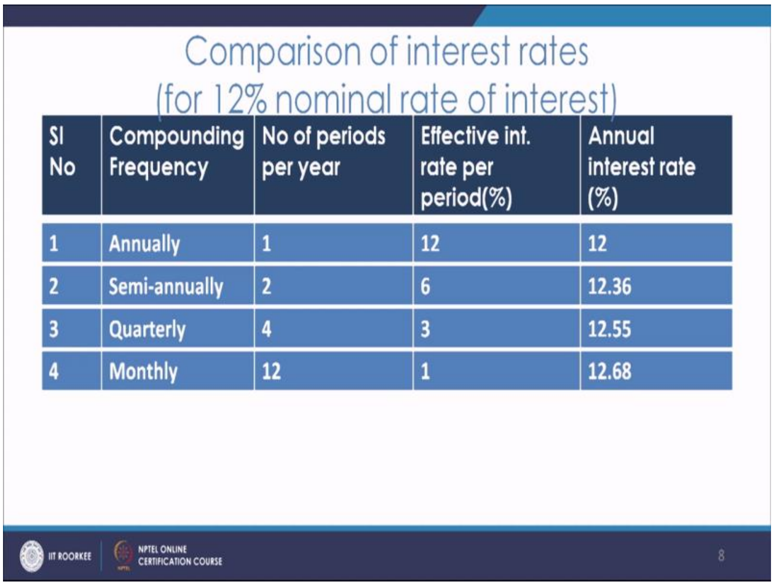

Sl No	Compounding Frequency	No of periods per year	Effective int. rate per period(%)	Annual interest rate (%)
1	Annually	1	12	12
2	Semi-annually	2	6	12.36
3	Quarterly	4	3	12.55
4	Monthly	12	1	12.68

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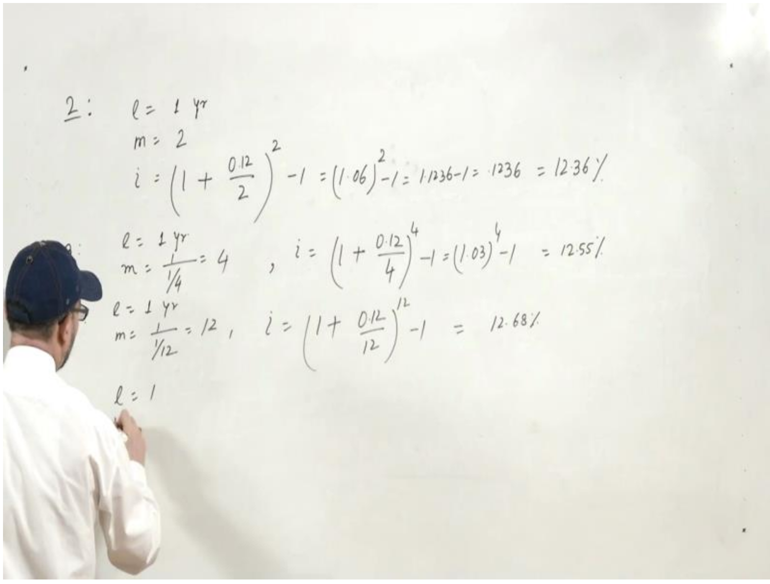

Further if you go for monthly, in the case of monthly calculation, l being 1 year, m now becomes the reciprocal of 1 by 12 years so it becomes 12. So effective interest rate for one year period becomes 1 + .12 by 12 raised to the power 12 - 1. And this if you calculate, it will come out as 12.68%.

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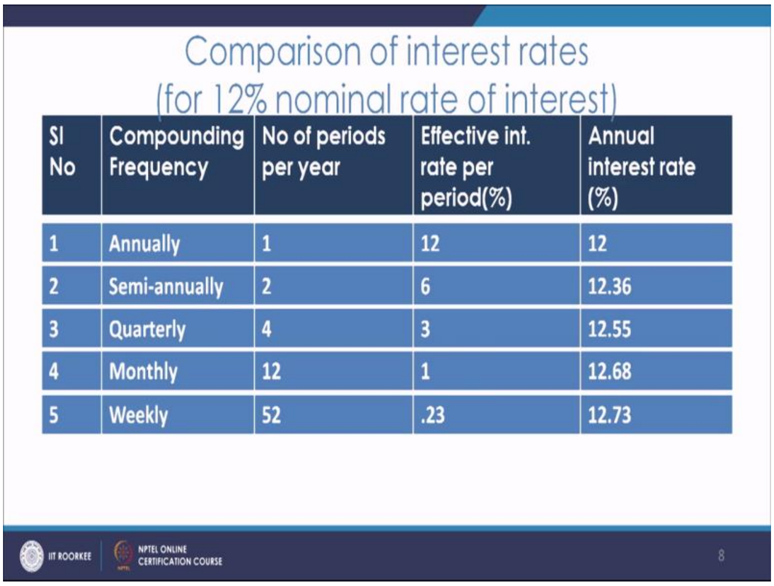

Sl No	Compounding Frequency	No of periods per year	Effective int. rate per period(%)	Annual interest rate (%)
1	Annually	1	12	12
2	Semi-annually	2	6	12.36
3	Quarterly	4	3	12.55
4	Monthly	12	1	12.68
5	Weekly	52	.23	12.73

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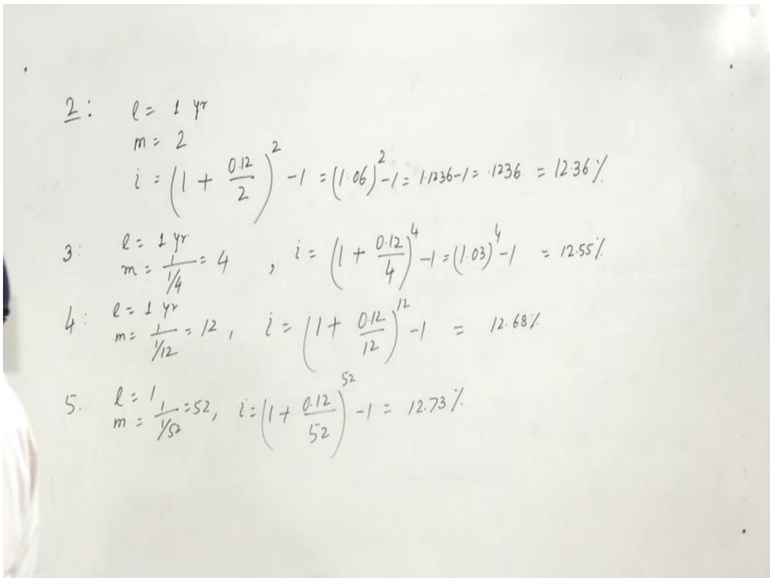

In this way if you go to calculate on weekly basis, on weekly basis, l being 1, m becomes 1 by 1 by 52 that is 52. So i will be 1 + .12 by 52 raised to the power 52 - 1 and this comes out to be 12.73%.

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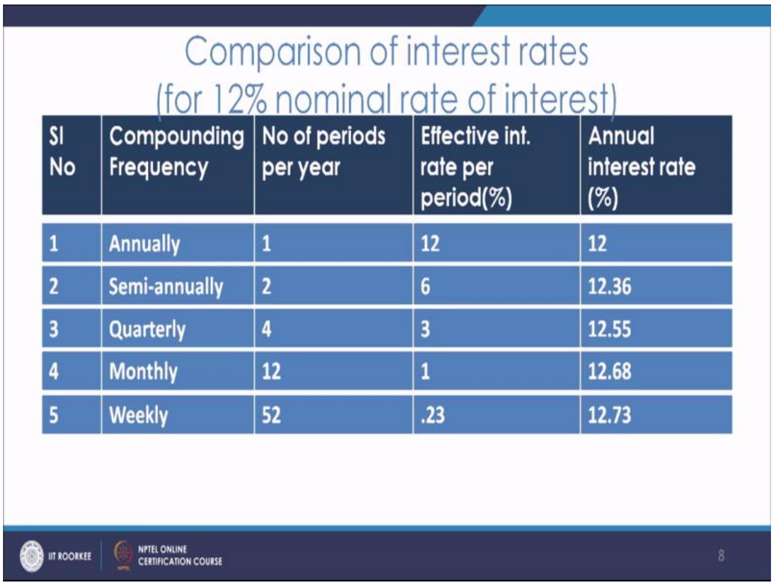

Sl No	Compounding Frequency	No of periods per year	Effective int. rate per period(%)	Annual interest rate (%)
1	Annually	1	12	12
2	Semi-annually	2	6	12.36
3	Quarterly	4	3	12.55
4	Monthly	12	1	12.68
5	Weekly	52	.23	12.73

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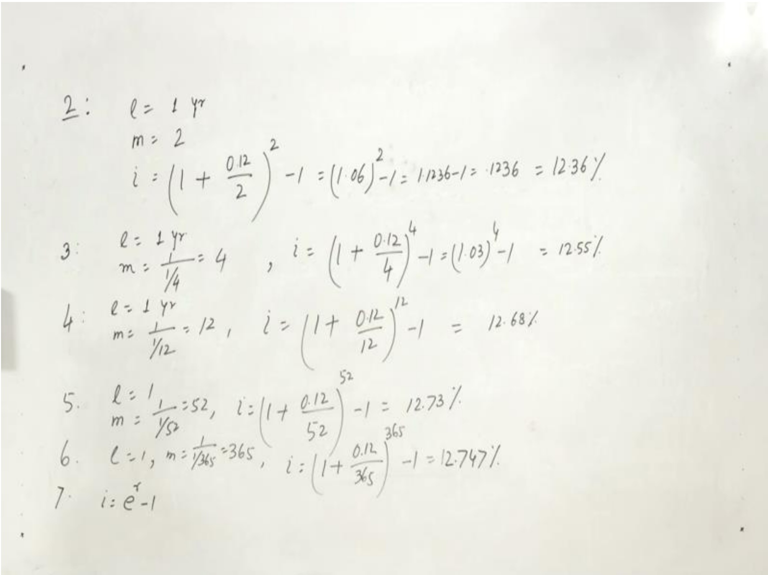

Similarly if you go for daily, in that case your l is 1, m is 1 by 1 by 365 years so it is 365 and in that case i becomes 1 + .12 by 365 raised to the power 365- 1 and this comes out to be two be 12.73%, 747% sorry.

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Comparison of interest rates (for 12% nominal rate of interest)

Sl No	Compounding Frequency	No of periods per year	Effective int. rate per period(%)	Annual interest rate (%)
1	Annually	1	12	12
2	Semi-annually	2	6	12.36
3	Quarterly	4	3	12.55
4	Monthly	12	1	12.68
5	Weekly	52	.23	12.73
6	Daily	365	.0328	12.747
7	Continuously	Infinity	0	12.749

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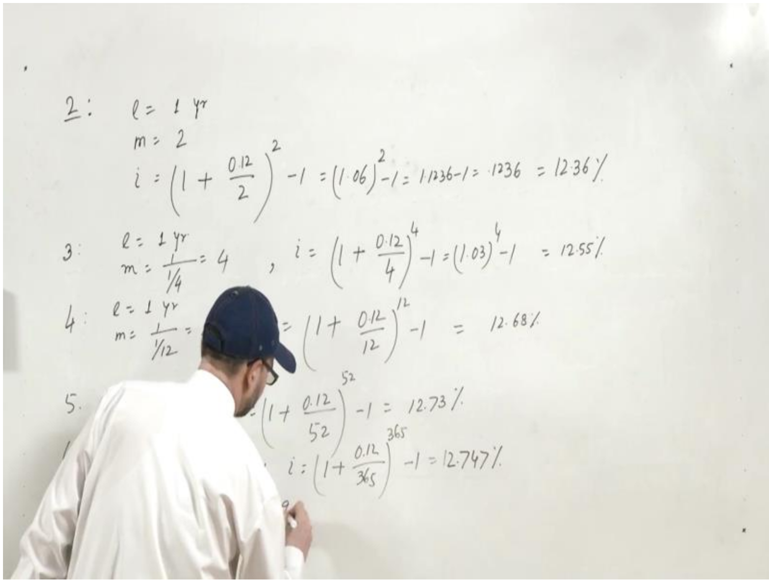

Now if you go for continuous basis, in that case we know that it is since we are doing on annual basis, annual effective interest rate we know that the formula is i equal to e raised to the power R - 1. So exponential of .12 - 1 and this comes out to be 12.749%.

So what we see this we have shown how you can calculate the different value of effective interest rate and annually the effective interest rate how it changes when the compounding frequency is changing. So this we can practice.

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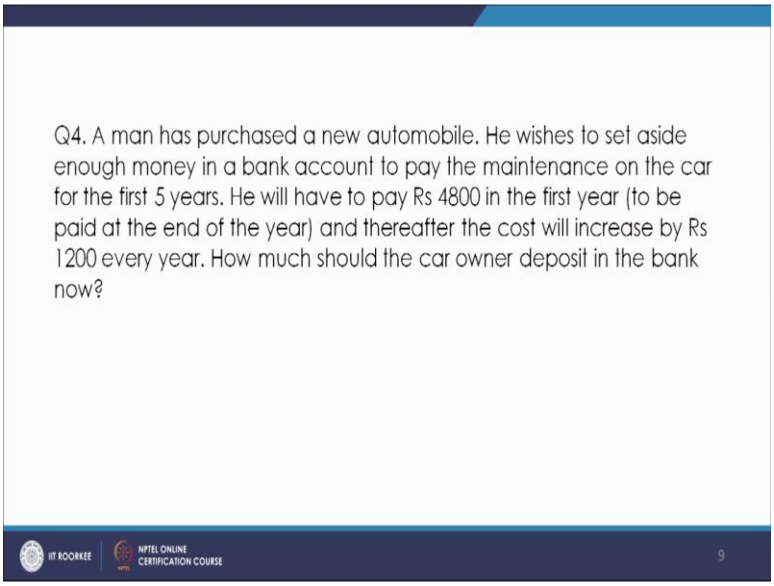

Next there is a question which talks about again the use of gradient series factor where basically a man is purchasing a new automobile and he has to put aside some money in the bank so that he can pay the maintenance fee of the car every year end for the first 5 years. He has to pay 4800 in the first year and thereafter the cost will increase by Rs. 1200 every year so how much should he deposit in the bank.

So basically this problem tends about the finding the present worth, finding the present amount of gradient series. We have already discussed how to calculate the value of the annual equivalent for the gradient series that can be used further to find the present amount for this gradient series. You can treat this problem as then assignment problem. The answer to this question you can have the help of some of the tables.

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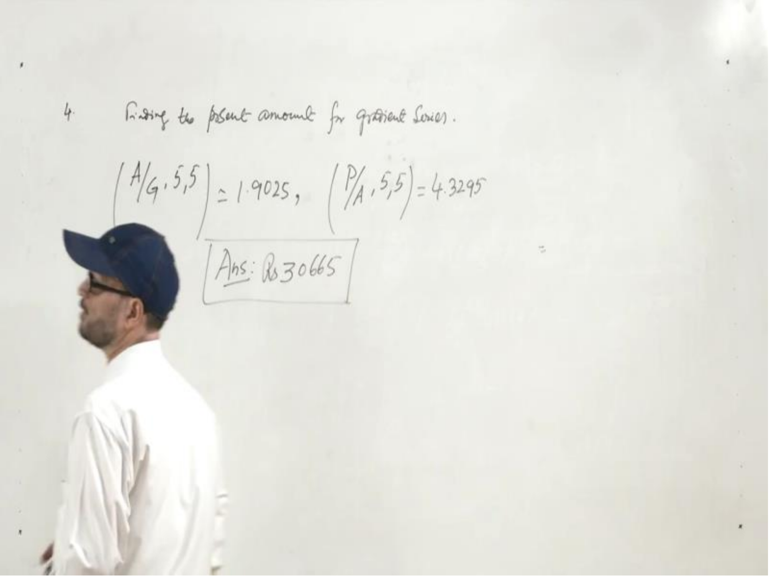

A by G 5 5 is given as 1.9025 and P by A 5 5 you can have it as 4.3295 and the answer is 30665. So treat this as the assignment problem and we can discuss it in the coming classes. Thanks.