Two Phase Flow and Heat Transfer

Dr. Arup Kumar Das
Department of Mechanical and Industrial Engineering
Indian Institute of Technology, Roorkee

Lecture No: 09
Droplet Annular Flow and Stratified Flow Model

Hello, welcome in the ninth lecture of Two Phase Flow and Heat Transfer. In this lecture you will be learning about droplet annular flow and we will be later on discussing about stratified flow models. So let us see the outline of the lecture. At the end of this lecture we will understand the following points.

We will be understanding the methodologies for generation of droplets from annular flow and how that transforms into droplet flow. We will be correcting Two Phase friction multiplier for calculation of friction factors whatever we have given in your last presentation.

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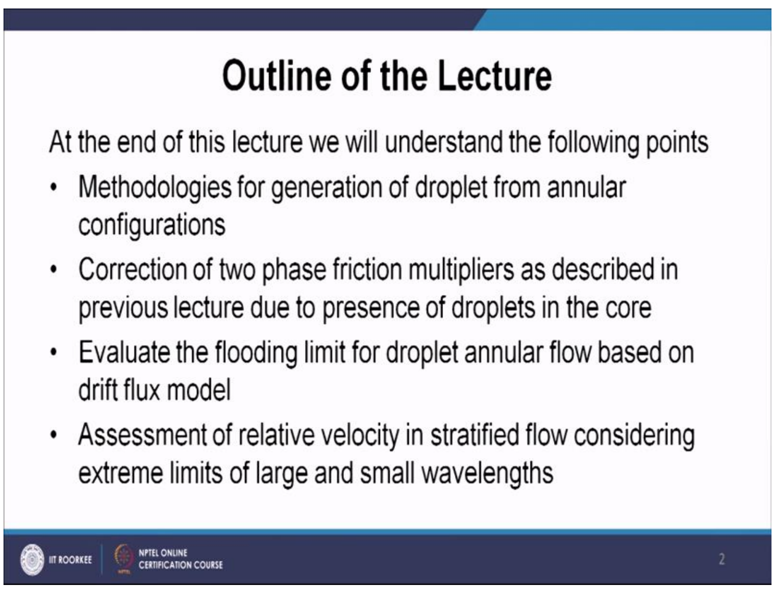

That we will be correcting based on the presence of droplet in the core. In case of droplet annular flow, you will be finding out that the annuli is shedding some droplet and which is being carried in the core right. So that will be actually changing the friction factor at the wall and accordingly the frictional force will be modified. So those issues we will be tackling in this lecture.

We will be also evaluating flooding limit for droplet annular flow based on drift flux model. So we know that there will be flooding by gaseous phase whenever there is very high velocity of the gas okay.

So in the extreme upside direction whenever droplets are flowing inside the pipe you will find out flooding can occur. So you will find out that what is the limit beyond which flooding can occur from the droplet flow. So for that we will be using drift flux model our earlier knowledge of drift flux model and at the end section of this lecture we will be discussing about stratified flow.

We will be showing that how wave structure in stratified flow can be clubbed in horizontal orientation and then we will be extracting the limits of large and small wavelengths. So both the extremes we will be see the large and small wavelengths okay. To begin with, first we will see how droplet annular flow actually generates. So we will start with annular flow what we have discussed in the last lecture.

We know in case of annular flow around the wall of the tube you will be having a liquid film. Now here I have shown one side of the tube wall with which the liquid film is added okay. So I have explained the gaseous flow in the core as ug over here.

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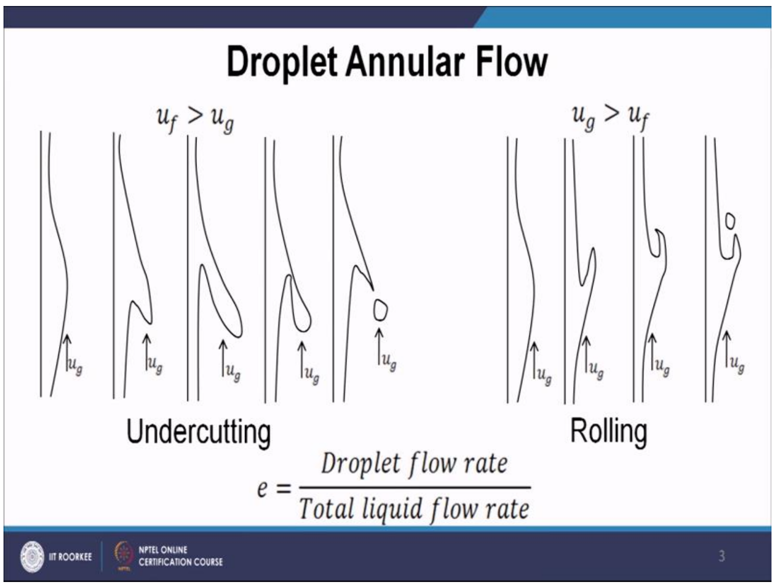

Now if you see that in magnitude the liquid velocity uf is higher than ug then you will be finding out that due to the liquid, higher liquid velocity there will be bulge formation in the film which is added with the wall. And it continues further, you will be finding out that this film is forming a bag kind of thing over here for the air okay. So air can accumulate over here inside the extrusion of the liquid which will be acting as a bag right.

This continues to grow further and you can find out that there is lengthy protrusion of the liquid film which further can pinch off at some point over here and form a droplet. Now here we are having continuous flow of gas which will be carrying forward those droplets which are being formed along with it in the core okay. So in this fashion, inside the tube we will be finding out lots of droplets are being created of different size depending on the flow velocities and we will find out in the core we are having lots of droplets right.

So that is characterized by droplet annular flow. Next another mechanism I will be showing you for which droplet is also created. Here you see we have started from similar configuration; we are having liquid film added with wall. So this vertical line is actually the tube wall let us consider one half of the tube wall we have shown over here. So here we are having the core gas flow characterized by its velocity ug.

Now if we consider the magnitudes of the flow velocities are such that ug is greater than uf then you will be finding out due to this flow velocities okay. A lamella, fluid lamella is actually being sheared off by the fast moving gaseous stream. So you can find out this lamella can pickup in length and we will be finding out that at some point of time whenever lamella length is becoming very high.

There will be chances that surface tension will increase in magnitude and you will find out a small droplet can be shed off from that lamella okay. Now this fast moving gas over here, we will be actually picking up this droplet and take it in the core okay. So these 2 phenomena are actually important for generation of droplet from the annular flow.

The first one we name as undercutting okay and the second one is called is rolling okay. These two phenomena are actually very, very important for generation of droplet annular flow. Now to characterize that what amount of droplet you are having in the core we define a parameter called e . So this e is actually the ratio of droplet flow rate in the core. So whatever volume of droplets you are having in the core or mass of droplet you are having in the core because density is not going to change.

So that we find out over here droplet flow rate divided by total liquid flow rate. So total liquid flow rate will be actually your droplet flow rate plus whatever film we are having over here with the wall added with. So that flow rate also will be coming into picture. So this total liquid flow rate will be film flow rate + the droplet flow rate. So the ratio between these two is actually called e. So e can be 0 whenever you are having perfect annular flow okay. That means there is no droplet in the core right.

And in case of your droplet annular flow, you will be finding out that e is lying in between 0 and1. Usually typically we will find out that this e value will be definitely less than 0.15 okay. Next let us see that how this ratio of the volume, droplet volume and the total liquid volume can be found out okay. So there are number of correlations available for this but best suited correlation has been given by Ishii and Mishima in 1982. What they said that this e can be characterized as tanh parabolic [7.25 * 10 to the power -7 jg 2+.

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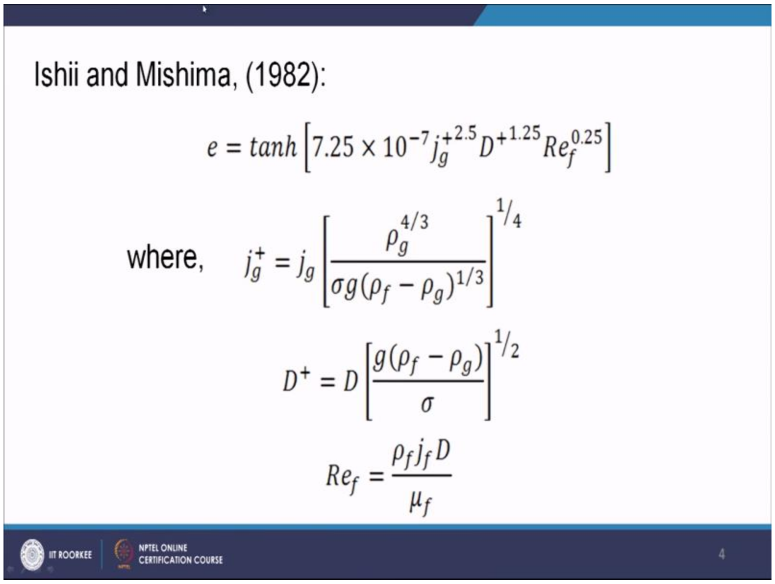

So this not jg this is modified jg actually. I will be telling you next. So, jg + to the power 2.5D+ to the power 1.25. So this is also modified diameter or non-dimensionalized diameter we can say multiplied by Re f, so this is liquid Reynolds number, modified version of liquid Reynolds number, no this liquid Reynolds number actually. So Ref to the power 0.25 okay. So let me quickly explained that what are these terms jg, D and Ref. So jg +, you can write down this is actually non-dimensionalized version of jg.

So it is jg multiplied by [Rho g to the power 4 /3/ sigma * g (Rho f - Rho g) to the power 1/3] whole to the power 1 / 4. So this is actually giving you the unit of meter per seconds and this is also meter per seconds. So you will be finding they are canceling out and this will be non-dimensional number okay. In a similar fashion D+ can be written as D [g (Rho f - Rho g) / sigma. So this terminology this numerator and denominator they are coming from the balance of buoyancy force and your surface tension force.

So we are non-dimensionalizing the diameter of a tube by this non-dimensionalized length square ½, to the power half of that will be coming into picture for non-dimensionalizing the tube diameter. So D plus = D* this one okay. And all of we know that film Reynolds number will be nothing but Rho f *jf *D / Mu f okay. So all the fluid properties will be taking into picture. So once you know, you see in these expressions all the values mostly are from fluid properties.

And only D is actually from your system whatever pipe diameter you are having and jg will be dependent on your ug, so you can find out once you know the flow parameters and the velocities you can find out what is the value of e okay. The ratio between the droplet mass and the total liquid mass you can find out in case of your droplet annular flow. So that is why this correlation is very, very important okay.

Next in the last lecture we have talked about that how to calculate the friction factor for annular flow and there we have seen that Two Phase multiplier phi will be very, very important, Two Phase multiplier phi g square will be very, very important. So here I will be showing you whenever we are having droplets in the core of the annular flow how the values of phi g square will be changing okay.

And once you calculate the phi g properly, so your calculation of the frictional force will not be difficult and the approach for finding out the friction factor using phi g square already I have shown you in case of your homogenous or separated flow models okay. So let us try to see how this phi g square is changing with the respect to the presence of droplets.

So here we are considering that we are having entrainment by the way this liquid whatever it is coming in the gaseous phase liquid droplets those are called entrained droplets so let us see that what is the effect of liquid entrainment okay, on the pressure drop that means on the frictional pressure drop. So first, we will be trying to explain what is the interfacial shear stress Taui. So Taui, we have earlier shown this will be 1/2f * Rho g (ug –ui) whole square.
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Effect of liquid entrainment on pressure drop:

$$\tau_i = \frac{1}{2} f_i \rho_{co} (u_g - u_i)^2 \qquad \text{Where, } \rho_{co} = \left[\frac{w_g + e w_f}{w_g}\right] \rho_g$$

$$\text{Assume, } u_i = 2u_f \qquad \text{Then, } \tau_i = \frac{1}{2} f_i \rho_{co} (u_g - 2u_f)^2$$

$$(u_g - 2u_f)^2 = u_g^2 \left[1 - 2\left(\frac{u_f}{u_g}\right)\right]^2$$

$$(u_g - 2u_f)^2 = u_g^2 \left[1 - 2\left(\frac{1-x}{x}\right)\left(\frac{\rho_g}{\rho_f}\right)\left(\frac{\alpha}{1-\alpha}\right)\right]^2$$

$$(u_g - 2u_f)^2 = u_g^2 \left[1 - 2\frac{w_f(1-e)}{w_g}\left(\frac{\rho_g}{\rho_f}\right)\frac{\alpha}{1-\alpha}\right]^2$$

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Earlier all those i's were replaced by ug's. So this was the g , this was Rho g and this was uf okay. So now here in the core what we will be finding out it is not only the gaseous phase we are having liquid droplets also. So the core density if you try to find out this will be not essentially the gaseous phase density, this will be modified by the liquid entrainment okay.

So that we will need to take care of. So here what we have written that what will be the core density now Rho co, so Rho co in ideal case, annular flow case that will be definitely equivalent to Rho g but as we are having droplet entrained in that. So what we are doing over here we are increasing the density by some factor which is nothing but wg + e wf / wg.

Now it is very good factor to add why because if e = 0 then you will be finding out that Rho co is becoming Rho g that means if there is no entrainment then you will be finding out it is becoming Rho g okay. Now once we have found out that this Rho co, next task to assess what is the value of interfacial velocity that means wi okay. Now for this derivation I am considering that ui is known to us and that is equivalent to 2 *uf. uf is essentially the liquid velocity so this is the assumption.

So if you are having some another value of ui accordingly you need to change. I have taken this one as 2* uf just to simplify our calculations okay. Now you see if you add all these things in the expression of Taui, the Taui will be changing to ½* fi Rho co (ug - 2 uf) whole square. Though I have not placed this Rho co expression from here to this one but one can do that one. So now I am interested in this (ug - 2 uf) whole square part. So I will be explaining that one separately.

So please remember that whatever I will be deriving now from now onwards this 1/2fi Rho co will be coming as multiplier to that. So here you Rho g -2 uf ug - 2 uf whole square what we can do we can take common ug square. So it will be 1 - 2 [uf /ug] whole square. Now all of we know and we have also shown you in the first lecture that uf/ug can be written as (1- x) / x* Rho g / Rho f *alpha / (1 –alpha) okay. In the first lecture, introduction lecture I have shown the proof of this one.

So we can replace uf /ug by this expression. Now next as we are having over here 1 - mass quality by mass quality. So what we can do depending on whatever masses we have in the core. And the wall we can replace this mass quality/ the liquid mass flow rate and the gaseous mass flow rate. So we will be finding out over here liquid mass flow rate added with the wall will be wf (1 – e).

So because wf*e portion is always with the core. So that is why we are writing wf (1- e) okay and obviously wg will be remaining same. So here we find out that (ug -2uf) whole square becomes this full expression. So what we can do finding out Taui, we can hook this expression with the multiplier ½ fi Rho co okay. So let us see that.

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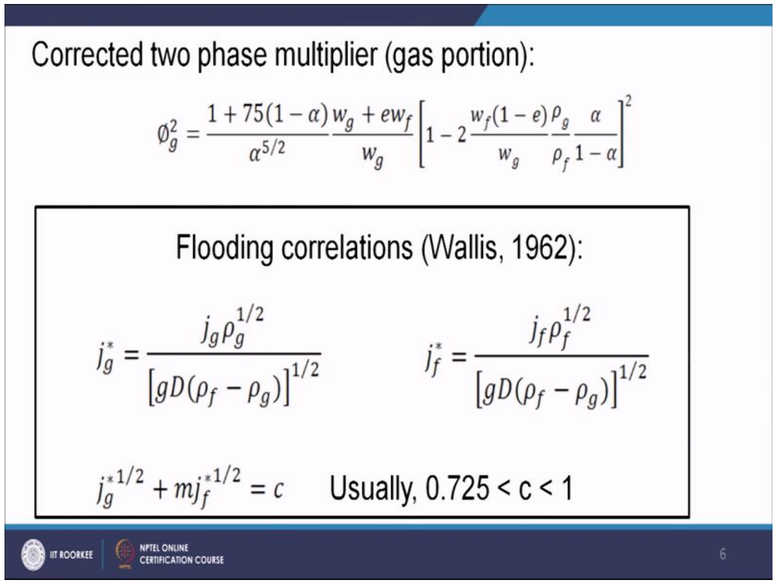

So if you proceed in this fashion. Already in the last lecture if you remember, I have shown you based on Wallis correlation that phi g square can be written as 1+ 75(1 – alpha) /alpha to the power 5/2 okay. Where you know fi is not equals to fTP, Wallis has given some correlation and based on that we have explained this one.

So what will be finding out that with this gas entrainment in the core, you will be finding out that along with this term there will be one multiplier. That multiplier is nothing but essentially this multiplier whatever you have, so you will be finding out over here [1 - 2 wf (1 –e) /wg (Rho g /Rho f)(alpha /1 –alpha). Whatever I have shown you in the previous slide along with this, what is this; this is nothing but your Rho co value.

You see Rho co; I have already shown you over here. Rho co is wg + ewf / Rho g. So that multiplier will be coming also over here because Rho is changing Rho g is changing to Rho co and you are ug square which was earlier in the Wallis criteria. That is changing to (ug -2) whole square that means this whole square expression. So these terminologies will be coming as multiplier over here with the phig square.

So in this way of phig square can be modified based on the knowledge of e okay. e, once again how to calculate I have already shown you. So based on the knowledge of e, you can modify this phig square value and calculate the pressure drop accordingly okay. Next let us move to the flooding criteria. So as we know that we are having, we are moving towards the droplet flow.

So what we need to do, we need to find out that what is the flooding condition or limit given for the tube. So what we will be following over here. We will be following one correlation given by Wallis once again. So what he has mentioned, he has mention jg* to the power of 1/2 + mj * jf* to the power of ½ = c.

Once again if you remember this is actually coming from our drift flux model, where we have shown that some multiplier into jg + another multiplier jf equals to constant which is a straight line manner and we have shown some examples also of pipe flow, adiabatic pipe flow where we have found out the operating points using jg and jf values.

So similar type of equation Wallis has also proposed over here but what he has done over here in place of jg and jf. He has given some non-dimensionliazed jg and jf to make generalized. So let us see jg*, jg* is actually involving jg and then he has actually multiplied Rho g to the power ½ / gd multiplier * Rho f – Rho g to the power ½. So as he has included this diameter and the density, fluid density.

So it will be valid for any fluid pairs and as well as your pipe diameters okay. Similarly jf also defines, so jf* is nothing but jf * Rho f to the power ½ gD (Rho f – Rho g) to the power 1/2 okay. So this is a very important criteria. So using this equation you can find out for given value of jf what will be the value of jg. So that you can avoid flooding okay. Now very important thing over here in this correlation, to find out the value of m and c. So Wallis has suggested that c will be lying in between .725 to 1.

So this will be depending on the pipe material whatever material you are having as well as it also depends on the pipe diameter. So you have to pick up. So he has given a chart actually for picking up the values of c in his research paper. So one can pick up the correct value of c by knowing the condition of your experimental setup and evaluate the value of c. Now the rest part whatever is remaining actually m. So let us see how m can be found out.

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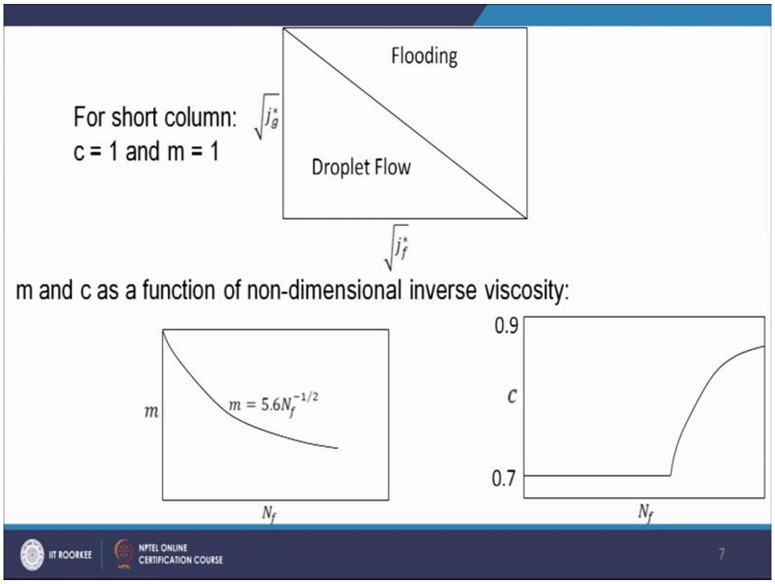

So what Wallis has mentioned. He has actually given 2 curves like this okay, for finding out the value of m and c once again. Though what he has related he said this both m and c they are actually dependant on your non-dimensionalized inverse viscosity.What we have defined in case of your dispersed flow model okay.

So this, once you know the fluid properties, definitely will be finding out the non-dimensionalized inverse viscosity and once you know the viscosity so using these two curves you can find out the value of m as well as you can find out the value of c. It is interesting to note that upto certain limit of non-dimension inverse viscosity, the value of c remains constant okay. After that it takes some variation.

So you have to take the values of m and c accordingly right. Wallis is also propose that if you are having a short pipeline that means the droplet annular flow is happening in a short tube. So in that case you can go for simpler value because finding out m and c in this manner is little bit tedious. So you can go for very fast and calculation, short calculation by taking m and c = 1.

So that means essentially it becomes jg* to the power ½ + jf* to the power 1/2 = 1 which is an equation of straight line having 45 degree inclination. So if you see, I have shown here that curve that this is nothing but your jg* to the power 1/2 and this is your jf* to the power ½. So if you plot this is the 45 degree line okay. So below this you will be finding out that droplet flow can exist over there okay.

And if you go beyond this one that means keeping a liquid velocity, same if you go for higher gas velocity willing leading towards flooding. So these types of curves are very, very helpful for finding out the flooding limits okay. Next let us move to our next part where we will be mentioning about stratified flow. So stratified flow all of that we know part of her horizontal channel and you know that in case of horizontal channel you will be having wave stratification between the liquid phase and the gaseous phase.

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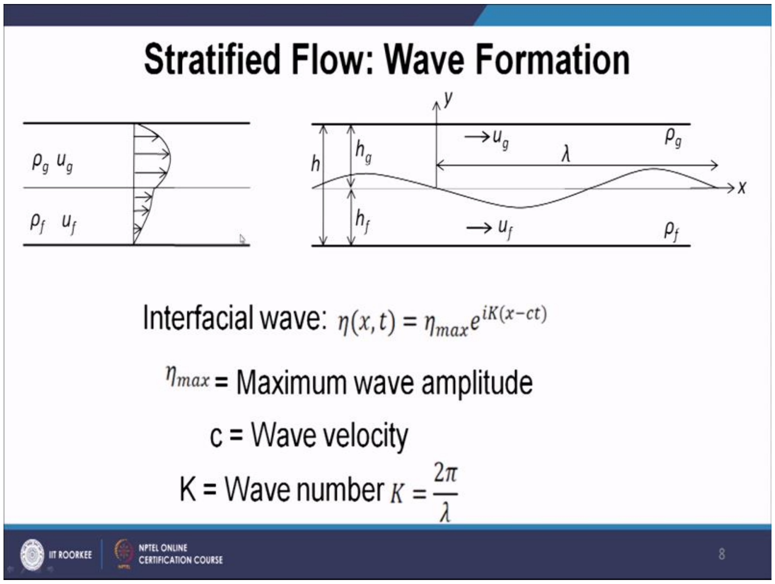

Here I have shown a figure where he can find out liquid phases flowing at the bottom and gaseous phases flowing at the top right. So to show the velocity vectors over here we have shown that incase of gaseous phase. Where will be higher moving velocity compared to liquid part over here? So that means in case of Two Phase your parabolic velocity profile exactly will not be valid. So you can find out the maximum of the parabola has shifted towards upward side okay but there will be no discontinuity of velocity at the interfacial point okay.

So if you consider that one in some cases, people consider the slip but here we are not considering that slip. Anyhow so here Rho g and rg are the corresponding density and velocity for the gaseous phase and Rho f and uf are the corresponding velocities for the velocities and densities for the liquid phase. Now what we will be doing over here. In this stratified flow we will be imposing the waves okay. So all of us know that let us say this is an undulated stratified phase interface okay somehow lying over here okay.

At a height of hg for the gas and hf for the liquid whereas total height for the channel height is actually h. Now if you impose the wave over here okay let us take the waves equation is etta xt is = etta max* e to the power ik (x-ct) where we know that in this type of waves k is actually the wave number presented by 2 pi / lambda. Lambda is the wavelength okay. c is the wave velocity, etta max can be the maximum wave amplitude okay. If we impose this wave, so this will be looking like this okay.

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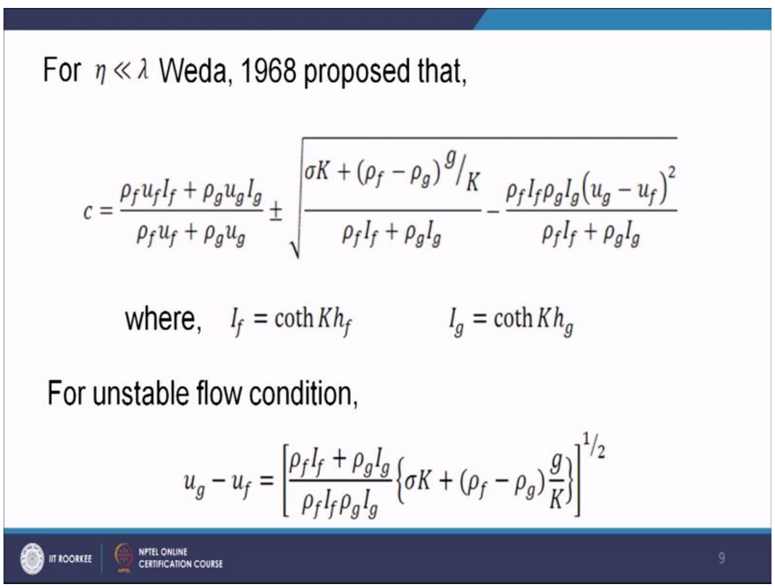

So let us try to assess what happens if we impose this wave on this non-undulated structure of the stratified flow. So first if we want to calculate the wave speed, we will be taking one important work done by Weda. He has given in 1968 that his wave velocity can be approximated as Rho f uf. If Rho g ug ig / rho f uf + Rho g ug + minus root over of you are having sigma k which is the wave number + Rho f – Rho g *g /k once again / Rho f if +Rho g ig – Rho f if Rho g ig *ug- uf whole square / Rho f if + Rho g ig.

So this is actually you know your equation of a quadratic in nature. So from the roots he has found out. Anyhow so will not be going detail of that one but using this one will be trying to prove the extreme limits. So here by the way have not mention the if is nothing but your coth high parabolic wave k into hf where hf, is height but you coth high parabolic wave k into hf where hf, is the height. I have shown in the previous figure and ig = coth high parabolic*khg. Now if you first try to find out what is unstable flow condition?

Unstable flow condition means as the wave, we are imposing on the stratified flow. So if the wave is very unstable, so then there will be chaotic situation inside the pipeline. So to check that we have found out what is the unstable flow condition so for unstable flow condition, definitely the wave velocity will become imaginary. So for making it imaginary we know if we are having something under root negative than that will be imaginary. So the limit will be finding out this one is equals to 0. So if we make this one is equals to 0 from there, we can get what is the relative velocity (ug –uf).

So that (ug – uf) by making this one is equals to 0 will be coming as Rho f, if +Rho g ig * sigma k + Rho f –Rho g*g/ k to the power1/2. So this is very, very important equation okay. Next let us see the extreme limits. The first one what will be showing you hf /lamda or hg /lamda that is higher than 0.25 that means the wave is very steep okay. So the amplitude is small but the wave is very steep okay. So the height is actually very big compared to this lambda okay. So let us see now what is the consequence.

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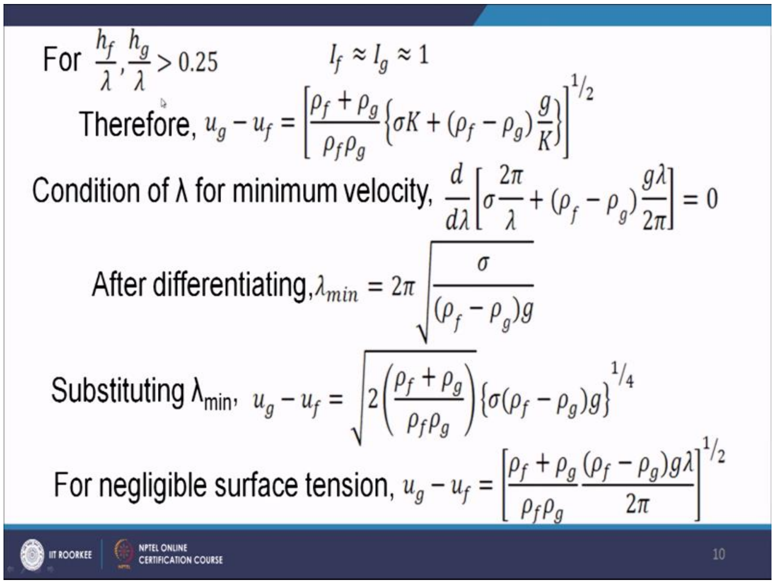

So if you see already I have shown you that if and ig is got high parabolic of khf and khg. So if hf and hg they are very big values then you will be finding out that this if and ig will become nearly equals to 1 right. If I put this values of if and ig once again back to the limiting (ug – uf) that you will be finding out (ug – uf) becomes Rho f+ Rho g / Rho f Rho g * sigma * k + Rho f – Rho g *g / k to the power ½. It is interesting to note that in this equation; still we are having the wave number value k okay.

So let us try to find out that for what amplitude this relative velocity becomes your minimum okay. So let us try to differentiate this whole equation with respect to your lambda. So if you do so then will be finding out d/ d lambda of this part is equals to 0 for the minimum velocity because these are constant terms. So this will not be coming into differentiation. So eventually this differentiation of this will be giving as the minimum conditions okay.

Once we do the differentiation, we will be getting that minimum lambda comes out to be 2 pi root over of sigma / Rho f – Rho g *g okay. So this the minimum lambda for which we will be getting the stable solution okay at minimum velocity okay. Now if you put this value of lambda means once again back to (ug -uf) then you will be getting expression like this. That (ug –uf) is limiting equals to root over of 2* Rho f + Rho g / Rho f * Rho g * sigma Rho f – Rho g * g to the power ¼.

This is very easy to calculate just you put the value of lambda mean over there in place of 2 pi /lambda and here. 2 pi by lambda; you will be getting this equation okay. Now here if you keep negligible surface tension okay. So surface tension is very negligible then you will be finding out this (ug – uf) will be turning out this one okay. So here you will be finding out that in this. We are not having surface tension, so let us take this equation.

So over here if you nullify this surface tension values let us say it is not surface tension, it is not there really this is not for the minimum velocity this is only for the no surface tension case. Then you will be finding out (ug-uf) is equals to this factor Rho f + Rhog / Rho f Rho g multiplied this one okay. So that same thing we have found over here okay. Let us see the other extreme case. So if we are having Rho f / lambda hf / lambda hg / lambda is 0.1 okay.

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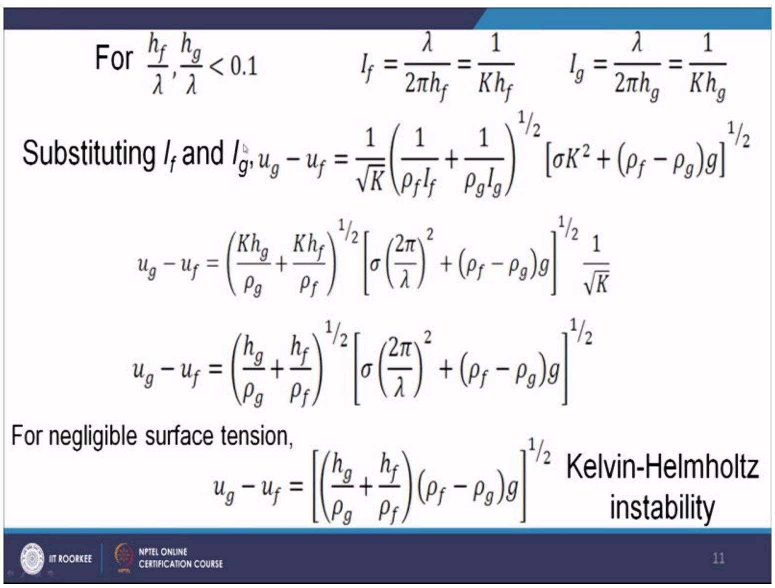

That means you are having a very long wave length okay. In that case we will be finding out that if and ig those are turning out to be 1/khf and 1/khg. So essentially that lambda by 2 pi hf and lamba by 2 pi hg okay. So let us do one thing put this values of if and ig back in the (ug-uf). So if you put this value (ug-uf) over here, so I will be finding out that. Once I put this values this will be simplifying okay.

So and it is very interesting to note over here that we have having k to the power 1/2 and here we are having 1/ k to the power 1/2 and eventually (ug-uf) will become one expression involving sigma Rho f Rho g*g but interestingly there is no lambda value over there or wave number value there. So this is irrespective of any wave number okay. If we now go for negligible surface tension so surface tension is 0, we will be leading towards hg/ Rho g + hf/ Rho f * Rho f- Rho g* g to the power ½.If you remember your knowledge of fluid mechanics this is typically the expression we see in case of Kelvin Helmholtz instability. So basically for long wavelength that will be dominated by Kelvin Helmholtz instability right.

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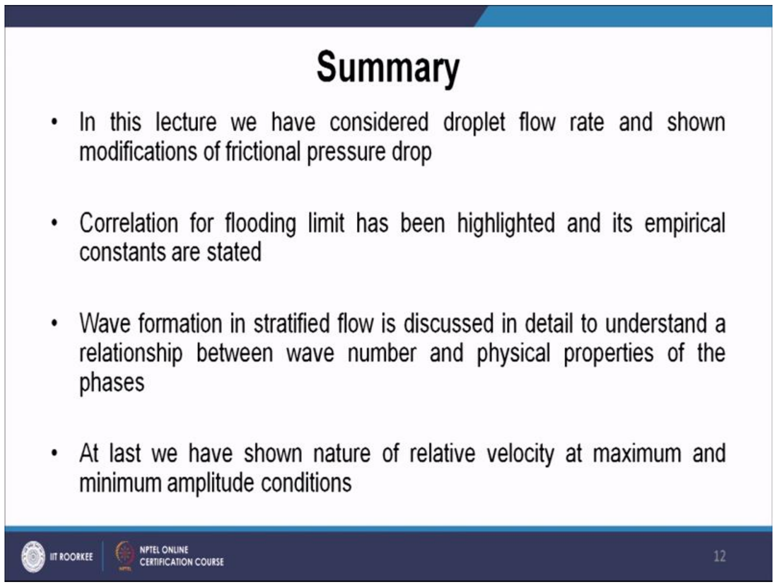

So let us summarize this lecture. In this lecture we have considered droplet flow rate and shown the modifications of frictional pressure drop. We have correlated flooding limit okay and highlighted its empirical constants and we have also shown you how empirical constants can be found out. We have found out the wave formation in stratified flow okay.

And discussed what are the wave number value relationships okay. And physical properties involved in that and at the end we have seen the extreme limits for the wave amplitudes that means maximum and minimum extreme amplitudes okay. So those things we have seen and found out that whether it is dependent on the surface tension or not okay. So with this let us test your understanding after the end of this lecture.

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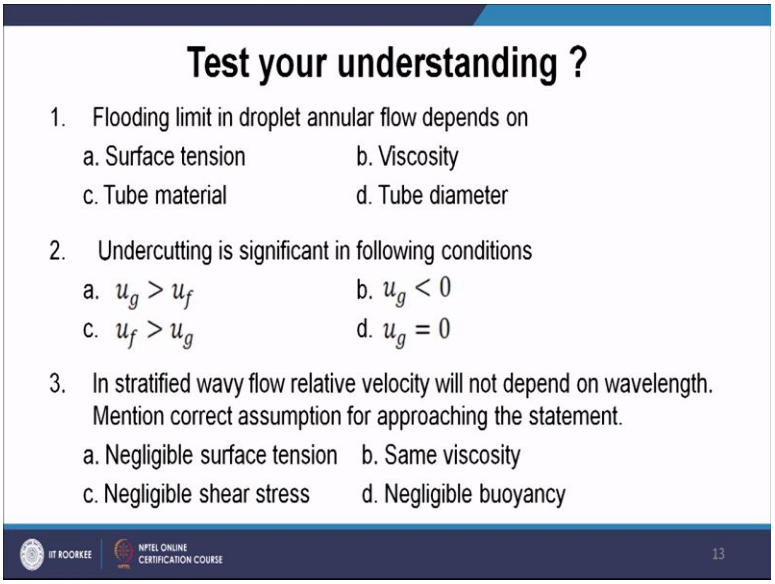

So first question goes like this. Flooding limit in droplet annular flow depends on, 4 answers we are having. Surface tension, viscosity, tube material, tube diameter. So I think all of you guess the answer is viscosity and tube diameter because the expression if you see, we are having mu as well as the capital d okay. Next question undercutting is significant in following conditions. In the first slide I have shown you, so probably it is very easy to get that answer.

So we are having 4 answers ug > uf, ug < 0, uf > ug and ug = 0. So correct answer is the answer c. In stratified wavy flow relative velocity will not depend on wavelength. Mention correct assumption for approaching the statement. We are having 4 answers. Negligible surface tension, same viscosity, negligible shear stress and fourth one is negligible buoyancy. So the correct answer will be negligible surface tension. So with this I will be ending this lecture, thank you.