Two phase flow and heat transfer
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Lecture No.08
Annular Flow

Hello, welcome in the eighth lecture of the course Two Phase Flow and Heat Transfer. In this course you will be understanding annular flow models. Annular flow model is a typical flow pattern in gas liquid Two Phase Flow where we will be finding out liquid film is added with the pipe and in the core of the tube you will be finding out gaseous phase.

So in this lecture we will be stressing about the calculation of film velocity in both laminar and turbulent regime. We will be evaluating film flow rate in the Two Phase Flow configuration, annular Two Phase Flow configuration.

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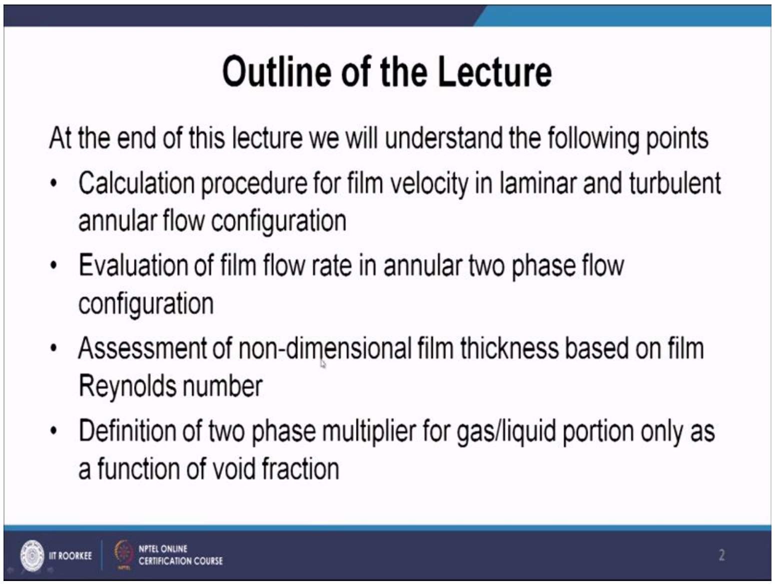

We will be assessing non-dimensional film thickness based on film Reynolds number. As well as we will be defining 2 phase multiplier for gas or liquid portion only as a function of void fraction. So first let us see that how annular flow configuration can be shown here in pictorial view. Here I have shown you a pipe inside which annular flow is occurring. You see inside this, we are having first film added with wall and inside the core we are having gaseous phase okay.

Now here, you can see in the film we can consider a small element like this where the small element will be getting shear stress Tau from the adjacent liquid layers.

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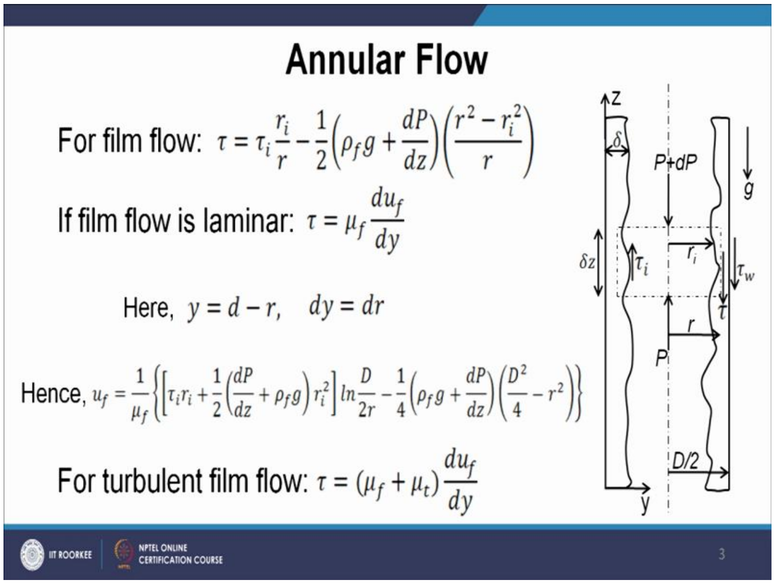

And in the interface it will be getting the shear stress Tau i in the opponent direction. Now near the wall this shear stress Tau is being converted to Tau w which is nothing but the wall shear stress okay. And if you think about this fluid element the fluid element is having pressure from the bottom as P and from the top it is P+dP okay.

This element length we have considered over here delta z. Now if you try to have a force valence then you will be finding out that we are having over here the shear stress at the interface. The shear stress at the outer layer of this fluid element as well as we are having some weight of the fluid element acting in the downward direction.

Apart from that you will be having also the pressure forces P+dP and P over here okay. So if you write down the force balance equation over here, you can see the first term whatever I have written this is nothing but 2 pi r* dz that is the area whatever through which this Tau or shear stress from the other liquid layer is being acted okay. So this is in the downward direction because Tau is in the downward direction okay.

In a similar fashion here we have written the shear stress, what it is getting the liquid layer is getting from the gaseous interface which is in the upward direction.

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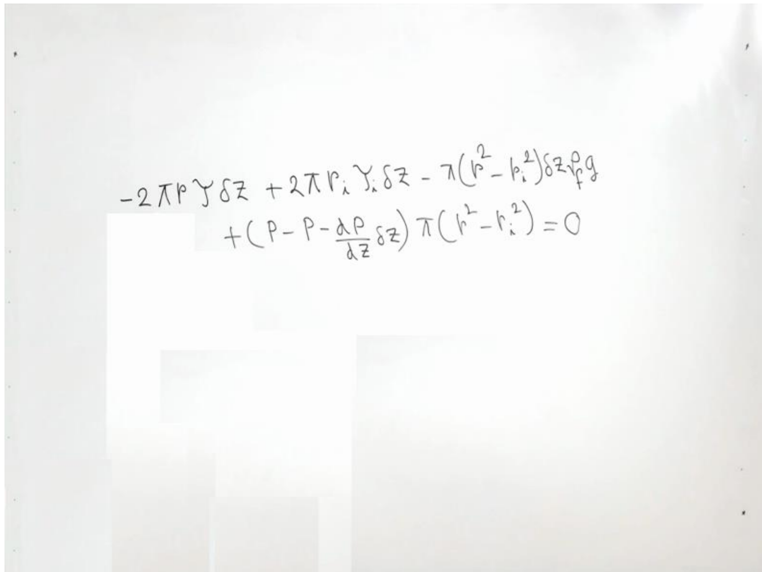

So that is why here you are having a +sign the area will be once again 2 pi r * dz and the magnitude of shears interfacial shears stress is nothing but Tau i. So these 2 are coming from the shear stresses and this is actually your interfacial shear stress. Now as I have told that we are having over here some sort of weight of the fluid element. So the weight can be found out by volume *density. So for the volume pi * r square *ri square* dz.

So this is the volume multiply by density of the liquid Rho f and the g which is acting in the downward direction. And from the pressure force you see here we have pressure P from the downward side and from the upward side P+dP. So this will be P – (P+dP). Now this dP, if you consider for unit length dz, so we can write down P -dP /dz, dz* dz so this delta z cancels out. But for facilitating this calculation of dP/dz, I have written like this.

This pressure is acting on the area pi* r square - r i square which is the bottom area or you can say the top area for the liquid film. So from here if you simplify this one and try to calculate the value of the Tau then you will be getting the Tau. As Tau = Tau i * r i / r – (1/2 * Rho f+ g + dP/ dz, dP / dz comes from here multiplied by r square - r i square /r.

A simple calculation from these equations, if you try to figure out what is the value of this Tau, you will be getting in this equation after cancellations. Now we are having different types of a film flows. So for film flow that I have showed this is a equation. Incase of laminar film flow, we can write down Tau = Mu f * du f /dy okay. So this further can be calculated by some transformation, the transformations are like this okay.

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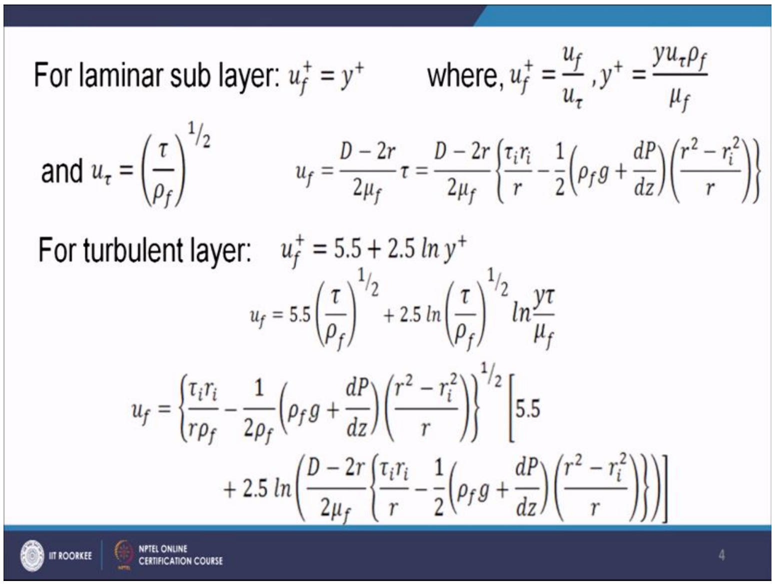

So let us try to see what happens in case of turbulent film flow. So in case of turbulent film flow, we know that there are several layers we have to consider. First one is laminar sub layer then we will be having buffer layer and we can also have turbulent layer. So I have given here the example of 2 layers only, laminar sub layer and the extreme one turbulent layer. So we know from fluid mechanics that in the laminar sub layer uf+ = y+ where uf, is nothing but uf / uTau and uTau can be written as shear stress by density of the fluid to the power ½.

And y+ is a characteristic length you can write down that one as y *uTau /Rho f *Mu Rho f / Mu f right. So what you can do just in a similar fashion if you write down that what is uf+ = y+ over here using this equations inside this then you will be finding out uf can be written in the form of Tau okay. And that comes as uf = d- 2r / 2 Mu f * Tau okay. Already we have expressed what Tau is over here for the film flow okay.

So we can replace the Tau over here and write down uf for the turbulent, laminar sub layer zone d - 2r / 2 Mu f* Taui r i / r – (1/2 * Rho f* g + (dP /dz) * (r square - r i square / r) okay. Similar thing can be also done in case of turbulent layer. For turbulent layer we know uf+ = 5.5+2.5 * ln (y+). This is typically coming from fluid mechanics. So what we can do in a similar fashion as we have done in laminar, laminar sub layer, we can also perform this one for turbulent layer.

We can find out that uf can be written as 5.5 Tau/ Rho f to the power ½ +2 .5 ln (Tau/ Rho f to the power ½) ln (yTau/ Mu f). So once again what we can do Tau, we can replace from the previous equation what we have found out for the film flow and finally obtain the film velocity in this fashion okay.

So for both the regimes laminar and turbulent we have shown how film velocity can be obtained. This is very, very important for an annular flow regime okay. Next as we have found out the flow velocity, laminar film flow velocity, it will be very critical to find out what is the film flow rate. So what we have done over here wFf which is the film flow rate we have found out by integrating 0 to delta pi capital D small dy * uf * Rho f.

So this is basically pi D dy uf Rho f so the limit for y is from 0 to delta okay.

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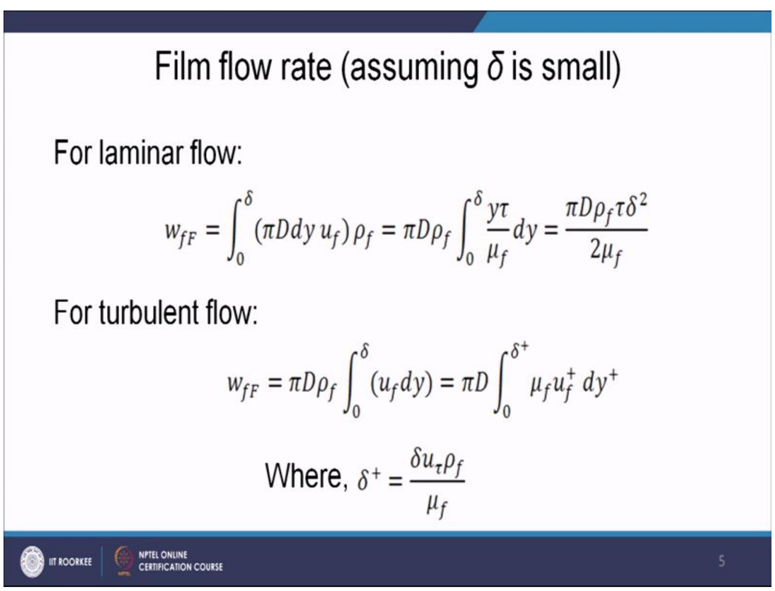

So you can write down this one for laminar domain that this uf will be written as yTau /Mu f okay. And if you do this derivation of the integration from 0 to delta, you will be getting the final expression for the laminar film flow is pidRho fTau delta square b/ 2 Mu f where Tau, is the shear stress we have already derived. Similar thing can be done for turbulent flow.

In case of turbulent flow it will be pi d Rho f 0 to delta uf dy and here uf will be replaced by uf+ okay and dy will be replaced by dy+ at the same time your limit will be transforming from delta to delta+ okay. Already we have said what is the relationship between y and y+. Here delta+ will be delta *uTau * Rho f /Mu f okay. So once you know the expression for this uf+ depending on which layer you are in, you will be finding out the integration value and you can get the film flow rate for the turbulent regime okay.

Next let us see some other extend if in the previous case we have shown the film was moving up okay. So let us say in this case we are taking you are having falling film so the film is actually falling down and the gas is moving up.

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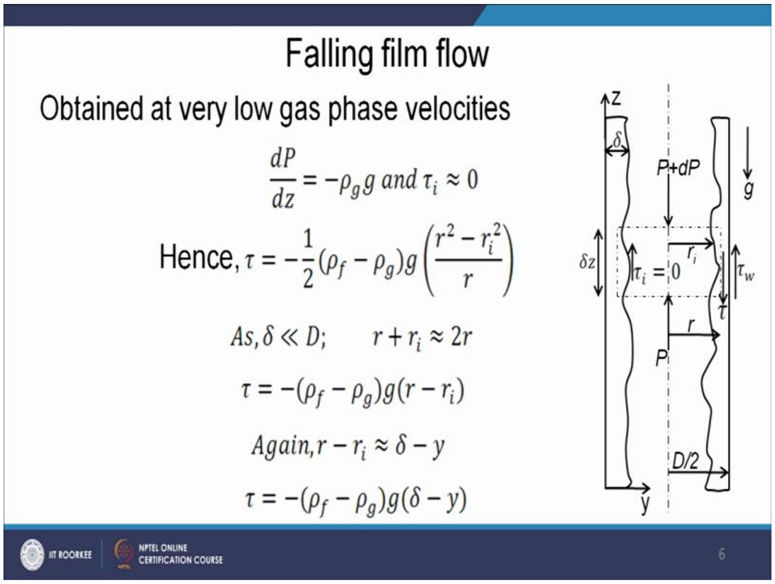

So in this case what we can do, we can typically find out similar figure okay whatever we have shown in the last 1 only thing that this Tau w is in the upward direction because the film is now falling down okay. Now let us try to see that what happens for the gaseous velocities. So if we obtain if you consider that very low gas velocities then we can write down dP/dz =- Rho g* g okay. And in this case if you are having very low gas velocity, you can assume that at the interface there is no shear stress okay. So Tau i is nearly equivalent to 0 okay.

So Tau i is nearly equivalent to 0 over here okay. So if you apply the previous equation whatever we have derived over here for Tau i and make this Tau i= 0, you will be getting something like this Tau = - 1/2 *Rho f - Rho g * g * r square - r i square / r okay. Now let us try to convert this one in easier forms. So what will be doing, we will be taking assumption that film thickness is very small compare to the pipe diameter.

So delta is less than d. Under this assumption, we can write down r+ri. So r was the arbitrary cross sectional radius and ri was the interfacial radius that will be more or less equivalent to 2r because delta is very small. So under this we can write down if you see over here (r square - ri square) so that will be r+ri *(r –ri). So r+ri we can write down as 2r so that r and this r will be canceling out. Finally, we will be getting - Rho f - Rho g *g*(r- ri) okay.

Once again we can write down (r-ri) is nearly equivalent to (delta- y) remember y is being calculated from the wall so (r-ri) will be (delta- y). So what we can do (r-ri) we can replace in terms of (delta- y) and final expression for the falling film will be coming as Tau =- (Rho f- Rho g)*g * delta- y right. So this is the shear stress expression for falling film flow. Next let us try to understand if this is laminar flow then what happens.

In a typical fashion, if you replace this Tau as Mu* duf/ dy and once again integrate with respect to y then will be getting uf = - g /Mu f * (Rho f- Rho g)and integration of this term (delta- y), we will be giving you delta y- y square /2 okay.
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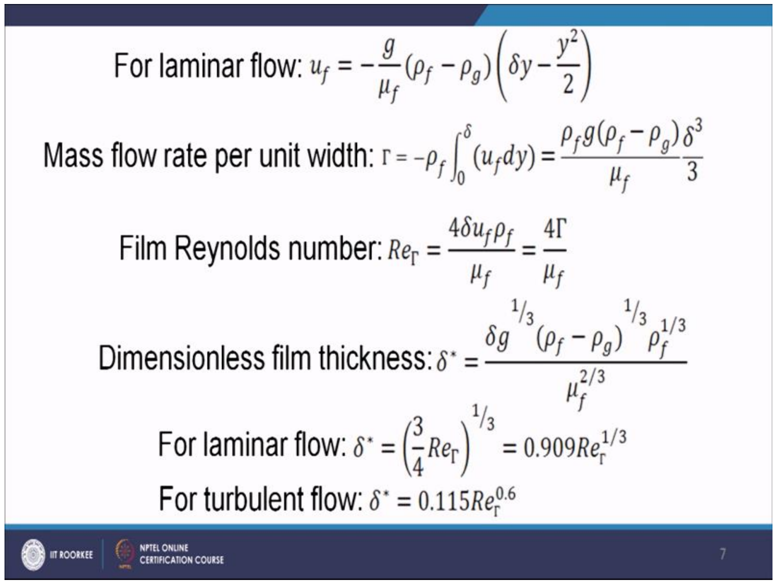

So if you go further and try to find out the the mass flow rate per unit width, so gamma can be written as - Rho f 0 to delta uf * dy okay. So this is the mass flow rate per unit width. So you can write down this uf as this function and once again integrate from 0 to delta. You will be finding out the final expression as Rho f g (Rho f- Rho g)/ Mu f* delta cube / 3.So you see the mass flow rate per unit width is actually dependent on the delta cube okay.

Going in this fashion what we can do, we can define a typical non-dimensional number which is called film Reynolds number. So here we are defining film Reynolds number as Re gamma okay. So Re gamma will be dependent on the film thickness. So the length scale will be typically the film thickness. So what we have done, we have define the film Reynolds number as 4 duf Rho f/Mu f okay.

So once you get the expression for this mass flow rate per unit width and you know uf already we have seen over here so if we plug both this uf and gamma over here. We can write down this film Reynolds number as a function of gamma and uf only okay. So we find out film Reynolds number will be nothing but 4 gamma / Mu f where gamma, is the mass flow rate per unit width okay.

Then we try to find out what is the dimensionless film thickness and non-dimension film thickness. So that will be delta star. We can write down delta star as delta *g 2 the power 1/3

(Rho f- Rho g) to the power 1/3 * Rho f to the power 1/3 /Mu f to the power 2/3 okay. Okay with this definition what we can find out, we will be getting relationship between these delta star and Re gamma which is nothing but film Reynolds number and little bit of derivation between these 2 can be proving you that gamma delta star is becoming 3/4 Re gamma to the power 1/3.

Please try to practice this one; this is not very difficult task only replacement of 1 term in the another one will be giving you the correct expression. So we can find out the delta star for laminar film flow comes out to be 3 /4 Re gamma to the power 1/ 3. So if you do 3 /4 to the power 1 /3 so delta star typically becomes .909 Re gamma to the power 1 / 3.

Follow the same procedure for turbulent layer. We will be getting some another expression. The expression comes out to be delta star = .115 Re gamma to the power .6 okay. Please practice this turbulent one. It you will be getting the same expression.

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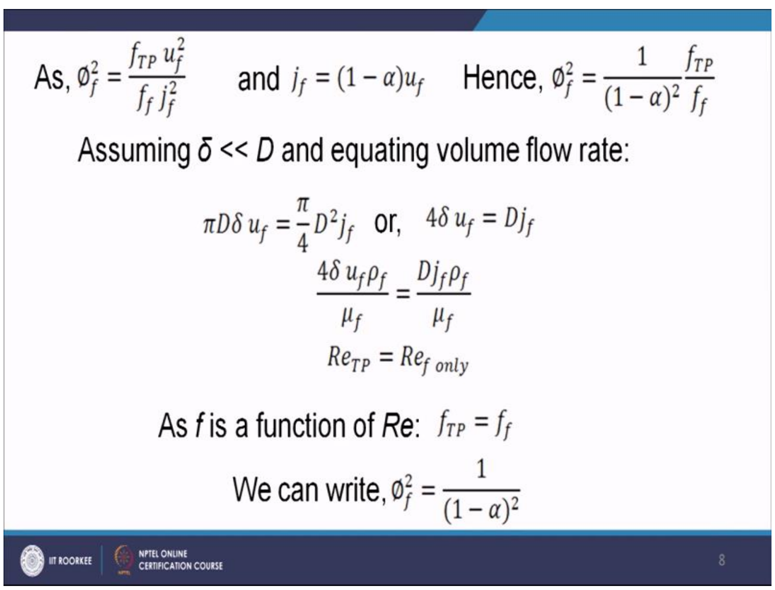

Okay, next let us try to find out what happens for the friction factors. So if you see Two Phase multiplier for fluid portion only so here. phi f square you know that fTP Two Phase friction factor by ff fluid portion, friction factor * uf square /jf square, this we have defined in the drift flux chapter and then we know that jf can be written as (1- alpha) uf.

So I can replace this jf and uf in terms of (1- alpha). So here you will be getting phi f square becomes (1- alpha) whole square using this equation over here. So we will be getting phi f square is (1- alpha) whole square fTP/ ff okay. Now if you assume that delta is very thin, so the film thickness is very small compared to the pipe diameter then I can write down that the volume for the film is pi D delta *uf.

On the other hand if you try to compare the superficial velocity of the film the volume comes out to be. pi /4 D square* jf. So if you equate this 2 then we will be getting a relationship between uf and jf in this fashion 4 delta uf = Djf right. Little bit of multiplication and division and it will be giving you 4 delta uf Rho f / Mu f = Djf Rho f / Mu f. Now these are nothing but Reynolds number expression. This is for Two Phase and this is for fluid portion only.

So you find out Reynolds number for Two Phase and Reynolds number for fluid portion only they are same. Assuming delta is very small than d okay. So if Reynolds number is the same obviously friction factors will be the same, so you can write down phi f square = 1/(1- alpha) whole square. From this equation if we cancel both these terms considering the delta less than d okay.

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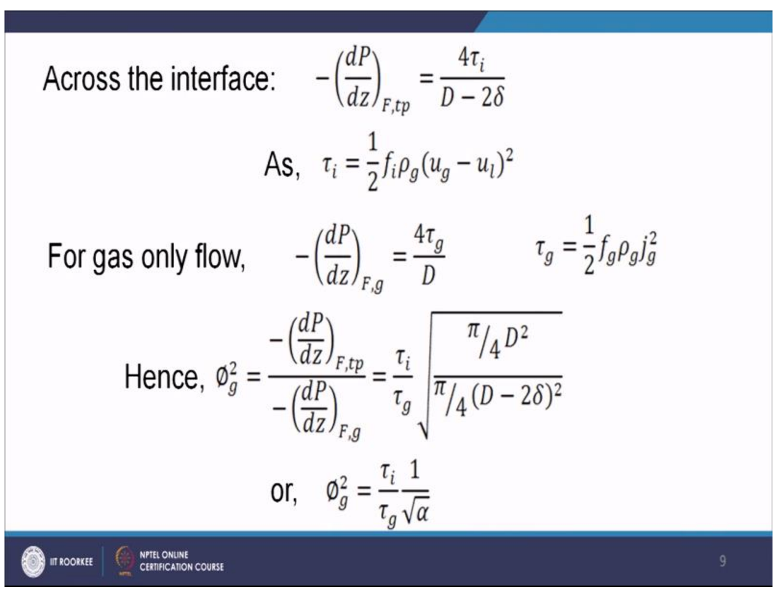

Now let us try to see the gas portion only. So for the Two Phase you can write down across the interface – dP/ dz friction factor = 4 Taui/ d- 2 delta okay. And Taui can be written as this. We have already explained earlier 1/2 fi Rho g* ug- ul square and for gas portion only dP/ dz friction factor for gas is nothing but 4 Taug/ D and Taug will be ½(fg* Rho g* jg square). Now if we plug all these things that means this dP/ dz Two Phase friction factor dP/dz is gas only fraction factor, gas portion friction factor in the Two Phase multiplier phi g square then we will be finding out we are getting Taui / Taug root over of pi /4 D square /pi/4 (D- 2 delta whole square).

Now this gives me 1 /root alpha and Taui /Taug comes over here. So we find out that Two Phase gas portion only friction factor is multiplier is actually Taui/Taug *1/ root alpha.
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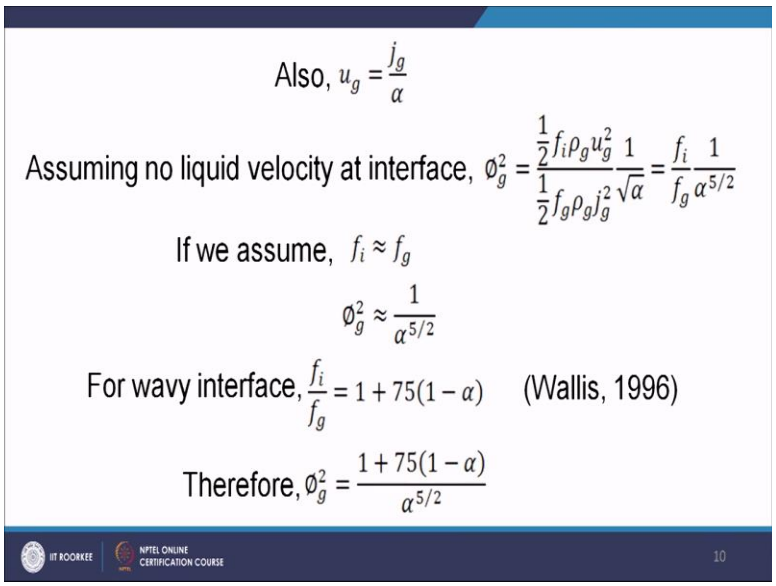

Now let us try to find out how root alpha can be calculated. So if you see if we give this ug= jg/ alpha. Then we can write down this Taui/Tauj from here. Okay over here like this considering no liquid velocity. So in this expression you see here you are having ul. So if you cancel this one then you are getting 1/2 fi Rhog j g square and this one is1/2, If g Rho g jg square.

So you will be finding out relationship between ug and jg in this fashion. So if we put all this Taui and Taug in this expression okay with assumption of no liquid velocity, we will be finally leading to fi/fg *1/alpha to the power 5/2. Now there are several options for this ratio of the friction factors. So if we first assume that both the friction factors are same interfacial friction factor and gas only friction factor then we will be getting that gas portion multiplier is nothing but 1/alpha to the power 5 / 2 okay.

Now Wallis, he has given another opinion in case of this equal friction factor he has given that at for the wavy interface fi /fg = 1+75(1- alpha). If you hook into this fi /fg in this equation then you will be getting phi g square = 1+75(1- alpha) / alpha to the power of 5 /2 right.

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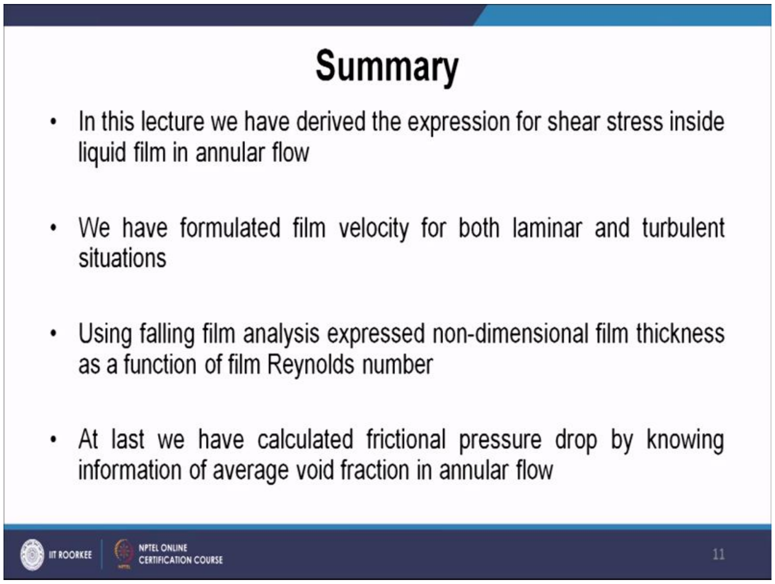

So let us summarize this lecture. So in this lecture we have derived the expression for shear stress inside a liquid film in the annular flow okay. We have formulated the film velocity for both laminar and turbulent regime okay.

Using falling film analysis we have expressed the non-dimensional film thickness as a function of the film Reynolds number and at the end we have calculated the friction factors okay or Two Phase multipliers for gas portion and liquid portion only as a function of your void fraction okay for consideration of the annular flow. Right, so these we have summarized in this lecture let us find out some questions to test our understanding.

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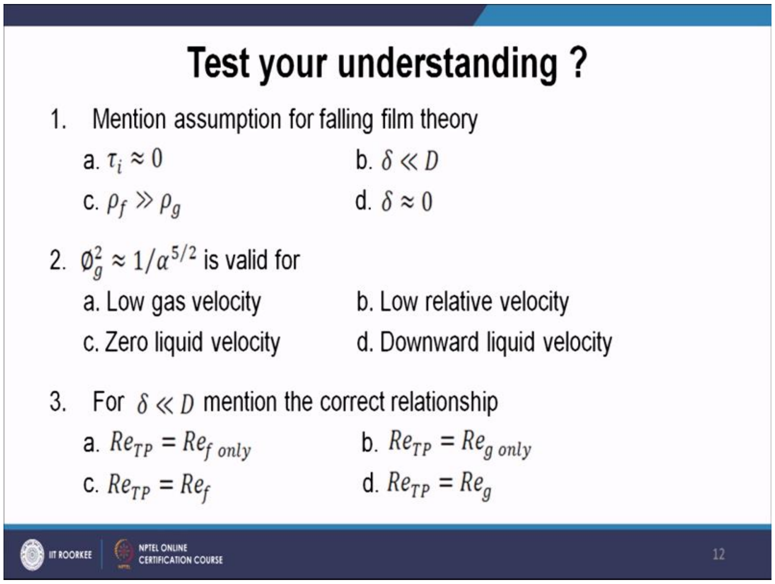

So we are having 3 questions over here first one. Mention assumption for falling film theory. So we are having 4 options over here Taui nearly equals to 0, delta << D, Rho f >> Rho g and finally delta nearly equals to 0. So I think all of you got which one is the correct answer. So you can get first 1 and second 1 Taui nearly equals to 0 and delta << than d is actually the correct answer okay. Then this we have already discussed during the derivation in this lecture okay.

Then second question phi g whole square is nearly equal to 1 /alpha to the power of 5 /2 is valid for. 4 options you are having. Low gas velocity, low relative velocity, zero liquid velocity and finally downward liquid velocity. Answer is obvious for zero liquid velocity. So without using zero liquid velocity (ug- ul) whole square will not be converting into ug square. You cannot cancel this things okay.

Then the third question for delta << D, mention the correct relationships we are having Reynolds number for Two Phase = Reynolds number for fluid portion only, Reynolds number for Two Phase = Reynolds number for gas phase only, Reynolds number for Two Phase = Reynolds number of fluid portion, Reynolds number of Two Phase = Reynolds number of gas portion okay.

Obviously the answer is part a Reynolds number of Two Phase = Reynolds number of fluid portion okay. So with this we will be concluding this lecture. Thank you.