Two Phase Flow and Heat Transfer
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Lecture No: 07
Slug Flow Model

Hello welcome to the seventh lecture of Two Phase Flow and Heat Transfer. So, today's topic is slug flow model. So, at the end of this lecture you will be understanding the following points, you will identify the important non-dimensional numbers to specify the velocity of the slug bubble.

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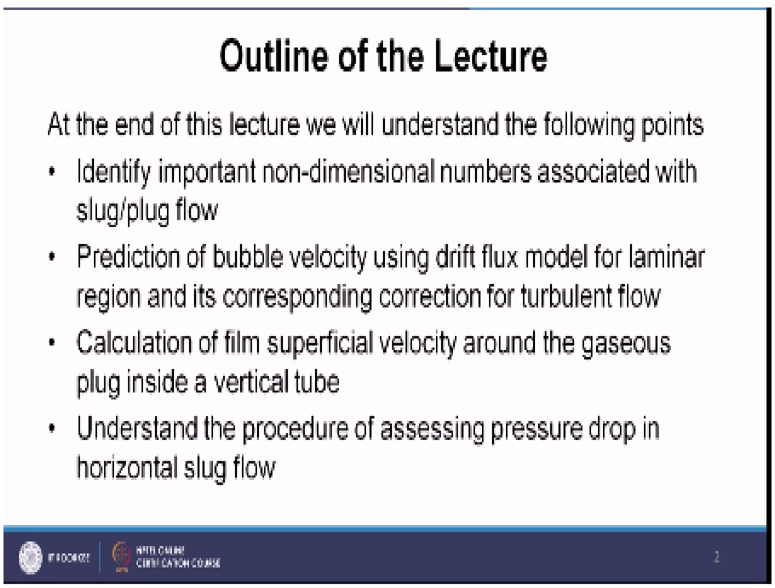

We will be predicting the velocity of the slug bubble using drift flux model and laminar regime, we will be finding out how the velocity is changing for turbulent flow. We will be calculating the film superficial velocity around the gaseous plug inside a vertical tube. And we will be understanding the assessment of pressure drop inside the horizontal slug flow.

Now to give you better understanding what is stuck slug flow, I have shown you here a schematic. So you can find out that in this schematic we are having a Taylor bubble as gaseous plug over here.

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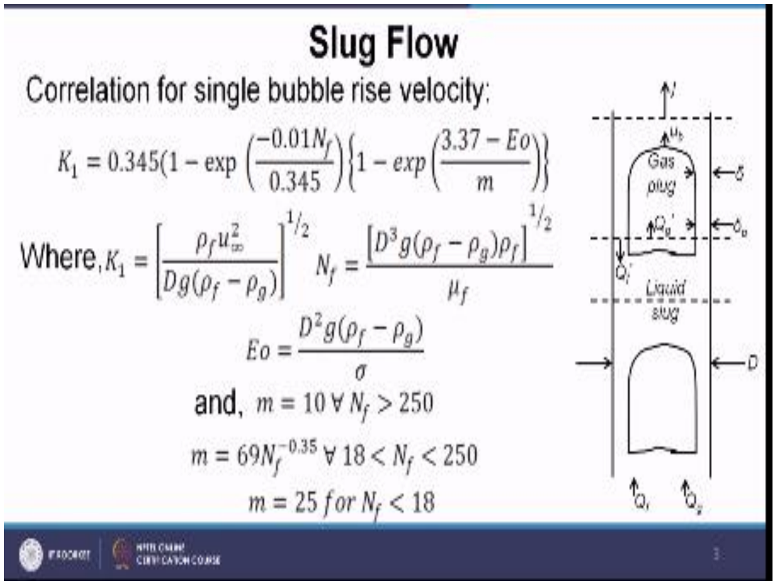

And in between 2 gaseous plugs, we will be having liquid slug. So in real slug bubble regime you will be finding out lots of satellite bubbles are here proceeding this gaseous plug but for simplicity and purpose of analysis we have eliminated those satellite bubbles. We are only dealing with the gaseous plug over here and followed/ A liquid slug.

So it will be some sort repetitive pattern of gaseous slug and gaseous plug and liquid slug. So what we have considered over here. We have considered that this is a unit cell where we will be finding out a gaseous plug is over here. At the top of that we are having a portion of the liquid slug from the previous gaseous plug and at the bottom of that we are having another portion of the liquid slug for this gaseous plug. So this cell will be actually continuing in the bottom side as well as top side right.

So our point of concern is over here, this unit cell okay now let us consider this unit cell if you see minutely this unit cell, we are having gaseous plug but around that we are having liquid film also okay. So here we are having liquid film, here also we are having liquid film. Now this gaseous plug will be moving up with the velocity ub and to accommodate that movement of the gaseous plug, the liquid needs to come down okay.

So here we will be finding out the liquid velocity volumetric flow rate of the liquid. We have considered over here qf` right. And to recognize the film thickness over here, we have considered the film thickness is delta and in the vertical portion of the slug bubble we have considered that the film thickness is becoming uniform and that thickness is actually d infinite right.

And obviously we know that pipe diameter, we always represent with D and the overall flow rates for the liquid and gas that is qf and qg. Remember though the liquid is actually coming down over here with a volumetric flow rate of the qf `, actually the liquid overall is movement moving up because this liquid slug is being pushed by the bottom Taylor, bottom gaseous plug and overall movement of the liquid is also in the upward direction along with the gas.

So you can find out qf is also in the upward direction along with Qg right. So at first let us try to find out that how this rise velocity of this gaseous plug can be calculated. So for that there are lots of correlations available, experimentally observed correlations available. The best one is this 1. K1 = 0.345(1 -e to the power- 0. 01 nf / 0 .345) {1- e (to the power 3.37 - Eotvos numbers by m)} right.

Let us find out what are these terms first. K1, so K1 is actually Rho f u infinity square. So where Rho f is actually the liquid density, u infinity is the terminal velocity of the bubble of the gaseous plug. And we are having D pipe diameter*g (Rho f- Rho g to the power 1/2 okay. So we can put this whole expression over here which will be getting as a function of u infinity.

Then in the expression if you go further, we are having another non-dimensional number which is Nf, this is actually inverse non-dimensional viscosity. So you can find out this Nf is actually represent as [D cube g (Rho f – Rho g) Rho f to the power]1/2/ Mu f. So as Mu f is in denominator. So we call this one as inverse viscosity non -dimensional viscosity. On the other hand if you (()) 5:36 further, we are having Eotvos numbers.

So the Eotovos numbers all of we know once again to represent that the Eotovos numbers will be D square g (Rho f- Rho g) / sigma. So here we get the ratio between the buoyancy force and surface tension force. Here we get the ratio between your buoyancy force and viscous force and here inertia and your buoyancy force right. Now only empirical constant over here left is m.

So m will be 10, if your non-dimensional inverse viscosity is higher than 250. m can be represent as a 69 Nf to the power -.35. If Nf varies in between 18 to 250 and if Nf < 18 that means for very viscous flow you will be finding out m =25 right. So if I put all these non -dimensional numbers over here, so all other parameters will be known. Only unknown will be u infinity. So one can find out the velocity of the gaseous plug right.

Next once we know the gaseous plug velocity then we have to find out what is the superficial velocity and how this can be found out for some different situations like laminar and turbulent zones. So lets us try to see that how in a slug plug flow bubble velocity varies.

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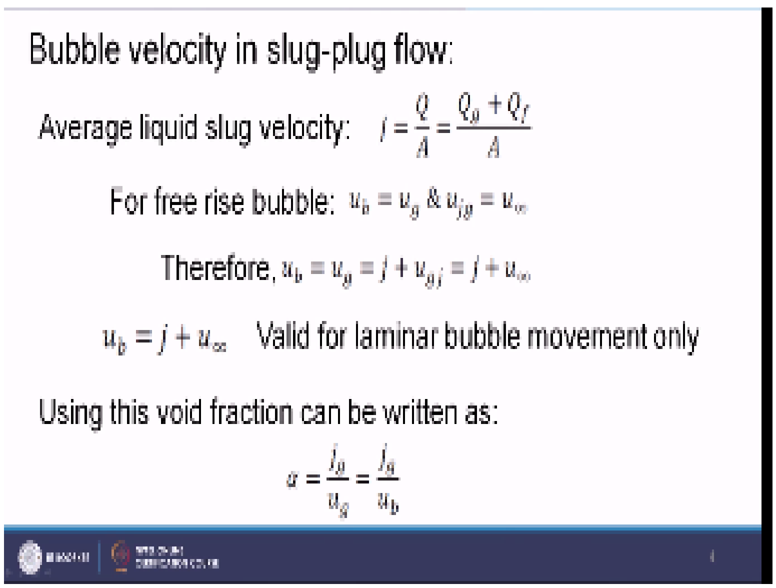

So first we have shown you over here average liquid slug velocity will be nothing but j okay which= all of we know that j = Q/A and Q can be written as Qg + Qf okay. So Qg and Qf already I have shown you in the figure here, this Qf and this Qg okay. So you will be finding out that j is actually Qg + Qf /A. Now for a free rising bubble, so let us say the bubble is moving freely. So we know that ub will be actually symbolizing the gas velocity.

So bubble velocity eventually will become the gas velocity. So we can right down ub = ug right. And if we see that what is the velocity in comparison to the average. Overall velocity that means the velocity is sleep out in comparison to the average overall velocity that will be ujg that is actually equivalent u infinity which we have calculated in the previous slide okay. Next over here you see in this equation if we put ub which is equivalent to your ug, so here we will be finding out j+ ugj right.

Now ugj just now we have shown that is actually u infinity. So we will be founding out ub = j+u infinity okay. So this new expression, we get ub = j +u infinity this is valid for a laminar bubble movement in a slug plug flow right. So this u infinity is the unconstraint bubble velocity. So if you do not consider the walls inside the slug plug flow then this u infinity can be found out from the previous expression over here.

And a bubble velocity in slug plug flow can be found out by adding j along with the u infinity okay. Now already we know that if we have to find out that what the average void fraction is, average void fraction alpha can be written as jg /ug okay. Now we know that ug will be ub. So actually, the alpha in this slug plug flow case we can be write this one as jg /ub okay.

So ultimately, we get for laminar flow ub = j+u infinity and alpha average void fraction= jg /ub. Remember in this type of slug plug flow case the void fraction at different cross section will be changing with respect to time of the flow progresses. Because here, whenever we are seeing through this liquid slug obviously void fraction will be 0. And here if you see over here you will be having a finite amount of void fraction.

So we are always interested to find out the average fraction in the cell okay. Now here we have talked about the laminar flow. Let us see what happens if we are having fully developed turbulent flow okay. So if we are having fully developed turbulent flow, we will be finding that the bubble velocity is changing little bit. So what we can do empirically we can add 1 constants over here, 2 constants over here.

C1and C2 along with the expressions whatever we have found out for ub in the laminar regime. So already we have shown in the laminar regime ub is j+ u infinity for turbulent regime.
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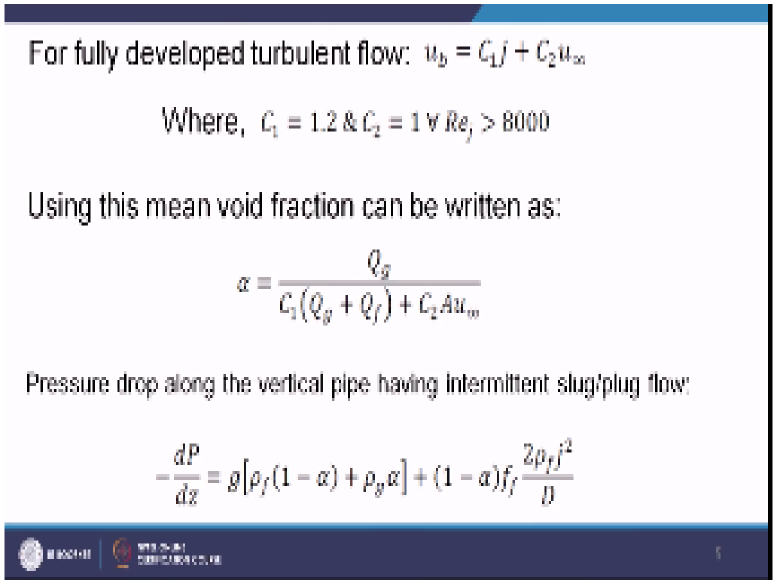

We are writing down ub = C1j+C2u infinity right. Now various researchers they have proposed the values of C1and C2. Wallis has proposed that you can take for slug plug flow C1 = 1.2 and C2 = 1 if your overall Reynolds number is in is actually greater than 8000 okay. If your Reynolds number is in between 2000 to 8000, this expression will not be valid. If it is more than 8000 then only you can apply C2 = 1.2 and C2, C1 = 1.2 and C2 = 1 right.

Now using this mean void fraction what we can written down. Alpha what I have earlier showed you over here which is nothing but jg /ub. So same thing we can write down over here. Alpha = Qg now this jg can be converted to Qg /A okay. Qg/ Ag. So that can be written as Qg and Ag will be absorbing over here. So in place of your ub, we are writing down C1j + C2 *u infinity.

So C1j, now j can be once again written as jg +jf and jg can be written as Qg/ Ag and Qg / A and jf can be written as Qf /A. So you can find out here we are having Qg / A. Here we are having Qg/a and here we are having Qf /A. So A can be cancelled out and A can be absorbed in the last term which was C2*u infinity. So C2*u infinity*a. So this becomes the expression for alpha for turbulent flow right.

Now as we know that not only the value of alpha and the velocities of bubble will be important. Important will be to know what is the pressure drop. So here I have given you the expression for pressure drop in slug plug flow also. You see –dp/dz can be written as the buoyancy pressure drop. So this is g* Rho f (1-alpha). Now this alpha will be actually used from the previous calculation.

The average alpha whatever we have found out so Rho f (1- alpha) + Rho g*alpha. This is from the buoyancy and then (1-alpha) ff. So we are using fluid part only assumption okay. So ff*2* Rho f j square/D okay. As you are dealing with the slug flow finding out the liquid superficial velocity and gas superficial velocity will be difficult. So what we do, we find out the overall superficial velocity j and we express the frictional pressure drop in terms of fluid part okay.

So ff we are using over here this frictional portion we have already discussed in a previous lecture okay. Now after finding out the friction factor it is also very important to know that what will be the liquid flow rate okay. What it is coming out in downward side okay. So to assess that first let us get what is the gaseous flow rate. So the gaseous flow rate will be flow of the gaseous plug.

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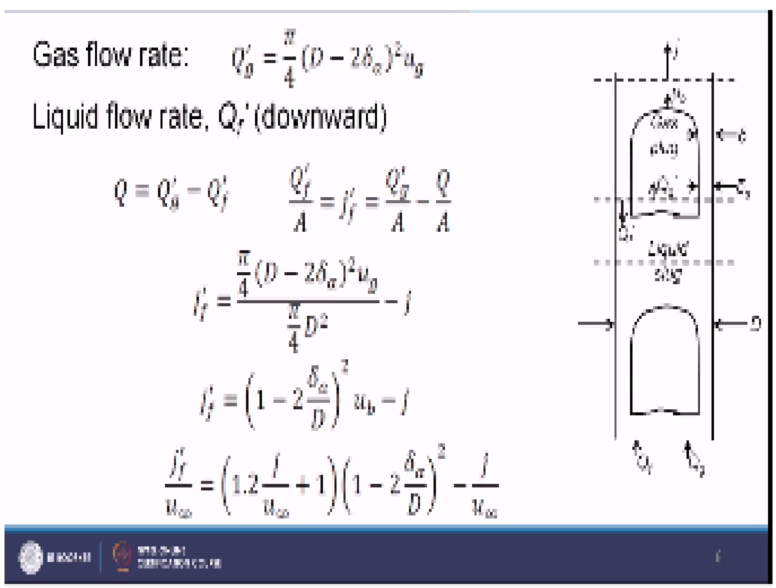

So this is Qg` and liquid flow rate would be Qf`. So Qg` can be written as the area occupied by the liquid sorry, gaseous plug multiplied by the velocity of the gas. So here what we have considered the pipe diameter was D and we have considered that this thickness of the film uniform thickness of the film around the gaseous plug is D infinity. So overall we will be finding out the diameter of this gaseous plug is D-2*delta infinity.

So we will be finding out the area of this gaseous plug, if you consider a perfect cylinder as this gaseous plug. So it will be pi/4 (D - 2 delta square) okay. So this is the area of the gaseous plug considering it as a cylinder multiplied by its velocity ug. So will be finding out this is the gas flow rate in this cell right. Now let us right, to find out what is a liquid flow rate. So what will be doing? As we know the overall flow rate we know from summation of Qf + Qg.

So I can write down Q= in situ Qg` to - Qf`. Now this Qg` is will be never equivalent to this Qg okay because they are different because in this case we are having no gaseous phage right. In the liquid slug we are having no gaseous phase. So here we can find out this Q as Qf + Qg and that can be written as Qg`- qf`. Why this Qf `- because in this cross section if you see this cross section, this dotted line here Qf is in the now negative direction right.

So we can write down Qf`/A = Qg`/A – Q/A. So from this expression I can write down Qf` = Q-Qg` and if you divide it by A, we get this expression. The left hand side can be written as jf` okay because Qf` / A is nothing but jf` okay. So we get jf`= Qg`/ A. Now what is Qg` once again we have written over here or assessed found out the value of Qg` gaseous flow rate. So this will be written over here and A which is nothing but the area of the pipe pi.

So pi /4 *D square. So you can write down over here in terms of a pi/4 D square and obviously we know that Q/ A is actually j right. So if we simply, this first term in the right hand side we will be getting something around (1- 2 *delta infinity /D) whole square *ub now this ug we are converting to ub because we know here this gaseous velocity a gaseous plug velocity will be same as the bubble velocity okay.

So we get jf`= (1- 2 into delta infinitive /D) whole square ub- j okay. So already we have found out what is ub in the previous slide. For turbulent cases so same formula will be using over here (1.2* j/u infinity + 1). So that was actually (1.2* j + of u infinity). If you will take u infinity common and then divide the whole expression by u infinity. So this left side, so left hand side will become jf`/u infinity, right hand side will become (1.2*j/u infinity + 1) and then this expression multiplier will be this is coming from the area ratios.

So this will be staying over here and the last term will remain as -j/u infinity. So in this way we can find out the liquid flow rate. Liquid superficial velocity in the in the cross section where gaseous plug is present right. Next if we consider the falling film theory then this jf` /u infinity can be predicted from falling film theory. So that has been discussed by Miosis and Griffith. I will not going into detail of this one.

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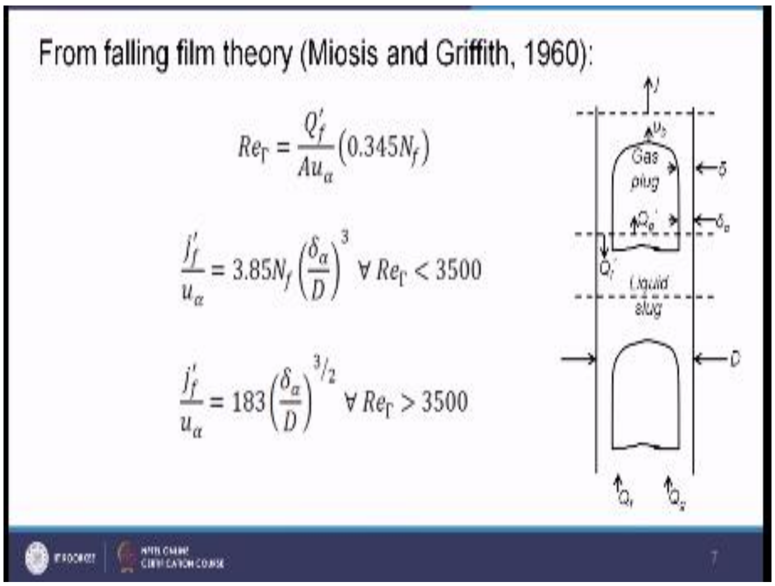

But I will be showing that how empirically can be found out using falling film theory how the liquid velocity superficial velocity can be found out. So here you see this velocity will be nothing but jf` /u infinity will be 3.85 Nf. Nf already we have discussed that is non-dimensional inverse viscosity into delta infinity, uniform film thickness / D whole cube. This expression valid is valid for Reynolds number, film Reynolds number less than 3500.

The definition of film Reynolds number, I have given over here. So film Reynolds number is nothing but Qf`/ A* u infinity (0.345Nf). And if this film Reynolds number is larger than this 3500 then you can use this expression jf`/u infinity= 183(d infinity/ D) to the power 3/2 right. Next let us shift to horizontal slug flow from the vertical one. So here I have shown once again the schematic of horizontal slug flow.

So you can find out that in case of horizontal slug flow this is once again like a Taylor bubble. So this is actually a gaseous plug in between we are having in between 2 gaseous plug, we are having a liquid slug.

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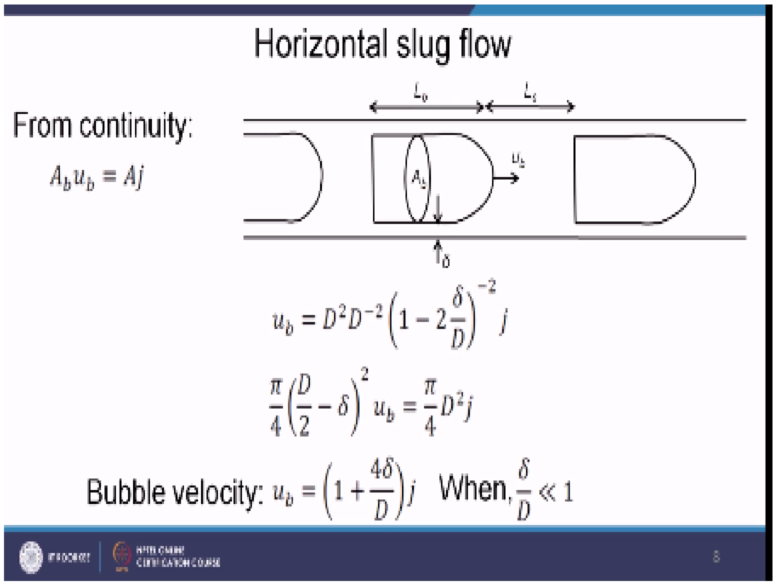

Once again the same situation this is having bubble velocity. The film thickness we have represented as delta okay. And here I have shown the length of the bubble as lb and the length of liquid slug as ls. Now if you start from the very basic continuity equation then I can write down Ab to into ub = A*j. Why because you see in this horizontal slug flow actually the movement of the bubble will be causing the overall flow because otherwise it is not assisted by buoyancy.

The flow is not assisted by buoyancy. So you will be finding out when the bubble is moving that will be only causing the flow. So it you see from the continuity side if you find out what is the volumetric flow rate for this bubble so which is nothing but Ab * ub that will be responsible for the overall flow A* j right. Now from here if we try to find out that what is the value of ub, so will be Ab/ A*j right.

Now A once again we know that will be pi/ 4 D square, D is the tube diameter and Ab, I can write down as pi /4 (1- 2) pi / 4 D square (1- 2 delta /D) whole to the power- 2 which I have already shown you in the vertical case okay into j will be remaining from here. So here we are having j so the same j will be remaining over here okay. So already I have shown you the expression over here. So you can find out so this from this we are getting this expressions.

So ub finally after cancelling this D square and D to the power -2 comes out as (1 + 4 delta/D) j okay. Now this we can write down whenever we considered delta /d < 2. Why because this is a polynomial. So if we find out this delta /d is very small than 1 then only I can write down this is actually (1- 2)(-2* delta / d). So this is actually 1 + (4 delta / d) okay. So this expression bubble velocity is valid only for very small film thickness compared to the pipe diameter.

If it is not small then you have to go with the overall polynomial and we have to evaluate the value okay. Now let us see just like our vertical case what are the important non-dimensional numbers in case of horizontal slug flow.

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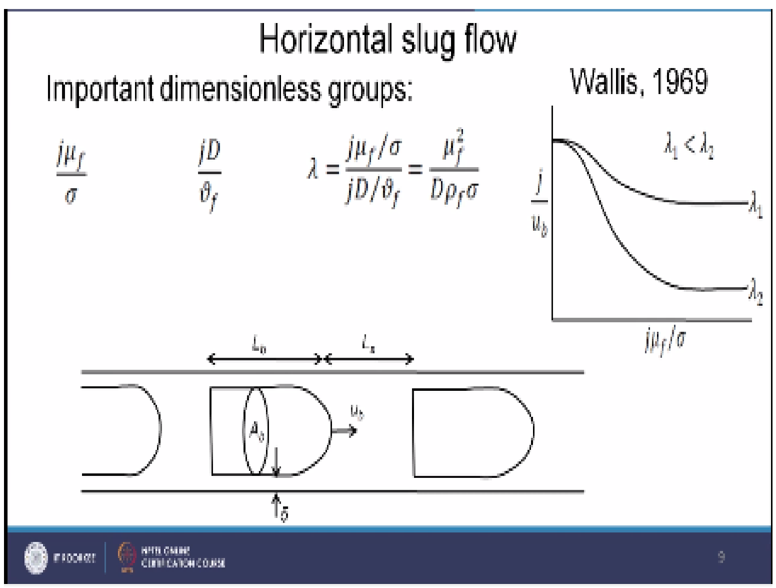

So important non-dimensional numbers first one is jMu f/ sigma okay. Second one we can take as jD/Mu f. So this is having the viscous effect this is having the surface tension effect compare to viscous effect. And another one we can derive as lambda which will be the ratio between these 2 okay. So here you will be finding out that in this we are having 3 parameters lambda, jMu f/ sigma and jD /Mu f.

If you try to find out that how this parameter are actually interlinked then we can plot something like this. So in the abscissa we are having the first non-dimensional number jMu f /sigma. And in the ordinate we are having the second one which is actually j /ub. So here this is actually non-dimensional bubble velocity, non- dimensionalised by the overall superficial velocity and here will be finding out several curves having different lambda value.

So lambda is actually the ratio between these 2 non-dimensional numbers and it is values Mu f square / D Rho f *sigma okay. So here we can find out that the curve will be varying like this. For a low viscous fluid will be finding out that bubble velocity is high okay. And we will be finding out that for different lambda. As lambda increases we will be finding out that the curve is falling. Actually, in the downward side right this has been proposed by Wallis.

Next let us try to find out what is the bubble velocity in this case and the overall void fraction. So as we know already that ub is nothing but C1 j + C2u infinity for turbulence situations.

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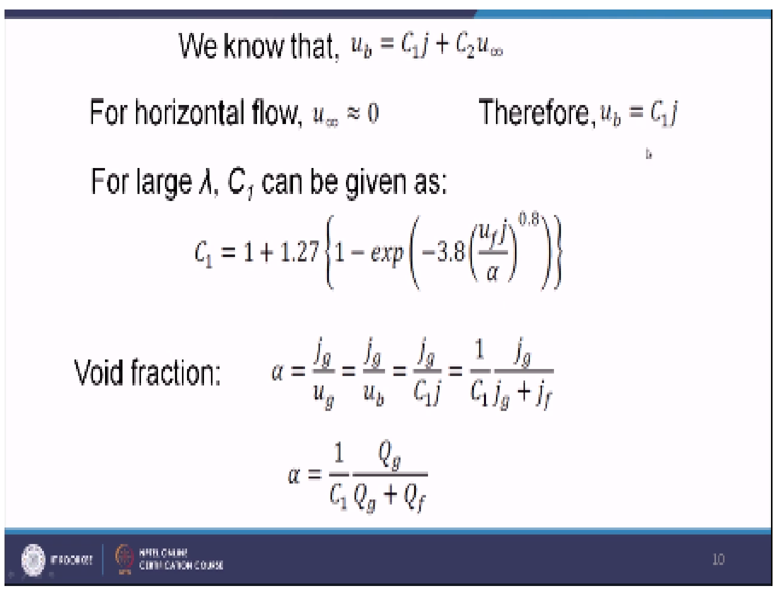

So here what we can do, u infinity we can make 0 because it is in horizontal pipeline. So there will be no velocity, unconstraint velocity of the bubble. So you can write down u infinity = 0. So we get ub= C1j, over here C1j right. Now once again various correlations are there. 1 very important correlation is for finding out the C1 for horizontal slug flow C1 will be 1 + 1.27 {1- e to the power (- 3.8 (ufj / sigma) to the power. 8)} right.

Okay now if we want to calculate the void fraction also, you see void fraction alpha will be jg /ug. So jg/ ug, we know that ug will be actually your ub because gaseous phase is only limited in bubble. So we will be finding out ug /ub okay. And already I have shown ub = c1j. So what we can write down jg /C1 *j. So ub has been replaced by C1j. So you get 1/ C1 and then jg /j. j can be written as jg + jf okay. In the first lecture I have shown you this part right.

And once again js can be converted into corresponding Qs. So you can write down that alpha = 1 /C1 Qg /Qg + Qf. So over here Qg / Qg + Qf right. Now let us try to assess that what will be the pressure drop across this slug plug flow. So here we are writing first or we assessing first the volume of the bubble.

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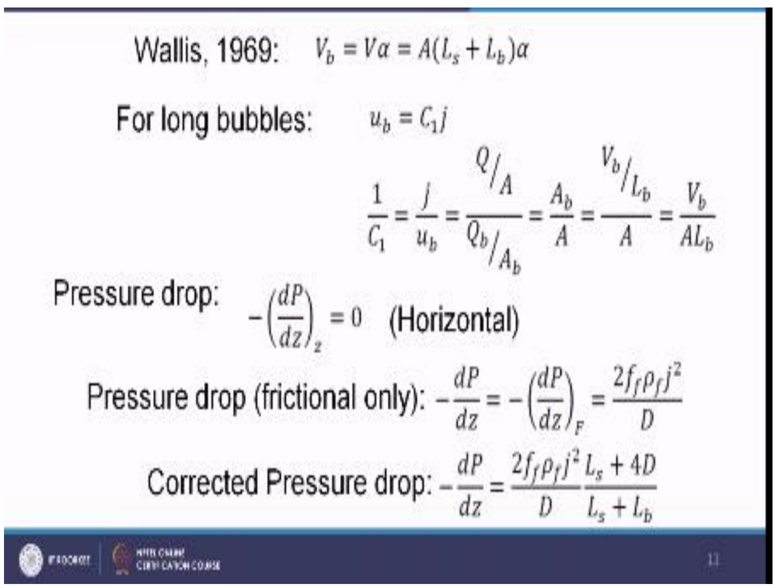

So we can find out that volume of the bubble will be v *alpha and this volume of bubble, this volume can be written as a into ls + lb. Now this A* ls + lb is the volume for the (()) 25:26 width of the cell. Whatever we have shown over here that we are having a gaseous plug and liquid slug. So if you consider a unit, so this unit volume is this 1 A is the pipe diameter and ls is the slug length and lb is the bubble length.

So we can find out that Vb turns out to be A(Ls + Lb) alpha. Now already we have shown that for long bubble. We will be finding out ub= C1 j in the previous slide we have shown ub= C1 j.

So what we can do we can write down C11/ C1 is actually equals to j/ ub okay. So now let us try to find out what is this j and u in terms of the volumetric flow rates. So j can be written as Q/ A and ub can be written as Qb/ Ab okay. And as we know that Q and Qb both will be same because the slug bubble is actually causing the flow inside the pipeline. So we can find out that 1 /C1 comes out to the ratio of Ab/ A okay.

Now what is Ab, Ab once again can be written as Vb / Lb because of bubble volume will be actually your area of the bubble and length of the bubble okay. So ultimately, I get 1 / C1 = Vb / A Lb right. Now related to pressure drop already we know as it is a horizontal pipe. So for horizontal cases your gravitational pressure drop will be 0. So – dp/dz for z gravitational head is actual buoyancy heat is actually 0. But there will be frictional head.

So frictional head I can write down – (dp/ dz) f will be 2 ff. Here I am considering fluid part okay. So 2ff Rho f j square /D okay. Now if we are considering that inside dynamics of the Taylor bubble that means the gaseous dynamics inside the Taylor bubble then we will be finding out 1 multiplier is necessary. So this multiplier has been proposed by Wallis over here in 1969. So this multiplier is Ls + 4D / Ls + Lb.

So dependent on the slug length and the bubble length and the tube diameter okay. So next let us try to find out that this pressure drop how that can be simplify and found out in the form of volumetric qualities.

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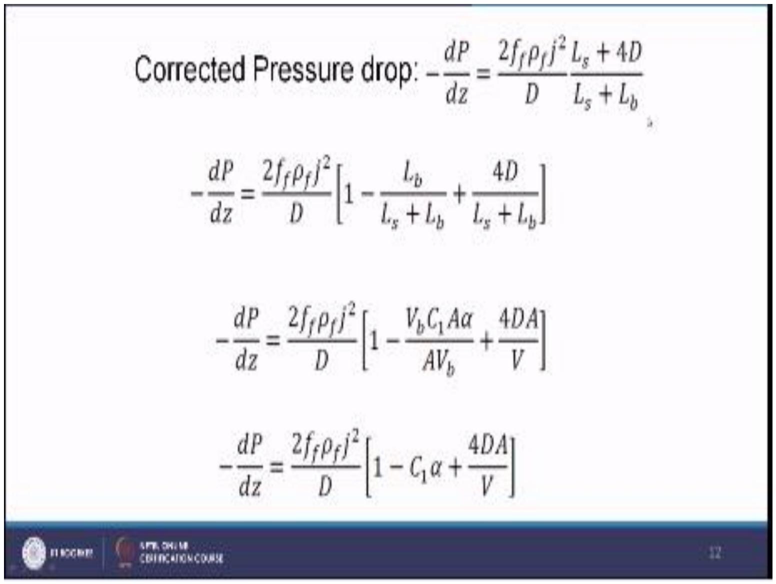

So already we have shown that – dp/ dz will be 2ff Rho f * j square /D Ls + 4D /Ls + Lb. Now let me simplify this part. So this, I can write down easily 1- Lb/ Ls + Lb + 4D/Ls + Lb. Basically this first 2 terms gives me Ls /Ls + Lb okay. Then this Lb and Ls + Lb, I can replace so Ls + Lb quickly I can write as A * Vb because already I have shown Vb = A* Ls + Lb *alpha right. So Ls + Lb will be Vb/ A alpha.

So what I can do here, I have written Ab /alpha right and for Lb, I have written bb C1 A okay Bb C1 ;A why because this Lb is actually the length of the bubble. So length of the bubble will be actually merged with the volume of the bubble over here. So volume of the bubble into the area of the bubble will be coming out as the length of the bubble okay. So here we can find out that both this Lb and Ls + Lb has been replaced by this factor and here once again this Ls + Lb has been done by same (()) 29:15.

So this will be V/ A once again V/ A. So if you simplified further ultimately, I will be getting – dp/ dz = wff Rho f j square / D (1- C1) alpha + 4 DA /V. Now here in this expression you see C1 is actually empirical parameter. So using this can be found out from Wallis correlation whatever I have explained you earlier okay. And the rest things we can find out from the slug plug flow and can be evaluated okay.

To summarize in this lecture, we have understood the procedure of calculation for terminal velocity of a Taylor bubble from non-dimensional numbers.

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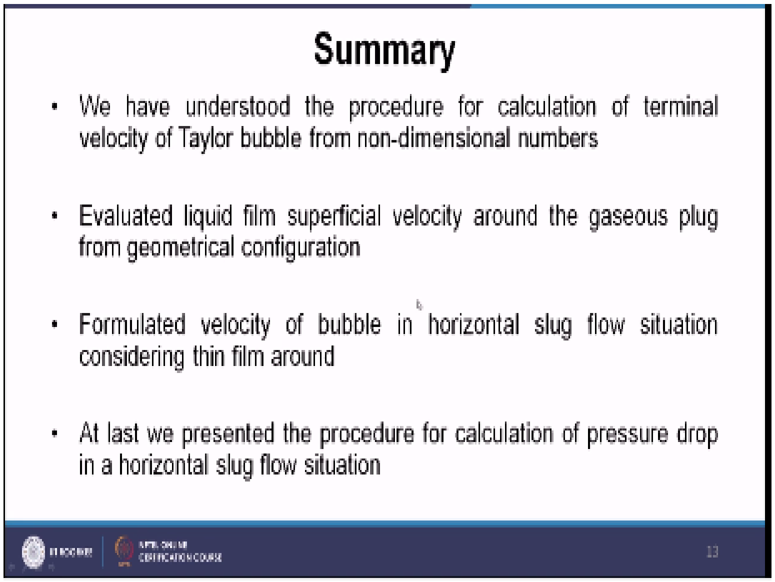

We have evaluated the liquid film superficial velocity around the gaseous plug from geometrical configurations. We are formulated velocity of bubble in horizontal slug flow situation okay. And in that we have considered that we are having thin film around the bubble and at the end we have presented the procedure for calculation of pressure drop inside a horizontal slug situation okay.

Next let us test how you have gone through this lecture. So we are having once again 3 questions.

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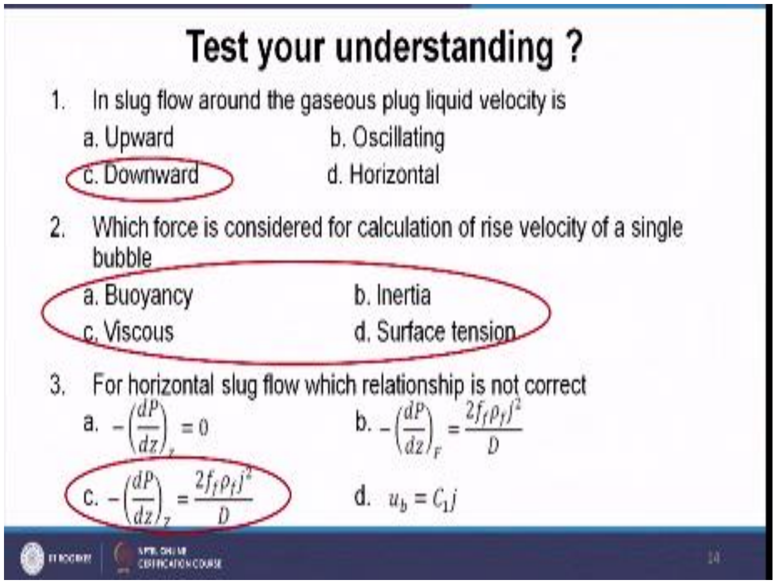

First, in slug flow around the gaseous plug liquid velocity is 4 options we are having upward, oscillating, downward and horizontal. So mostly you have understood what is the answer correct. Answer is downward because liquid will be having downward velocity around the slug. Okay second question which force is considered for calculation of rise velocity of a single bubble okay.

We are having 4 options buoyancy, inertia, viscous and surface tension. Which force is important for calculation of terminal velocity rise velocity of a single bubble answer all? So all the forces are equally important for calculation of right rise velocity okay. Last question for horizontal slug flow which relationship is not correct.

So we are having 4 equations, gravitational pressure drop= 0, frictional pressure drop= 2ff Rho f j square /D, once again gravitational pressure drop= 2ff Rho f j square /D and ub= C1j. Probably you have understood which one is the correct one, obviously part c is not the correct answer. Hope you have like this lecture. Thank you.