Two Phase Flow and Heat Transfer
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Lecture No: 06
Dispersed Flow Models

Hello welcome to the sixth lecture of Two Phase Flow and Heat Transfer. Today we will be dealing with dispersed flow models. So, at the end of this lecture you will be knowing the applicability of dispersed flow models in gas liquid Two Phase Flow.

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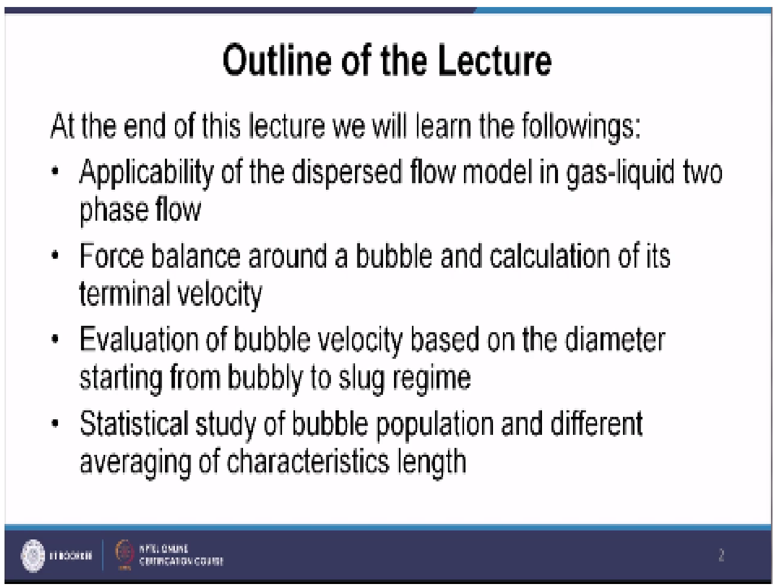

We will be also understanding force balance across a bubble; we will be calculating the terminal velocity of the bubble. We will be finding out velocity based diameter in the slug regime, bubbly flow slug regime and finally we will be finding out statistically how bubbly flow can be evaluated in terms of the volume and number density ratio. So let us discuss what is dispersed flow?

So here dispersed flow means we have not kept our periphery limited into the bubbly flow regime. What we can do if we are having dispersion of gas in the liquid that we will be calling us bubbly flow and on the other hand if you are having dispersion of liquid droplets in gas we will be calling that one as droplet flow. So both the conditions can be tackled using dispersed flow model whatever I will be dealing now.

Though I will be stressing only on the terminal velocity of the bubble but in a similar fashion terminal velocity of a droplet can be found out and similar type of analysis can be done for droplet flow.

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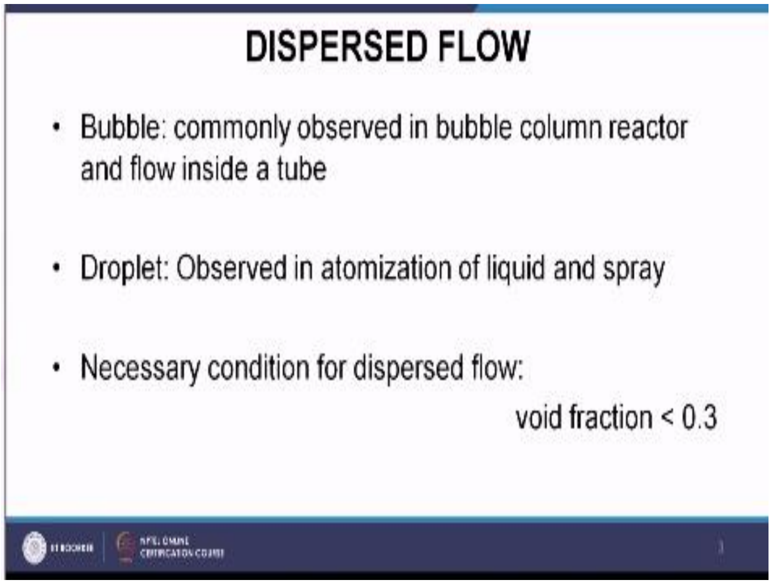

Here I have shown you that bubbles will be commonly observed in bubble column reactor and flow inside a tube where, bubbly flow is very common for droplet. We have seen in case of atomization in spray we will be finding out that droplet flow is present. So necessary condition for dispersed flow is that obviously void fraction will be very small the terminal void fraction or limiting void fraction is less than 0.3.

So if you are having void fraction less than 0.3, we will be calling that one as dispersed flow and we can apply dispersed flow model for its analysis. So we will start with derivation of the motion of a single bubble.

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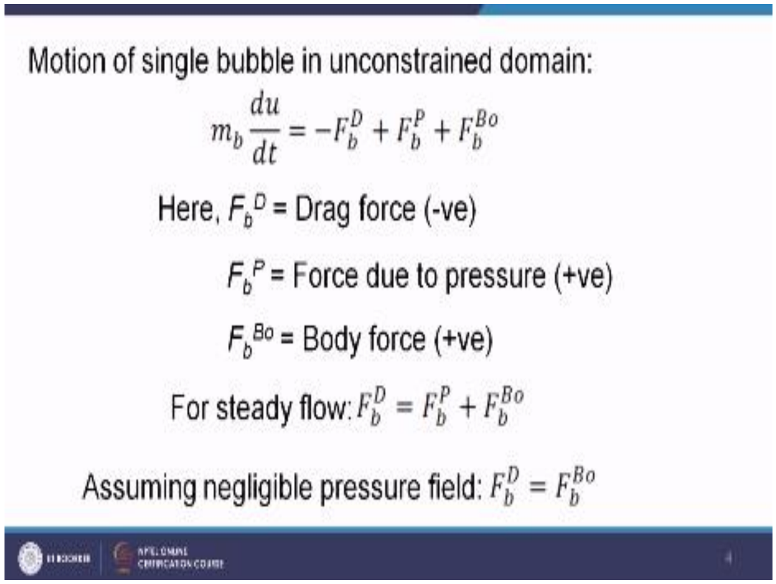

So here we have taken motion of a single bubble in unconstraint domain. Unconstraint domain means you have a very big pool and the effect of wall is not coming into picture whenever the bubble is moving up. So in that situation we will be finding out that here mass of the bubble we have signified as mb, the acceleration of the bubble is du/dt. So, essentially bubble velocity is u that we can find out from the force balance across the bubble.

Here we have taken mainly 3 forces, though apart from these other forces will be also coming into picture. We have taken first the drag force whenever the bubble is moving in the upward side obviously drag will be in the downward side. So we have taken -fDb symbolize bubble we are having pressure force over here fbP so if you are having pressure driven flow, so some pressure forces will be also applicable on the bubble obviously.

Whenever bubble is moving up, the major cause of upward movement will be buoyancy. So if b buoyancy will be over there okay. So let us see all these things individually. So here I have shown drag force is negative and pressure force and body or buoyancy force will be actually positive. Both the things will be positive. Positive means it will be aligned with same direction of u and drag force will be negating that 1 or in the opposite direction.

Now if we talk about steady flow of the bubble let us consider that the bubble is moving up at A, at a constant rate. So in that case we can find out there is no acceleration. So you will be finding out this side is become 0, left hand side is becoming 0 and we will be finding out drag force is equals to pressure force + body force right. Now let us consider further simpler assumptions. So let us consider that we are having negligible pressure field as it is unconstrained bubble.

So we are having a big pool where only due to the body force the bubble is moving up and it is experiencing the drag force. In that case, we can consider that we are having negligible pressure field and we can cancel this first term in the right hand side and we can write down fbD is equals to fb body. So that means body force is equivalent to your drag force. So let us see what are the expressions for both these body force and drag force?

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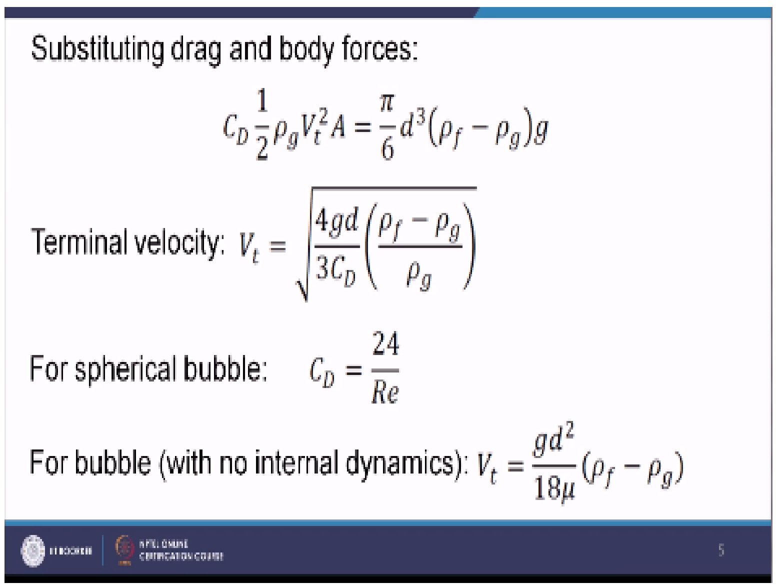

So if you consider about drag force, you see CD ½, CD is the drag coefficient ½ Rho g vt square*A. So this vt is nothing but terminal velocity of the bubble and Rho g is the gas density right on the other hand side. If you talk about the buoyancy force or body force this will be pi/6 d cube where, d is the diameter of the bubble Rho f- Rho g. So, that much amount of body force it will be experiencing due to the density difference of the liquid and gas.

So Rho f is density of liquid Rho g is density of gas multiplied by acceleration gravity gravitational acceleration. So if we equate this 2 from here, we can find out what is terminal velocity. So terminal velocity vt comes out to be root over of 4gd / 3CD (Rho f- Rho g/Rho g) right. Now if we think about very, very small velocity of the bubble that means the bubble is moving very slowly in the upward direction that can be taken as equivalent to creeping flow.

So in case of creeping flow, we know that CD or drag coefficient can be written as 24/Re this comes from the balance of viscous force and your drag force. So if you equate the viscous force and drag force, you will be finding out that necessary condition comes out as CD which is nothing but 24/Re. So once you put this value of CD over here then we will be finding out that the terminal velocity comes out to be gd square /18 Mu (Rho f minus Rho g) right.

So here you see in this condition. We have in this condition or in the previous equation where, we have shown the single bubble motion. We have never consider internal dynamics of the gaseous mass that means we have considered that the bubble is perfectly spherical gas mass and there is no internal dynamics of the gas right. So without considering any internal dynamics we have found out that terminal velocity is gd square/18 Mu (Rho f – Rho g) right.

So it is specifically dependent on the gravitational acceleration diameter is the major factor over here. And viscosity of the fluid will be opposing this one. So that means if you are having high viscous fluid, so terminal velocity will be lessening down okay. Next let us see if we consider internal dynamics of the bubble, so what we will be finding out that? Whenever we are considering internal dynamics, so inside the bubble whenever it is moving up, you will be finding out lots of vortices are generated in the gas.

Now to tackle that one and to find out what is the expression for terminal velocity, probably we need to go for computational fluid dynamics or some sort of analytical correlations.
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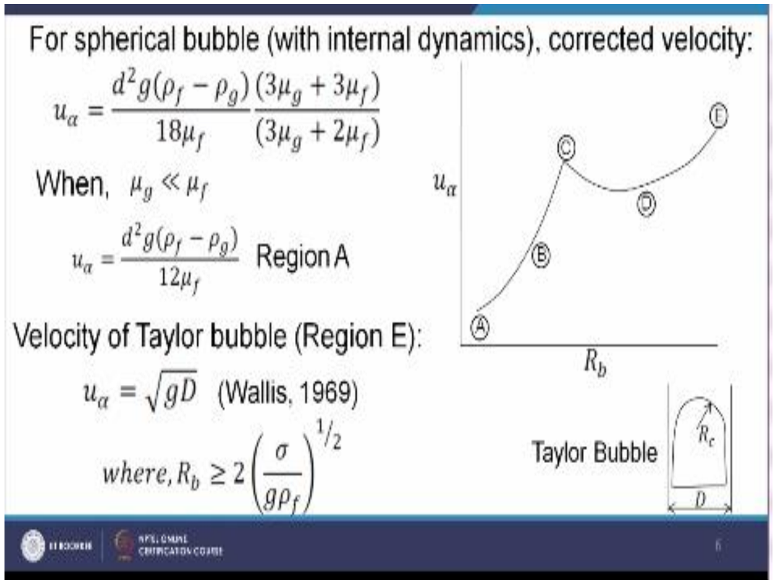

Similar correlation has been given by people. So here we will be showing you 1 correlation given by Wallis. So you see what he has done with the terminal velocity whatever we have obtained in the previous one. So he has obtained over here that we are having a multiplier 3 Mu g+ 3 Mu f/ 3 Mu g+2 Mu f right. So this multiplier, he has given just to accommodate the internal dynamics of the bubble.

So we have found out what is the terminal velocity u, infinitive or ut whatever you call for a bubble spherical bubble in unconstrained domain. Now if let us say in this expression if gaseous phase viscosity is far lower than the liquid phase viscosity that means Mu g < Mu f then in that condition you will find out that this whole expression turns out to be d square g (Rho f- Rho g)/12 Mu f okay.

Now here we have considered that the bubble is actually spherical mass. Now spherical mass of a bubble or gaseous bubble will be only staying in the pool or inside the pipeline. Whenever, the size is very small so actually this is for a small diameter bubble okay. So terminal velocity for a small diameter bubble if you try to plot A, a curve in between the terminal velocity and the bubble radius, we will be finding out that this expressions is valid for a very small diameter.

So we can say somewhere over here this expression is valid so that means this expression can be written somewhere over here in region A okay. So whenever the bubble grows in size, we will find out that spherical nature is not keeping constant. You will be finding out the shape is changing okay. The extreme shape we know that it will be a Taylor bubble. So Taylor bubble already we have seen in case of the flow regime description.

We have shown that it will be a bullet shaped very long bubble okay. Where the frontal side is actually blunt and at the ends at the lower side, you will be finding out the lots of vortices are generating satellite bubbles okay. So the extreme end of this single bubble is the Taylor bubble and the lowest end is actually is a spherical bubble right. So if you see the velocity of the Taylor bubble. So we will be finding out velocity of Taylor bubble is actually dependant on only the pipe diameter.

So we can write down u infinity is equals to root over of grd okay. So this has been given by Wallis. So we will be finding out that this is only dependant on the pipe diameter not on the bubble diameter right. So here this Taylor bubble is regime is somewhere over here which is the largest size of the bubble and here this is a smallest size for which we have found out the terminal velocity in this fashion u infinity = d square g (Rho f – Rho g /12mut okay.

Now we will be having multiple things in between. For example, here you see in between A and E we are having few more regimes okay. If we try to plot the velocity with respect to the radius of the bubble, we are having few more regimes like B, C and D. Now I will show you that what will be the configuration or velocity for those bubbles. Here you see I have given a Taylor bubble shape where, Rc is the critical radius of the no shape over here over the Taylor bubble and this can be some time used as critical diameter for the Taylor bubble.

And predict velocity can be predicted based on that also okay. Now for the rest domains that means B, C and D these are the transition spherical bubble to your Taylor bubble. So velocity finding is once again empirical in this case.
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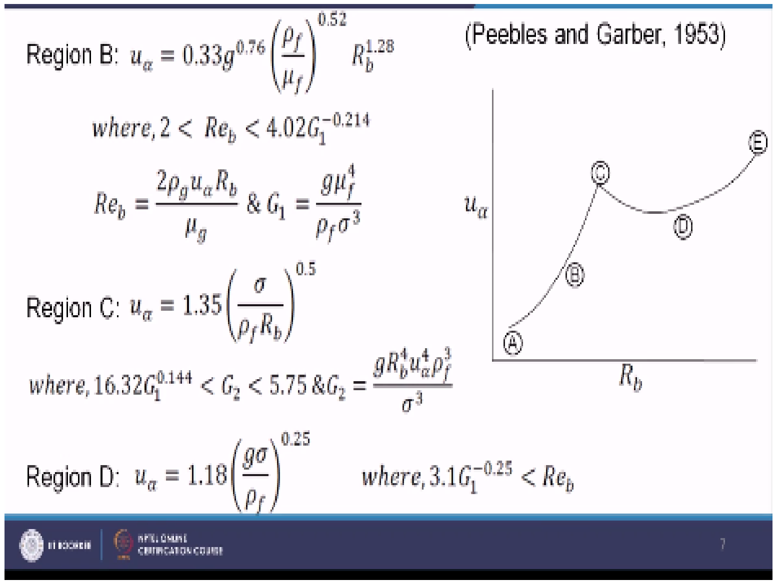

So Peebles and Garber they have given in 1953 some correlations for finding out the velocity in this domains okay. So for first 1 let us say this B. So this B region is actually more near to the spherical region. Though it is not spherical, you can say that is Toroidal bubble. So in this domain you will be finding out velocity u infinity is equals to 0.33 g to the power 0.76 Rho f by Mu f to the power 0.52 into Rb to the power 1.28 right.

Now this Reb is the Reynolds number this is only valid whenever it is in between 2 and 4.02 into g1 to the power -0.214 right. What is this g? g is nothing but once again calculated from the liquid viscosity and the surface tension g Mu f to the power 4/ Rho f into sigma which is the surface tension between the gas and liquid to the power q right. So if this condition for the bubble Reynolds number is applicable then only we will be finding out the terminal velocity using this one.

In a similar fashion for the regime C which is further elongated bubble not a Taylor bubble but highly deviated from a spherical one. You will be finding out the terminal velocity is 1.35*sigma by Rho f*Rb to the power 0.5 where, Rb is the bubble radius. So for this also we have the zone of applicability. So you can find out g2 should be in between 16.32 *g1 to the power 0.144 and it will be less than obviously 5.75.

Now what is this g2 once again it is dependent on the surface tension and physical properties. So g2 is gRb to the power of 4 u infinity to the power 4 Rho f cube/sigma cube right. And for the last 1 which is very near to the Taylor bubble domain okay. So they here I will be finding out that u infinity is 1.18 g sigma/Rho f to the power 0.25 okay. Important thing here you see in this domain also the velocity is not dependant on the bubble diameter right.

So the applicability for domain, this domain is this 1. So Reb, Reynolds number for the bubble should be actually greater than 3.1*g1 to the power -25. Where g1, I have already defined in this place okay. Now let us see when what happen if you are having bubbly flow inside a tube.

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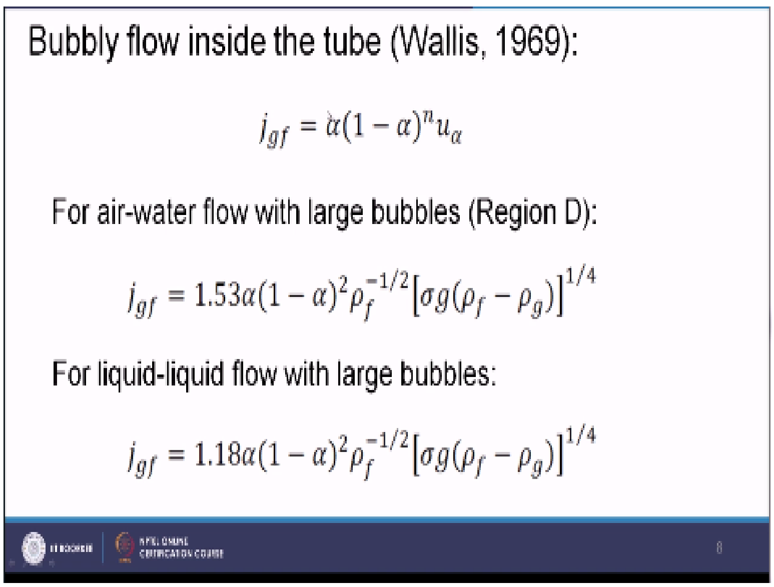

So already we have seen that in case of a bubbly flow jgf which is the drift flux will be alpha (1-alpha) to the power of n into u infinity. Now this u infinity is once again freely raising terminal velocity and alpha (1 – alpha) that comes as actually pre-factor. Already we have seen gfg = alpha (1- alpha) ufg. Here this ufg is nothing but u infinity*(1-alpha) to the power n-1. So if you club this 2, you will be finding out this expression. So this has been proposed by Wallis okay.

He was also proposed further for air water flow with large bubbles which is actually the region D over here. This 1 is very large bubble. So you will be finding out jgf = 1.53 alpha (1-alpha) square Rho f to the power -1/2 *sigma * g Rho f – Rho g to the power 1 /4 okay. Now this is for air, water, gas, liquid if we go for liquid, liquid only the factor 1.53 will be changing to 1.18

right. Okay now as we have talking about bubbly flow we will be having a cluster of bubbles. So not only the velocity and reflux will be important.

We need to also find out that what is the number distribution or volume distribution of the bubbles inside the domain right. So, to know the number distribution inside the bubbly flow for different size bubbles, what we have to do. We have to go for size distribution in the cross section of the tube okay.

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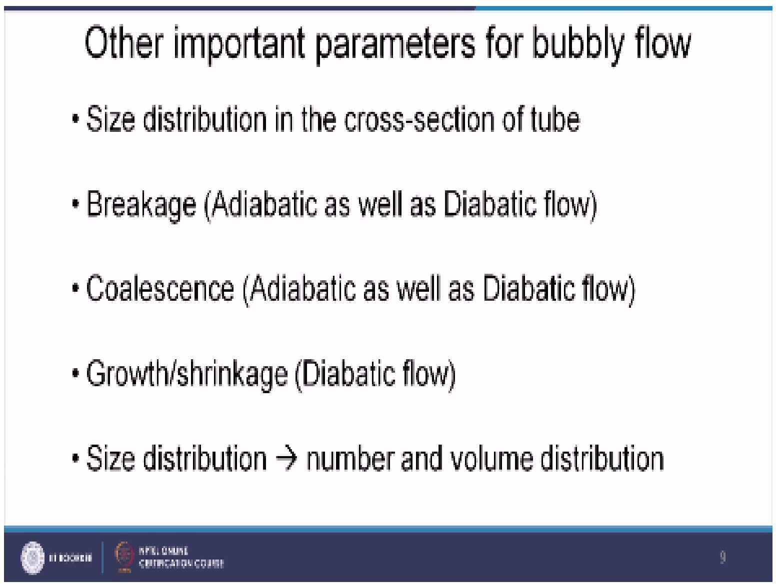

Now to tackle the size distribution in a proper way, we have to see the bubble dynamics. So what is bubble are doing bubbles can collide among themselves and break into further smaller sizes. And during collision what it can do it can merge with some other bubble and form a bigger bubble due to collisions. It can also nucleate from a surface during phase change basically. It can also growth or shrinkage if you have heat transfer inside this okay.

So all this phenomenon it can do breakage collisions nucleation growth shrinkage due to this we will be finding out number density is being changed okay. Size distribution and number distribution we have to take care of, so let us see first how size distribution varies. So if you try to see in a typical bubble population based on the diameter, how number distribution varies. We have to go, we have to take some distribution and typically we take normal distribution.

So if you see the curve of the number distribution, you will be finding out the curve is like this.

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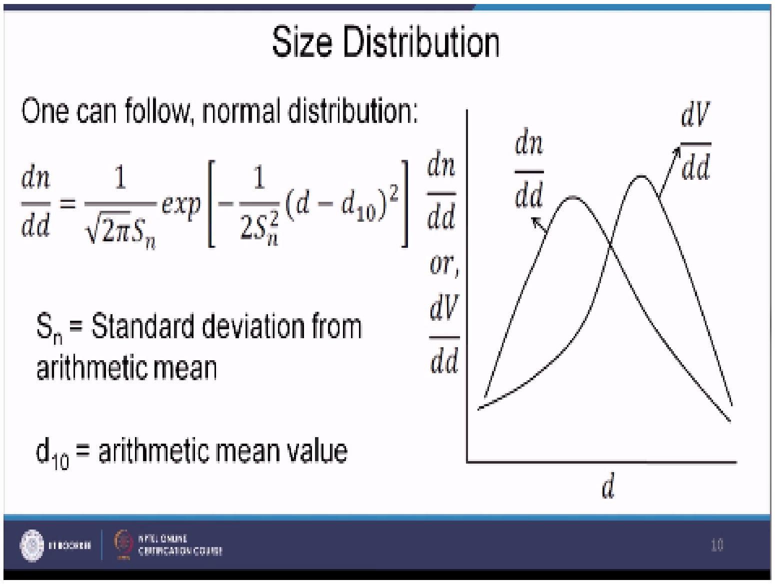

So for the intermediate domain you will be finding out a large number of bubbles that means intermediate size will be getting more in number right. So we can we can replicate this 1 as normal distribution. So normal distribution one can write as dn/dd. So this d is a diameter, this n is a number is equals to 1 /root over of 2 pi Sn. Where Sn is the standard deviation from arithmetic mean e to the power -1 /2 Sn square (d -d10) whole square.

Now this d10 is the arithmetic mean of the bubble diameter right. Now in this same figure I have shown you the volume distribution also. So obviously we know as number distribution is giving you only the number count volume distribution will be actually shifting towards right okay. So here this is the number distribution. So, if you because volume is actually to the power cube.

So length square length to the power cube so you will be finding out the curve will be shifting this side. So we get this curve of dv/dd with respect to d. So let us see if we are having some other options for the number density distribution. So already you have shown the normal distribution. Here I am going to show you the log normal distribution. This is also being applied for multiple cases.

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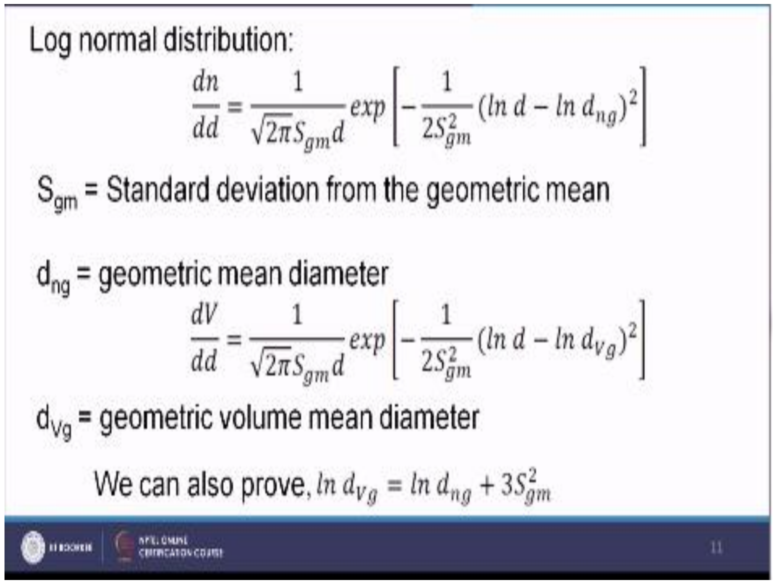

So you can find out dn/dd is = 1 /root over of 2 pi Sgm. So Sgm is a standard deviation from the geometric mean. In the previous case that was from the arithmetic mean this is from the geometric mean*d *e to the power -1/2 * Sgm square into ln d- of ln d ng. Now this dng is nothing but geometric mean diameter.

Earlier in case of normal we have taken d10 which was the arithmetic mean right. So this also we can use log normal distribution. I have already told about volumetric distribution. So dv/dd = 1 /root over of 2 pi sgm d *the power -1 /2 is sgm square (lnd -lnd vg square). Now this dvg is nothing but geometric volume mean diameter. So earlier I have shown you dng, this is geometric mean diameter based on the number account. Here it is geometric volume mean diameter.

So if you just playing with this parameters sgm, dvg and dng, you can show this expression ln, dvg= ln dng+3 * sgm whole square this is coming from mathematics okay. Now for bubble mass it is very important to know what is the mean diameter. So what we do there are several ways for you defining mean diameter.

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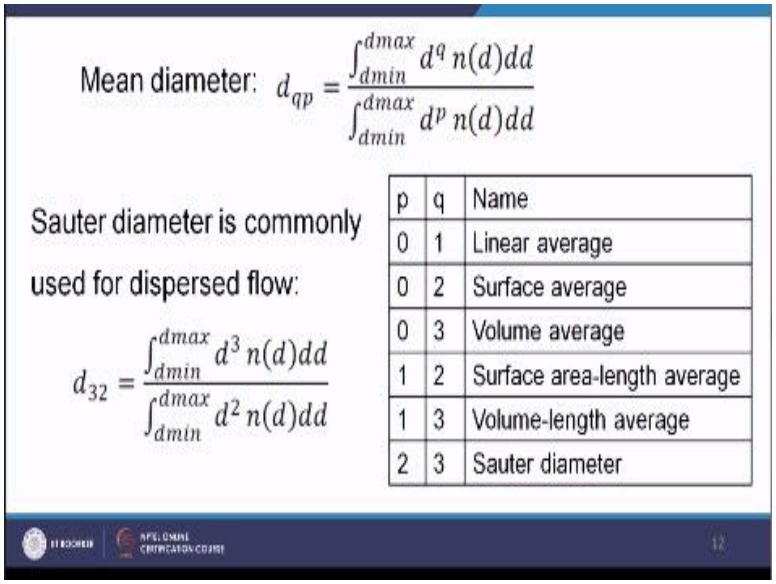

If you generalize that we can write down that mean diameter dqp = dmin to dmax, this dmin is the minimum size of the bubble available in the population and dmax is the maximum 1. So this integration will be doing from dmin to dmax and the integration will be d to the power q n (d). So n(d) is the number count of dx size of the bubble into dd. So the second d is for diameter and first d for the derivative and then dmin to dmax once again integration d to the power p n (d) dd okay.

So this p and q is a power over here. So as a result this is dqp right. So several possible values can be there for q and p. So for example 0 and 1 if we take that will be over here, p0, q is 1 that will be linear average. If 0 and 2 that is surface average, if you take 0 and 3 that is volume average, if you take 1 and 2 that is surface area length average. So that means if I take 1 over here so n (d) dd and d square n (d) dd that is surface area length average.

If you go for 1 and 3 that means dq n (d) dd and d n (d) dd. So that is actually volume length average and finally the most important is Sauter diameter which is nothing but p = 2 and d = 3.So I have shown over here what is Sauter diameter this basically we used for our bubbly flow calculations. So Sauter diameter can be written as dmin to dmax dq n (d) dd and dmin to dmax d square into n (d) dd right.

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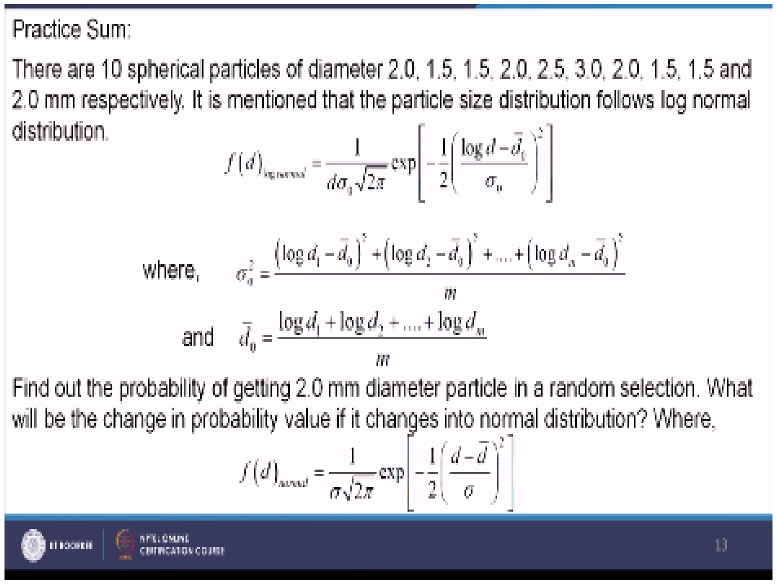

Let us practice a sum, so what we will be considering that we are having population 10 spherical particles of diameter 2, 1.5, 1.5, 2, 2.5, 3, 2, 1.5 and 1.5. So 10 diameters are there and it is mentioned that the particle size distribution follows log normal distribution. So the expression for log normal distribution is this 1 where, sigma 0 square is can be found out in this way log (d1-d0) whole square and then we can put a summation for all the particle diameters divided by m number of particles.

And d0 can be found as log d1 summation of log d1/m okay. So we have to find out the probability of getting 2 millimeter diameter particle in the random selection and we have to also find out what will be the change in probability value if it changes from norm changes into normal distribution from this log normal distribution. Normal distribution is also given over here.

Let us see how this sum can be solved. So first this d0, so you can find out using this expression. I will be calculating the d0 value so if I put all the particles diameter over here d0 comes out as 0.47872 in a similar fashion. Let me calculate sigma 0 square using this expression so that would be finally coming out to be sigma 0 square is 0.07809 now once.

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Solution:

Log Normal distribution:

$$\bar{d}_o = [ln2.0 + ln1.5 + ln1.5 + ln2.0 + ln2.5 + ln3.0 + ln2.0 + ln1.5 + ln1.5 + ln2.0]/10$$

$$\bar{d}_o = 0.47872$$

$$\sigma_o^2 = \frac{[4 \times (0.6931 - 0.47872)^2 + 4 \times (0.4055 - 0.47872)^2 + (0.9162 - 0.47872)^2 + (1.0986 - 0.47872)^2]}{10}$$

$$\sigma_o^2 = 0.07809$$

$$\sigma_o = 0.2794$$

$$F(z) = \frac{1}{2 \times 0.2794 \times \sqrt{2\pi}} exp\left[-\frac{1}{2}\left(\frac{ln2 - 0.47872}{0.2794}\right)^2\right]$$

$$F(z) = 0.532$$

I put this d0 and sigma 0 square in fz okay. So fz is nothing but this expression if I put over here so I will be getting it as 0.532, here important thing is that this d, I will be putting whatever diameter I want to get which is 2 mm right. So this is coming out from the log normal distribution. Same calculation I will be repeating for normal distribution. Here I will be finding out d bar which is the arithmetic mean. So it will be coming out to be 1.95.

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Normal Distribution:

$$\bar{d} = \frac{(2 + 1.5 + 1.5 + 2 + 2.5 + 3 + 2 + 1.5 + 1.5 + 2)}{10}$$

$$\bar{d} = 1.95$$

$$\sigma^2 = \frac{[4 \times (2 - 1.95)^2 + 4 \times (1.5 - 1.95)^2 + (2.5 - 1.95)^2 + (3 - 1.95)^2]}{10} = 0.2225$$

$$\sigma = 0.4717$$

$$F(z) = \frac{1}{0.4717 \times \sqrt{2\pi}} exp\left[-\frac{1}{2}\left(\frac{2 - 1.95}{0.4717}\right)^2\right]$$

$$F(z) = 0.8348$$

I will be calculating the sigma square for the arithmetic mean ways. So that is actually deviation from d bar and whole Square of that 1. So that will be coming out as 0.4717. So if I use the normal distribution then I will be getting this fz comes out to be 0.8348. So from log normal

0.532 it is transforming into normal .8348 for getting 2 millimeter diameter particle. Now to summarize in this lecture, what we have done.

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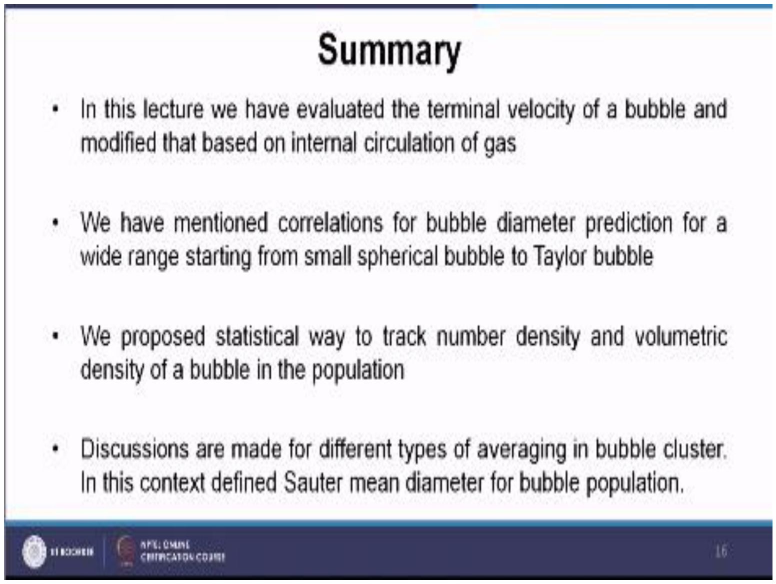

We have evaluated the terminal velocity of a bubble and modified that based on internal circulation of the gas. We have mentioned correlations for bubble diameter prediction starting from small spherical bubble to larger Taylor bubble regime. We have also proposed statistical way to track the number density and volumetric density and at the end we have practiced a sum right.

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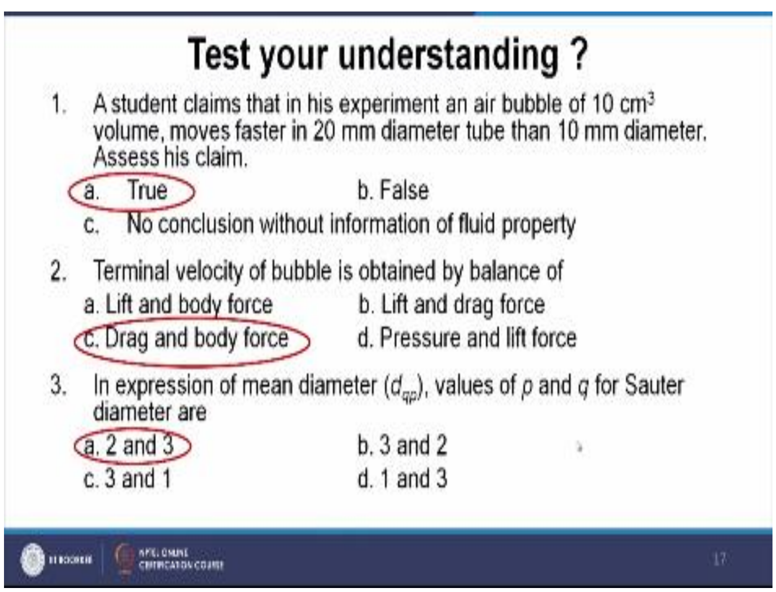

So at the end of this lecture let us test your understanding we are having 3 questions over here. The first question goes like this, if a student claims that in his experiment an air bubble of 10 cm cube volume moves faster in 20 mm diameter tube than 10 mm diameter. You have to assess whether he is claiming correct or not. We are having 3 answers over here true, false and no conclusion.

Without information of fluid property here you see the correct answer obviously will be true because you see, you will be finding out here that volume is over here. The tube diameter is 20 millimeter and here tube diameter is 10 millimeter. So obviously, his claim will be true. In the next question we are having terminal velocity of bubble is obtained by balance of lift force and body force lift force and drag force drag force and body force and finally pressure and lift force.

So the correct answer you know will be drag force and body force. In expression of mean diameter dqp values of p and q for Sauter diameters are 4 options we are having 2 and 3, 3 and 2, 3 and 1 and finally 1 and 3 the correct answer is 2 and 3 okay. Hope you have enjoyed this lecture. Thank you.