

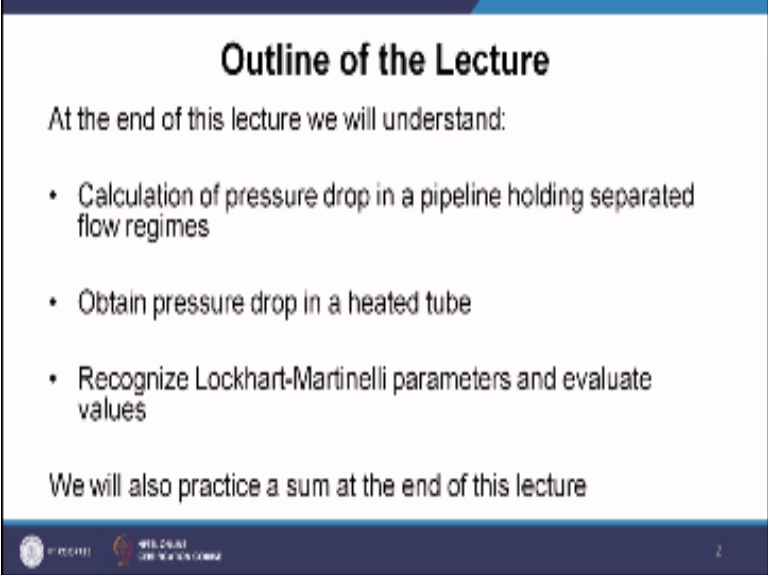
Two Phase Flow And Heat Transfer
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Lecture No: 5
Separated Flow Model

Welcome to the course Two Phase Flow and Heat Transfer. Today we will be dealing with the fifth lecture of this course and this course this lecture is about separated flow model okay. Now if you remember in our first lecture in our first lecture wherever we have given the nomenclatures we have told you about the drift flux velocities drift velocities and that we have used in our last lecture drift flux model.

Here in a separated flow model we will be considering that the both phases are actually flowing separately and there is relative velocity existing between these 2. Unlike your drift flux model in separated flow model we will be trying to capture the mass momentum and energy equations for the phases separately. So let us first see that what we will be learning in this lecture.

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Outline of the Lecture

At the end of this lecture we will understand:

- Calculation of pressure drop in a pipeline holding separated flow regimes
- Obtain pressure drop in a heated tube
- Recognize Lockhart-Martinelli parameters and evaluate values

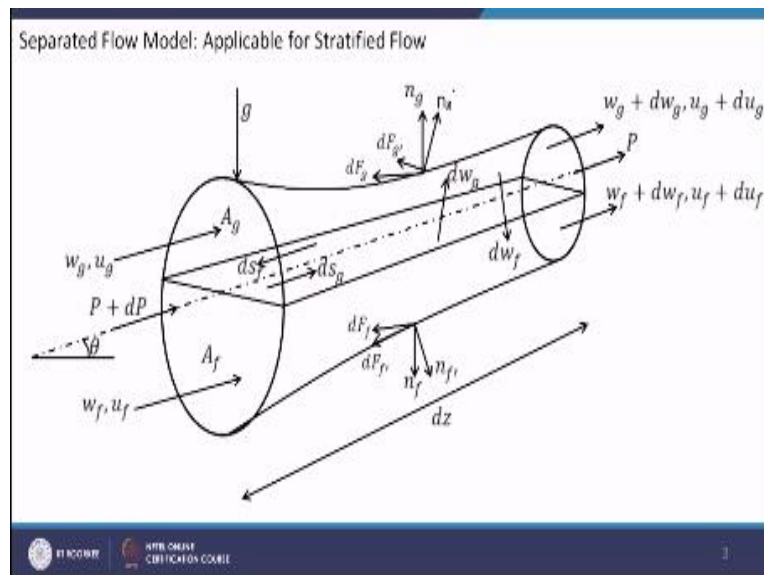
We will also practice a sum at the end of this lecture

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So at the end of this lecture we will understand the calculation of pressure drop in a pipeline holding separated flow. So basically, we will be giving you how to calculate the pressure drop as we have shown you in case of homogeneous flow. We will obtain pressure drop in a heated tube. So from the adiabatic situation will be converting into heated tube situation. So heat is coming from the periphery of the tube.

So in that case phase change how it can be taken care of in separated flow model that we will be learning. We will recognize Lockhart-Martinelli parameters and evaluate its values from Martinelli Nelson charts. Also, we will be practicing a sum at the end of this lecture. So let us now go to a situation where separated flow is occurring. So here once again just like your drift flux model I have shown you a schematic diagram of a pipe carrying separated flow.

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So basically, you can find out separated flow is applicable for situations like stratified flow so here I have shown you a stratified situation for liquid and gas. So liquid is at the downward side and gas is at the upward side. And here to make it generalized we have considered the pipe is changing its diameter as well as we have considered that this is making some angle with horizontal which is theta right.

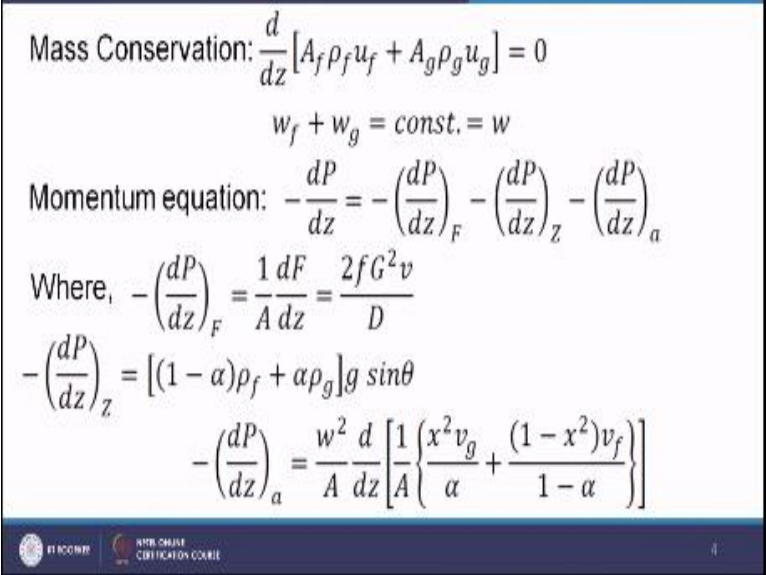
So let me explain the other terminologies over here. So you can see here we have considered that mass flow rate for the liquid is w_f and velocity for the liquid is u_f . So whenever it is exiting from the pipe the mass flow rate changes to $w_f + dw_f$ and velocity changes to $u_f + du_f$. Similarly, for the gaseous phase w_g and u_g is at the entry and $w_g + dw_g$ and $u_g + du_g$ is at the exit. Now let us talk about pressure as we know that pressure is dropping down so at the inlet we will be finding out that the pressure is $P + dP$ and obviously at the exit it will dropping down to P .

Here, what we have considered that the area occupied by the liquid phase is A_f and area occupied by the gaseous phase is A_g right. So this is the interface between the phases. So for across this interface you will be finding out that we are having the interfacial forces ds_f in the liquid phase and ds_g in the gaseous phase. Also we are having mass transfer due to phase change.

So what we have considered that dw_g amount of mass is actually being accepted by the gaseous phase and dw_f amount of mass is actually accepted by the liquid phase. Apart from that we have also considered the frictions. So here we are having frictions dF_f and its perpendicular direction is nf dashed and if we considered the gravity in the vertical direction so you will be finding out its components are nf and dF_f .

Similarly for the gaseous phase we are having the friction factor as dF_g okay. And its component in the horizontal and vertical directions is dF_g okay. So with this let us try to construct the momentum and continuity equations for separated flow. So first I will be showing you the mass conservation equation.

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Mass Conservation: $\frac{d}{dz} [A_f \rho_f u_f + A_g \rho_g u_g] = 0$

$$w_f + w_g = \text{const.} = w$$

Momentum equation: $-\frac{dP}{dz} = -\left(\frac{dP}{dz}\right)_F - \left(\frac{dP}{dz}\right)_z - \left(\frac{dP}{dz}\right)_a$

Where, $-\left(\frac{dP}{dz}\right)_F = \frac{1}{A} \frac{dF}{dz} = \frac{2fG^2v}{D}$

$$-\left(\frac{dP}{dz}\right)_z = [(1 - \alpha)\rho_f + \alpha\rho_g]g \sin\theta$$

$$-\left(\frac{dP}{dz}\right)_a = \frac{w^2}{A} \frac{d}{dz} \left[\frac{1}{A} \left\{ \frac{x^2 v_g}{\alpha} + \frac{(1 - x^2)v_f}{1 - \alpha} \right\} \right]$$

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So you see here in the mass conservation equation we have added up both the mass conservation equations for gaseous and liquid phase so $d/dz [A_f \rho_f u_f + A_g \rho_g u_g] = 0$. Here what we have considered dw_g and dw_f in the individual equations, $d/dz [A_f \rho_f u_f]$ will be

equals to $w_f dw_f$ and $\frac{d}{dz} [\alpha g \rho_f + (1-\alpha) g \rho_g]$ will be equivalent to your $\frac{d}{dz} w_g$ right. So whenever we add those 2, then you will be finding out these 2 term will be canceling out and finally you will be getting the mass conservation equation like this okay.

Now all of we know that $A_f \rho_f u_f$ is nothing but w_f and $A_g \rho_g u_g = w_g$. So you can write down $w_f + w_g$ is equal to a constant term and we write down that one as overall mass flow rate w right. Now let us go to the momentum equation, so just like our homogenous flow model we will try to write down the momentum equation as $-\frac{dP}{dz}$ is equivalent to addition of 3 pressure drops.

So first one is occurring due to the friction and second one is occurring due to the accelerational due to the gravity or buoyancy and third one is occurring due to the accelerational a right. Now already in a homogenous flow model we have described that how the frictional pressure drop comes into picture. Here also I have shown that $-\frac{dP}{dz}$ at for the friction can be written as $\frac{1}{A} \frac{dF}{dz}$ where, A_f is nothing but the frictional force okay.

And if you try to write down this frictional force in terms of the shears stress and the perimeter then we have already shown this can be converted to a simplified equation like this $2\tau G$ square specific volume divided by the tube diameter okay. This already we have proved in the homogenous flow model lecture.

Now for the buoyancy part that means gravitational part you can write down $-\left(\frac{dP}{dz}\right)_z = [(1-\alpha) \rho_f + \alpha \rho_g] g \sin \theta$. The first part, $(1-\alpha) \rho_f g \sin \theta$ that comes from the liquid momentum equation. And $\alpha \rho_g g \sin \theta$ that comes from the gaseous momentum equation. The most important and vital term is $\left(\frac{dP}{dz}\right)_a$, so you see $\left(\frac{dP}{dz}\right)_a$ will be this expression.

So let us try to understand how this expression came. So let me show you that how this accelerational part comes into picture.

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$$\begin{aligned}
& \frac{1}{A} [A_f \rho_f u_f^2 + A_g \rho_g u_g^2] \\
& \frac{1}{A} \left[\frac{A_f^2 \rho_f^2 u_f^2}{A_f \rho_f} + \frac{A_g^2 \rho_g^2 u_g^2}{A_g \rho_g} \right] \\
& \frac{1}{A} \left[\frac{W_f^2}{A(1-x)\rho_f} + \frac{W_g^2}{A x \rho_g} \right] \\
& \frac{1}{A} \left[\frac{W^2 (1-x)^2}{A(1-x)\rho_f} + \frac{W^2 x^2}{A x \rho_g} \right] \\
& \frac{W^2}{A} \left[\frac{(1-x)^2}{A(1-x)\rho_f} + \frac{x^2}{A x \rho_g} \right]
\end{aligned}$$

In your momentum equation if you see the accelerational part due the inertia, will be like this $1/A [A_f \cdot \rho_f \cdot u_f^2]$ this is for the liquid part and $[A_g \cdot \rho_g \cdot u_g^2]$ this is for the gaseous part. Now let us see how this can be simplified. So what we can do $1/A$ here, $A_f^2 \rho_f^2 u_f^2 / A_f \rho_f + A_g^2 \rho_g^2 u_g^2 / A_g \rho_g$ right. So here you see $A_f \rho_f$ and u_f that can be written as G_f^2 and A_f we can write as $A(1-\alpha) \rho_f$ + this one.

Once again $A_g \rho_g$ and u_g can be written as G_g okay* A_g can be written as $A \cdot \alpha$ okay. So this we get multiplied by $1/\alpha$ right. Now here we know G_f can be written as $G^2 (1-x)$ whole square okay. Okay, in a similar fashion G_g can be written as $G^2 \cdot x^2$ from the definition of the mass quality okay. This becomes $A \cdot \alpha \cdot \rho_g \cdot 1/A$ okay. So we get over here you see G^2 we can take common okay.

G^2/A and then $(1-x)^2 / A(1-\alpha) \rho_f + x^2 A \alpha \rho_g$ right. So this type of term we will be getting from the inertial pressure drop. So here similar type of things I have shown you over here you see this one is W^2/A okay. By the way a small nomenclature problem is here. This will be actually w okay. This will be actually w so it can be replaced by w . Please make the necessary corrections so this can be replaced by w .

So, here this will be coming as w . So you can find out it is becoming $w^2/A (1-x)$ whole square / $A (1-\alpha) \rho_f + (x^2/A * \alpha * \rho_g)$. So same term I can show you over here. So you see $w^2/A d/dz$ because this dp/dz . So d/dz will be remaining over here. $1/A * x^2$, now this ρ_g has been written as $1/v_g$ and over here ρ_f has been written $1/v_f$. Okay, so this is the necessary term for accelerational pressure drop okay.

So, all these 3 terms will be coming into picture in the momentum equation. And here you can find out the momentum equation can be written as $-dP/dz$ equals to summation of the frictional gravitational and accelerational pressure drop right. Next let us try to find out the frictional pressure drop.

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Assuming single phase flow through tube:

$$-\left(\frac{dP}{dz}\right)_{F,tp} = -\left(\frac{dP}{dz}\right)_{F,SF} \quad \phi_{fo}^2 = \frac{2f_{fo} G^2 v_f}{D} \quad \phi_{fo}^2 \quad \text{Liquid phase throughout tube}$$

$$-\left(\frac{dP}{dz}\right)_{F,tp} = -\left(\frac{dP}{dz}\right)_{F,SG} \quad \phi_{go}^2 = \frac{2f_{go} G^2 v_g}{D} \quad \phi_{go}^2 \quad \text{Gas phase throughout tube}$$

$$-\left(\frac{dP}{dz}\right)_{F,tp} = -\left(\frac{dP}{dz}\right)_{F,gas} \quad \phi_g^2 = \frac{2f_g G^2 x^2 v_g}{D} \quad \phi_g^2 \quad \text{Gas portion only}$$

$$-\left(\frac{dP}{dz}\right)_{F,tp} = -\left(\frac{dP}{dz}\right)_{F,liquid} \quad \phi_f^2 = \frac{2f_f G^2 (1-x)^2 v_f}{D} \quad \phi_f^2 \quad \text{Liquid portion only}$$

From above, we can get:

$$\phi_{fo}^2 = \phi_f^2 (1-x)^2 \frac{f_f}{f_{fo}} = \phi_g^2 x^2 \frac{v_g}{v_f} \frac{f_g}{f_{fo}} = \phi_{go}^2 \frac{v_g}{v_f} \frac{f_{go}}{f_{fo}}$$

So already we know that frictional pressure drop can be written as $-dP/dz$ okay. Frictional for Two Phase, now what we can do? There are 4 different situations, what we can assume. We can assume that in place of the 2 phase inside the pipeline. Only single phase flow is occurring sf means single phase and we can calculate the value of the friction factor considering the single-phase fluid flow okay.

sf mean single phase fluid flow okay and then multiply with a parameter over here which is ϕ fo square. Now f_o symbolizes fluid only okay. What we can do the Two-Phase friction factor, Two Phase pressure drop? We can calculate using the single-phase liquid only or fluid only

pressure drop multiplied by a parameter which is ϕf_o square okay. Now calculating why we are doing, so because calculating single phase liquid friction factor is very easy because we know, what are the parameter that means density and viscosity for single phase liquid.

So quickly we can calculate what the Reynolds number is and based on the Reynolds number we can go for either $64/Re$ or Blasius equation okay. So we can find out for turbulent and laminar regime right now. We need to know these parameter ϕf_o square okay. To relate this Two phase friction factor with the single phase fluid only friction factor, so as I have told you that single friction factor can be written as 2 into F_{fo} fluid only friction factor.

So this you need to calculate using the liquid properties only and (ϕ) 13:37 will be calculated based on liquid properties only okay and then rest things will be similar $G^2 \cdot v_f / d$ okay. Now this function, already I have shown you in the previous slide. You see here, I have shown you $2f \cdot G^2 \cdot v / d$. So in case of single phase liquid only this f will be converted into f_o , f_{fo} and in case of this v , we will be writing v_f okay.

So same thing we have written over here and this multiplier is remaining over here okay. Somehow, we need to know this multiplier then using this single-phase liquid only friction force. We can find out the Two-Phase friction force. So this is 1 idea that liquid phase through the tube if we consider we can find out the Two Phase friction force also okay. Similarly, we have considerations like gas phase through the tube.

So what we have done over here? You see here Two Phase friction force can be calculated using a single-phase gas only friction force multiplied by a factor ϕg only okay. ϕg only square, so this is the multiplier somehow, we need to find out this ϕg okay. There are so many things I will be telling you later on how to find out this ϕg okay. Now if we are considering that the whole pipeline is occupied by the gaseous phase.

So obviously, the friction factor or friction force will be $2 \cdot f_{go}$ gas only. So friction factor will be calculated based on the gas properties, densities and viscosities. As well as we are having the gaseous density over here or a gaseous specific volume over here in the picture okay multiplied

by ϕ go square. Now 2 more considerations are there. Also here in this first 2, we have considered the whole pipeline is occupied by the liquid and gas here.

In the second, third and fourth, we will be considering that Two Phase Flow is there inside the pipeline but we are only interested in the gaseous phase or liquid phase. So here the consideration is not like this. That the whole pipeline is occupied by the gas Two Phase is there but we are only calculating from the gaseous portion. So let us see what happens over here. So, friction force for the Two Phase can be written as dP/dz friction force for the gas part only.

So remember, this is not gas only, this is gas part only and then you have a multiplier ϕ G square okay. So this is not ϕ g only, this is ϕ G square okay. Here you can find out that we have to find out the friction factor as $2fg$. Now the mass whatever we have to written down that now will not be coming G because we are not considering the whole pipeline is occupied by the gaseous phase over here.

We are only considering the gaseous portion. So gaseous mass we need to take. So, gaseous mass is nothing but capital g, small gg. So, that gg can be written as $g^2 \cdot x^2$ because we know that $gx = gg$ right. So this gives you the friction factor for the gaseous phase, gaseous portion only multiplied by this factor remember this fg will be also considered based on the gaseous phase properties.

So Reynolds number you have to calculate based on the gaseous phase properties. Similarly, we can go for liquid portion. So here you see, we will be only considering the liquid portion over here. So $-dp/dz$ friction factor at liquid portion only then ϕf square, ϕf is the liquid portion only. This is not only considering that the whole pipeline is occupied by liquid this is only the portion of the liquid we are considering right.

So the friction factor will be $2 \cdot ff$. So all these ff will be calculated based on the Reynolds number calculated with the liquid properties and then gf will be written over here. So gf square, so gf we can write down as $g^2 (1-x)^2$ whole square multiplied/ $v_f / d \cdot \phi f_0$. If you compare these equations so you can correlate between the multiplier ϕf_0 , ϕ go, ϕ g and ϕf . So this

is very simple, so if you just compare then you will be finding out ϕ_{fo}^2 will be actually $\phi_{go}^2 \cdot v_g / v_a \cdot \phi_{fo} / f_{fo}$ right.

In the similar fashion we can also equate this fluid only your gas only parameters with gas portion and liquid portion multipliers. So ϕ_{fo}^2 can be written as $\phi_f^2 \cdot (1-x)^2 \cdot f_f / f_{fo}$. Similarly, ϕ_{fo}^2 can be written as $\phi_g^2 \cdot x^2 \cdot v_g / v_f \cdot f_g / f_{fo}$. So these equations will be coming just by comparing these sides. So left hand sides are all equal, so if you compare the right hand side, you will be getting these equations right.

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$$-\frac{dP}{dz} = \frac{H_1}{H_2}$$

Where,

$$H_1 = \frac{2f_{fo} G^2 v_f}{D} \phi_{fo}^2 + G^2 \frac{dx}{dz} \left[\left(\frac{2xv_g}{\alpha} - \frac{2(1-x)v_f}{1-\alpha} \right) + \left(\frac{\partial \alpha}{\partial x} \right)_P \left\{ \frac{(1-x)^2 v_f}{(1-\alpha)^2} - \frac{x^2 v_g}{\alpha^2} \right\} \right]$$

$$- \frac{G^2 dA}{A dz} \left[\frac{x^2 v_g}{\alpha} + \frac{(1-x^2)v_f}{1-\alpha} \right] + [(1-\alpha)\rho_f + \alpha\rho_g]g \sin\theta$$

$$H_2 = 1 + G^2 \left[\left\{ \frac{x^2}{\alpha} \frac{dv_g}{dP} + \frac{(1-x^2)}{1-\alpha} \frac{dv_f}{dP} \right\} + \left(\frac{\partial \alpha}{\partial P} \right)_x \left\{ \frac{(1-x)^2 v_f}{(1-\alpha)^2} - \frac{x^2 v_g}{\alpha^2} \right\} \right]$$

Next let us try to see that if we try to put all these 3 factor so friction factor, frictional pressure drop, gravitational pressure drop and accelerational pressure drop in the final equation then we will be getting $-dP/dz = h_1 / h_2$. Now let us try to identify different portions in this h_1 and h_2 . So in h_1 obviously you can find out this is coming from the friction factor. We have only considered the fluid only assumption.

So if you are going for gas only or liquid portion or gas portion, accordingly this portion needs to be modified okay. Any option you can take okay. Then here you see the last term is due to the accelerational, due to the buoyancy part $[(1-\alpha)\rho_f + \alpha\rho_g]g \sin\theta$. So this term I have already shown you over here, $(1-\alpha)\rho_f + \alpha\rho_g \cdot g \sin\theta$. Now rest terms whatever we are seeing over here these are actually coming from your accelerational part.

Now let us discuss about the accelerational part here you see the accelerational part we are having d/dz of some term. Now here we can find out we are having 4 different types of terms. So 1 is A. So obviously, A is varying with respect to z. Another 1 is alpha. So obviously alpha is varying with respect to z. We are having x quality, mass quality this is also varying with respect to your z. As well as we can have variation of v_g and v_f with respect to not z with respect to p.

As p is varying inside the pipeline whenever you advance forward, we will be finding out the v_g and v_f value will be also changing. So we will be having partial derivative of 4 different terms okay. Now as I have mentioned that this v_g and v_f is actually functional pressure. So whenever we are doing d/dz , basically we have to multiply dP the denominator and numerator we have to get d/dP of and then dp/dz as multiplier.

So once you have dp/dz in the right hand side, left hand side also we are having dp/dz . So dp/dz will be coming at the bottom side. Whenever we are finding out the dp/dz so, you will be finding out in the last expression. If you see in h_2 we are having 1 okay, plus this term. So this term is actually due to the differentiation with respect to pressure. You see this x^2/α remains over there.

So this is d/dP of v_g okay. So we had v_g once we do d/dp . So you will be having d/dp of v_g . Similarly, here we are having d/dp of v_f okay. Now whenever we are making the derivative with respect to pressure, we know that alpha is also a parameter. So you have to find out $d/d\alpha$, d/dP of alpha and then once again $d/d\alpha$ of this term. So here what we have done? Kept the $(1 - x^2) * v_f$ has constant and we have made the derivative of $(1 - \alpha)$. Here $(1 - \alpha)$ with respect to alpha.

So it becomes $(1 - \alpha)$ whole square with minus sign okay. And then $(1 - \alpha)$ if you make the derivative with respect to alpha, this will give another minus sign. So minus becomes plus. So you will be getting + sign over here right. On a similar fashion if you make the derivative of this term. This term means $x^2 \alpha * v_g$ okay with respect to alpha then you will be getting $-\alpha^2$, $-1/\alpha$ and $x^2 v_g$ will be remaining like this okay.

So this term is actually coming for the adhesion of d/dP of the accelerational part. Now rest terms that means you are having few new terms also over here. So apart from vg , we are also having x , a and α . So those variations are over here. First 1 is over here which is nothing but derivative of the x . So what we have done we had d/dz of the terms. So we have done dx/dz and then d/dx of these terms.

So if you do d/dx of x square, you will be getting $2x$. If you do d/dx of $(1-x)$ square, you will be getting $2(1-x)$ with the minus sign because $(1-x)$ will be giving you minus 1. And then finally if you do this $d/d\alpha$, so this is the variation of α . So if you see over here $(1-x)$ whole square into vf and then once again for $(1-\alpha)$ square what we have done it was $(1-\alpha)$. So $(1-\alpha)$ whole square with a minus sign and $(1-\alpha)$ will be getting once again differentiated.

So that gives another minus sign okay. So minus becomes plus over here and for this term okay x square vg/α square, it becomes x square vg/α square with a minus sign right. And the last term over here this is due to the d/dz of A okay because area also will be changing with respect to z right. So these 3 terms are not having dp/dz involved in this but this term is having dp/dz involved.

So once we write down or add all those terms, so this term will be going in the left hand side. And dp/dz , dz if you take common then it will be 1 plus this 1. So as a result the overall dp/dz we can write down as h_1/h_2 okay. So this is the expression with h_1 and this is the expression with h_2 . So here in h_1 this is once again from the friction, this is once again from the buoyancy. Rest 3 terms are coming from your accelerational and in h_2 this last term involving G square and this and this term it is coming from once again accelerational right.

Next let us try to see what happens. If we go for a uniformly heated tube of diameter D okay, so what we have done from this expression? We will try to find out what does the pressure drop for finite amount of length wherever heat is given from the periphery okay. So let us try to see over here. So what we have done basically before coming over here. I will be showing you that in h_2 .

This term is actually equals to or nearly equals to 0 for most of the liquids. Whatever, we have in daily day life like water air and all these things. So what we have done? Whenever we have derived the pressure drop in a heated tube, we have actually neglected this term. So h_2 becomes 1 okay. As well as you see in this in this case you see this term whatever we have $d\alpha/dx$ and then this 1. This also goes to 0 okay. So what we have done?

We have also made this term is equals to 0. We have made these terms also equals to 0 right. We are having over here $d\alpha/dz$. So in case of pipeline, we will be find out this can be also made to 0 because in case of a circular pipeline without any cross sectional change we will be finding out $da/dz = 0$. So we left to first term, this term and this term. So we have to integrate these things.

So let us integrate for a finite length. So, finite length that can be a pipeline having L length so we are giving over here the L length where, the quality mass quality changes from 0 to x okay.

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For an uniformly heated tube of diameter D :

$$\Delta P = \frac{2f_{fo} G^2 v_f L}{D} \left[\frac{1}{x} \int_0^x \phi_{fo}^2 dx \right] + G^2 v_f \left[\frac{x^2 v_g}{\alpha v_f} + \frac{(1-x^2)}{1-\alpha} \right]_0^x$$

$$+ \frac{Lg \sin\theta}{x} \int_0^x [(1-\alpha)\rho_f + \alpha\rho_g] dx$$

$$\Delta P = \frac{2f_{fo} G^2 v_f L}{D} \left[\frac{1}{x} \int_0^x \phi_{fo}^2 dx \right] + G^2 v_f \left[\frac{x^2 v_g}{\alpha v_f} + \frac{(1-x^2)}{1-\alpha} - 1 \right]$$

$$+ \frac{Lg \sin\theta}{x} \int_0^x [(1-\alpha)\rho_f + \alpha\rho_g] dx$$

So for the first term the frictional part you can find out this is a constant term. So this is not varying with respect to your x because we have taken fluid only assumptions over here okay. Then here you see but the friction, but the multiplier ϕ_{fo} will be dependent on x. So $1/x$ into integration of 0 to x $\phi_{fo}^2 * dx$ that remains in the integration okay. This is somehow we need to find out okay.

Then, for the accelerational term if you see we had this term so if you do the integration with respect to x , this becomes $2x$ integration dx so that means x square okay. So here we have got x square v_g , we have taken v_f common. So if v_f came over here okay on the other hand side second term will be giving you $2(1-x)$ integration and dx if you write down the after performing integration, you will be getting $(1-x)$ whole square with positive sign okay.

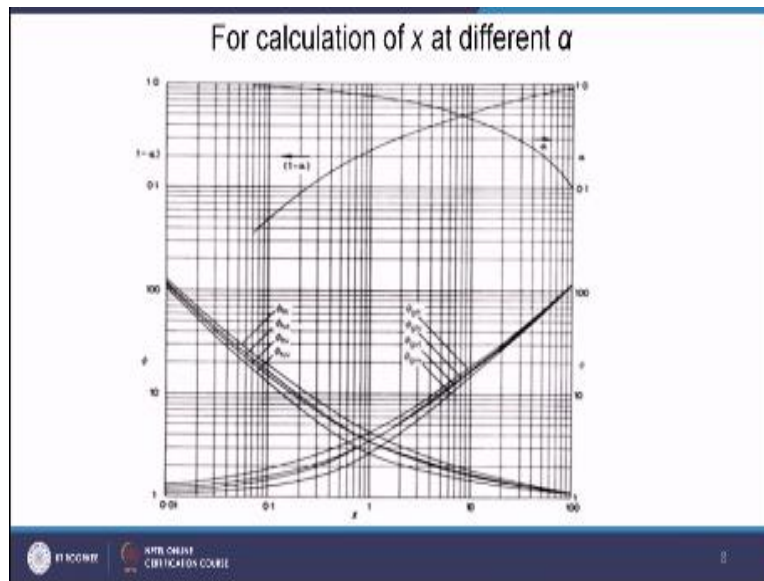
So, that we have kept over here $(1-x)$ whole square. As we have taken v_f common we cannot see v_f over here okay. v_f multiplier over here right. Then for the accelerational part you see $L g^* \sin \theta$. Now where from this L is coming? You see if we integrate it over dx , so we will be finding out that we are having the α inside this right. So α is vary when you progress in the tube.

So what we need to do? We need to go for variation of x okay. In place of z , so what we do $\frac{dz}{dx}$ * $\frac{dL}{dx}$. So we give dx over here and $\frac{dz}{dx}$ we keep outside now as I have told that pipeline is starting from length = 0 to length = L and whenever the quality is changing from $x = 0$ to $x = L$, so you will be finding out this $\frac{dz}{dx}$ becomes $L - 0 / x - 0$. So this is L/x is coming over here in picture right.

So this is also another term which we need to find out somewhere okay. So the final expression if we see over here after putting this limit in the second term, accelerational term, we will be getting -1 over here. Because if you put $x = 0$, so this term cancels but this term remains okay. So this $x = 0$ remains so we will be findings out this is becoming -1 okay. So this is the final expression for uniformly heated tube diameter, uniformly heated tube of having diameter d in case of the pressure drop.

So pressure drop you see still we are having 2 integration okay which we need to take care of now how to take care this integrations. So first I will be showing you different charts over here. All these charts are actually given by Martinelli and Nelson. As I have already told you different parameters are obtained by Lockhart Martinelli. So here we are we are finding out the values using Martinelli Nelson chart.

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So let us see first 1, we will be finding out if we know the value of x , how to find out α okay. You see in this chart what we are having in this chart in abscissa of we are having x okay. And in ordinate we are having 2 things actually in the lower portion we are having the value of ϕ . ϕ is nothing but the Lockhart Martinelli parameter ϕ whatever we have seen in case of the fluid only, liquid only, fluid part and gas part okay.

And here we are having in the upper part we are having the value of $(1-\alpha)$ in this side and α in this side okay. Now let see you know your value of x . Somehow you know your value of x . So what you will be doing, you will be following from here. What is the value of x , you will be following from here?

First if we move up and if you intersect this line okay in this curve and from here, if you read in the left-hand side, you will be getting the value of $(1-\alpha)$. Similarly, from a particular value of x if you move up and intersect this curve which is actually α curve and then move in the right-hand side direction, you will be getting the value of α . So this chart is made in such a fashion that this side and this side they are actually summation will become always 1 right.

Now after getting the value of α , next task is to get the value of the Lockhart Martinelli parameters. So what we have done over here from this side. We are having 4 different lines from

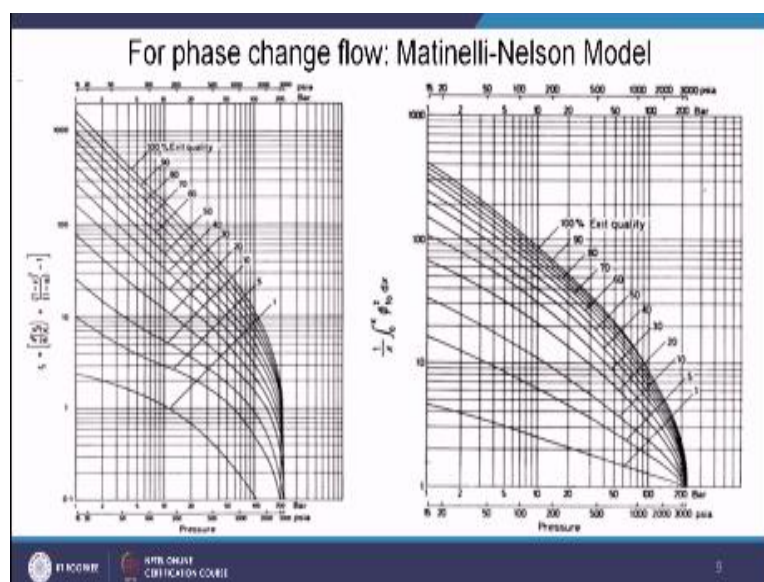
this side also we are having 4 different lines okay. Now these lines are actually for different combinations okay. So you can see the lines are like this first one, the upper one is phi gtt, second one is phi gtv, third one is phi gvt and fourth one is phi gvv okay.

Now what is this tt vv gas g symbolizes usually of gas we can understand then tt symbolizes both the phases gas and liquid are in turbulent situation okay. Similarly, tv symbolizes liquid is in turbulent situation but gas is in laminar situations. Similarly, vt symbolizes liquid is in laminar situation but gas is in turbulent situation and finally phi gvv symbolizes that both liquid and gas are in laminar situation right.

So once we know your value of x and you know that what are the individual conditions for both the phases liquid and gas. So you can choose which curve you need to take and find out the value of the friction Lockhart Martinelli parameter from the ordinate right. Now if you are applying the gas only or gas portion equations then you will be taking these curves and if you are taking liquid portion friction factor Lockhart Martinelli parameter then you will be taking this curve.

So here we are having phi ftt, phi fvt, phi ftv and phi fvv respectively right. Okay then let us discuss the next chart. So if you are having phase change, so Martinelli Nelson model what will be understanding over here. You can see this is actually gamma.

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So what is this gamma? Gamma is nothing but whatever you have seen in this (()) 32:53 okay. So $x^2 \cdot v_g / \alpha \cdot v_f + (1 - x^2) / (1 - \alpha) - 1$ okay. So this has been given over here in the ordinate and in abscissa. We are having the pressure okay in the pipeline. We are having some average pressure so that pressure will be giving over here right. And here we are having different lines for exit qualities.

So at the pipeline exit if you know the exit quality so by knowing the pressure value and exit quality value, you can find out what is the magnitude of this term which will be coming into picture while calculation of the pressure drop. So this term will be coming into picture right. In the similar fashion, in the right-hand side curve if you see we are having in ordinate $1/x$ 0 to x integration of $\phi^2 dx$ if you remember your Δp .

They are we had in this bracket $1/x$ 0 to x $\phi^2 dx$ right. So this term is actually can be found out using that curve Martinelli Nelson curve once you know the pressure and the exit qualities. So these are the exit quality lines varying from 1 percent to 100 percent okay. So both these unknown in the iners in the in the accelerational part and the frictional part you can find out using Martinelli Nelson model right okay.

Next let us try to see if you are having some adiabatic situation okay. So in case of adiabatic that means phase changes not involved over there, in case of adiabatic situation you can find out these curves will be important. So if you have to find out the Lockhart Martinelli parameter for fluid only then you will be using this curve in the abscissa of this curve.

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Problem

A vertical tube of 3 m length and 10 mm diameter is carrying water-vapour mixture. The inlet quality of the water-vapour mixture is 0.05. Total mass flow rate through the tube is 0.01 kg/s, the two phase mixture pressure is 46.941 bar. Initial 2 m of the tube is heated with wrapped electrical coil which supply 10 kW of heat to the tube. Rest 1 m is adiabatic. Calculate the pressure drop using Martinelli Nelson correlation.

Solution:

P = 46.941 bar			T _{sat} = 260 °C		
μ_l (Ns/m ²)	μ_g (Ns/m ²)	V_l (m ³ /kg)	V_g (m ³ /kg)	i_l (kJ/kg)	i_g (kJ/kg)
104.8×10 ⁻⁶	17.90×10 ⁻⁶	1.2755×10 ⁻³	0.042149	1135	2796

The problem is like, this a vertical tube of 3-meter length and 10 mm diameter is carrying water vapor mixture. The inlet quality of the water vapor mixture is 0.05, total mass flow rate through the tube is 0.1 kg per second, the 2-phase mixture pressure is 46.941 bar initial 2 meter of the tube is heated with wrapped electrical coil while which supply 10 kilo watt of the heat to the tube. Rest 1 meter is adiabatic.

So we are having 2 sections heated section and adiabatic section. Calculate the pressure drop using Martinelli Nelson correlations okay. So first what we will be doing at this pressure? We will be finding out the properties like saturation temperature, liquid and gas viscosity, liquid and gas specific volume and the enthalpies okay. Now as we have done in homogeneous flow model, let us find out what is the quality at the exit of 2 meter of the pipeline.

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Quality at 2 m, $x_{2m} = \frac{Q/w}{i_{fg}} + x_i = 0.652$

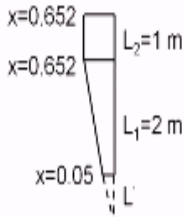
Assuming linear quality increase,

$$\frac{L'}{L_1 + L'} = \frac{x_{inlet}}{x_{2m}}, \text{ Therefore, } L' = 0.166 \text{ m}$$

Now, $G = 4w/(\pi D^2) = 127.4 \text{ kg/m}^2\text{s}$

$Re_{fo} = GD/\mu_f = 1.22 \times 10^4$ Turbulent

$f_{fo} = 0.079 Re_{fo}^{-1/4} = 0.00752$



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So I will be finding out by $q/w / i_{fg} + x_i$ because here at the inlet we had some quality okay 0.05. So we can find out that at 2 meter it is becoming 0.652. Now if you assume that linear variation of the quality then what we can do we can find out what was the length responsible for increasing the quality from 0 to 0.05 okay. So that if you find out over here, let us take that length as L' . So $L'/1+L' = x_{inlet} / x_{2m}$. So you will be finding out $L' = 0.166$ meter okay.

So let us find out few important parameters like G . G will be $4w/ \pi d^2$ 127.4. And then Reynolds number for fluid only portion GD/μ_f so this is coming as turbulent okay. Friction factor will be calculating using Blasius like this okay this is coming as 0.00752.

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For 0.652 quality and 46.941 bar pressure, $\gamma_2|_{x=0.652} = 15$

For 0.05 quality and 46.941 bar pressure, $\gamma_2|_{x=0.05} = 1.2$

$$\Delta P_a = \Delta P_a|_{x=0}^{x=0.652} - \Delta P_a|_{x=0}^{x=0.05} = G^2 v_f [\gamma_2|_{x=0.652} - \gamma_2|_{x=0.05}] = 286 \text{ N/m}^2$$

For 0.652 quality and 46.941 bar pressure, $\alpha = 0.93$

For 0.05 quality and 46.941 bar pressure, $\alpha = 0.55$

$$\Delta P_z|_{x=0}^{x=0.652} = \frac{Lg \sin \theta}{x_{2m}} [(1-\alpha)\rho_f x + \alpha\rho_g x]_0^{0.652} = 1509.4 \text{ N/m}^2$$

$$\Delta P_z|_{x=0}^{x=0.05} = \frac{Lg \sin \theta}{x_{inlet}} [(1-\alpha)\rho_f x + \alpha\rho_g x]_0^{0.05} = 595.5 \text{ N/m}^2$$

$$\Delta P_z|_{x=0.05}^{x=0.652} = 913.9 \text{ N/m}^2 \quad \Delta P_{L1} = 1064 + 286 + 913.9 = 2263.9 \text{ N/m}^2$$

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For 0.652 quality and 46.941 bar pressure,

$$\frac{1}{x} \int_0^x \phi_{fo}^2 dx = 16$$

$$\Delta P_{F, x=0}^{x=0.652} = \frac{2f_{fo} G^2 v_f (L_1 + L')}{D} \left[\frac{1}{x} \int_0^x \phi_{fo}^2 dx \right] = 1079 \text{ N/m}^2$$

For 0.05 quality and 46.941 bar pressure, $\frac{1}{x} \int_0^x \phi_{fo}^2 dx = 2.9$

$$\Delta P_{F, x=0}^{x=0.05} = \frac{2f_{fo} G^2 v_f (L_1 + L')}{D} \left[\frac{1}{x} \int_0^x \phi_{fo}^2 dx \right] = 14.99 \text{ N/m}^2$$

$$\Delta P_{F, x=0.05}^{x=0.652} = \Delta P_{F, x=0}^{x=0.652} - \Delta P_{F, x=0}^{x=0.05} = 1064 \text{ N/m}^2$$

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Now at 0.652 quality let us find out from the charts what are the values of the $1/x \int_0^x \phi_{fo}^2 dx$. We require the pressure information for that we get the value will be from 16 so this chart will be using okay. This chart will be using for this 1. So you finding out the value from the chart as 16. Similarly, you will be finding out what is the frictional force for 0 to 0.652.

Remember our pipe is varying from .05 to 0.652. So this we were finding out for the overall. So if you multiply this 1 with this multiplication factor 16, we get this one 1079. Now we have to subtract the initial portion that will 0 to 0.05 okay 0 to 0.05 which is the hypothetical length. So we have found out this fraction factor once again with the exit quality of 0.05 at the same pressure that comes out to be 2.9.

Once again it has been found out from Martinelli Nelson chart. So you will be finding out delta pf for the fictitious pipe is becoming 14.99. So ultimately for our real pipe which is varying from quality 0.05 to 0.652 that becomes 1064 Newton per meter square. Okay now let us see for the accelerational part. So with 0.652 quality at this pressure we will be getting gamma is equals to 15.

So that we are calculating using this chart okay. So gamma we are calculating using pressure and exit quality parameters. So you can find out that is becoming 15 and for the hypothetical part it is

becoming 1.2. So for the real part if you see the accelerational pressure drop that becomes g square v_f multiplied by this subtraction between these 2 factor 286 Newton per meter square. Then for the gravitational part we can find out that for this exit quality and this pressure α will become 0.93.

So those we will be calculating using this curve okay adiabatic situation curve. So you will be getting that for those conditions for the whole pipe fictitious pipe + real pipe this becomes 1509.4 okay. This portion we are getting from α we are getting from your chart which is 0.93. And for the fictitious part α is 0.55 so for the both parts fictitious part and the whole part we are getting these 2.

For subtraction between these 2 will be my actual pressure drop due to accelerational. So altogether we are getting this is a total force total frictional pressure drop for the heated pipe length. Okay now this is for the adiabatic session this will be very simple. So we have to once again find out ϕ_{fo} using your ϕ_{fo} and α using your Martinelli Nelson chart using these 2 charts ϕ_{fo} and α we will be finding out okay.

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For adiabatic section, $\phi_{fo}^2 = 34.3$ $\alpha = 0.93$

$$\Delta P_{L2,F} = \frac{2f_{fo} G^2 v_f \phi_{fo}^3}{D} L = 1068 \text{ N/m}^2$$

$$\Delta P_{L2,a} = G^2 v_f \left[\frac{x^2 v_g}{\alpha v_f} + \frac{(1-x)^2}{1-\alpha} - 1 \right] L = 327.72 \text{ N/m}^2$$

$$\Delta P_{L2,g} = g[(1-\alpha)\rho_f + \alpha\rho_g]L = 679.54 \text{ N/m}^2$$

$$\Delta P_{L2} = 1068 + 327.72 + 679.54 = 2075.26 \text{ N/m}^2$$

$$\Delta P = \Delta P_{L1} + \Delta P_{L2} = 4339.16 \text{ N/m}^2$$



Then finally we will be finding out the friction factor accelerational factor and (ϕ) 40:28 factor. So this is 1068 coming out to be because we have found out ϕ_{fo} and α okay. Accelerational part comes out to be 327 gravitational part comes out to be 679. So altogether for the non-heated

part it becomes 2075. So if you add delta p L1 and delta p L2 it becomes 4339.16 Newton per meter square for the whole pipe. Okay let us summarize. In this lecture, we have discussed about the pressure drop how to obtain the pressure drop for separated flow.

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Summary

- In this lecture we have discussed about the pressure drop occurring in separated flow patterns
- We introduced friction factors for fluid only, gas only, gas and liquid situations and derived their relations with two phase friction factors
- We have discussed about Lockhart Martinelli charts for obtaining friction factors and different terms required in pressure drop calculation
- For adiabatic condition also we have shown the use of Lockhart Martinelli charts for evaluation of fluid only friction factors ratio and void fraction provided quality is known
- At the end we have practiced a sum for pressure drop calculation inside a pipe carrying separated flow with phase change

We have also shown how to get the Lockhart Martinelli parameters and the use of Martinelli Nelson chart and at the end of this we have practiced a sum where we have shown the adiabatic situations as well as the heated situation. Right, to test your understanding let us see the questions. Identify the correct relations.

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Test your understanding ?

1. Identify the correct relation

a. $\phi_{fo}^2 = \phi_f^2 (1-x)^2 \frac{f_f}{f_{fo}}$

b. $\phi_{fo}^2 = \phi_g^2 (1-x)^2 \frac{f_f}{f_{fo}}$

c. $\phi_{fo}^2 = \phi_f^2 x^2 \frac{f_f}{f_{fo}}$

d. $\phi_{fo}^2 = \phi_f^2 (1-x)^2 \frac{f_f}{f_{go}}$
2. Separated flow model is valid for

a. Bubbly flow

b. Slug Flow

c. Stratified flow



d. Droplet flow
3. Which graphs will be helpful to derive the friction factors in separated flow model

a. Baker Plot

b. Martinelli-Nelson

c. Hewitt and Roberts

d. Moody's Chart

You are having 4 relations I think you can identify the correct 1. The answer is part a.

Similarly, separated flow model is valid for. 4 options

you are having bubbly, slug, stratified and droplet. Correct answer is stratified. Third 1 which graph will be helpful to derive the friction factors in separated flow model baker, Martinelli Nelson, Hewitt and Roberts and Moodys chart.

By now obviously you have understood that correct answer is Martinelli Nelson right. With this I will be ending this lecture. Thank you.