Two phase flow and heat transfer
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Lecture No: 4
Drift Flux Model

Hello, welcome to the course Two Phase Flow and Heat Transfer. Today we are in fourth lecture. Today we will be discussing about drift flux model. So let us first see what the outline of this course?

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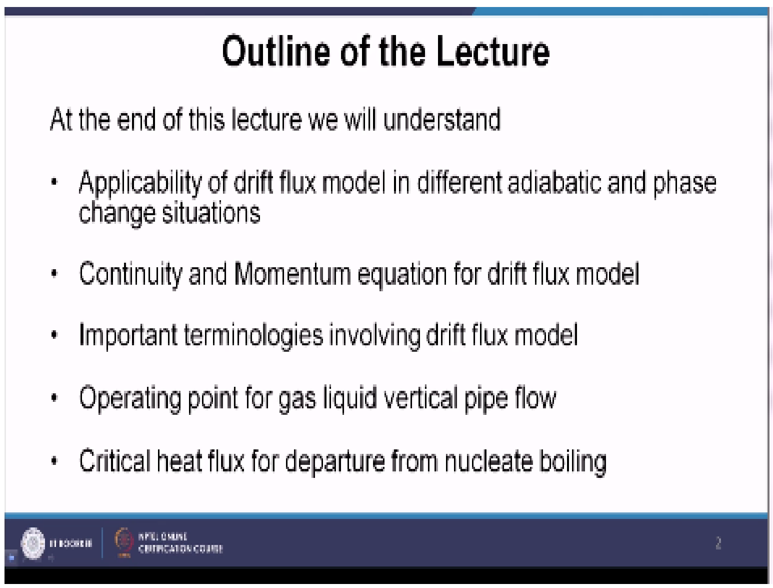

At the end of this lecture we will understand the applicability of drift flux model in different adiabatic and phase change situations. We will be also deriving continuity and momentum equation applicable for drift flux model. Important terminologies involving drift flux model will be discussed in this lecture. We will also try to see for gas liquid vertical pipe flow, what are the different operating points for upward and downward movements of gas and liquid.

And at the end we will be seeing how critical heat flux can be calculated for departure from nucleate boiling. Now to give you a brief idea about drift flux model, we will recap our last lecture homogeneous model. So in homogeneous model we have seen that both gaseous phase and liquid phase are having same velocities but here in drift flux model we consider that there is significant drop between the velocities that means there is relative velocity between the gas phase and the liquid phase.

Now as it is having 2 different velocities ideally, we should consider 2 separate sets of equations. That means in case of adiabatic flow, 2 continuity equations and 2 momentum equations and whenever we are having heat transfer into consideration we will be having 2 energy equations also. But in drift flux model we are actually considering not only the individual phases, we are actually considering the relative motion.

So here what we consider once again we give mixture kind of assumption over here for both the phases. And we are interested in gas relative velocity. So altogether you will be finding out that we are having in case of adiabatic situation, 1 mixture continuity, 1 mixture momentum equation as well as we will be having gas relative velocity equation. So if you consider both the phases separately, you will be having altogether 4 equations, 2 continuity and 2 momentum equations.

But here in drift flux model as we are only considering the relative velocity between the gas and liquid. We will be having 3 equations the mixture momentum equation, mixture continuity equation and gas relative velocity equation. If heat transfer comes into picture, we will be having 1 mixture heat transfer equation also. So let us see that drift flux model where it applies.

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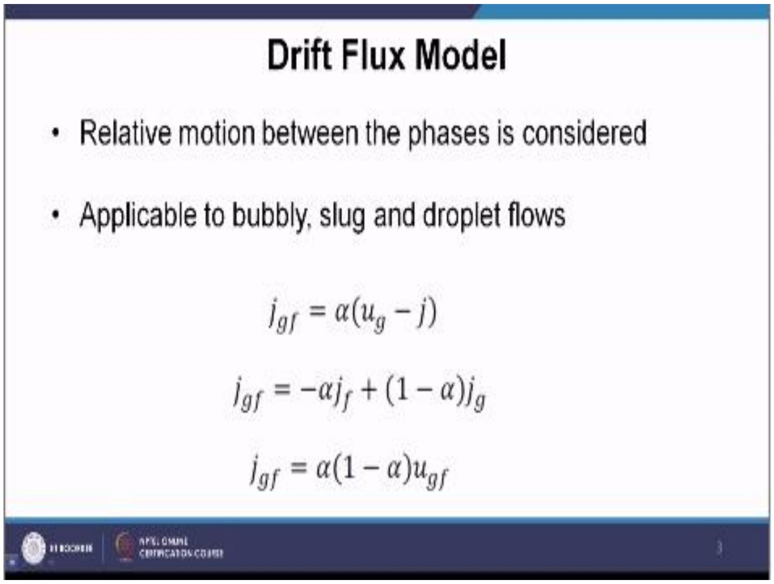

Basically, we will be finding out drift flux model is applicable for bubbly flow, slug flow and droplet flow. So, wherever interface is not clearly distinct just like your bubbly flow and droplet flow and its immediate transition slug flow. We will be finding out drift flux model is applicable.

Here already I have told that relative motion between the phases is considered. Now these 3 equations already we have shown in the first lecture these are very important equations. So you can find out jgf, this is the drift velocity.

So you can find out drift velocity is nothing alpha (ug- j). So this already we have shown you the definition this is actually how gas velocity is drifting from the overall relative velocity. So you can find out jgf=alpha (ug -j). Then we have also shown you the proof of next 2 equations in the first lecture. jgf is equals to alpha*jf +(1- alpha) jg. This comes from the first equation once again.

Here you are having alpha into ug if you try to write down ug in terms of jg, you will be finding out from this equation we are getting this 1. On the other hand if you start from this equation and try to reduce this j in terms of the individual phase velocities ug and uf, you will be finding out we are coming to this third equation. So those things we have already shown you in the first lecture.

Let in this lecture we will be taking the clue from the first lecture and these 3 equations we will be using jgf = alpha (ug – j), jgf = -alpha*jf + (1- alpha) jg. By the way this jf and jg these are gas and liquid superficial velocities and jgf = alpha (1-alpha)ugf where, ugf is the relative velocity between the phases okay. Next let us try to see a schematic of situation where drift flux model can be applicable. So you see what I have shown you over here this is the pipeline having variable cross sections.

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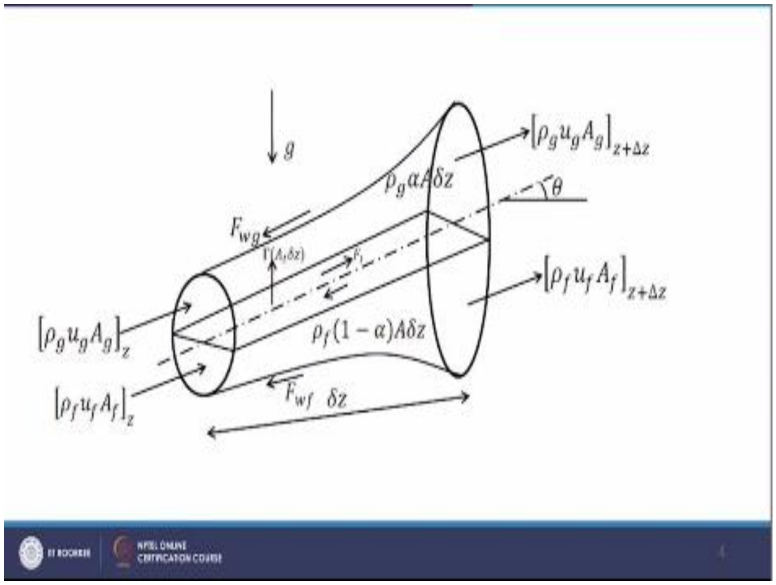

We can find out this is smaller cross section compared to this 1. So we are having variable cross section and to show the relative velocities between the phases we have considered both phases are separate. So you see at the bottom we have given liquid phase and at the top we have given the gaseous phase. So liquid phase is signifying f and gaseous gas phase is denoted by g. So you can find out that the as the flow is flowing from left to right.

So we are having the mass of the liquid as Rho f uf and Af at the entry and mass for the gas is Rho g ug and Ag at the entry right. On the exit side you can find out Rho f uf and Af at z + delta z where, we have considered delta z is actually the length of the pipeline. So this is some sort of indefinite decimal length. We have considered for finite length. We have to go for integration of this one and on the other hand side for the gas ug Rho g Ag at z + delta z.

To make it generalized, we have considered that the pipeline is making angel theta with a horizontal direction. Apart from that as we are having gravity in the downward direction, you will be finding out that we are having mass over here. Rho g alpha a *delta z for the liquid inside that pipeline and sorry, inside the pipeline for gas. And Rho f (1 - alpha a) delta z for the liquid in the downward side of the pipeline.

Now as we are considering the relative velocity between the phases, this interfacial force will be very important over here in drift flux model. You will be finding out the 2 interfacial forces. We have given a phi for the liquid side and –a phi for the gaseous side. Apart from that if there is sort of mass transfer that means buoyancy, (()) 7:36 if there is some sort of mass transfer, we have also given mass transfer gamma over here which is actually occurring due to this interfacial area.

So this interfacial area is Ai and the delta z is the length. So you can find out that gamma *Ai * delta z will be mass transfer. We have also considered in this module the wall friction force. So you can find out the gaseous phase is actually finding out wall friction force Fwg, on the other hand liquid side is getting Fwf okay.

So with this idea let us now try to see how the continuity and momentum equation, mixture continuity and momentum equation can be constituted okay. Next let us try to see the individual components first. So here I have written the continuity equation for the liquid side.

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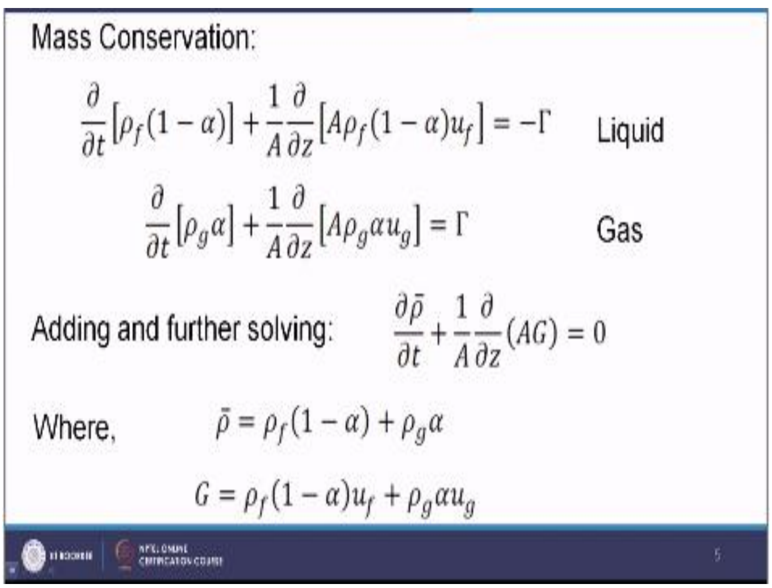

So you can find out this the unsteady part which we have already seen in our fluid mechanics. So this is the unsteady part delt/ delt t [Rho f (1- alpha)] and then we are having the inertia part 1/A delt/delt z [A Rho f (1- alpha)uf. So (1 –alpha) is coming into picture as we considering the liquid part. Now this - gamma is actually due to the mass transfer. So some liquid is converting into vapor due to phase change.

We have considered over here to become generalized. So that is why minus gamma we have added as the source term. On the other hand, in the gaseous phase momentum equation it will be taking similar form of the liquid. So in place only (1-alpha), we have to consider alpha because we know that alpha will be associated with the gaseous phase.

So this is first unsteady part and then we are having inertia part 1 /A delt /delt z [A Rho g alpha *ug] and yes, gas has actually acquired mass due to phase change. So we will be finding out gamma we have kept over here as positive. So you see this is actually giving the mass balance because these 2 will be canceling each other. Now if we try to add these 2 equations okay, if we try to add then we will be finding out both these unsteady parts can be added.

So delt/ delt t [Rho f (1 –alpha) + Rho g*alpha which is nothing but we know Rho bar or average density. So this also we have derived in case of your homogenous flow equation if you remember. So there also we have shown that Rho f (1 – alpha) + Rho g *alpha will be Rho bar, this also we have proved over there. So we are considering after adding we are having the unsteady part as delt Rho bar/delt t right.

On the other hand side for the inertia part you see, we are having 1/ A delt/delt z of now here if we take a common that it will be Rho f(1- alpha) uf + Rho g *alpha *ug. So that is nothing but your G okay. So here we can right down the mixture momentum, mixture mass conservation equation as delt Rho bar/ delt t + 1/ A delt/ delt z (AG) is = 0 because both the sources terms will be canceling from each other okay.

So this is the mixture continuity equation. Already I have shown you over here what is Rho bar and G.
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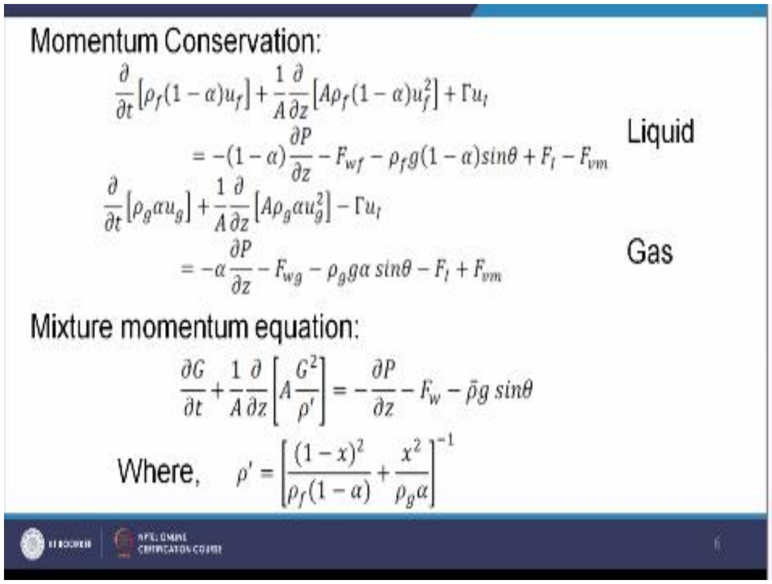

Next let us try to see the momentum conservation equation. In case of momentum conservation equation, you see first I have written for liquid as well as gas. So liquid, let me explain the terms once again. So first term is due to the unsteady behavior. So delt/ delt t[Rho f (1– alpha)uf and then we are having inertia term 1 /A delt/ delt z [A Rho f (1- alpha) u f squared. So this is basically the inertia.

We can also see from the figure that we are having some force over here okay. gamma *Ai *delta z. So those components will be coming over here. gamma into ui where, ui is the interfacial velocity okay. Then we are having the force due to pressure drop – (1 – alpha) delt p /delt z. We are also having the frictional force in the negative direction this is the force including the buoyancies.

So Rho f*g (1 –alpha) sin theta because the pipe is making theta degree angle with the horizontal and then we have included 2 terms, this is interfacial force Fi and then virtual mass force Fvm okay. So this is the liquid momentum equations. Similarly, we can write down the gas momentum equation delt/ delt t (Rho g alpha*ug). So this first term is actually your unsteady term and then we are having 1/A delt/ delt z[A Rho g alpha* ug square].

So this is the gaseous inertia. We are having –(gamma *ui). You see here in liquid we have given +(gamma* ui), here it will be –(gamma * ui). So due to mass transfer whatever velocity we obtained that is ui, so this is coming as positive over here in liquid term and coming as negative in the gaseous term. Right on other hand side we are having the pressure drop okay.

And the wall friction force due to gaseous momentum Fwg, also we are having the buoyancy component Rho g g alpha* sin theta and these 2 components the interfacial force and virtual mass force but in the opposite sign as we have seen in case of the liquid so - Fi + Fvm. Now once again just like here continuity equation we are going to add it over here. So if we add you, will be finding out all these terms involving opposite signs in both these equations will be canceling out.

Only in the first term you see delt/delt t if we take common then we will be having Rho g alpha ug + Rho f (1- alpha) uf this is nothing but your G which we have shown you in the previous slide.

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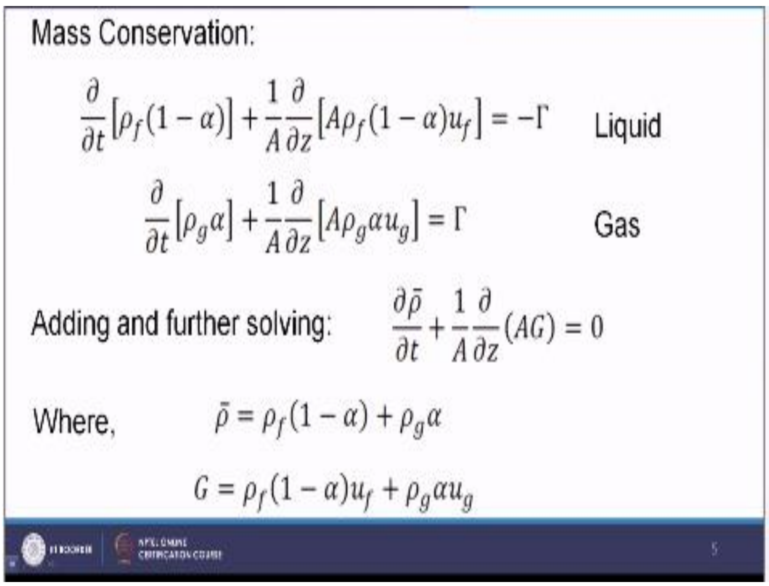

So G is nothing but Rho f (1 – alpha)* uf + Rho g *alpha *ug okay. So we will be writing down this term, unsteady term as delt g/delt t right. Then in the second term we are having 1/ A delt/ delt z of here we are getting 1/A delt/delt z[A Rho f(1- alpha) uf square + A Rho g alpha *ug square. Now here we need to do little bit of derivation. So let us see how this term actually changes into the mixture momentum equation.

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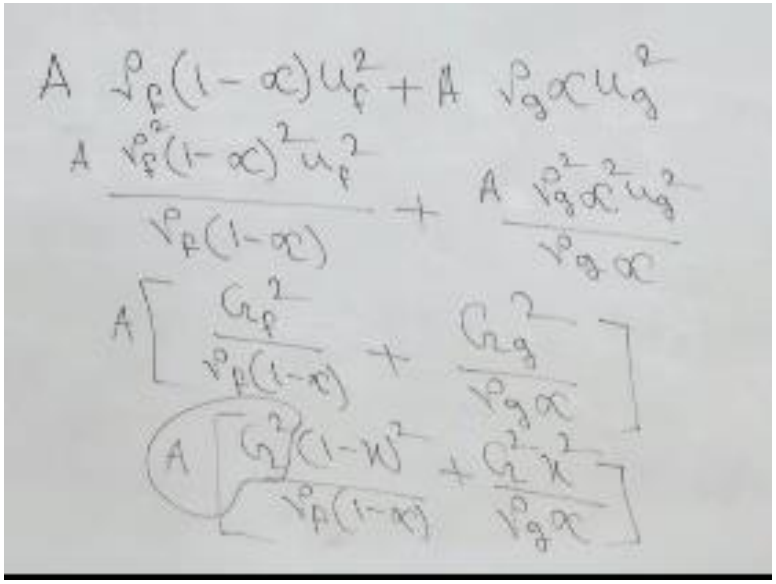

So we are having Af Rho f and then (1-alpha) * uf square okay. + Ag Rho g alpha* ug square right. So this term, what we can do over here, say Rho f (1-alpha) whole square uf square* Af /Rho f (1- alpha) + on the other hand side you can find out we are having Ag Rho g square alpha square ug square/Rho g *alpha right. Now you see over here, we can write down this Af (1 – alpha) uf. This is nothing but your Gf okay divided by Rho f (1- alpha).

Also we will be finding out okay. 1 probably this Fs will not be there so this is A okay. So A, we can take common okay and here we will be having Gg whole square / Rhog *alpha okay. Now you see this Gf can be written as G square *(1 –x) whole square / Rho f (1 –alpha) and here this one can be written as G square* x square from the definition of quality, mass quality Rho g* alpha okay multiplied by A.

So you can find out, we can take Ag square common and we can we can define this (1-x) whole square / Rho f (1 – alpha) + x square* Rho g * alpha as a new term. So what we have done over here, you see we have called Rho dashed as (1- x) whole square / Rho f (1- alpha) + x square / Rho g * alpha to the power whole to the power 1 reciprocal of that as Rho dashed. So ultimately you will be getting Ag square which is this term Ag square which is this term and then divided by Rho.

That means if I consider this 1 as reciprocal of Rho so then you will be finding out Ag square by Rho dashed over here in our mixture momentum equation. So these 2 terms actually gives us 1 /A delt/ delt z(A *G square/ Rho dashed right. Next in the right hand side, let us see in the right hand side if you see, you will be finding out here we are having delt p/ delt z (1- alpha).

Here we are having alpha* delt p/ delt z which will be canceling out each other and will be finding out – (delt p /delt t) will be remaining right. Similarly here, what we have done the frictional force Fwf for the liquid phase and Fwg for gaseous phase. We have added up and we have written Fw overall frictional force in the mixture momentum equation for the buoyancy force also.

Same thing will happen as we have seen in case of the pressure force Rho f*Rho g (1-alpha) sin theta and here Rho g* g alpha into sin theta if we add then will be finding out Rho bar where, Rho bar definition already I have shown in the previous slide Rho bar is Rho f (1 –alpha) + z Rho g *alpha. So we will be finding out that these 2 term if we add up, we will be getting Rho bar *G * sin theta right.

So these become my mixture momentum equation. So we are talking about drift flux model. So here the mixture continuity equation and mixture momentum equation will be coming into picture. Now apart from that as I have mention that we will be also considering the third equation because the relative velocity comes into picture. Basically, we take in reflex model the gas relative velocity.

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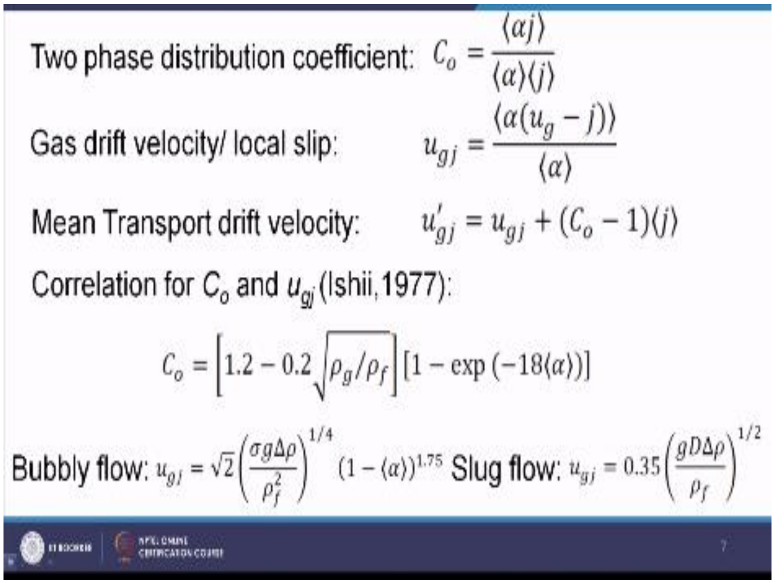

So you see here the gas relative velocity is Ugj dashed okay. So which we call mean transport drift velocity okay. Now this mean transport drift velocity is actually given by Ishii as Ugj + C0 - 1* j, this is actually area averaged of the j right. Now this mean transport drift velocity which is very essential for the drift flux calculation is dependent on 2 parameters which one is Ugj and c0 okay.

What are these Ugj? Let us see over here Ugj is actually gas drift velocity or local slip. So how much the gaseous phase actually slipping from the overall averaged velocity okay. So that is Ugj equals to this symbol using this symbol, whatever I have done that is actually area average quantity. So area average quantity of alpha* Ug - j divided by area averaged of alpha. So this is the definition of Ugj.

On the other hand c0 which is nothing but 2 phase distribution coefficient. So that definition I can write down c0 is equals to area average of alpha * j divided by area average of alpha and area average of j right. So once I know this quantities that what is the area averaged alpha* j, what is the area averaged of alpha, area averaged of j area average quantity. This one we can find out what is C0 and Ugj.

Once we get Ugj and C0, we can put it over here to get the mean transport drift velocity. So this mean transport drift velocity will be very important. This will be signifying that what is the relative velocity between the phases specifically for the gaseous one. Now there are several correlations for finding out this C0 and Ugj. Major correlation has been proposed Ishii. So here Ishii has given in 1977 that what is the value of C0.

He has given this empirical correlation 1.2- .2 root over of Rho g / Rho f where, Rho g and Rho f are the gaseous phase and liquid phase densities into 1 -e the power -18 of area averaged of alpha or void fraction. Now for Ujg, we are having several correlations. Once again now Ishii has given the flow regime based correlations. So for bubbly flow he has mentioned Ugj = root 2 (alpha*g*delta Rho/Rho f square) where, delta Rho is nothing but the Rho f- Rho g to the power 1 /4 *1 -area averaged alpha to the power 1.75.

Similarly for slug flow, he has given this type of equation right. So using this type of equation you can find out C0 and Ugj and you can put it over here to get the mean transport drift velocity okay. Now let us try to see that how this drift flux module is helpful. So for finding out the configurations inside the pipeline. So here we will take the help of 2 equations which I have derived in the first class, first lecture.

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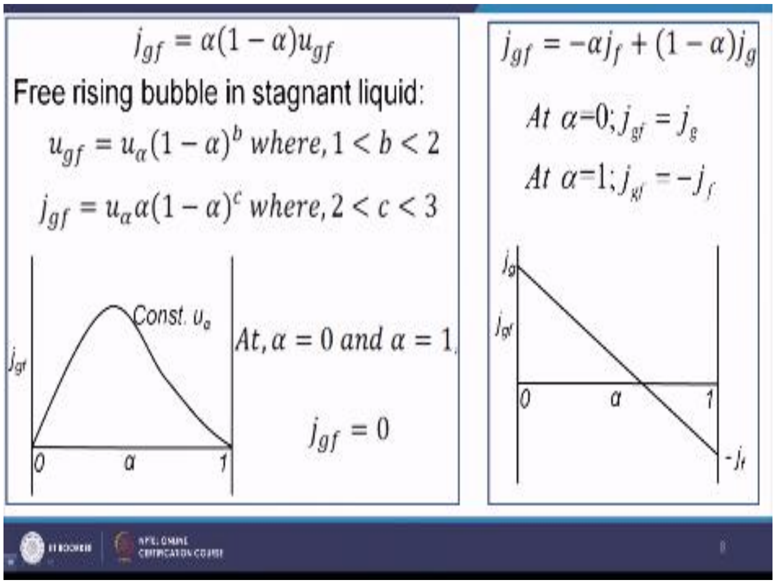

So you can find out first equation which I will be using jgf = alpha (1 – alpha) ugf okay. Now if we talk about bubbly flow situation that means lots of bubbles are there. We will be finding out that Ugj is actually a function of velocity of free rising bubble okay. So you can write down that

Ugf is actually u infinity* (1-alpha) to the power b okay. So you see here alpha is the void fraction, u alpha is the free rising bubble velocity and b is actually a constant which values in between 1 to 2.

So depending on the fluid parameters and the flow velocities, this b parameter actually varies but its range will be in between 1 to 2. So if I put this Ugf in this equation jgf = alpha *(1-alpha) Ugf. We will be getting finally jgf = u infinity *alpha (1–alpha) to the power c where, c is nothing but b + 1 okay. So that means the range of c may be in between 2 to 3 as b was in between 1 to 2. Now this is very important equation.

You see here if I try to find out what is the value of jgf, whenever alpha is 0 and 1 you will be getting that at alpha = 0 and 1. jgf goes to 0. So obviously, the nature of the curve if I try to plot in between jgf and alpha, this will be something like this getting 0 at 0 getting 0 at 1 right. So this is the typical curve between jgf and alpha. If I consider that free rising bubble is there in stagnant liquid okay. Now let us see the other extent.

So if we are having jgf = -alpha*jf + (1 –alpha) jg. So this equation we also we have proved in the first lecture. So you can find out from this lecture we are getting. At alpha = 0, jgf becomes jg and alpha = 1 if you put it over here then we will be getting jgf = - jf. Now if I try to plot it over here once again in jgf and alpha plane. Now remember these 2 things are not similar because this is for free rising bubble, this is not for rising bubble.

This can be for any other configurations. So here we get you see if alpha is 0 then we are getting jgf = jg. So in the positive let us this is the jg okay and whenever alpha = 1, jgf will come negative of the velocity magnitude for the liquid phase. So negative side we have come and then you see this is –jf. We have notified over here so if you try to join this 2 then we will be getting this straight line for jgf and alpha equations.

Now these 2 curves are very, very helpful for finding out the operating point in case of the pipeline when we are having bubbly flow or slug flow into consideration okay. So let us try to see what happen.

So first I have shown you adiabatic vertical pipe. If you see in case of adiabatic vertical pipe, I have here plotted jgf verses alpha already this curve.

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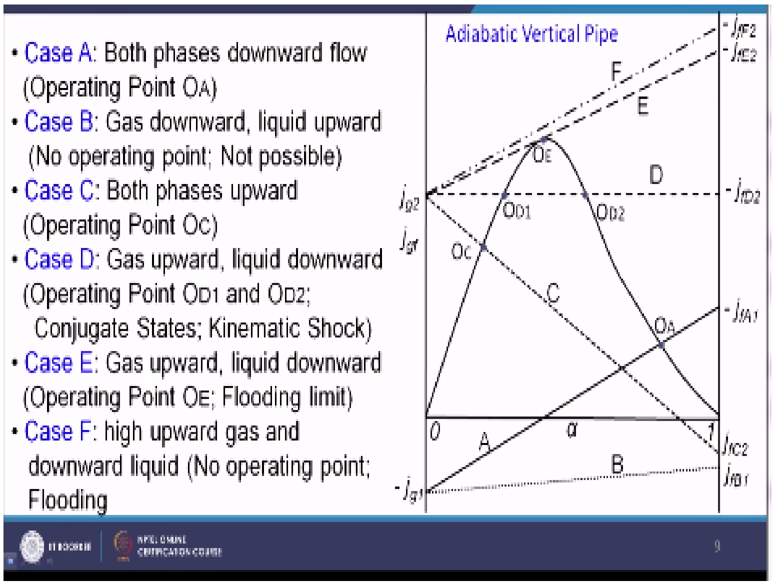

You know we have shown you in the previous slide. So it gets 0 over here. At alpha = 0 and alpha = 1 right. Now let us take several cases. So first one we will be taking case A, which is both the phases are having downward flow. So you if both the phases are having downward flow that means both jg and jf they are negative. So if they are negative then will be finding out at alpha = 0. Negative sides we need to go for jgf = jg so that will be becoming minus jg1 right.

So this is the operating point whenever alpha = 0. On the other hand whenever we are going for alpha = 1 that is – (jf) and jf is already negative so it will be coming positive side. So you will get the operating point somewhat in the positive side let us say this 1. So if you join this line between these 2 points, so this will be might lying whenever I have plotted between jgf and alpha.

Now the operating point will be the intersection between the curve and the line whatever we have found out so this is my operating point OA. So in this operating point at this alpha the adiabatic vertical pipeline can stay. So we can find out operating point is OA. Continuing like this, let us take the second case we are having gas downward and liquid upward. So if gas is downward once again, at alpha= 0 the jgf will become –jg.

So we have taken the same point for example okay. So this is my alpha = 0 point and alpha = 1 that becomes – jf. Now jf liquid velocity is upward. So it will become positive okay. So positive multiplied by minus. So it will become negative. So we will be having operating point at alpha = 1. Somewhat in the downwards side let us say this point okay. So if we join between this point and this point this is my operating line.

Now interestingly you see the curve has never intersected this dotted line. So that means in this case you will find out there is no operating point. So you will get this type of flow gas downward movement and liquid upward movement is never possible okay. Next let us see Case C. So in case C both the phases we have taken upward. So if both the phases are upward so that means jg and jf both the phases upward so if jg is upward then I will be finding out at alpha = 0 we have to movie in the upwards side.

So let us take that point as this one jg2 okay. In the upward side at alpha = 0 and as liquid is in the downward side let us take the points somewhere over here okay. With a negative liquid velocity because negative into positive it becomes negative. So this is the operating points for case C right. So if we join between these 2 points then you will be getting operating point is over here. So this is the operating point for both the phases upward motion okay.

Now continuing like this, if we will go for gas upward and liquid downward, so gas upward liquid downward means obviously the gaseous points will be lies over here and liquid downward point will becoming somewhere over here okay because liquid downward means negative velocity. Negative it becomes positive. So it is comes somewhere over here. So you can find out if you join these 2, we are getting 2 points as operating point OD1 and OD2.

So these 2 are actually called conjugate states or kinematic shock. So it will be changing the position from 1 place to another place in this cases right. Next Case E, gas upward and liquid downward but at special case let us say the gas the liquid velocity is taking such a fashion that this line between joining between jg2 and jfe2. We are getting such a fashion that this line is getting tangent of this curve. So this point we can call that starting from kinematic shocks.

It is converting into a single operating point this is actually called a flooding limit. So beyond this we will be having always flooding by some liquid phase okay. Next Case F, high upward gas and downward liquid both the velocities are very high. So you can find out gas velocity lies somewhere over here and liquid velocity is further up. You will be finding out there is no operating point between this curve and the line okay.

So you will be finding out in this case we are having no operating point and this actually is a typical example of flooding. Now you see here in the Case E is a typical point where this line is actually the tangent of this curve. Now if we take different gaseous velocities and liquid velocities and try to draw this operating point OE okay, then we will be finding out that this curve whatever we have drawn from this jgf = u alpha* alpha (1- alpha) to the power c, is actually the low cause of all the flooding limits okay.

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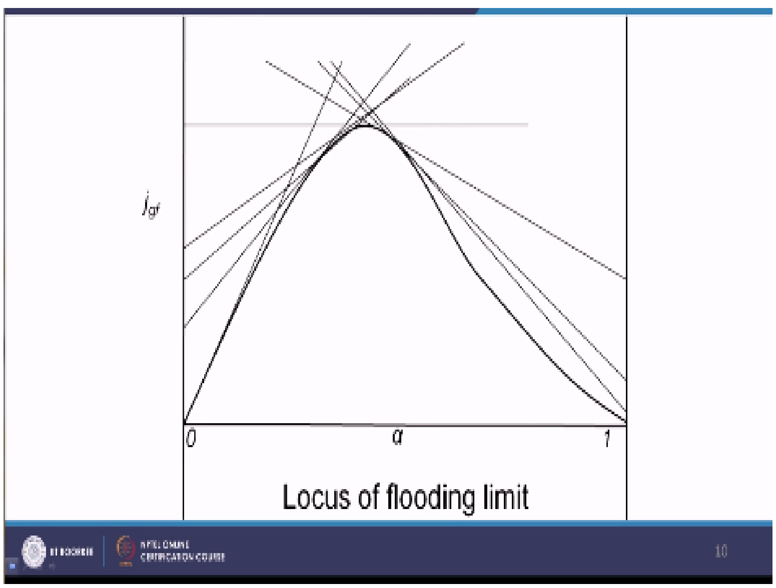

You have to do the experiments for different liquid velocity and gas velocities that means jgf = jf, jg at alpha = 0 and equals to -jf at alpha = 1. And we can find out this type of flooding limit points and if we find out the low cause of this flooding limit that will be the curve right. Next let us try to see another important case which is called pool boiling crisis.

This is having heat transfer into consideration. So let us try to find out the situation over here. So in case of nucleate boiling already we have leaned heat transfer. So we know that there will be a surface through which will be applying the heat flux.

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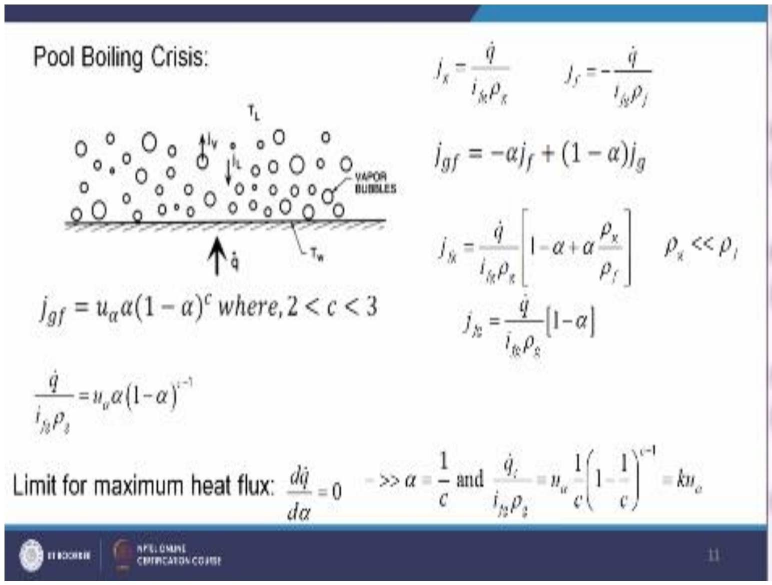

So as we are having the heat flux so we can find out the lots of bubbles will be generated over here. Bubbles will be moving up and liquid will be coming down for replenishment right. So we are having 2 different situations over here bubble velocity up and liquid velocity down. So we have taken jf, jv and jl into consideration. Now this jv is signifying the vapor gaseous phase. We have written here as jg and liquid phase jl we have written here as gf.

Now we know that jg can be written as q`/ ifg which is nothing but latent heat into Rho g. Similarly, jf can be written as - q dot ifg * Rho a because these 2 are not having similar magnitude on as well as direction okay. Now we know already this equation. So what we can do, we can put the value of jf and jg over here. We get equation like this jfg = q dot ifg *Rho g (1 – alpha)* alpha* Rho g / Rho f.

Now if the density ratios are very high so Rho f is very, very higher compare to Rho g. Then I can neglect this term. So I will be getting jgf = qdot / ifg *qg (1- alpha) right. Already I know that for this type of bubble free rising bubble jgf =u alpha *alpha (1-alpha) to the power c whereas c is nothing but in between 2 to 3. So what we can do? We can equate this jgf and this jgf and will be finding this equation. Now to get the pool boiling crisis what we need to do? We need to maximize the heat flux for a given alpha.

So delt q `/ delt alpha = 0. Once you do delt q` /delt alpha of this equation, you will be getting optimum point alpha will be actually 1/ c from here. And using this we can have q dot c / ifg * Rho g = u infinity* 1/ c (1-1) / c to the power of c-1 which is equals to k*u infinity whereas this term is actually a constant we can take this 1 as k right. Now if we try to find out the values of k, will be finding out that for c = 2 because c, the c limit is between 2 to 3. For c = 2, k = 1 /4 and for c = 3, k = 4 /27. So k will be varying in between 4 / 4 /27.

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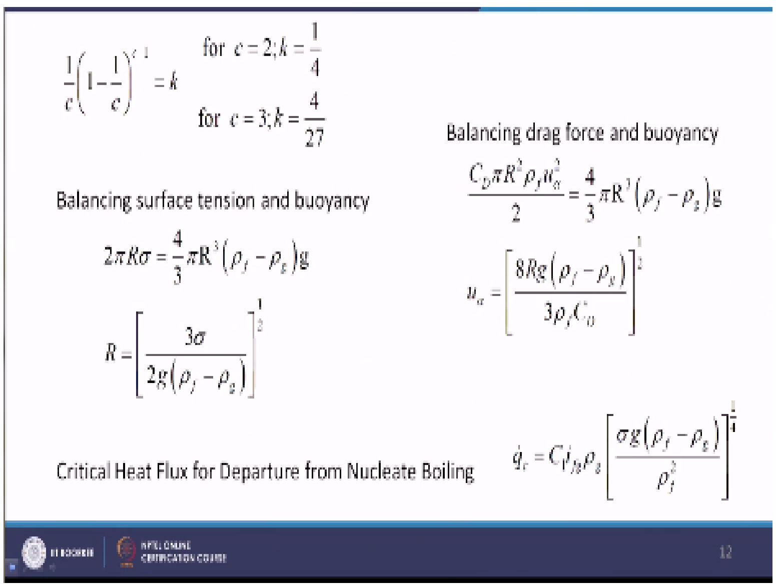

Then to obtain the final value of the critical heat flux let us balance the drag force and buoyancy. So if you balance the drag force and buoyancy so you will be getting this is the drag force cd *pi / pi r square * Rho f *alpha square/ 2. And this is the buoyancy 4/ 3 pi rq with the volume multiplied by density ratio into g. So you will be getting the terminal velocity u alpha in this fashion where r is also there.

Now to obtain the r at which the bubble has released from the surface, we will be balancing surface tension and buoyancy. So this is the surface tension force 2 pi r*sigma and buoyancy force, already we have discussed in the last one. So here from you will be getting the departure radius of the bubble. Once you put these departure radius of the bubble over here, we will be getting u alpha and if we put the value of u alpha over here.

In this critical heat flux term we will be getting the final expression for the critical heat flux for departure from the nucleate boiling. So here we have seen that using the drift flux model how we can predict the critical heat flux for drift flux model okay. To summarize what we have done, we have drive the continuity and momentum equation applicable for drift flux model. Defined 2 phase distribution coefficients and local slip for different flow regimes.

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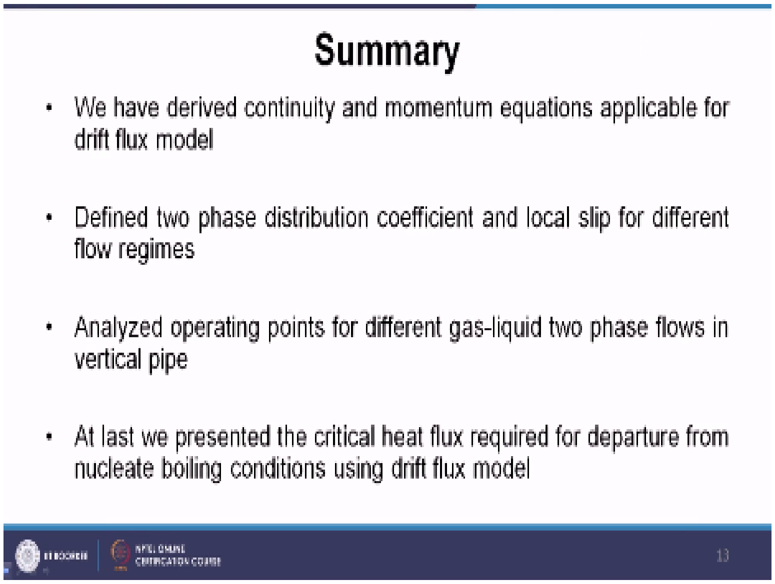

We have also analyzed different operating point for gas liquid Two Phase Flow and finally we have shown you the critical heat flux for departure from nucleate boiling. Let us have some practice. So we are having 3 questions over here.

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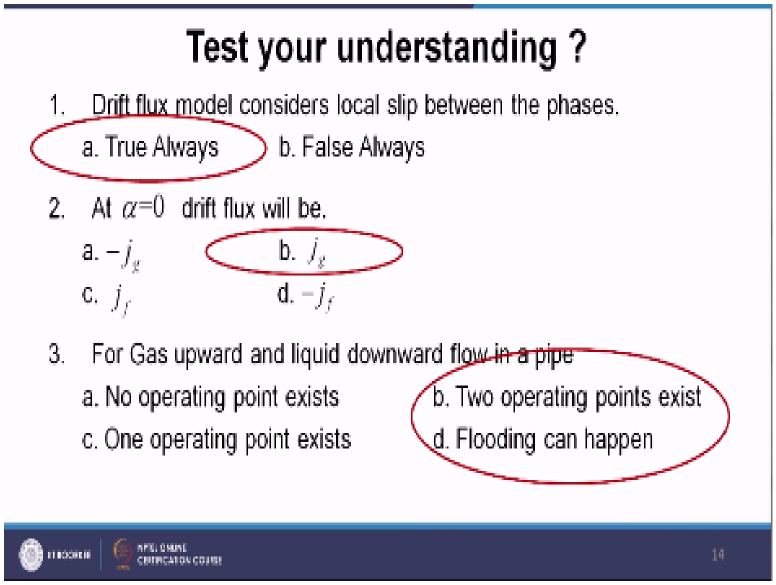

So drift flux model considers local slip between the phases. So you answer in between true and false obviously the correct answer will be true always. Next at alpha equal to 0 drift flux will be jgf will be 4 options were having – jg, jg, jf, - jf the answer is jg. Then for gas upward and liquid downward flow in a pipe we are having 4 options, no operating point exist, 2 operating point exist, 1 operating point exist and flooding can happen.

So which is the correct option? So both these things can happen 2 operating point can exist for a particular velocity and flooding can happen at higher liquid velocities. So with this I will be ending this lecture. Thank you.