**Two Phase Flow And Heat Transfer**
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**Module No: 01**
**Lecture No: 03**
**Homogeneous model**

Welcome to the third lecture of 2 phase flow and heat transfer. In this lecture we will be discussing about homogeneous model. So at the end of this lecture you will be understanding applicability of homogeneous model. What are the basic assumptions of this homogeneous model that also you will be understanding in this one.

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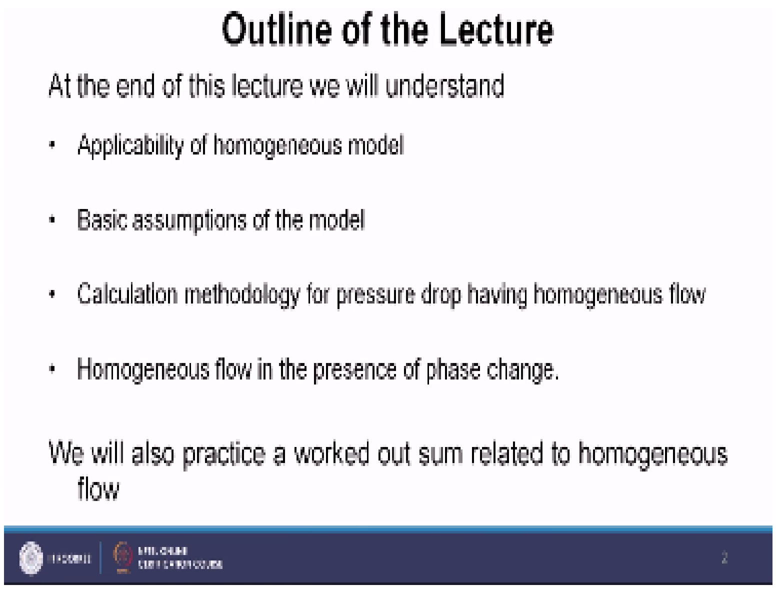


We will be calculating the pressure drop inside a pipeline having homogeneous flow. Also we will be seeing that homogeneous flow in the presence of phase change. And at the end of this lecture we will be also practicing 1 sum where, you will be finding out pressure drop in case of homogeneous flow.

Now to give you a basic outline of this homogeneous flow here I have shown a figure where you can find out that lots of bubbles of gaseous phase are actually homogeneously dispersed in the pipeline in the liquid vicinity.

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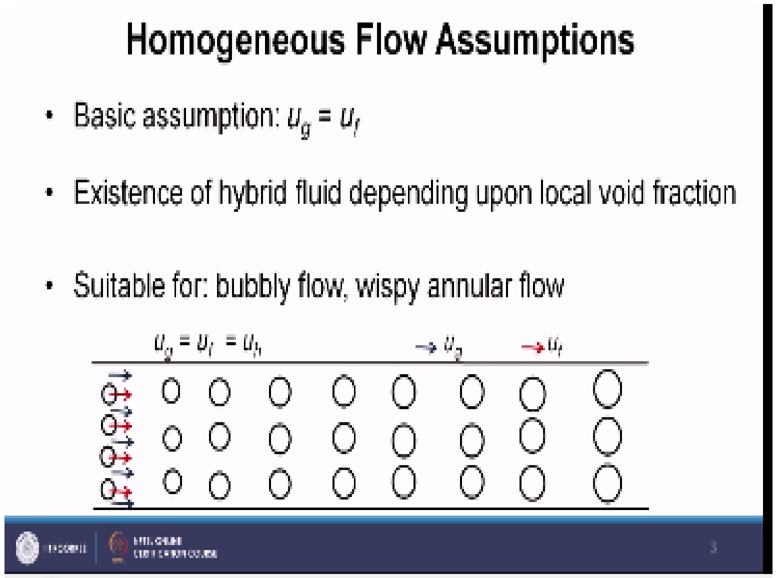


So here you will be calling the liquid velocity as Uf and gaseous velocity as Ug. Now based on the assumption of homogeneous flow we consider Ug=Uf. So that means in homogeneous flow we consider the gaseous phase velocity is equivalent to your liquid phase velocity. So here we write down this velocity as uh, h signifies homogeneous model.

So you will be finding out Ug=Uf=Uh. This type of flow regime is very common for bubbly flow situation wispy annular flow situation and so on. Next as it is homogeneous flow first we need to understand, what are the different properties we can have for homogeneous phase?

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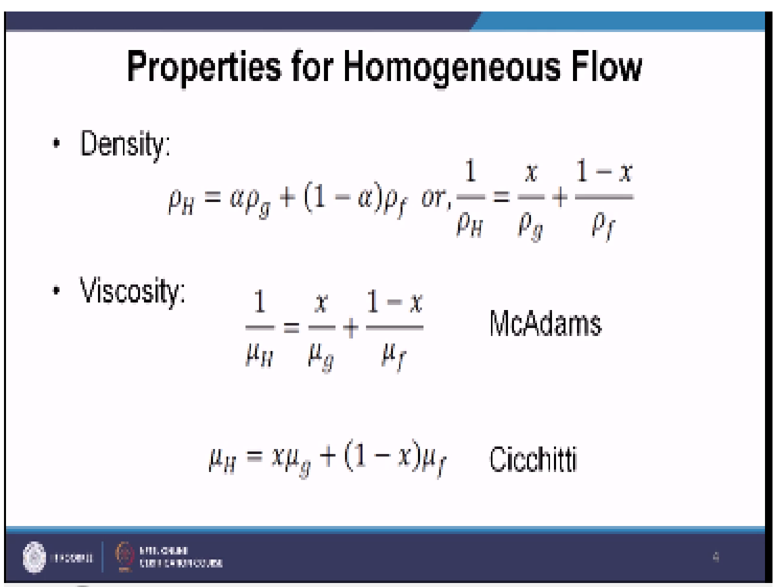

So here first one I have shown you as density so, density Rho h for the homogeneous flow can be written as alpha*Rho g+ (1-alpha)*Rho f. So let us see how that can be proved. So we know that from continuity equation W total flow rate=Wg + Wf right.

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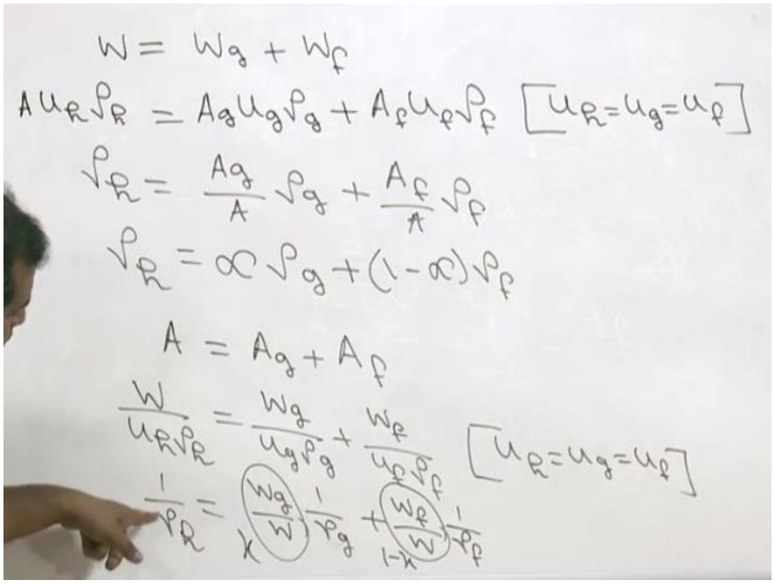


Now in case of homogeneous flow this W can be written as A Uh*Rho h so Rho h is the homogeneous density and Uh is the homogeneous velocity. On the other hand Wg can be written as Ag Ug and Rhog from continuity equation in a similar fashion Wf can be written as Af Uf and Rho f. Now yes, we know from homogeneous model that Uh=Ug=Uf. Uh which is homogeneous velocity=Ug and Uf. So this U can be cancelled from both the sides.

So I will be finding out Rho h = Ag /A *Rho g + Af /A * Rho f. Now we know Ag/A is nothing but our void fraction alpha. So we can write down this is alpha* Rho g and similarly Af / A this is nothing but 1-alpha*Rho f so we get that Rho h is actually alpha Rho g+1-alpha*Rho f. The equation whatever I have shown you over here. In a similar fashion if you go from any aside so total area of the pipeline is occupied by the gaseous phase and liquid phase.

So I can write down A = Ag / Af. So A, I can write down W/Uh*Rho h from continuity equation. In a similar fashion Ag can be written as Wg / Ug*Rho g and subsequently Af can be written as Wf / Uf*Rho f okay. So here once again by taking the homogeneous assumption the velocities can be concerned Uh = Ug = Uf. So we can write down 1/ Uh = Wg /W * 1 /Rho g + Wf / W * 1

/ Rho f right. Now we know Wg/ W from thermodynamics this can be called as x okay and Wf by w can be called as (1-x) right.

So using this I can write down 1 / Rho h=x / Rho g + (1-x) / Rho f.  So homogeneous property density can be written as in these 2 form Rho h = alpha*Rho g + (1-alpha)*Rho f 1 / Rho h = x / Rho g+ (1-x) / Rho f right. Now in the similar fashion viscosity of the fluid can also be defined. You see I have shown 2 important viscosity correlation McAdams correlation and Cicchitti correlation over here.

So viscosity can be obtained by several correlations just like here I have mentioned McAdams correlation which is nothing but homogeneous viscosity 1 / Mu h=x quality/ Mu g gaseous phase viscosity + (1-x) / Mu f which is the liquid phase viscosity. Similarly, Cicchitti correlation also we use observations so Mu h=x * Mu g+ (1-x) * Mu f. So in this way we can obtain the viscosity for homogeneous phase.

Now these are correlations because as we have developed over here the densities homogeneous densities finding out homogeneous viscosity is not easy. So we have to rely on the correlation. Okay next let us try to find out how the pressure drop in a homogeneous pipeline can be found out. So here I have shown you a typical example of a small cross section of the pipeline.
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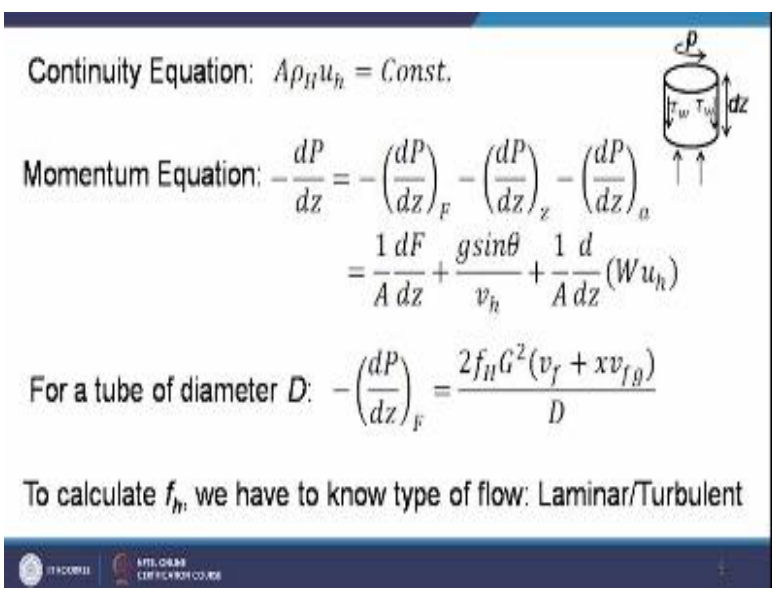

So in this pipeline you can find out the length of the pipeline is dz okay. The diameter of the pipeline let us say there is capital d so the perimeter of the pipeline will be p which is pi*d and we have considered that homogeneous flow is from bottom to top okay. As the flow is from bottom to top in the wall of the pipeline there will be shear stress Tau w. Now let us try to first figure out that how different fluid mechanics equation works over here.

So first the continuity equation you can see over here. Continuity equation is A * Rho h* Uh so you're A* Uh that is A volumetric flux multiplied by Rho h which is the homogeneous density that will be the mass flow rate which will be constant as per the continuity equation. Next the momentum equation from fluid mechanics let us try to write down over here. So -delt p / delt z – dp/dz is the pressure drop across the pipeline that will because due to 3 different reasons.

First one is friction so –dp/dz due to friction. Then –(dp/dz)z which is nothing but the potential heat and then finally –(dp/dz) due to acceleration because flow is there so there can be acceleration so pressure drop can be cause due to acceleration also. So all these 3 friction (()) (08:38) and acceleration all these 3 are actually giving you the overall pressure drop in the pipeline. Let us now try to see all these individual terms separately.

So first the friction comes into picture so see over here delt p /delt z friction that I can write down as 1 / A which is A is nothing but the area cross sectional area of the pipeline multiplied by df/dz where f is the frictional force. So the frictional force I can write down 1 / A df/ dz. Now let us try to see how we can simply that frictional force over here.

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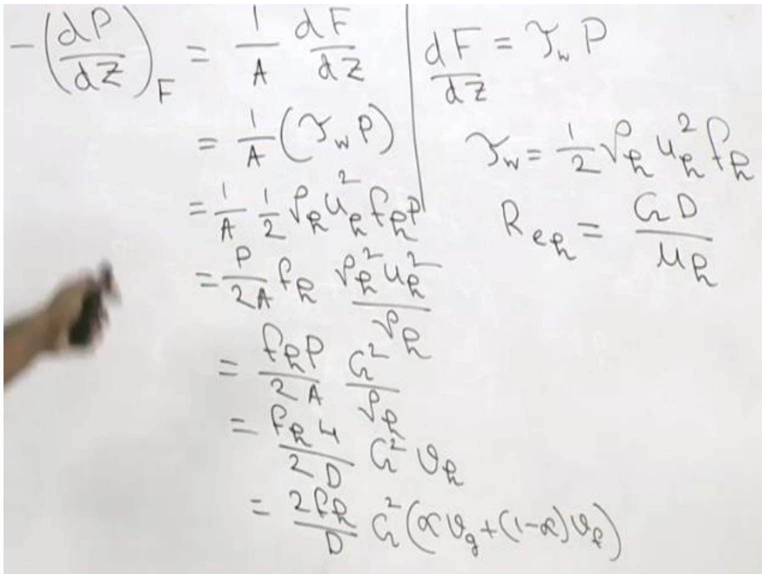

So we will be using the pipe cross section whatever I have shown in the slide so we find out that - (dp/dz) due to friction this is actually 1 /A df/dz right so this f is nothing but the frictional force A is a area okay. Now f is the frictional force and I know that frictional force will be depending on the shear stress. So I can write down df/dz is actually your shear stress multiplied by the perimeter p.

Perimeter p, I have already shown you in the figure which is nothing but pi*d where d is the diameter of the tube. So what I can do I can replace this one over here so you get 1 / a* Tau w*p right. Now what is Tau w shear stress we know from fluid mechanics that Tau w we can write down as ½ Rho h Uh whole square*friction factor Fh okay.

So this friction factor is homogeneous friction factor along with this you are having Rho h* Uh square multiplied with 1/2 right. Now, how to find out this friction factor? Now for this friction factor definitely you need to first identify our homogeneous flow is in which regime whether it is laminar or it is turbulent. So what we do we find out first the Reynolds number for the homogeneous flow which is nothing but Gd /Mu h okay?

So you can find out that from the homogeneous whether it is laminar or turbulent depending on that you will be first finding out the Reynolds number over here which is Gd /Mu h. Mu h already we have shown you can take either McAdams correlation or Chiccitti correlation. Now after finding out this Reh if you find out this is becoming laminar regime. So what will be doing we will be using fluid mechanics equation 64 / re which is very common.

And if it is turbulent you will be using Fh =0 .079 Reh to the power-1 fourth. So once you know the value of Reh finding out friction factor will not be difficult right. Next once we know this friction factor value over here we will be putting this one over here. So, 1/ A * 1/2 Rho h Uh square * Fh right. So this becomes 1/2 A*Fh now in place of Rho h Uh square let us write down Rho h square Uh square / Rho h right. So subsequently we get Fh / 2A now this Rho h*Uh which is actually your which is actually your g so you can write down this one as g.

So you see over here we are having by the way I have missed over here the p so let us keep also p over here. So we will be finding out that this becomes g square / Rho h having p over here.Now p / A so p is a perimeter pi d and a is pi d square/ 4 if it is cancels you will be getting d / 4. So you will be getting Fh and then 4 / d okay and G square and Rho h we can write as small vh which is nothing but specific volume for the homogeneous flow.

So here you see we are getting 2Fh /d*G square and vh can be written as alpha*vg+ (1-alpha)*vf right So this is the final equation for the frictional pressure drop we get which I have shown you over here. You see 2Fh G square vf+x*vfg) by the way this part what I have done alpha I have taken common so this will become (vf+x*vfg) / d, d is the diameter of the tube right.

Next let us go to the next component which is nothing but the acceleration component. So here we see that acceleration component will be 1/A so per unit area and this is a momentum W * Uh, W is the mass flow rate and Uh is the homogeneous velocity. So rate of change of momentum divide by area that will be your force. So let us find out how we can simplify these equations. Next let us go for acceleration part pressure drop due to acceleration.
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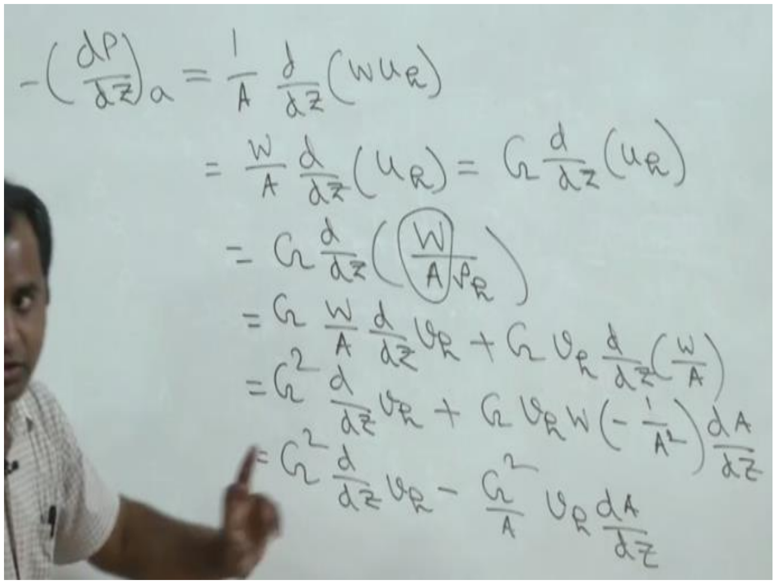


So there we will find out –dp/dz due to acceleration which is nothing but 1 / A d/dz (w*Uh) right. Now you see this W will not be changing with respect to your Z. So you will be finding out that W/ A d/dz (Uh) okay. So you can write down this 1 also W/A is nothing but G. So d/dz(Uh) right.

Now Uh once again let us try to convert it into mass flow rate because we can have occurrences where pipe diameter is not constant. So let us try to convert this 1*d/dz now Uh we can write down as W by A* your Roh h okay. So this is once again coming from continuity because we know W=A*Rho h*Uh.

So let us try to find out what is this differentiation is becoming so here we will be calling this one as first factor and 1 /Rho h which is nothing but vh small vh your specific volume for the homogeneous flow you will be calling a second factor. Now let us see how this by parts differentiation goes so first let us keep this one out and d/dz vh+ we are having G Vh and then d/dz (W/A) right.

W once again is not going to change so that can come out so it will be become d/dz that 1/ A right. So let us take these things separately so this is G square d/dz vh+ here I am getting G Vh W and then d/dz( 1 / A that will become -1/ A square right. So W/A 1 A I can take from here I can convert into G square so this will become G square d/dz of vh+this will become G square by A we can convert this 1 to -okay vh*by the way I missed over here da/dz.

So here we will be having G square by a vh*da/dz right. So we are having 2 terms over here first one is actually giving you if you are having some change of area then da/dz will be coming over here and next one, we are having d/dz (vh) which is the homogeneous specific volume. So let us now try to see how this d/dz (vh) comes out to be. So let me do it over here.

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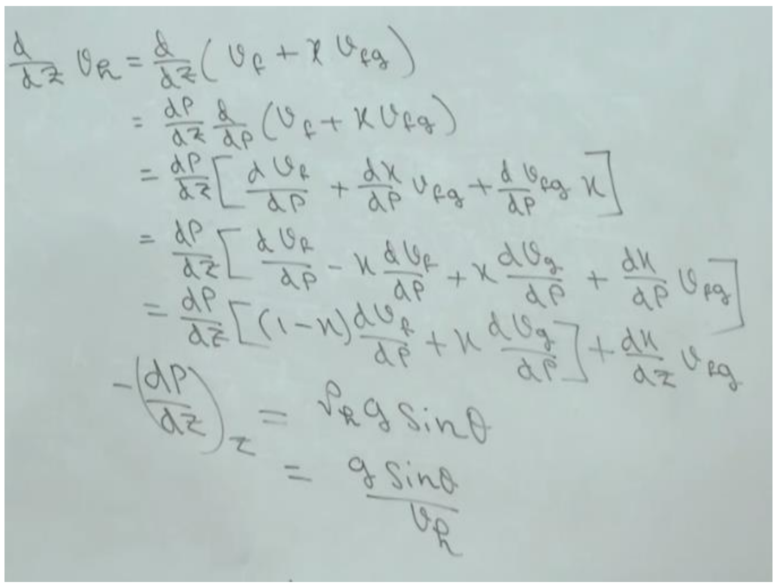

So this d/dz (vh) which is nothing but d/dz (vh) we can write down as vf+ (alpha*vfg) okay. So, all these things are actually function of p. So what will be doing dp/dz and then d/dp(vf + alpha*vfg) right. So let us try to do the differentiation first.So what you will be finding out over here? That dp/dz multiplied by so here also we have to do by parts. So we will be finding out that okay by the way this will be I think in terms of x not in terms of alpha okay.

So we will first write down dvf/dp + here we are having by parts so d/dp (x vfg) +d/dp(vfg*x) okay. So let us pass it further over here. So this becomes dp/dz over here dvf/dp and here you see once again you will be getting x*d/dp (vf – (x*dvg/dp+dx/dp*vfg). So what I have done essentially this terms I have written over here dvfg. I have subtracted like this. So ultimately you will be getting dp/dz and here you see this we take common so we can write down.

We can write down [(1-x)*dvf/dp+x*dvg/dp] and finally this is actually delt p/delt z* delt x/ delt p. So this will become delt x/delt z*vfg right. So all these terms simultaneously I can put in the equation of the pressure drop, over all pressure drop. Apart from that in the equation we are also having – (dp/dz) for potential heat. So that for an inclined tube to be generalized I can write down as Rho h G sin theta where, theta is the inclination with the horizontal.

So what you can do? This we can write down as G sin theta /vh so all these 3 terms, this one and finally this one and the previous one friction factor whatever we have seen, if we club together, then finally you will be getting equation like this. You see this is the final equation of dp/dz for acceleration and this is your dp/dz for potential heat okay gravity heat.

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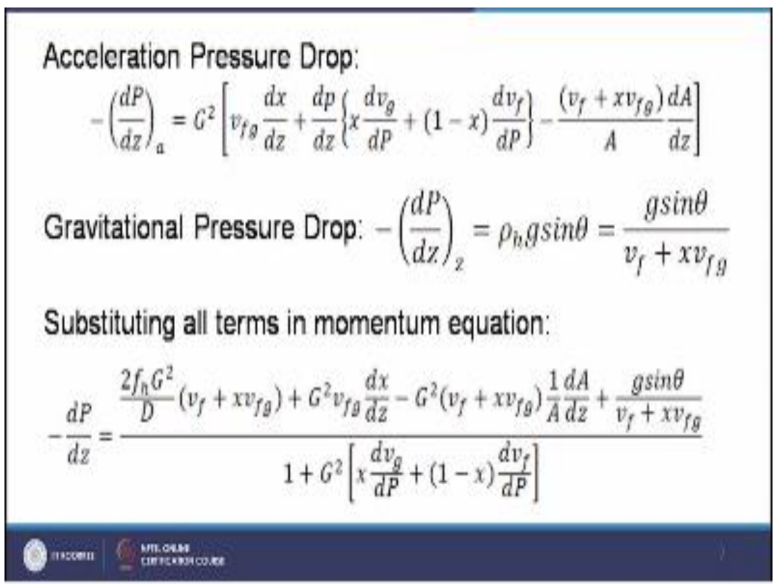


And finally if we club this one along with frictional heat like this, we will be finally obtaining – dp/dz as this equation. So, this one is coming from friction. This 2 terms are coming from your acceleration heat, this one is coming from gravitational heat and finally in the in the down side you were getting 1+ G square [x * dvg/dp + (1-x)*dvf/dp] okay.

Now for most of the fluids you can find out especially for air and water you can find out that this term whatever you are having in this brace is actually nearly equal to 0. So you can see that if we cancel this one, then you will be having – (dp/dz) equals to this term okay. Now if we consider inside a pipeline now for a pipeline which is a constant cross section you will be finding out dA/dz=0 so this term also we can drop.

So for a pipeline flow having common fluids like air and water you can find out- (dp/dz) will be frictional heat + your acceleration heat and finally the (())(23:07) or gravitational pressure heat over here okay. So this is the equation what you can find out for a uniform cross section tube – (dp/dz) = 2Fh G square / d* vf+ xvfg + G square* vfg *dx /dz +g sin theta / vf + x*vfg.

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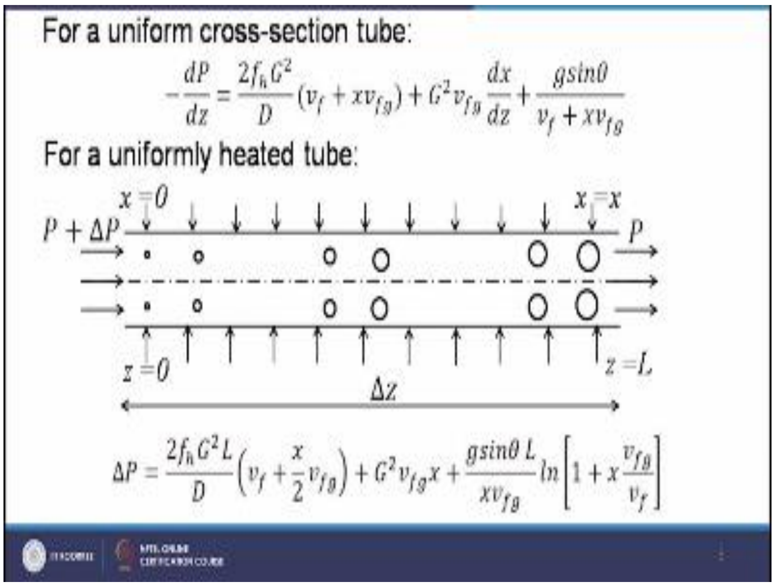


This is basically your Vh whatever I have shown in the calculations over here. Now let us try to see if you are having uniformly heated tube. That means there can be situations where facing the occurring due to the constant heat in tube along the pipe periphery. So here we are shown these arrows are actually for heating tube. So as results as you can find out x is changing from 0 to some arbitrary values let us say x okay.

And the pipeline length in the domain is changing from z = 0 to z= L. So for this type of situations what we need to do if we have to find out the pressure drop in the pipe, p+ deltz p to p. So deltz p is the pressure drop. So you can find out, we need to integrate this equation for this total length of the pipeline. So let us do that and find out how the pressure drop can be obtained. So in this equation I will take term by term and let us see that how dp/dz is changing. So first I will taking this frictional part.

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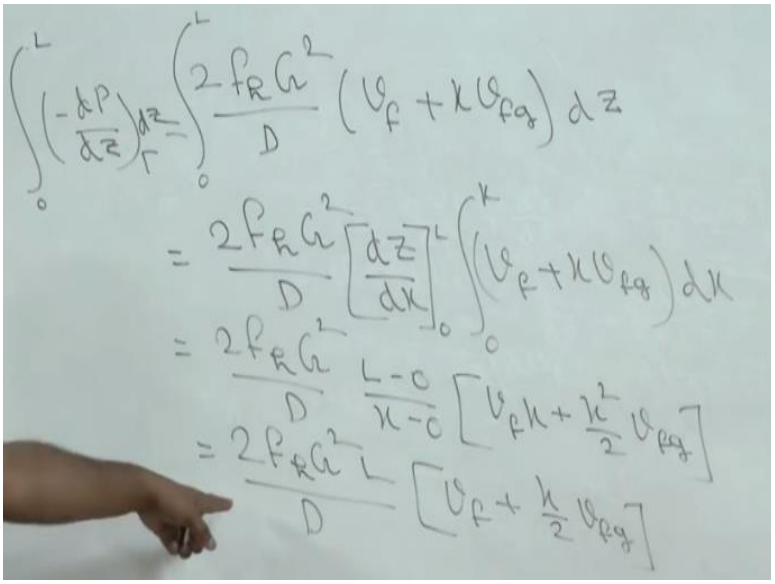

So we are having 2Fh*G square / d and then we are having vf+x*vfg. So this we have got as frictional part okay. Now so this is – (dp/dz) for frictional. So this if I have to integrate for the whole pipeline. So what I will be doing? Integration of 0 to z or 0 to L. L is the pipe length let us say this is for dz right. So we similarly need to integrate this side so 0 to L and here we are having dz over here right. So if I try to integrate then I will be finding out few terms will be coming out quickly.

So G square/d this can come out now here we are having function of x. So what we can do dz / dx. You can take out with the limit 0 to L and here we can keep the integration with respect to x and the limits will be found 0 to x right. So if we integrate, so this will be 2fh G square / d and here dz/dx from 0 to L. I can write down L-0 /x-0. So L / x.

And here this will become [vfx+ x square / 2*vfg] right. So I get finally 2*fh G square L d*x. I can subtract over here. So [vf+x /2 vfg]. So this comes from the frictional pressure drop. You can see over here. You can see over here this I have shown 2*fh G square L / d*vf+x / 2*vfg. So this is coming by integrating this term with respect to z from 0 to L okay.

Similar thing we can do for the rest terms also. So you can see G square vfg G square vfg and we are having dx/dz. If you integrate from 0 to L with respect to dz. So you will be getting x only.

So we are getting over here x and this term G sin theta vf + x vfg. If you integrate then we will be getting this term. So this is a situation where we are having gravitational pressure drop with respect to the phase change when whenever present right.

So this is the final one whatever we get for pressure drop in side a pipeline which is uniformly heated right. Next let us try to see or solve a problem. So here the problem statement is like this.

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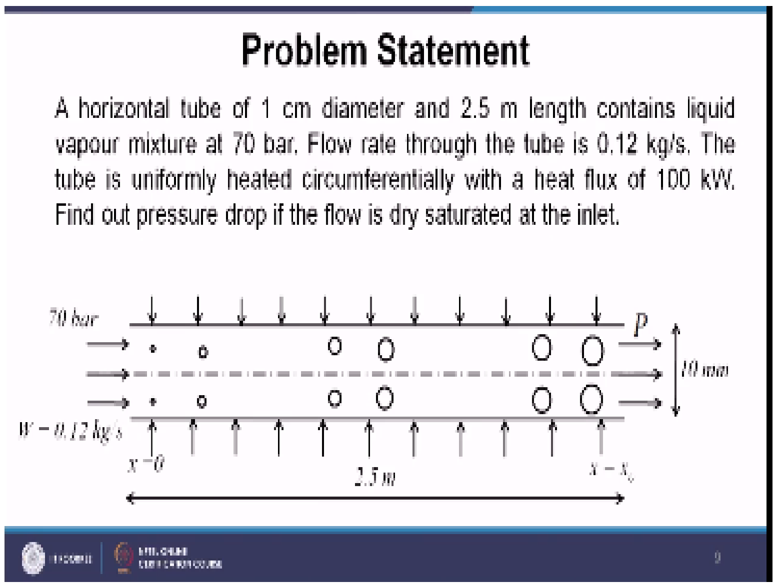


A horizontal tube of 1 centimeter diameter. So this diameter is 1 centimeter. So 10 millimeter over here and 2.5 meter length. So length is 2.5 meter contains liquid vapor mixture at 70 bar. So pressure is 70 bar.  Flow rate through the tube is 0.12 kg per second. So 0.12 kg per second is the overall flow rate W over here. The tube is uniformly heated circumferentially. So were having we are giving the heat from circumference okay.

With the heat flux of 100 kilo waltz so the heat flux whatever you are giving that is a 100 kilo waltz. You have to find out the pressure drop if the flow is dry saturated at the inlet so at the inlet we are having x = 0. So let us try to solve this problem. So first let us find out the at pressure equals to 70 bar. What are the properties we can find out from water that what a table of twin table that t sat = 285.7 degree centigrade?

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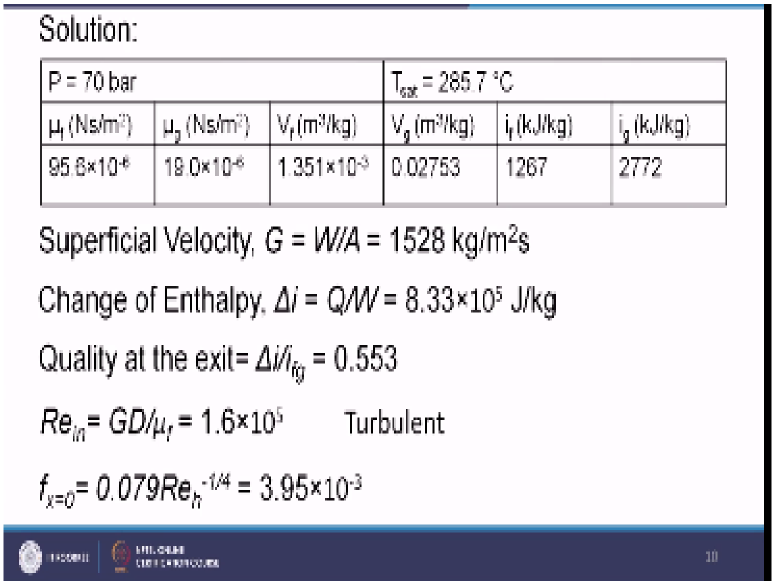


Mu f comes out to be 95.6*10 to the power-6, Mu g is 19*10 to the power-vf is 1.351*10 to the power-3, Vg is 002753 and enthalpy is 1267 kilo joule per kg for liquid and for gas 2772 kilo joule per kg right. Now first let us find out the superficial velocity g which is W by A. So W is given a we can find out because tube diameter is given.

So we can find out the superficial velocity comes out to be 1528 kg per meter square second. Now let us see what is the enthalpy change happening. So have you know the mass flow rate W so and whatever an amount of heat is given. So we can find out the change of enthalpy delta i will be q /W both q and W is given.

So we can find out delta i is nothing but 8.33*10 to the power 5 joule per kg. Now this heat is actually being observed for change of phase. So we can find out that quality at the exit will be this delta i/ iFg subtraction between these 2. So you can get at the exit these will be the quality. So it is starting from 0 and it is ending at 0.553. Let us now try to find out the Reynolds number at the inlet and exit so at the inlet the Reynolds number will be GD/Mu f.

So this becomes this one which is turbulent. So as per understanding what we have to do for finding out friction factor? We have to follow fx = .079*Reh to the power-1/4 and the friction factor is comes out to be 3.95*10 to the power-3 at the inlet. Similarly, at the exit we can find out Mu h=2.96*10 to the power-5.

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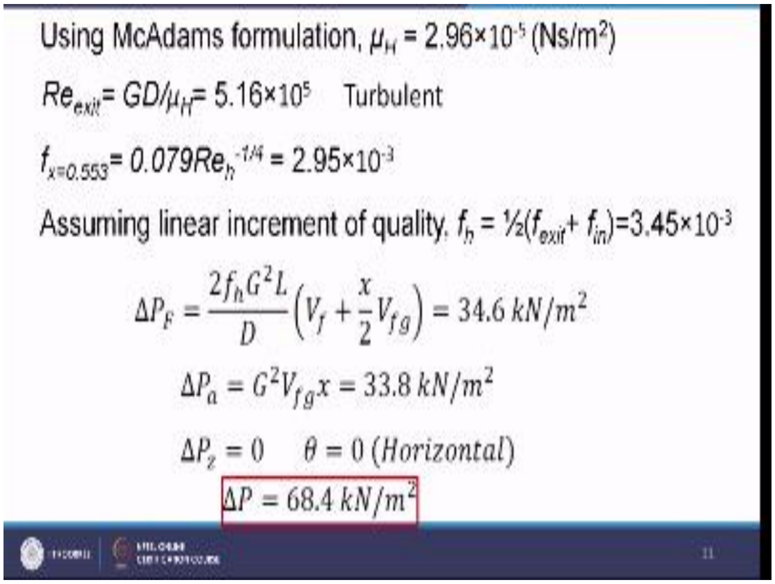


For this we have used McAdams formulation whatever I have told you using the value of x and Mu h and Mu g we can find out this Mu h. So at exit Reynolds number will become Gd by Mu h which is using this value I can find out 5.16*10 to the power 5. This is also coming in turbulent regime obviously.

So you can find out friction factor using same formulation and it comes out to be 2.95*10 to the power-3. So once we get both a friction factor it is very easy to find out the average friction factor which is nothing but fx = 1/2 f exit +f in. This comes out to be 3.45*10 to the power-3. Now already we have described in the previous few slides that what will be the frictional pressure drop?

We know that frictional pressure drop will be 2fh G square L /d*vf+x /2*vfg all the values we have already derived fh is this one. So we can find out that due to friction 34.6 kilo newton per meter square pressure drop will be there. On a similar note acceleration pressure drop will be G square*vfg*x. x we know at the exit. So we can find out this will be coming out to be 33.8 kilonewton per meter square.

And as the pipeline horizontal there will be no pressure drop due to the gravitational heat. So deltp z = 0 because theta is 0 right. So if you add these things total overall pressure drop will be

68.4 kilo newton per meter square okay. So in this way we can find out pressure drop inside a pipe which is uniformly heated. Okay to summarize so what we have done, we have discussed the model equation for homogeneous flow over here.

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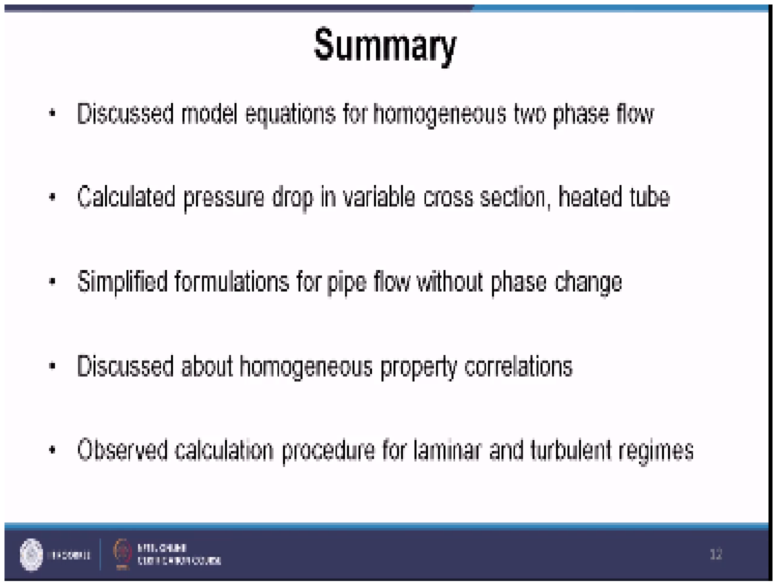


We have also shown you how to calculate the pressure drop in case of phase change in case of variable cross section heated tube okay. We have also seen some calculation procedure some laminar and turbulent regime, how to find out the friction pressure drop for laminar and turbulent also we have seen. Last to test your understanding at the end of this lecture let us take some questions.

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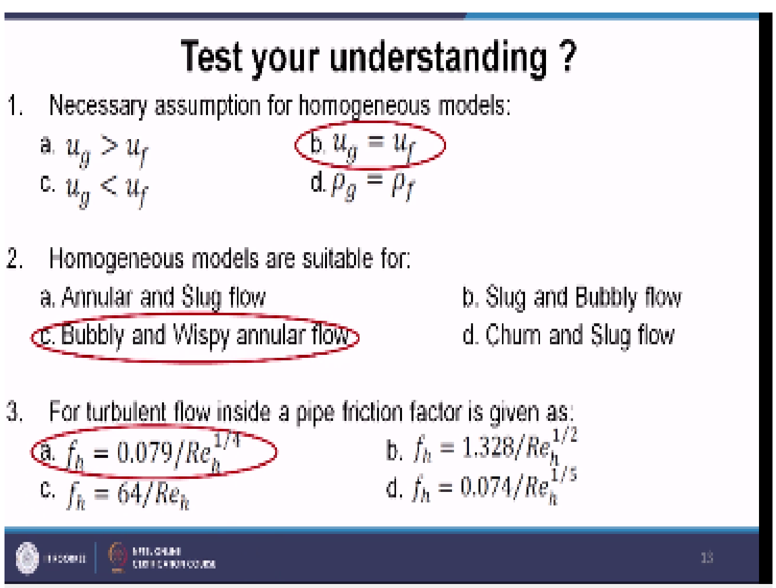

So first one necessary assumptions for homogeneous flow whether it is Ug > Uf, Ug = Uf, Ug < Uf and Rho g = Rho f. So obviously, all of us are getting the answer your correct answer is Ug = Uf. Second question homogeneous models are suitable for annular and slug flow, bubbly and wispy annular flow, slug and bubbly flow and finally churn and turbulent flow.

So which 1 is the answer the applicability of the homogeneous flow is for bubbly and wispy annular flow. Let us see the third question for turbulent flow inside a pipe friction factor is given as so 4 different equations we have given for turbulent flow. So which one is the correct answer all of us know from the fluid mechanics knowledge. So correct answer is Fh = 0.079 / reh to the power ¼. Okay so with this knowledge we will be ending this lecture. See you in the next one. Thank you.