Two Phase Flow and Heat Transfer
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Lecture No: 19
Solid-Liquid Flow

Hello, welcome to 19th lecture of Two Phase Flow and Heat Transfer course. Today's topic will be discussing about solid liquid flow okay. So at the end of this lecture you will be understanding the following topics.

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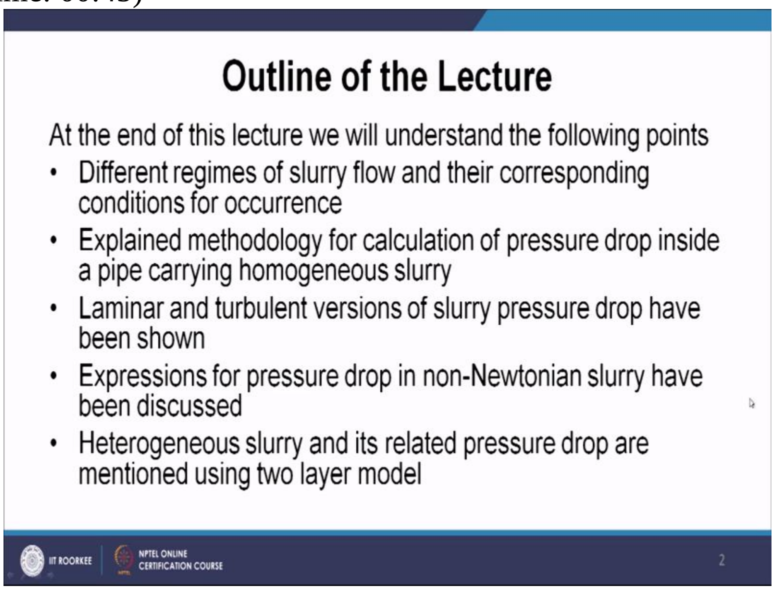

You will be understanding different regimes of solid liquid slurry flow and their corresponding conditions for occurrences. We will be explaining methodologies for calculation of pressure drops inside a pipe carrying slurry flow. We will be seeing in laminar and turbulent regime what will be the slurry pressure drop. We will be expressing the pressure drop for non Newtonian slurry.

And finally we will be showing you that how heterogeneous slurry can be tackled and its related pressure drop can be calculated. In this context we will be introducing 2 layer models. Okay, so let us first see what this solid liquid is and how that can be characterized. Solid liquid flow mainly it is classified as slurry flow okay. So in slurry you will be finding out suspension of solid particles okay in liquid okay. Now the mixture of solid and liquid can have a different format that means it can be homogeneous mixing, it can be heterogeneous mixing and so on.

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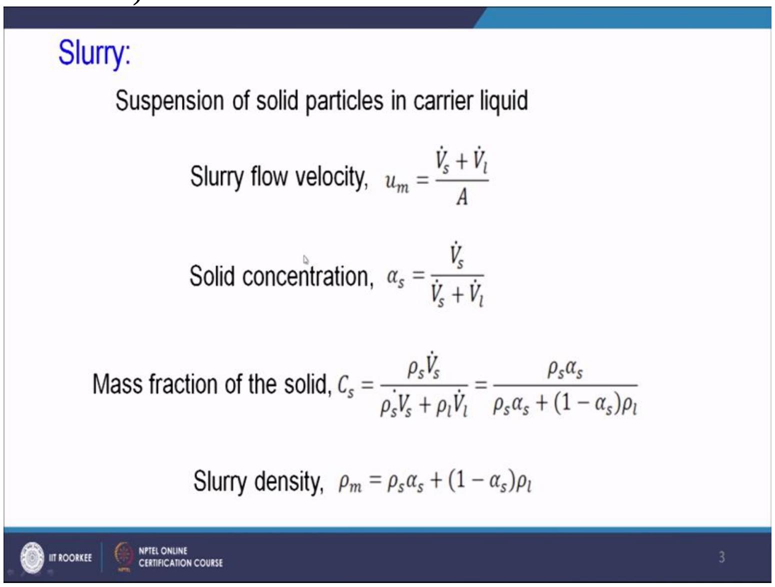

And overall whenever it is homogeneous mixing there can be fluid properties dependent on that we can classify this one as Newtonian and Non Newtonian slurry. So let us first start to understand that what are the different characteristic parameters for slurry flow. So first we will be see what is slurry flow velocity. So in case of slurry flow velocity, we write down the velocity as um so which signifies that the mixture velocity. Remember this velocity is for a homogeneous mixture okay. So this velocity um can be characterized as volumetric flow rate of solid + volumetric flow rate of the liquid / the pipe cross sectional areas.

So that means v dot s which is the volumetric flow rate of solid +v dot l / A that is your flow velocity, slurry flow velocity okay. Immediately next we need to know what is the concentration of solid in the slurry. So that concentration solid concentration we express in the form of alpha s. So, alpha s is a solid concentration. That can be written as obviously the ratio between the solid flow rate / the overall flow rate. So here solid flow rate is v dot s and overall flow rate is v dot s + v dot l okay. So this is actually your solid concentration alpha s. In a similar fashion we can define the mass fraction of the solid.

So mass fraction means here solid concentration is in the term of the volume fraction. So once we find out the mass fraction of the solid then you will be finding out that we have to multiply this alpha s expression with their respective densities. So you will find out that Cs is nothing but here we have v dot s. So it will be v dot s * Rho s to get the mass of the solid / whole mass. So, whole mass will be nothing but Rho s * v dot s + Rho l * v dot l okay. Now little bit of modification of this one, if I divide this one v dot s the whole equation, if I divide then you will be finding out that we can replace this volumetric flow rate in terms of your alpha.

So we can write down alpha is 0 is nothing but your solid concentration previously explained. So you will be finding out that it will be this expression will turn out to be Rho s alpha s / Rho s alpha s + (1 - alpha s)Rho l okay. So, just division of this v dot s by v dot s+ v dot l will be giving you this one. So denominator and numerator you divided v dot s/ v dot l okay. Next if we try to find out that what is there at the bottom side of this solid concentration? This we can define as mean slurry density okay. So Rho m mean slurry density will be equals to alpha s, solid density multiplied by your solid density is Rho s and alpha s is your solid concentration +(1 – alpha) which is actually your liquid concentration multiplied by Rho l okay.

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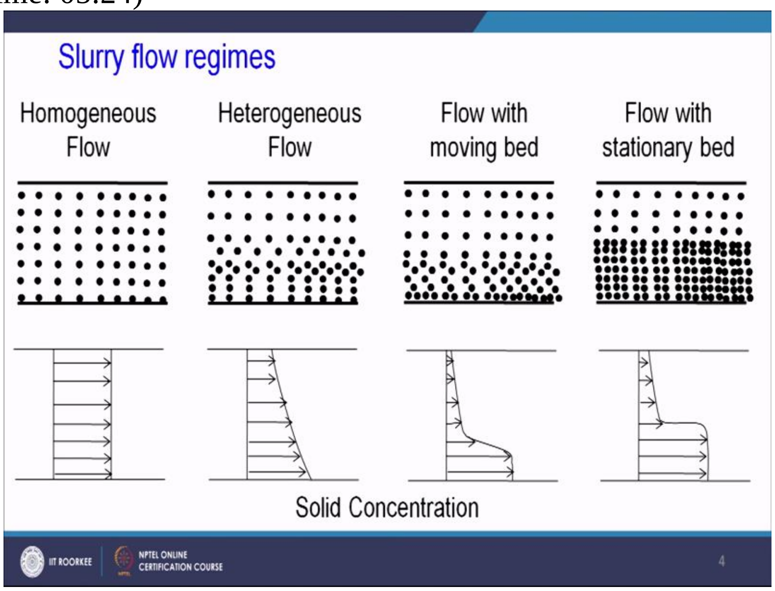

Next let us see that while we are having slurry flow what different flow regimes are possible okay. So we will be starting with very low liquid velocity, will be starting with the velocity contour over here. So what will be finding out, that slurry flow regimes depending on the dispersion of the solid in the liquid can be classified into several categories. Okay starting from this one, which is homogeneous flow, here you can see inside the pipeline we are having homogeneous dispersion of the solid okay.

So everywhere the solid concentration will be more or less similar okay. So you see over here particles will dispersed okay. So if we try to plot it, solid concentration you will be finding out solid concentration across the pipeline is constant curves. So that means everywhere we are having same solid concentration alpha s. So that you characterization of this homogeneous flow okay. Next if the solid concentration increases then you will be finding out that some amount of solid is getting settled down okay. So we go for different solid concentration at different radial location. So that is called heterogeneous flow.

In case of heterogeneous flow in the schematic, I have shown you over here at the lower side of the pipeline. You can find out more amount of alpha s or solid concentration, whereas in the upper side we are having very low amount of solid concentration. So you will be finding out solid concentration curve along with the radial direction. So you will be finding out that it is increasing whenever we go down okay. Next let us see whenever we are having flow with moving bed. So in this case you will be find out that there is some sort of deposition on the bed of the pipeline okay.

At the bottom side of the pipeline the solid has deposited and formed bed and the bed is actually moving along with the flow okay. So here you will be finding out and this bed whenever it is moving you will be finding out this is very thin in nature that is why it can be transporting with the overall flow. So you will be finding out over here that. Though we are having precipitation or you know down flow of the solid particle but along with the flow, carrier flow liquid flow. You will be finding out this solid will be also transporting. Okay, if we find out the solid concentration, will be finding out a curve like this.

Here we are having overall moving particles and here we are finding out that solid concentration is very high but still that is moving okay. But it is characterized by a thin layer of this higher solid concentration okay. Next you will be finding out still if we increase the solid loading. Then you will be finding out we are getting flow with stationary bed. Now what will happen you will find out though there is a solid bed at the bottom pipeline but due to it is thickness you will be finding out that the liquid incoming slurry mixture will not be able to transport the whole amount of solids okay.

So you will be finding out this bed is more or less stagnant or stationary okay. But if we try to plot the solid concentration, will be finding out that it is having a very high concentration at the bottom which is stationary actually and here we will be having moving slurry okay. So you can find out the thickness of this one okay with the higher concentration is far higher compared to this one where we are having lower one. So this 4 regimes we can see starting from homogeneous, heterogeneous your moving bed and stationary bed in case of your slurry pipeline.

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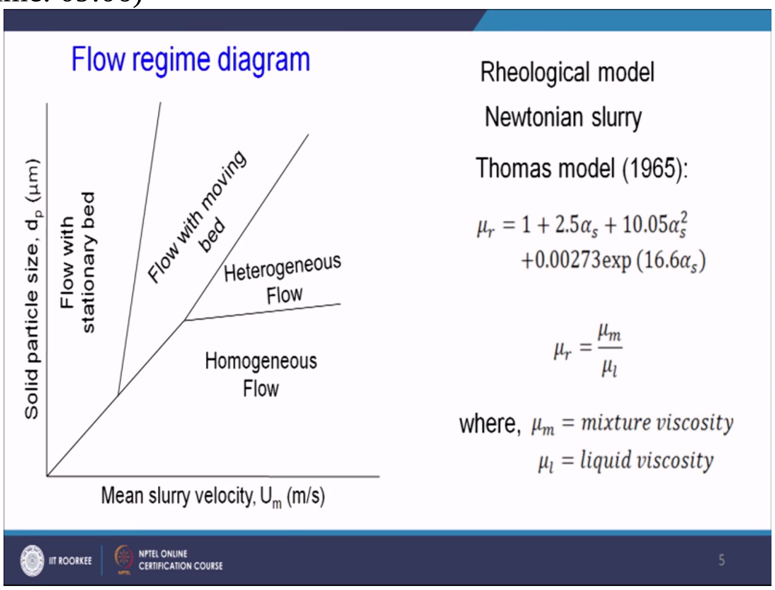

Let us try to see; now how we can characterize these flow regimes depending on the velocity, mean velocity of the slurry and the particle diameter, because these are 2 important parameters on which the regime will be depending. So let us see over here. So here you see in the abscissa we have plotted the mean slurry velocity and in the ordinate we have plotted the solid particle size dp. So you can find out over here whenever the liquid velocity is actually increasing very high, liquid velocity or you can say at lower size of the solid particles always will be getting homogeneous mixture of the solid and liquid, which can be characterized as you know homogeneous flow okay.

So that means there we are having (())(09:28) dispersion of the solid particles and the carrying liquid. So if the particle sizes are small then we can find out that it is having a homogeneous mixture. On the other hand if the flow velocity is high also then also you will be getting that it is having homogeneous mixture. Next if you increase the solid particle size that means the particles if those are getting bigger than there will be role of gravity which will be allowing the solid particles to fall down and you will be finding out that there is some sort of non-homogeneous mixture okay. Or we can say heterogeneous flow okay inside the pipeline.

So this heterogeneous flow actually occurs for larger size of the particles okay. Now if we go for further higher particle sizes then you will be finding out that not only the heterogeneous flow is actually balancing that one, you will be finding out there is deposition of the solid on the bed of the pipeline. And you will be finding out we are having flow with moving bed okay. And whenever the liquid, over all mixture velocity is reducing further then you can find out that the bed is getting thicker and thicker and finally the bed will become stationary and you will be obtaining the flow with stationary bed okay.

So flow with stationary bed happens at very high particle size or very low mixture velocity okay. Now let us see how this homogeneous flow can be tackled rheologically okay. That means how to find out the mixture properties. Already I have shown you the density how to find out the mixture density okay for solid liquid flow. Here I will be showing you how to find out the mixture viscosity because these are 2 important rheological parameters we need to take care of. So here let us see rheological model the first I will be showing you for a Newtonian slurry.

That means homogeneous but still the fluid is behaving as Newtonian that means your shear stress and strength are proportional to each other. So we can write down in that case that your viscosity follows a model, famous model called Thomas model. So Thomas has given this model for finding out the viscosity in 1965. What he proposed that, Mu r, Mu r is actually relative viscosity of the mixture and the liquid. So Mu r is actually Mu m / Mu l okay. That is equals to 1 +2 .5 alpha s+ 10.05 alpha square +0.00273 e to the power 16.6 alpha s.

That means this relative viscosity is actually a function of your solid loading alpha s okay. So once we find out what is the loading then you will be getting the value of Mu r and once I know the value of carrying liquid viscosity which will be pure fluid we will be getting what is the mixture velocity over here okay. So finding out rheology is not that much difficult now. So next as we have done in case of your gas solid pipeline. Here also let us do the same thing find out the pressure loss okay. Whenever we are having slurry flow and let us considered that the pipe is having a steady section.

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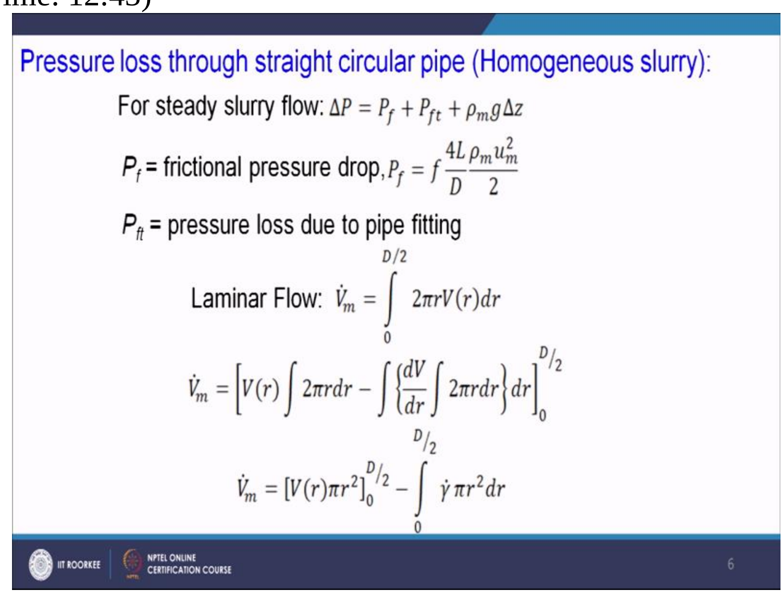

So let us now go for you know critical things like bend or something t joint or something like that. Let us considered we are having straight circular pipe and we are finding out the pressure loss. Now let us see by the way if we are doing it for you know some bend section or something like that then you will be finding out a function of the pressure drop will be coming over here which is Pft, which we have mentioned over here as pressure drop due to pipe fitting. So pipe fitting can be bend, pipe fitting can be t junction or something like that. So those empirical relations are already there through which we can find out the pressure drop in case of the pipe fittings.

So I will not be going in detail of pipe fitting pressure drop but let us see what the other part are. So other parts as we know that in the steady slurry flow will be having the frictional pressure drop as well as we will be having the buoyancy pressure drop or gravitational pressure drop. So, gravitational pressure drop is very simple. We know the mean slurry density already I have shown you how to calculate. So that will be alpha s Rho s + (1 -alpha s) Rho l. So once we know the mean slurry density * g and then the potential head what the pipeline is actually occurring inside the pipeline it is occurring.

So that will be giving the potential head or buoyancy head okay. So we are mainly interested in the pressure drop due to frictions. So Pf so all we know that is that pressure drop due to friction Pf is = f * 4l / D *Rho m um squared / 2 okay. So you see in this expression which is coming from fluid mechanics. So, all other parameters will be known including the Rho m and um okay. But only unknown will be f. So main target of this derivation is to show how f can be calculated. So let us see how f can be calculated but before going that let us try to find out what is the mixture volumetric flow rate.

First, will be starting with laminar 1 okay. So then quickly will be going for the turbulent 1. So first will be showing with laminar slurry. So in case of laminar slurry you will be finding out v dot m, which is the mixture volumetric flow rate is nothing but 2 pi r v r * D r. So v r is the let us say at particular radial location what is axial velocity. So this will be multiplied by with 2 pi r * dr. So 2 pi r is actually the circular r and multiplied by dr is actually infinitesimal small circular ring and if we integrate, if from 0 to D /2 pi diameter then you will be getting the overall pipe area multiplied / v r, which is dependent on the radial location will be getting the volumetric flow rate v dot in okay.

So if we do the integration by parts because here we are having r. Here we are also v is function of r then we get v(r) integration of 2 pi r dr - dv dr integration of 2 pi r dr* dr okay limits stays over here from 0 to D /2. Now if you come over here for the first term you see this is coming as vy v(r) into 2pi r dr comes out to be pi r square and the limit is 0 to D /2 now. Interesting thing over here is that v(r) will be becoming 0 at D / 2 and at r = 0 the whole expression gets 0. So at upper limit and lower limit this expression will be vanishing. So this goes to 0 but here in the other hand side you see we are having dv/ dr.

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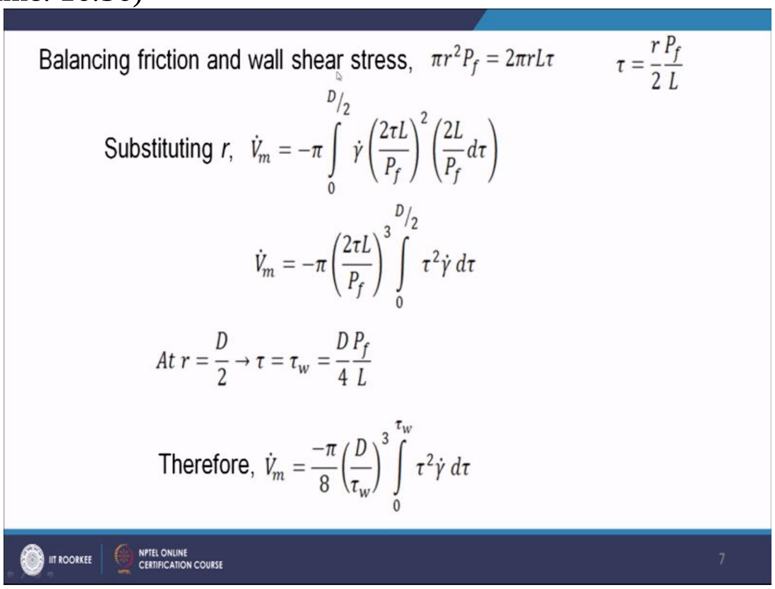

So dv/ dr is nothing but your string. So I can write down gamma dot okay. And here 2 pi r dr integration of 2 pi r dr this gives me pi r square and overall dr remains over here with the limit of 0 to D /2 okay. So if we go further. We can write down that this vm is actually a function of wall shear stress. So let us see how we can do that. So let us first balance the frictional force and the wall shear stress. So already we have done this kind of analysis in case of your pipeline flow gas liquid pipeline flow. So pi r square into pf, which is the frictional force is actually equal to 2 pi r l.

So 2 pi r is actually your perimeter and then l is the length of the pipe multiplied by the wall shear stress Tau okay. So if you balance this one you can write down Tau is nothing but r / 2 Pf / l. So what I can do, I can replace always Tau in term of r /2 pf/ l. So already we had over here you see gamma dot pi r square dr. So what we can do, we can replace over here that what is the value of gamma dot over here. So here we have, you see from here if you finding out what is the value of r. r will be 2l Tau /pf. So we have given 2l Tau /pf whole square. So this is gamma dot r square.

And if you do the differentiation of r with respect to Tau you will be getting dr = 2l / Pf * D Tau. So here I have written that one in place of dr. Now once you simplify this one you will be getting that v dot m i = -pi 2 Tau l /pf whole cube because here we have whole square and here we have single pf. So it becomes whole cube and then 0 to D /2 inside the integration. It is Tau square gamma dot *delta Tau okay. Now for the wall, we know that r = D /2 Tau will be going to wall shear stress Tau w. So Tau w can be written as once again by following this equation it can be written as D /4 Pf / l.

So what we can do this expression Pf. I can replace using Tau w. So this is comes out to be - pi / 8* (D / Tau w) whole cube and then the integration 0 to Tau w Tau square gamma dot del Tau right. So let us continue further. So if you see, in case of Newtonian slurry we know laminar Newtonian. So we know the Tau = Mu m * gamma dot. So let us replace this gamma dot over here with this t au /Mu. So we will be getting something like this. So Tau cube Mu m delta Tau okay. Once we go for the integration give me Tau to the power 4 /4 Mu m okay.

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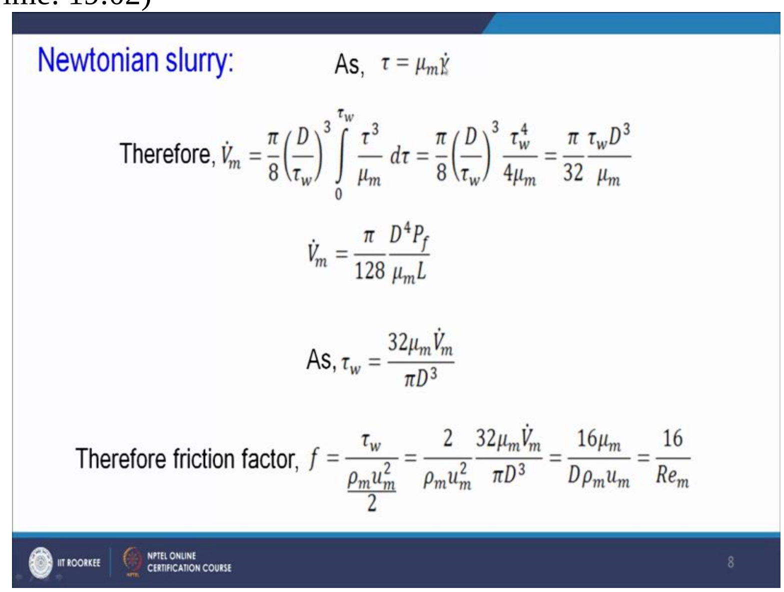

And you know canceling out Tau cube and Tau to the power 4. It gives me pi /32 Tau w D cube / mu m okay. So ultimately v dot m we get if we once again replace that this Tau w with respect to Pf. So we will be getting pi /128 D to the power 4 Pf mu m* l okay. You can get that the value of Tau w also from this expression in terms of vm. So Tau w comes out to be 32 Mu m vm / pi D cube okay. So this is expression for Tau w and vm okay. Then let us quickly calculate what is the friction factor. All of we know that friction factor Tau w a friction factor f is actually your Tau w by Rho m Mu m square / 2.

So little bit of simplification and putting the value of v dot m and Tau w form here. You will be getting that this comes out to 16 Mu m /D Rho m um okay. So ultimately it becomes 16 /mixture Reynolds number Rem okay. So this mixture Reynolds number will be following the density of the mixture and the viscosity of the mixture right. So if we go for the non-Newtonian version of this one so here I have shown you 3 different non-Newtonian models power law Bingham Plastic model and Herschel-Buckley model.

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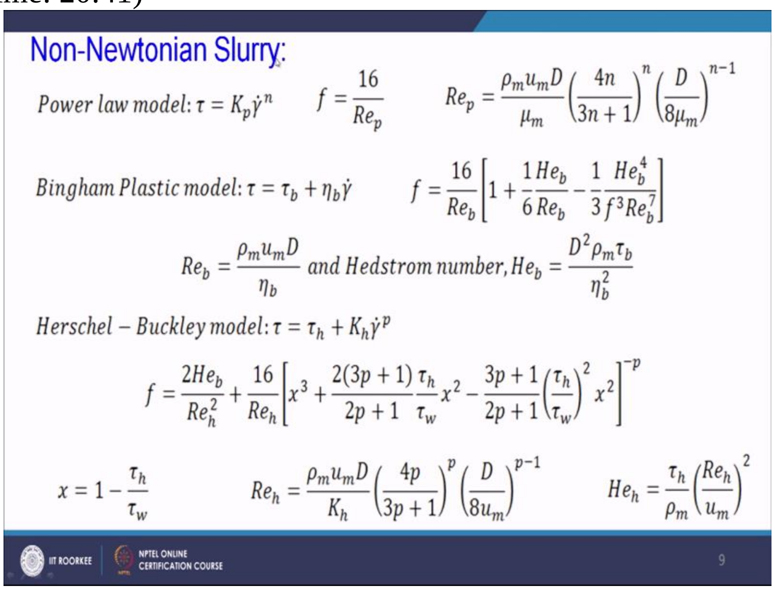

So all of we know from fluid mechanism that for power law Tau = kp gamma to the power of n. For Bingham Plastic, Tau = Tau b +eta b gamma to the power n. For Herschel-Buckley this Tau h + Kh* gamma to the power of p. So we put all these things in the previous integration. Whatever I have shown you ultimately will be getting that this f is nothing but 16 /Reb into this non-dimensional number ratio, where you see we are having your Hedstrom number and Reynolds number over here into picture okay.

Similarly power law we are getting Rep, which is nothing but your Reynolds number and here this is the function which is involving the term n which is the characteristics of the power law model. Similarly for Herschel-Buckley, you see will be finally getting f is actually this term, where you see p is one parameter which is coming from the model and Tau h will be also coming over here because these are modal parameters for Herschel-Buckley. Okay, let us go to next from laminar to turbulent. In case of turbulent flow will be finding out that the famous correlation Churchill correlation can be used for finding out the friction factor.

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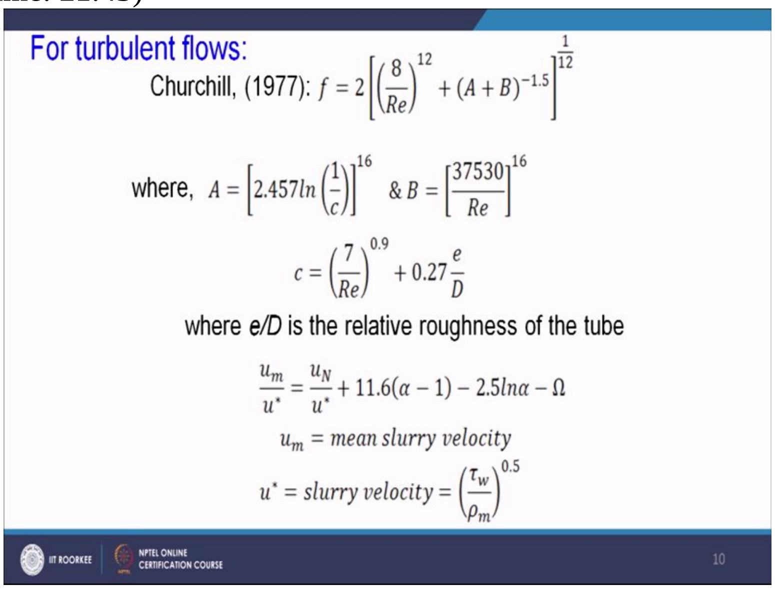

So Churchill he has mentioned that f = 2* 8 /re to the power 12 + a +b to the power - 1.5 and overall to the power 1 /12 okay. Here a and b these are actually given here like this A = 2.457 ln of 1 /c to the power 16 and b is 37530 /Re to the power 16 okay. Where c is nothing but dependent on the Reynolds number and the relative roughness of the tube e / d okay. Once you know the fraction factor it is also necessary to get the mean velocity. So which actually has proposed that mean velocity um / u star, where u star is slurry velocity. Slurry velocity can be written as Tau w /Rho n to the power half. So that is equals to un /u *+ 11.6 * (alpha-1) – (1–(2.5 *ln alpha – gamma).

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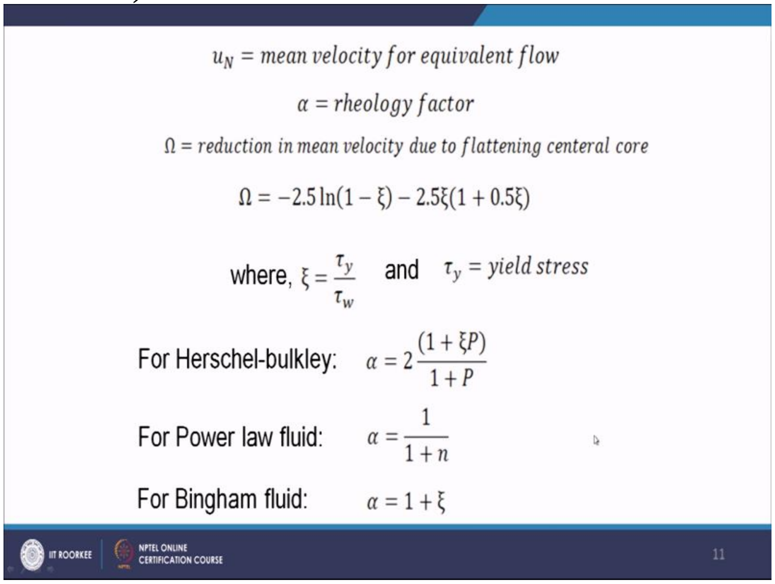

Now let us see what are this terms. So um is nothing but mean velocity of the equivalent flow okay equivalent flow of the liquid okay. Then alpha is the rheological factor okay. Alpha can be found out for different fluids. I will be showing you gamma is reduction in mean velocity due to flattening central core. So in case of non-Newtonian, you will be finding there is flattening of the central core. So that gamma is taking care of that one. So gamma can be written as - 2 .5 ln (1 – zeta) – (2 .5 zeta)(1 – 0.5 *zeta), where zeta is nothing but Tau y/Tau w.

Tau y is nothing but once again yield stress can be found out from the materialistic properties. And alpha whatever I have told, here the rheological properties or factors that can be that Churchill has proposed for different fluids; Herschel-Buckley power law fluid and Bingham fluid different functions of zeta and the rheological parameters okay. Then let us go to heterogeneous slurry. So after this homogeneous slurry all of we know that we will be having homogeneous slurry at a very high velocity. Then once the velocity decreases, we will be finding out that heterogeneous slurry comes.

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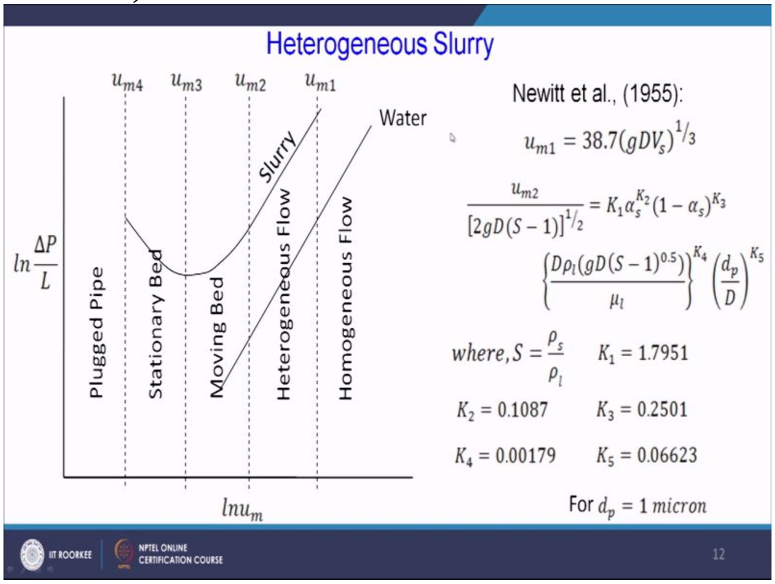

Then slowly further reduction will be giving you moving bed then stationary bed and at some point of time we had very low velocity, you will be finding out the pipe is totally plugged okay with solid okay. Now here we have some typical you know boundary velocities which will be differentiating between homogeneous and heterogeneous and heterogeneous and bed okay wherever the solid starts to drop down or settle down okay. So let us understand what is um1 and um2.

So Newitt in 1955, Newitt et al in 1955, they have given the idea what will be the um1 and um2. They proposed that um1 will be 38.7 *gDVs okay to the power 1/3.So this Vs is once again solid loading and he has also proposed that um2 can be written in terms of your you know density of the ratio of solid and liquid and some important factors k1, k2, k3, k4, k5 okay. And he has given this one for particle diameter 1 micron okay. He has proposed um2 as this correlation right, which is dependent on different Ks, 5 Ks and s okay.

So what we can do, we can characterize depending on the parameters and the properties that whether it will be homogeneous or heterogeneous slurry okay. Next let us see for heterogeneous slurry how to calculate the pressure drop already we have shown for you homogeneous. So you mainly we follow a model called two layer models. So two layer why because you can see here, we are having heterogeneous there some will be sort of deposition at the bottom of the pipeline bed. So this is the solid bed and here we are having more or less flowing slurry okay.

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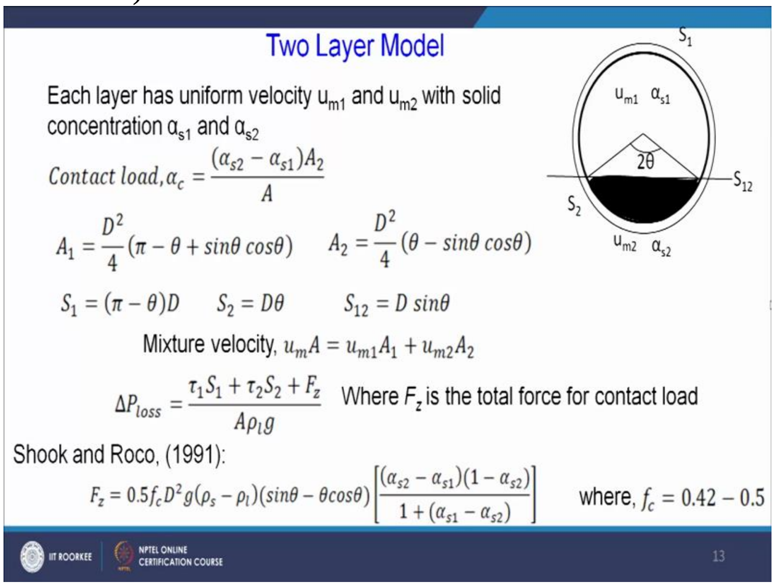

So we called this one as a you know 2 separate sections as it is 2 having 2 separate sections we are naming the velocity of this mixture as um2 and um1 respectively. Let say this bed which has already settled is making 2 theta angle okay from the centre of the pipeline okay. So here you see this bed is actually contacting this s2 amount of arc of the pipeline, where as the upper portion of the slurry is contacting s1 portion okay. So little bit of geometrical analysis if you do we can find out the contact length along the periphery by this 2 zones of 2 layers of slurry.

So what you can do, s1 you can write down pi - theta * D and s2 you can write down D*theta okay. Also we can show that the area which is occupied by this, you know upper layer of the slurry and lower layer of the slurry can be written as a1 * D square /4 pi - theta - of sin theta cos theta and a2 D square /4 theta - sin theta cos theta okay. So all these things can be calculated from you geometrical configuration of the pipeline. We can also that this line which is nothing but the junction between both the layers that is written as s12 the length of this line can be written as D into sin theta.

So first whatever will be showing is that contact load, contact load is alpha c. So contact load alpha c can be defined as alpha s2 - alpha s1* a2 /a okay all the parameters are known over here. So you can find out contact load. Then for finding out the mixture velocity and pressure loss you see mixture velocity um A can be written as um1A1 + um2 *a2. So these are the velocities for the layers okay um1 and um2 and corresponding to area we have already stated. Now pressure drop if you try to find out then it will Tau1 * s1 for the contact of the lower portion and Tau2 * s2 for the contact of the upper portion and then there will be a contact load called fz.

So that will be coming into picture / A Rho l * g. Now contacts load something which one need to find out empirically. So Shook and Roco in 1991, he has given this expression for contact load. So you see here everything is dependent on the value of alpha s1 alpha s2, which are know alpha different loading for 2 layers as well as we are having theta s parameters. So theta is 2 theta is these angles. So half of this one we need to take okay. So fc is very, very important over here. This is the friction factors for contact load. So usually we find out that fc is in between 0.42 to .5 okay. So this is the model, which using in which you can find out the contact load for 2 layer model.

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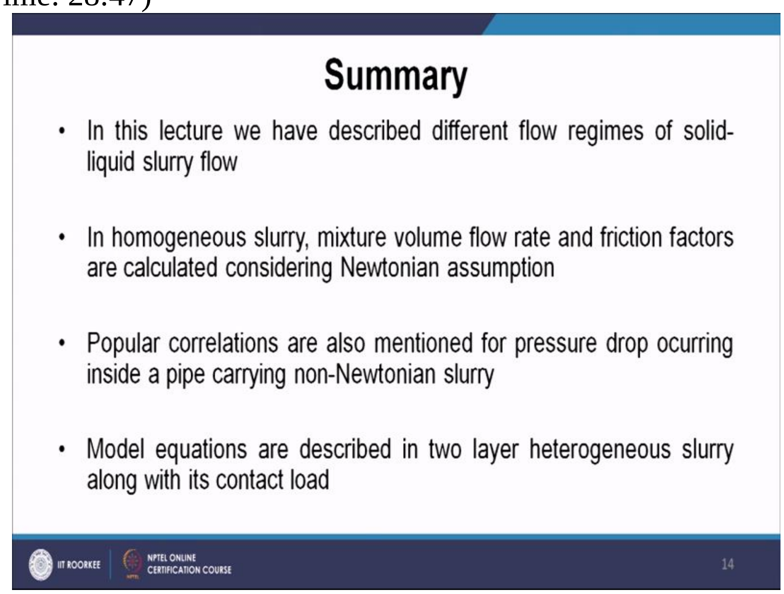

So let us summarize in this lecture. We have described different flow regimes of solid liquid slurry flow. In homogeneous slurry we have found out the pressure drop considering laminar, turbulent and considering Newtonian and non-Newtonian slurry. And finally for heterogeneous slurry using 2 layer model, we have shown how pressure drop can be calculated okay. Let us test your understanding at the end of this lecture. First question as velocity of carrier liquid increases which regime conversion we observe.

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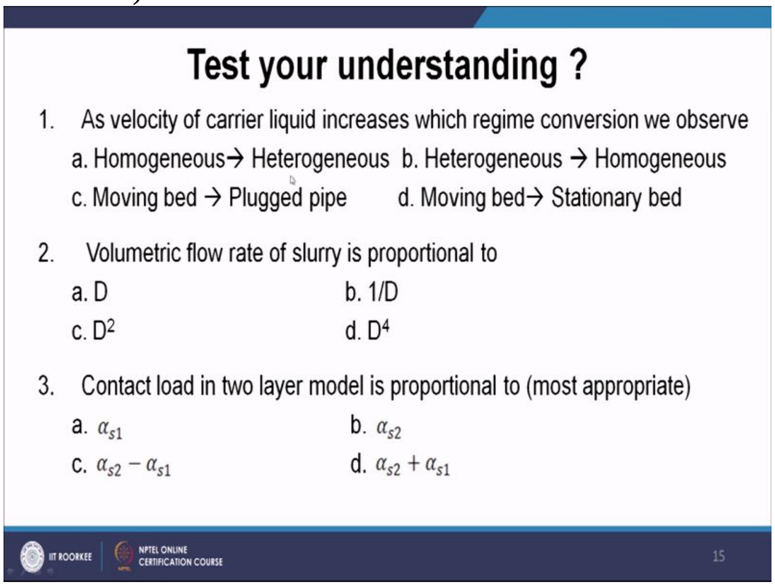

So carrier liquid is increasing its velocity. So first answer is homogeneous to heterogeneous, second one heterogeneous to homogeneous, third one moving bed to plugged pipe and fourth one moving bed to stationary bed okay. So correct answer is part b heterogeneous to homogeneous. Hope all of you have answered the correct one. Question number 2, so volumetric flow rate of slurry is proportional to. So volumetric flow rate is v dot m proportional to. We are having 4 answers D 1/d D square and D to the power 4 okay. So already we have shown the volumetric flow rate expression.

So you find out this is proportional to D to the power 4. So last question contact load in 2 layer model is proportional to okay. So you have to the most appropriate answer among all the options. So we are having alpha s1, alpha s2, alpha s2 - alpha s1 and alpha s2 + alpha s1. So these alphas are different loading fractions for different solid layers okay. In 2 layer model, so I think all in the last slide last to last slide I have shown you the correct answer all of you can you have guessed. So obviously the correct answer will be alpha s2 - alpha s1. So okay with this understanding I will be ending this lecture, thank you.