

Two Phase Flow and Heat Transfer
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Lecture No: 01
Introduction

Hello welcome to the Two-Phase Flow and heat transfer course. We are in the first lecture introduction to two phase flow. I am Dr. Arup Kumar Das. I will be sailing you through this course. Along with me I am having my teaching assistant Mr. Parmod Kumar.

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So if necessary you can contact both of us during this course.

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Syllabus					
Module	Module 1	Module 2	Module 3	Module 4	Module 5
Duration					
Week 1	Introduction	Flow Regimes	Homogeneous Models	Drift Flux Model	Separated Flow Model
Assignments *	Problem Solving (MCQ Answer) (Topics covered in 1 st week)				
Week 2	Dispersed Flow	Slug Flow	Annular Flow	Droplet Annular and Stratified Flow	Measurement of Void Fraction
Assignments *	Problem Solving (MCQ Answer) (Topics covered in 2 nd week)				
Week 3	Signal Analysis	Two Fluid and Population Balance Model	Interface Tracking	Lattice Boltzmann Method	Smoothed Particle Hydrodynamics
Assignments *	Problem Solving (MCQ Answer) (Topics covered in 3 rd week)				
Week 4	Molecular Perspective of Two Phase Flow	Boiling Heat Transfer	Condensation	Solid-Liquid Flow	Gas-Solid Flow
Assignments *	Problem Solving (MCQ Answer) (Topics covered in 4 th week)				

Okay, first I will be showing you the syllabus what we will be covering in this course. As you know that this course will be spanning 4 weeks and each week we are having 5 modules. So what we have done we have described or we have divided the syllabus into 20 lectures over here. So in the first week we will be giving you brief introduction on the Two Phase Flow we will be discussing about different flow regimes available in Two Phase Flow in the second module.

In first week third module we will be discussing about the easiest module possible which is called homogenous model. Then we will be going for drift flux model where sleep between the phases we will be considering. And finally, at the end of this week we will be finding out that how separated flow can model Two Phase Flow over here. At the end of this week we will be getting 1 assignment okay, mainly this assignment will be MSQ type.

In the second week we will be discussing the several regimes separately. For example, will be starting from the dispersed flow then will be talking about slug flow, annular flow and we will be also discussing about droplet annular and stratified flow together in 1 lecture. At the end of this week we will be also understanding how to measure experimental the void fraction. So at the end of second week, just like the first week we will be also having 1 assignment okay.

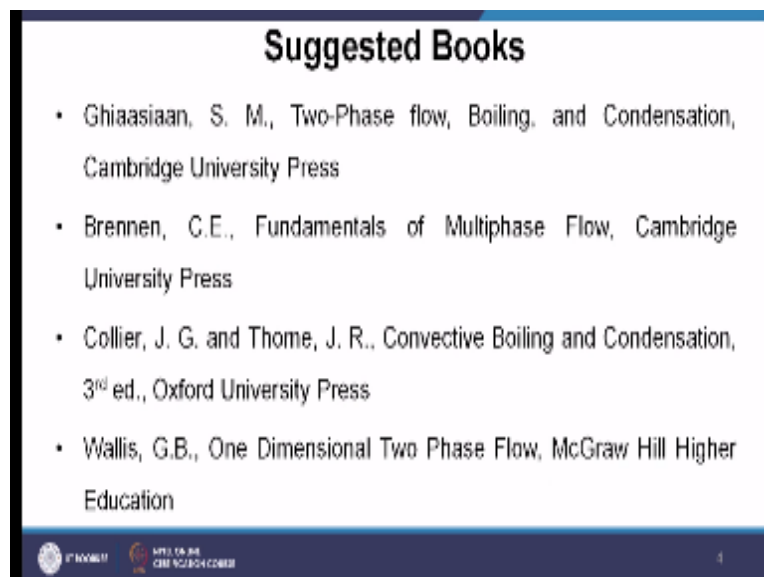
And in third week we will be starting with signal analysis mainly we will be doing here the measurement of void fraction whatever that we have received from there by doing signal analysis

will be trying to understand the flow regimes. Apart from that I will be showing you different numerical models also in this week. We will be starting with 2 fluid balance, population balance model then I will be showing you how to track the interface using volume of fluid.

And we will be learning some advanced methodologies like lattice Boltzmann smoothed particle hydrodynamics and molecular dynamics okay. So at the end of third week we will be also getting 1 set of assignment. And in the last week we will be seeing some specific cases like boiling heat transfer. So phase change then will be having the just to reverse 1 condensation and towards the end of this lecture we will be having solid liquid flow and gas liquid gas solid flow okay.

And at the end of fourth week also we will be having a set of assignments and finally it will be followed by 1 end semester examination.

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So some suggested book we are having some books as text book S M Ghiaasiaan, CE Brennen, Thome Collier, Wallis these books you can follow if necessary.

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Related Journals

- International Journal of Multiphase Flow, Elsevier Publication
- International Journal of Heat and Mass Transfer, Elsevier Publication
- Journal of Fluid Mechanics, Cambridge University Press
- Heat and Mass Transfer, Springer
- Langmuir, American Chemical Society
- Physics of Fluids, American Institute of Physics
- Journal of Computational Physics, Elsevier Publication

Some related journals where you will be finding out lectures of Two Phase Flow and innovations of Two Phase Flow is coming as published form. Now quickly I will be coming to the outline of the first lecture. So in this lecture we will be learning about what are the target areas of Two Phase Flow.

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Outline of the Lecture

- We will learn about the domain of target in this course
- Next, we will classify two phase flow in different categories
- Different applications and functionalities involving two phase flow will be discussed
- Importance of two phase flow in industrial perspective will be highlighted
- At the end of this lecture we will be able to understand different nomenclature of two phase flow

Also we will be learning what are the daily life and industrial applications of Two Phase Flow and towards the end of this lecture you will be learning about different terminologies to be used in the future course of Two Phase Flow and several useful notations for Two Phase flow and heat transfer. So to begin with let us understand what are different phases possible in real world? We

can find out solid phase, liquid phase as well as gaseous phase. We know that molecular structures of all these phases are not similar.

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• Commonly occurring phases of matter are:


Solid


Liquid


Gas

Change of Phase occurs due to increase/decrease of molecular spacing


Closely Packed


Moderately Packed


Loosely Packed

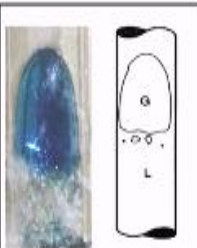
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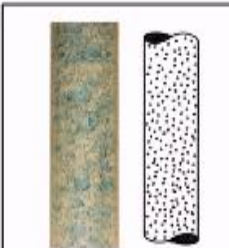
In case of solid phase we are having closely packed and in case of gaseous phase this is loosely packed and liquid is somewhere in between. Now if you are having association of 2 different substances or phases or 2 different components we will be calling that one as Two Phase Flow.

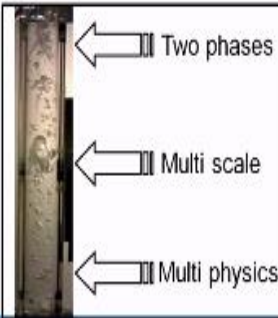
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Definition of Two Phase Flow

- Flow of two phases of a substance or two different components
- Breaking and making of time and space variant interfaces


**Well separated flow:
kerosene slug in water**


**Well dispersed flow:
kerosene bubble in water**


Two phases
Multi scale
Multi physics

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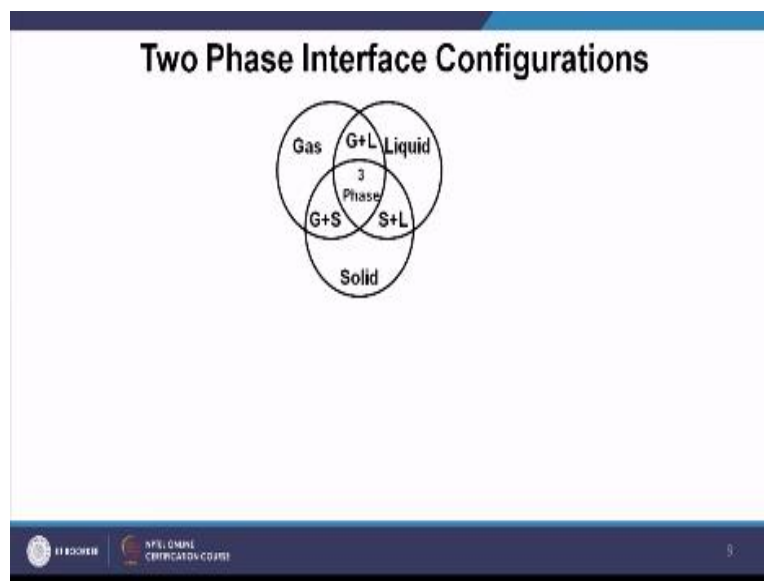
Now breaking and making of time and space variant interface is the common feature of Two Phase Flow. Here I have shown you well separated flow, you can see a kerosene slug in water blue colored thing is kerosene and around we are having white colored fluid which is nothing but

water. So you can see the well and distinct interface of the kerosene slug over here. But on the other hand in this figure, I have shown you well dispersed flow of kerosene in water.

So lots of blue color bubbles you can see in water flow. So these are 2 different horizons well separated flow and well dispersed flow. Now due to these things we are having 2 different types of features over here. Here we have shown a schematic video of bubbly flow in a channel. So you can find out existence of Two phases over here. So this is the gaseous phase and around we are having liquid phase.

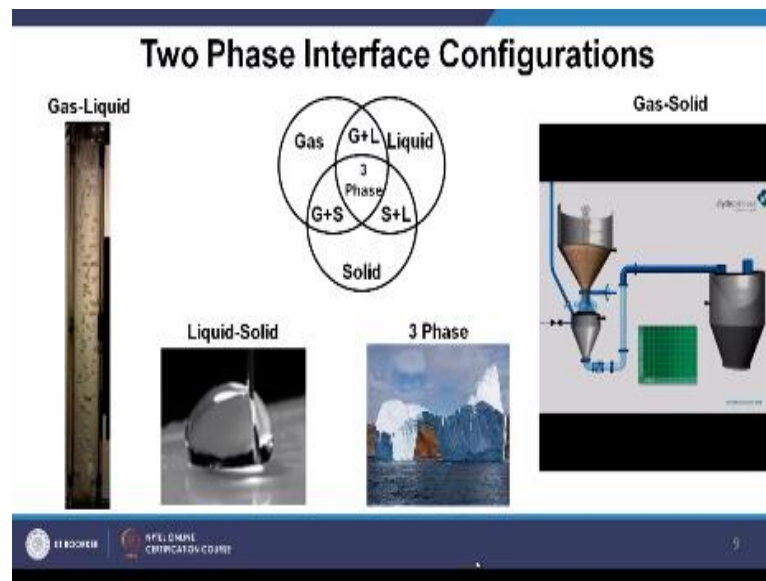
Apart from that if you see the video once again you will be finding out a well distinct interface, you can see which is a bigger in size and smaller bubbles are also passing. So that means it is an occurrence of multi scale. Side by side definitely as the scales are different, you will be having different physics for this type of problem. Next let us come to different domains of Two Phase Flow. So we are having single phase over here gas, liquid and solid.

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So if you are having occurrences where gas and liquid comes together that is gas liquid Two Phase Flow. Gas and solid comes together we are having gas solid Two Phase Flow. And simultaneously if solid and liquid comes together, we will be having solid liquid Two Phase Flow. There can be some sort of examples in real life and industry where all 3 phases come together and this will call as Three Phase Flow. Let me give you some examples over here.

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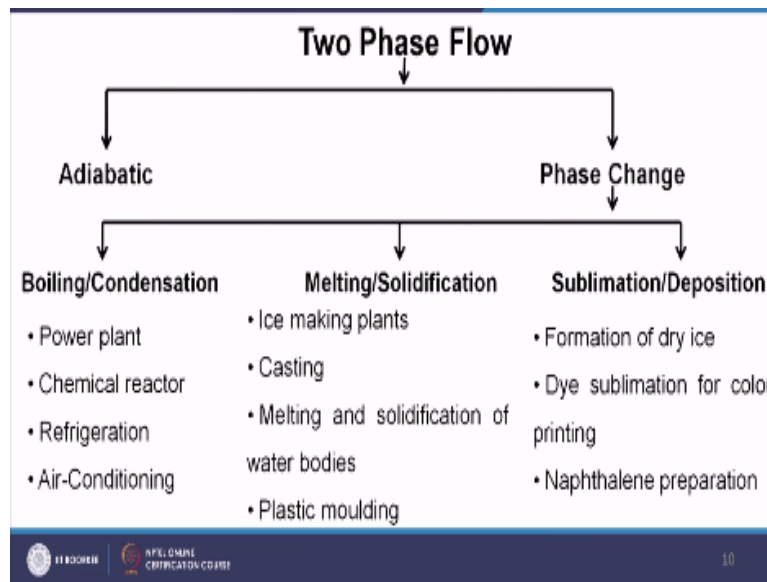


(Video Starts: 06:27) This is another time I am running that video where, you can find out gaseous here and liquid water in co-existence. So this is a typical example of gas liquid Two Phase Flow. (Video Ends: 06:38) Next I will be showing you another one where, you can find out liquid drop is situated on a solid surface and due to some liquid filled application it is changing its shape and size. So you can find out this is a typical example of liquid solid 2 phase flow.

Another one here I have shown gas and solid, so you can say these are a solid particle which is coming inside and you can find out here we are having association between gas and solid. So this is a typical example of gas and solid Two Phase Flow. We are also having examples of Three Phase Flow over here where, you can see solid ice liquid water and air in the vicinity. So this is a typical example of Three Phase Flow okay.

Let us now see that based on heat transfer how Two Phase Flow can be categorized. If there is no heat transfer associated, so you will be finding out adiabatic conditions.

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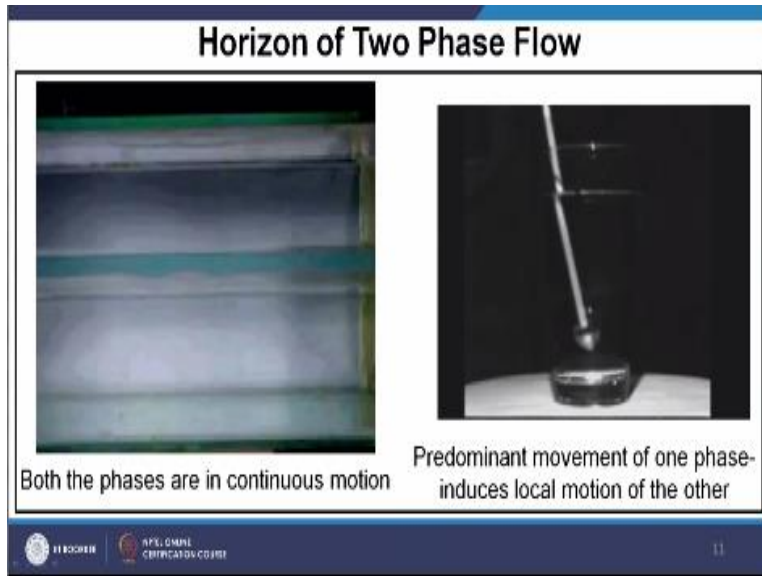


So pipe will be having no phase change inside this. But if you are having phase change or allowed heat transfer so you can have 3 different situations boiling or condensations, melting and solidifications and finally sublimation and deposition. So boiling and condensation is associated with gas liquid regimes, melting and solidification is associated with solid and liquid regimes and finally sublimation or deposition associated with gas and solid regimes.

So here I have noted down some of the important applications of all these 3 regimes. Boiling and condensation, melting and solidification and finally sublimation and deposition. Next let us see how Two Phase Flow horizon is now it is changing. So if we talk about flow, always it comes in on mind that both the phases will be flowing simultaneously. So here I have shown 1 video.

(Video Starts: 08:20) you can see kerosene is flowing from left to right and water is also flowing from left to right. So we are having well and defined interspacing between these two (Video Ends: 08:28). So this actually we can call as Two Phase Flow but you can find out some examples.

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Where predominant motion of the primary phase is not there only by impinging the (Video Starts: 08:41) secondary phase the motion is being generated in the primary phase so this kind of situation actually also a taking inside the horizon of Two Phase Flow. (Video Ends: 08:51)

So it is not only about the flow if the flow is being generated by the secondary phase that also we can tackle inside the Two Phase Flow horizon. Okay next without knowing the applications or you know daily life examples any subject we will not be studying. So here what I have done? I have given you some ideas about the applications of Two Phase Flow. So you see these are different occurrences of Two Phase Flow in nature.

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Starting from water wave gas liquid Two Phase Flow dew on leaves. So this is once again gas liquid Two Phase Flow. Here you can find out spitting cobra, a cobra is spitting so this is actually once again gas liquid Two Phase Flow geyser water fall. So downstream of waterfall geyser means we will be having hot bubbles blasting in the liquid surface, lots of droplets are being generated as well as daily life rain we can see all those are examples of gas liquid Two Phase Flow.

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Different occurrences in daily life sweating in our kitchen sink we can find out lots of bubbles are being entrapped in the kitchen sink while you are emptying a bottle we can find out gas liquid Two Phase Flow over here. You are sprinkling in your garden. So this is also 1 example of gas liquid Two Phase Flow.

If you are mixing your tea with your sugar or milk, so you will be finding out lots of bubbles are actually being entrapped over here. So this is 1 example of Two Phase Flow. Also we are having another example where, you can find out you are pouring water in a glass. So lots of bubbles are being entrapped so this is also example of Two Phase Flow. Some other examples in our daily in your toilet sink whenever you are taking bath in the soap foam, you are drinking buttermilk.

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So when your buttermilk is being stirred for mixing Two Phase Flow is being generated lots of bubbles in shower aquarium pump lots of bubbles are being generated very classy looking things we are also having some refreshment drinks at the top of the drink you can find a foam and at the bottom you can find out lots of bubbles so this is another example of Two Phase Flow. Also in the shaving foam which is typical example of Two Phase Flow once again.

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Few other examples we can see over here. So whenever you are swimming in different sports activities water polo, wind surfing underwater swimming and rafting all these are examples of gas liquid Two Phase Flow and in case of ice skating you can find out ice flakes in the form of solids is coming into picture in case of air so this is actually solid gas Two Phase Flow. Next few

other examples of Two Phase Flow whenever your car is splashing in water, so you can find out lots of droplets are being generated.

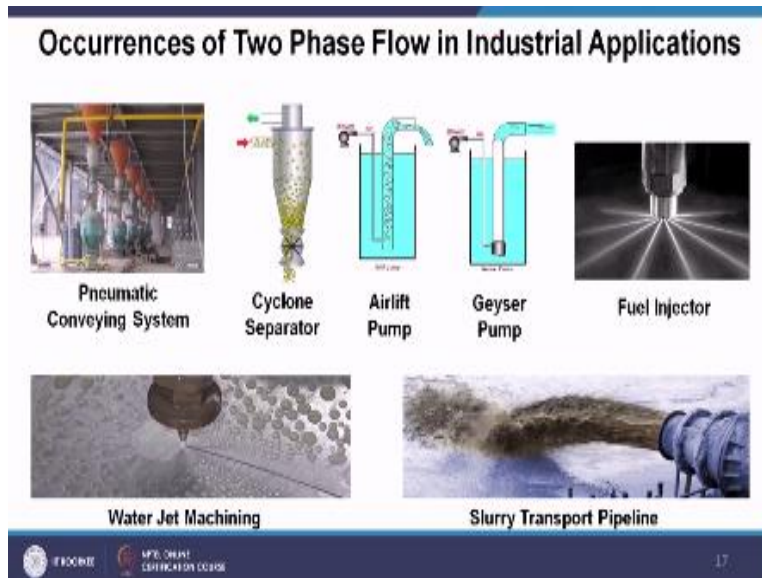
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While helicopter is landing so you can find out lots of gas solid flow is over here. Spiders web this once again solid liquid Two Phase Flow so you can find lots of droplets on the solid spider web. In case of dripping in hospital, we go for dripping so you can find out its nice droplets are being created over here once again Two Phase Flow. Sand storm natural phenomena gas solid Two Phase Flow over here if you go for fountain.

This is a famous fountain in Geneva, so we can find out lots of droplets are being created in the form of a mist once again Two Phase Flow. Cappuccino lots of nice figures we have seen in case of cappuccino. So you can find out here also we are having typical example of Two Phase Flow of liquid Two Phase Flow. Few examples over here for industrial applications because without industrial application we do not feel that subject is becoming interesting so few examples of industrial applications in case of pneumatic conveyer.

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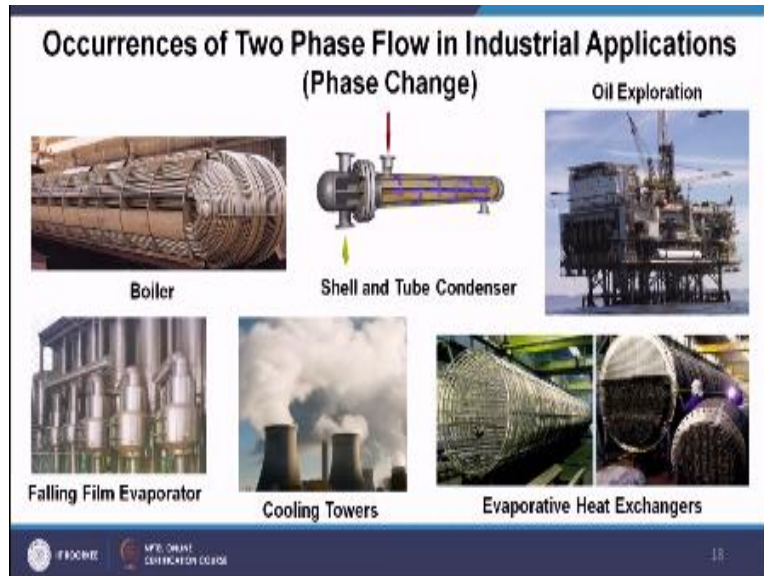


So you would be finding out gas solid Two Phase Flow is over here. In case of cyclone separator once again you will be finding out that using the cyclone separator we can separate the solid from the gas. So this is once again example of gas solid Two Phase Flow. Airlift pump and geyser pump these are being applied for pumping some liquid using gas. So here using bubbles we pump the liquid and here using a Taylor bubble or slug bubble we pumped the liquid from lower position to higher position. So these are typical examples of gas liquid Two Phase Flow.

In our car we can see fuel injector definitely that is atomizing the liquid fuel into droplets so, typical example of gas liquid Two Phase Flow. In machining, water jet machining you can find out a fast-moving water jet is over here which is actually cutting the metallic sheet so this is once again typical example of gas liquid Two Phase Flow. Here we have shown example of slurry transport pipeline, so you can find out here also we are having solid liquid Two Phase Flow into co coexistence.

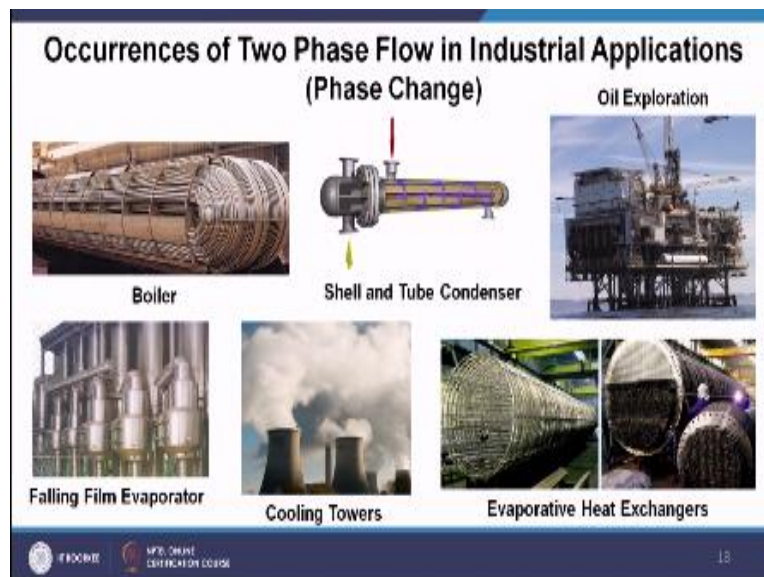
Few other examples over here, so these examples are associated with phase change so you can find out in boiler tube so liquid is being converted to gas.

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We are also having similar example in condenser where, gas is converting into liquid so you can find out these things in our power plant. Here I have shown you oil exploration you will be finding out lots of liquid Two Phase Flow in oil exploration plant once again example of industrial Two Phase Flow. Falling film evaporator, cooling towers these are also part of your power plant but everywhere we are having examples of Two Phase Flow.

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Evaporative heat exchanger, this is also typical example of heat exchanger where using heat, we are getting the change of phase and we are having Two Phase Flow. Next let us try to understand what are the different notations of Two Phase Flow? So first I will be showing you this figure

you can find out this is actually a cross section of a tube and here with this liquid color a blue color we have shown you 1 phase and with this white color you have shown you other phase.

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Notations and Relations

Void Fraction, α : $\alpha = \frac{A_g}{A} \quad 1 - \alpha = \frac{A_f}{A}$

Mass Flow Rate, W_g and W_f :

$W = W_f + W_g$
 $W_g = \rho_g u_g A_g$
 $W_f = \rho_f u_f A_f$

Volumetric Flow Rate, Q_g and Q_f :

$Q_g = u_g A_g = W_g / \rho_g$
 $Q_f = u_f A_f = W_f / \rho_f$
 $Q = Q_f + Q_g$

Mass Flux (or Mass Velocity): $G = W / A \quad G_g = W_g / A \quad G_f = W_f / A \quad G = G_g + G_f$



$A = A_f + A_g$

Let us consider that this blue colored thing is liquid and this white color fluid is actually gas. So fluid is being noted by f and gas is being noted by g. So the area if you try to find out which is occupied by liquid, we are calling that as A_f and the gas is occupying area which is nothing but A_g . So from the area average we can write down $A = A_f + A_g$ right. So from this figure you can find out this relation is valid $A = A_f + A_g$.

Now let us try to understand what are the different notations based on this figure we will be using in Two Phase Flow. First one we will be going for void fraction we usually write down void fraction as alpha. The definition of void fraction alpha is area occupied by the gas to the area occupied by the pipeline. So you see the whole pipeline is having area A and the gas is occupying this much area so that issue between A_g and A is called as void fraction.

So obviously using this relationship along with this one we can find out $(1 - \alpha)$ will be A_f / A okay. So if you add these 2, then you will be getting this equation once again back right. So after the void fraction which is very important parameter in case of Two Phase Flow let us see what mass flow rate is? So we are having next on as mass flow rate usually we write down mass flow

rate in Two Phase Flow as w . So W_g is for the gaseous mass flow rate and W_f is for the liquid mass flow rate.

Now as we know that continuity equation is valid from fluid mechanics we know so you can write down total mass flow rate $W = W_f + W_g$. Now if we try to write down individually this mass flow rate so we can write down from continuity equation once again w_g gaseous mass flow rate $= \rho_w u_g$ and a_g . So u_g and a_g is the gaseous volume flow rate multiplied by ρ_w is the mass flow rate of for the gas. Similarly, we can write down for the liquid $w_f = \rho_w u_f$ and a_f right. So after the mass flow rate let us try to see what is volumetric flow rate?

So volumetric flow rate usually we go for Q so we write down Q_g and Q_f okay. So you let us see first what happen for the volumetric flow rate. So Q_g we can write down obviously u_g into a_g here density will not be coming into picture like your mass flow rate over here. So q_g we can write down the volumetric flux is actually $u_g * a_g$. Now if you try to club these 2 equations, you will be finding out that $u_g * a_g$. From here can be written as w_g / a_g . So you can write down $q_g = w_g / \rho_w$ right.

In a similar fashion we can write down $q_f = u_f * a_f$ volumetric flow rate for your liquid and we can write down $u_f * a_f = w_f / \rho_w$ from this equation. So you can find out q_f times out to be w_f / ρ_w . We can write down by adding this 2 q equals to total flow rate, total volumetric flow rate is equals to $q_f + q_g$ right. So after the flow rates let us right to see how the flux is coming out first we will be going for mass flux sometime we call this mass velocity also.

We denote this as g . So, g is actually for the total mass velocity including both of the phases. We can write down $g = w / a$ right now g_g so this is for gaseous phase gaseous phase mass velocity is w_g / a . So whenever you are having w_g / a as the gaseous mass flow rate similarly we can write down for the liquid flow rate $w_f / a = g_f$. So adding these 2 you can write down total mass velocity g is actually $g_g + g_f$ right.

Let us try to see then the volumetric flux for in case of volumetric flux we will be writing that one as small g . So small g is actually total volumetric flow divided by the area so this is q/a .

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Notations and Relations			
Volumetric Flux (or Superficial Velocity):			
$j = Q/A$	$j_g = Q_g/A$	$j_f = Q_f/A$	$j = j_g + j_f$
Quality (or Mass Quality), x :			
$x = W_g/W$	$1 - x = W_f/W$	$x = (1 - i_f)/i_{fg}$	
Volumetric quality:		$\beta = Q_g/Q$	$1 - \beta = Q_f/Q$
Relative Velocity:		$u_{gf} = u_g - u_f$	$u_{fg} = u_f - u_g$
Drift Velocity:		$u_{fj} = u_f - j$	$u_{gj} = u_g - j$
Drift Flux:		$j_{gf} = \alpha(u_g - j) = \alpha u_{gj}$	$j_{fg} = (1 - \alpha)(u_f - j) = (1 - \alpha)u_{fj}$

So sometime we call this as superficial velocity also right. For gaseous phase if we try to find out j_g is will be equal to q_g / a corresponding counterpart for the gaseous phase and in the liquid phase we can write down $j_f = q_f / a$. Now if you add this 2, then you will be getting at the top q_g / q_f and at the bottom you will be having ag/af .

From this 2 and along with the previous slides equations $a = a_f + a_g$ and $q = q_f + q_g$. You can find out j becomes $j_g + j_f$ right. So after the flux let us try to find out few more quantities like mass quality or x . So x is already known to us from our thermodynamics knowledge. Let us recapitulate that one. So x , we know nothing but mass for the gaseous is divided by the whole mass.

So this is w_g / w definitely $1 - x$ will be w_f / w and do you know from thermodynamics that x can be written as 1 minus if where i is the enthalpy divided by i_{fg} right. In a similar fashion, we can go for volumetric quality. So this is mass quality. Next we are going to define the volumetric quality usually we write down volumetric quality as β . So β is actually q_g / q right.

So if this is actually gaseous phase, volumetric flow divided by the total volumetric flow ratio between these 2 is called as volumetric quality β in a similar fashion $1 - \beta$ can be written as q_f / q . Now as it is Two Phase Flow. So there will be something called relative velocity. Relative

velocity means the velocity between the gaseous phase and the liquid phase. So what we have done here, we have define relative velocity as u_{gf} .

So u_{gf} definitely it will be the subtraction between u_g gaseous phase velocity minus of u_f and simultaneously we can write down u_{fg} which is just opposite $u_f - u_g$. Right so you can find out over here that u_{gf} is equals to minus of u_{fg} because these expressions are actually having opposite signs to each other right. Now after this relative velocity let us try to find out what is drift, drift velocity.

So what is drift actually 1 phase how much it is lacking from the overall phase velocity. So that is actually drift so already we have seen the overall phase velocity we can write down our superficial velocity seen we can write down as j . So individual phase how much it is lacking we can find out by the subtraction of their velocity and superficial velocity. So you see $u_f - j$ is the velocity by which the liquid phase is lacking from the overflow.

So you can write down this one as drift velocity for the liquid $u_{fj} = u_f - j$ right. In the same fashion, we can write down for gas. So $u_{gj} = u_g - j$ how much the gas is actually drifting from the overall flow. Next let us define another term called drift flux. So drift flux is giving you some sort of idea that how much liquid is actually drifting from the gas earlier whenever we have described drift velocity that was how much it was drifting from the overall velocity.

So here overall superficial velocity here we are finding out how much the liquid is drifting from the gas velocity. So we you will be defining $j_{gf} = \alpha * u_g - u_j$. So you see $u_g - j$, it was u_{gj} . So I can write down $j_{gf} = \alpha * u_{gj}$. In the same manner just opposite version of this j_{fg} can be written as $(1 - \alpha) * (u_f - j)$ now already we have seen $u_f - j$ is u_{fj} . So we can write down $1 - (\alpha * u_{fj}) = j_{fg}$. Next here let us see some useful relationship for Two Phase Flow.

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Some Useful Relations

$$u_f = W_f / \rho_f A_f = Q_f / A_f = G(1-x)/(1-\alpha)\rho_f \qquad u_g = W_g / \rho_g A_g = Q_g / A_g = Gx/\alpha\rho_g$$

$$\frac{u_g}{u_f} = \frac{x}{1-x} \frac{\rho_f}{\rho_g} \frac{1-\alpha}{\alpha}$$

$$j_g = u_g \alpha = j \beta = Gx/\rho_g \qquad j_f = u_f(1-\alpha) = j(1-\beta) = G(1-x)/\rho_f$$

$$j_{fg} = \alpha j_f - (1-\alpha)j_g \qquad j_{gf} = -j_{fg} \qquad j_{gf} = (1-\alpha)j_g - \alpha j_f$$

$$j_{gf} = \alpha(1-\alpha)u_{gf}$$



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So first let us try to find out the liquid velocity u_f . So u_f from the continuity equation we can write down that is nothing but mass flow rate divided /density* area. So here we have written down $w_f / \rho_{wf} * a_f$. Here you see $w_f * \rho_{wf} * a_f$ this can be also written as w_f / ρ_{wf} is actually q_f . So we can write down $u_f = q_f / a_f$ right.

On the other hand you see already we have shown in the previous slide that $j_{gf} = \alpha * u_{gj}$. So what we can do here this a_f and q_f this we can write down in terms of this how lets us try to see. Let us try to see how we can write down this one.

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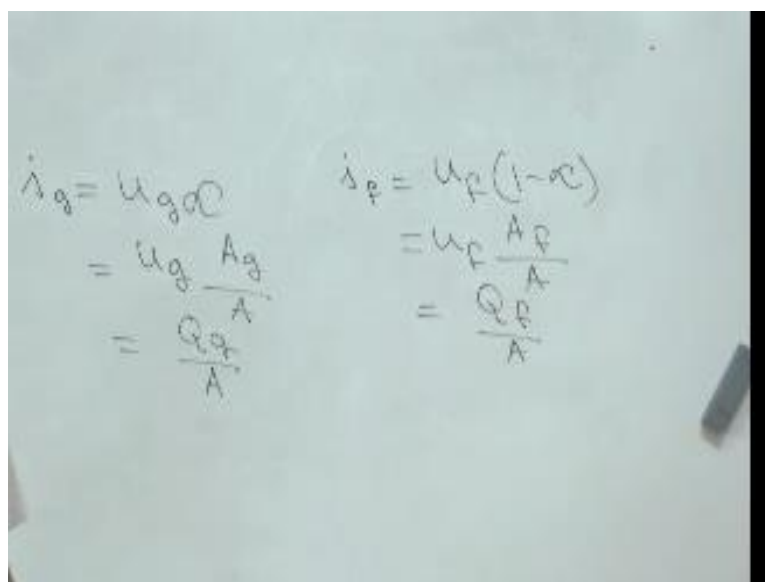
So first let us see u_f is actually we can write down this is $w_f / r_{owf} * a_f$ okay. So here you see we can write down this one as $w * (1 - x)$ and then r_{owf} in place of a_f we can write down $a * (1 - \alpha)$ right.

These 2 definitions already we have shown the w_f will be $w * (1 - x)$ similarly a_f will be $a * (1 - \alpha)$. Now here you see w / a . We can write down g which already we have shown. So this turns out to be $g * (1 - x) * r_{owf} * (1 - \alpha)$ right. Which I have shown you over here you see $g * (1 - x) / (1 - \alpha) * r_{owf}$ okay. In a similar fashion the counterpart can also be proved. So u_g is actually $w_g / r_{owg} * a_g$. So you can write down this one is $w * x$.

So w_g , i am writing $w * x$ r_{owg} and a_g we can write down $\alpha * a$. So you can write down this is actually $g x / r_{owg} * \alpha$. So second equation I have shown you over here u_g is $= g x / \alpha * r_{owg}$. Now if you make the ratio between u_f and u_g from this 2 equations, so you will be finding out g and g will be canceling out and finally these equation you will be getting $x / (1 - x) r_{owf} / r_{owg} * (1 - \alpha) / \alpha$ right.

So next let us try to see that how j_g can be written in terms of α and β . So let us see next how j_g can be written in terms of α and β . We will be starting from j_g we can write down in the form of $u_g * \alpha$ right.

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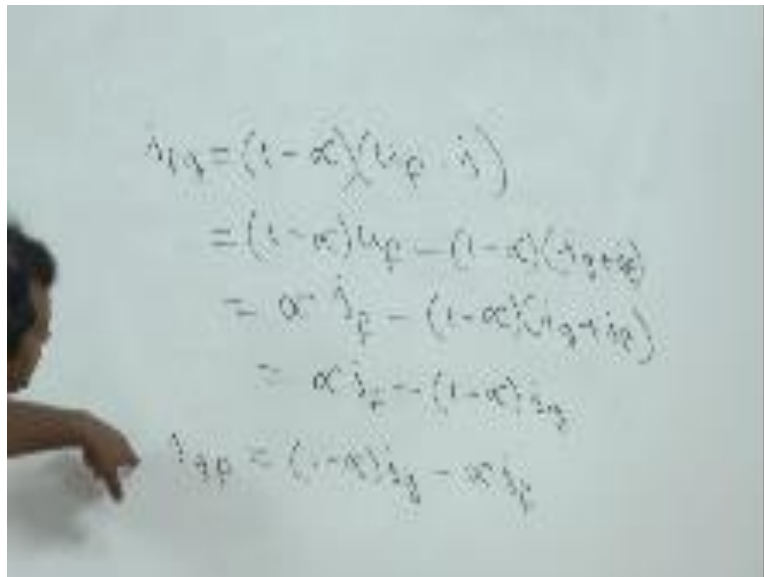


$$\begin{aligned}
 j_g &= u_g \alpha \\
 &= u_g \frac{A_g}{A} \\
 &= \frac{Q_g}{A}
 \end{aligned}
 \qquad
 \begin{aligned}
 j_f &= u_f (1 - \alpha) \\
 &= u_f \frac{A_f}{A} \\
 &= \frac{Q_f}{A}
 \end{aligned}$$

So how alpha is coming over here? Alpha is nothing but $u_g * a_g / a$. So you see over here $u_g * a_g$ that can be written as q_g / a which is the definition of your j_g . In a similar fashion we can prove that terminologies involving j_f . So j_f will be $u_f * (1 - \alpha)$. So you can find out u_f and $(1 - \alpha)$ can be written as a_f / a . So you can find out this is q_f / a which is nothing but the definition of j_f .

So after proving all these let us try to see 2 important relationships over here. You see we can write down $j_{fg} = \alpha * j_f - (1 - \alpha) * j_g$. Let us try to prove this, so we will be starting from this equation j_{fg} is $(1 - \alpha) * (u_f - j_g)$.

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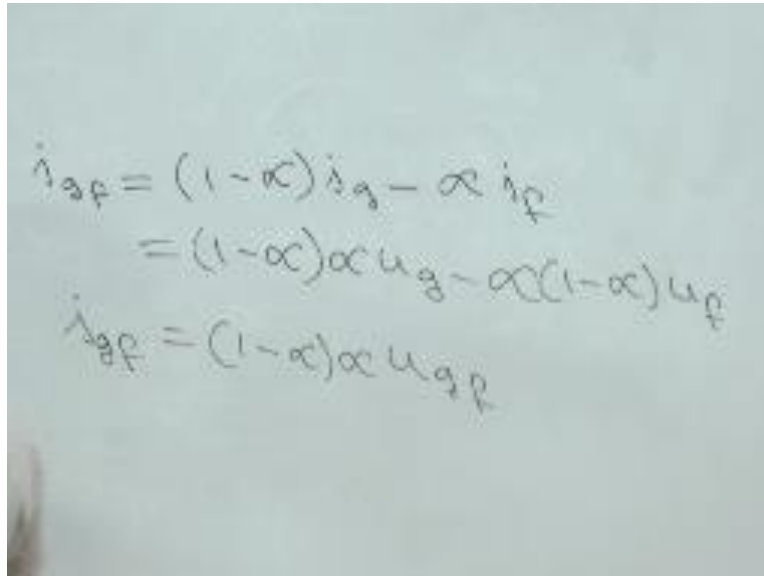
$$\begin{aligned}
 j_{fg} &= (1 - \alpha)(u_f - j_g) \\
 &= (1 - \alpha)u_f - (1 - \alpha)(j_g + j_f) \\
 &= \alpha j_f - (1 - \alpha)(j_g + j_f) \\
 &= \alpha j_f - (1 - \alpha)j_g \\
 j_{gf} &= (1 - \alpha)j_g - \alpha j_f
 \end{aligned}$$

So $j_{fg} = (1 - \alpha) * (u_f - j_g)$ right. So I can write down over here. $1 - (\alpha * u_f)$ and then $(1 - \alpha) * j_g$ can be written as $j_g + j_f$ right. So $1 - \alpha * u_f$ that will be $\alpha * j_f$ this I have shown you over here okay. So this I have shown you over here. So finally you will be getting $-(1 - \alpha) * j_g + j_f$. So ultimately if you take several terms common then you will be getting this final equation j_{fg} is equal $j_{fg} = \alpha * j_f - (1 - \alpha) * j_g$.

So this I get over here $\alpha * j_f - (1 - \alpha) * j_g$ right. In a similar fashion we can write down for j_{gf} which will be nothing but $(1 - \alpha) * j_g - (\alpha * j_f)$. So from these 2, from these 2 equations you can write down that. $j_{gf} = - (j_{fg})$ on the other hand if you write down j_{gf} in the

form of u_{gf} , you will be finding out the coefficient becomes $\alpha(1 - \alpha)$. This again can be proved once again from here.

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$$\begin{aligned}j_{gf} &= (1 - \alpha)j_g - \alpha j_f \\&= (1 - \alpha)\alpha u_g - \alpha(1 - \alpha)u_f \\j_{gf} &= (1 - \alpha)\alpha u_{gf}\end{aligned}$$

So I will be starting with j_{gf} . So j_{gf} you can see we can write down from here i_{gf} is actually $(1 - \alpha) * j_g - (\alpha) * j_f$. So $(1 - \alpha) * j_g - (\alpha) * j_f$ okay. So from here if you try to replace this j_g and j_f in terms of your u_g and u_f , so you will be getting $(1 - \alpha) * \alpha * u_g$ and $\alpha * (1 - \alpha) * u_f$. So ultimately you get $(1 - \alpha) * \alpha * u_{gf}$ okay. So this is the final equation. We have written down over here which will be very, very important for Two Phase Flow right.

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Summary

- Studied fundamental difference between different phases of a substance
- Discussed possible two phase configurations and their applications
- Detailed discussion of different two phase flow examples starting from daily life experiences to industrial applications
- Summarized different notations and basic relations; useful throughout the course

Next, let us summarize this lecture. So what we have done, we have studied different applications of Two Phase Flow over here. Discussed possible configurations in industrial examples and at the end we have summarized different notations, basic relations and useful correlations over here in Two Phase Flow. Let us try to find out how we have gone through in this lecture.

So some understanding we will be trying to test you first question.

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Test your understanding ?

- Which of the following is not an example of two phase flow.
 a. Swimming **b. Flow inside a pipe**
 c. Cooling Tower d. Boiling
- Recognize the correct one.
 a. $1 - x = Q_g / Q$ b. $x = W_f / W$
c. $1 - x = W_f / W$ d. $x = (1 - i_g) / i_{fg}$
- Choose the wrong expression(s).
 a. $u_f = G(1 - x) / (1 - \alpha) \rho_f$ **b. $u_f = Gx / \alpha \rho_g$**
 c. $u_f = W_f / \rho_f A_f$ **d. $u_f = \alpha W / \rho_f A_f$**

Which of the following is not an example of Two Phase Flow swimming, flow inside a pipe, cooling tower and boiling. So the answer is flow inside a pipe definitely this is a single flow single phase example. Recognize the correct one you have 4 equations over here $(1 - x) = q_g / q$, $x = w_f / w$, $(1 - x) = w_f / w$ and $x = (1 - i_g) / i_{fg}$.

So which one is the correct one. Answer is this 1. This 1 is not correct other 3 are correct. Choose the correct; choose the wrong expressions in the third question. So you are having 4 expressions over here $u_f = g^* (1 - x) / (1 - \alpha) \rho_f$, $u_f = Gx / \alpha \rho_g$, $u_f = w_f / \rho_f A_f$ and $u_f = \alpha W / \rho_f A_f$. So which are wrong? So let us see both b and d those are wrong. Hope you have liked this lecture. Please join us in the next one. Thank you.