Strength of Materials
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Lecture - 9

Hi good morning, this is Dr. S. P. Harsha from Mechanical and Industrial Engineering Department, IIT Roorkee. And I am going to deliver today lecture 9 on the subject of the strength of materials which is developed under the program of national program on National Technological Enhanced Learning.

If just I want to refresh those things like in previous lectures, we discussed about that what the strain is, what the basic concept of strains are when you apply the load, there is a deformation and if we want to measure deformation, there is a technical term known as the strain. And then, also we defined that actually you know like if we apply the load kind of deformation which we discussed, if it is under the elastic I means if you remove the load and body comes to its original state, then what the loss are there. Then you know like this established relation between the stresses and the strain within the elastic deformation that is based on the Hooke's law. And then based on the Hooke's law, we also defined one of the elastic constant that is the Young's modulus of elasticity. You see you know like it is defined for the proportionality between the stress and strains, and once we apply the load you see in the uniaxial directions like in x y or z direction, there is an impact is there on other directions like you see the y or z.

So, to measure the deformation in other directions, we want to you know like compute that actually what you know like the relation is there in between you see when you apply the load in x direction, that is an contraction is there. Let us say, if the tensile load is there, extension is there in the x direction, then there is a contraction in other two directions. So, if you want to set up the relation, there is one coefficient which we define and that is known as the Poisson ratio.

So, you see these are all the coefficients which we defined which is you know like which is always playing important role to measure the strain in respective directions like the three mutual directions. Also, you see after setting those things, we calculated that actually if there is a combination of the load like you see if that load is there, tensile load in x direction or y direction, then what will be the impact there. That means, how we can calculate you know like those strain component in the different parts of the material. If we cut the plane at an, inclined plane, we are saying that oblique plane, then what will be the strain components there at the normal component or the shear strain component.

So, this kind of you know like reactions, which we set up for when the axial loading is there directly or if you see the combined loading is there. That means, you see if you have the two mutually perpendicular axial loading is there, then what will be the relation. Just like you see we have already set up in the stress part. And similarly, you see the last part which you know like computed that was you see that if you apply the load, the two mutually perpendicular loads are which the tensile stress is there sigma x and sigma y.

In the same time, the shear stress is there, sigma tau x y and then, what will be the strain component at an inclined plane that epsilon theta and the sigma, this gamma theta epsilon theta which we are saying that there is a normal strain component in x and y direction. Well, gamma theta is the shear strain component, which we define separately. Then, you see you know like we got some of the relations based on those things that actually how you know like what will be the epsilon theta is there in terms of epsilon x, epsilon y or tau x y or gamma theta is there. What exactly the relation is there or we can say what the impact is there on those terms.

So, this kind of you know like relations, which we set up in the previous lectures and then, you see you know like we found that there is the symmetricity is there in between the stress component and the strain component. Like you see if you are talking about the stress, the similar you see the sigma theta or tau theta will give you the sigma you know like sigma theta. If you are talking about the normal stress, the normal stress component at the oblique plane we found that it was nothing but sigma x plus sigma y by 2 plus sigma x minus sigma y by 2 cos 2 theta plus tau x y by 2 sin 2 theta.

So, you see similarly we can also calculate the epsilon theta, the normal strain component under the effect of two mutually perpendicular stress with the shear stress and that you see we can calculate that epsilon x plus epsilon y by 2 plus epsilon x minus epsilon y by 2 cos of 2 theta plus gamma x y by 2 sin of 2 theta. Then, you see again if you want to calculate the shear strain, then like for symmetricity, what we did is we simply multiply by half. So, gamma theta by half will give you the epsilon x minus epsilon y by 2 plus you know like the square root of all that part epsilon x minus epsilon y whole square plus four times of gamma x y square divided by 2.

The meaning is pretty simple that you see you know like we have an analytical part which we discussed in the stress component. Similar kind of impact is there on the analytical part of the strain component. So, this is one solution when you see there in a combination of the stress components are there. We have you see the strains are there at a usual point, and we can get you know like the principle strains at principle planes. So, then also we set up the relation between the principle strains like principle strains are there at the shear stress component is not there or when the maximum shear stresses are there. Then, what will be the angular relation like 45 degree is there in the last part of our previous lecture. We discussed about those things.

So, today you see you know like in this lecture, we are going to discuss about the graphical solution as we discussed the same thing you know like in this for a stress component. That Mohr's circle will give you know like this geographical or we can say the geometrical component, and to get all those things by measuring simply you know like what the radius is there, what the coordinates are there, you will get all the answers by which generally you are calculating the analytical way. So, you see in this lecture, now we are this Mohr's strain circle is there, that is graphical solution. So, first to define those things you see there are some technical terms which you know like this general definitions are there of those terms. So, we would like to first concentrate on those things and then, we will go for the Mohr's strains circle.

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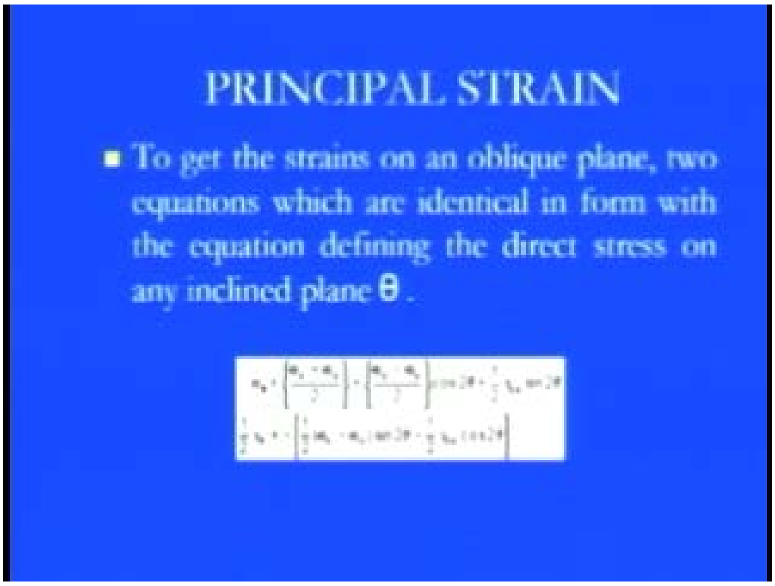

First of all the principle strain because you see the principle stresses are there, and we calculated the principle stresses just by keeping shear stress component 0. So, again you see we would like to calculate the principle strain by you know like that what exactly the impact is there of the two mutually perpendicular stresses with the shear stress component is.

So, first of all to get the strain at an oblique plane because you see you know like we have all the components at the extreme ends of the particular cube. So, to get a strain on an oblique plane, two equations always you see you know like the epsilon theta as well as the gamma theta. We need always to those values within the strain components in the object which are identical in form of the equations defining in the direct stress also on you know identical plane theta because we are always keeping our oblique plane at an angle of theta.

So, here you see these equations pretty you know like which we discussed recently that the epsilon theta is nothing but equals to epsilon x plus epsilon y by 2 which is independent of the theta angle plus the multiply of cos 2 theta. That is epsilon x minus epsilon y by 2 plus the gamma x y divided by 2 sin of 2 theta. So, you can get the direct normal strain component at the oblique plane if you know those two strain components, the mutually perpendicular epsilon x epsilon y with the tau x y and the gamma x y, which is the principle strain component. Sorry, the shear strain component.

Then, you have the shear strain at the oblique plane. That is the principle strain you see you know like we can say the gamma theta which is half to make the symmetricity. So, half is nothing but equals to you know like epsilon x minus epsilon y 2 into sin 2 theta minus, minus sign because you know like it is just trying to you know like make a contraction towards the other direction. So, we have this gamma x y divided by 2 cos of 2 theta.

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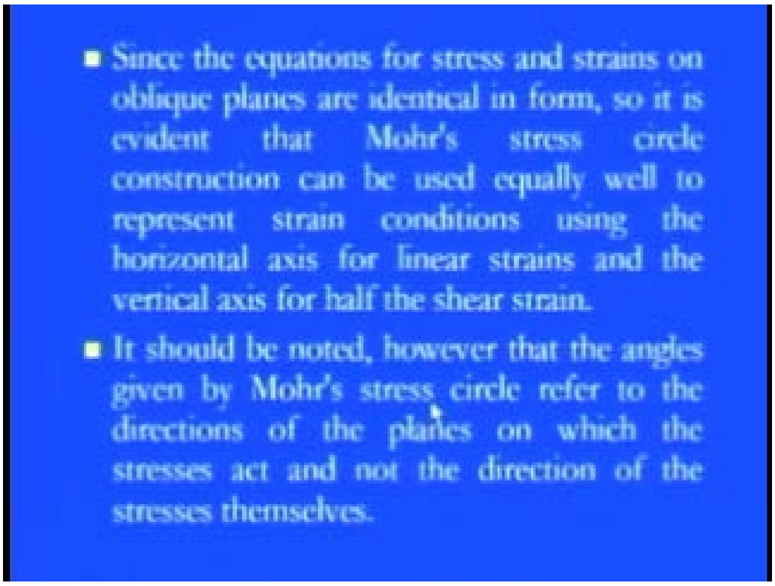

So, we have these two relations which is exactly similar you know like towards the stress component at the oblique plane. So, these equations for the stress and strain on oblique planes are identical as I told you in form. So, it is evident that Mohr's stress circle construction can be used on equally well you like to represent strain conditions using the horizontal axis for the linear strains. So, here you see you know like to make the Mohr's circle always we are taking the x axis. Just I am simply showing the linear strain or we can say the normal strain components and the vertical axis always.

So, in the shear strain, but since you have seen the numerical problems, we make it in the analytical solution. We just make it half of their. So, our vertical axis will show the half of the shear strain. So, now you see you know the basis you know the axis is. So, we can simply put the linear strain component towards the x axis, the half of the shear strain towards the y axis, and it should be noted that however the angles given by Mohr's stress circle, this is the stress circle refer to the direction of the planes on which the stresses act.

So, you see here not the direction of the stresses themselves means whatever the stress directions are there. The stresses we are not going to concern, but only we are checking that actually at what angle these stresses are acting.

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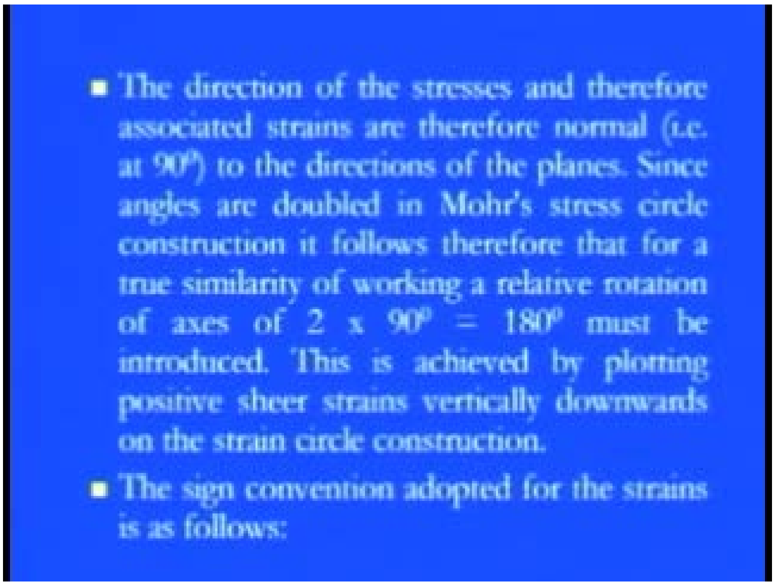

So, the similar things which we are going to discuss here also as far as the strain conditions are there, the direction of the stresses is because you see not taking the individual angle of the stresses. So, the direction of the stresses and therefore associated strains are therefore normal. You see if the bases of the direction of the stresses because they are coming from the direction of the force. Associated stresses are always therefore normal. That means, you see at the 90 degree because you see you know like whatever the angles they are making, there is a contraction and the extension are there to direction of the plane.

So, always we are taking normal to the direction of plane if the stresses are being associated with the strains. Since, the angles are doubled in the Mohr's stress circle always we are taking irrespective of 2 theta x 2 theta p whatever you see. So, always the Mohr's stress circle is constructed based on double of this angle. So, in here also you see you know like we are going to just go for the similar kind of thing with the working relations of the rotation of the axis by 2 times of 90 degree. That means, because it is a normal thing is there, so we have 180 degree which must be introduced in the strain circle. This is achieved by plotting positive shear stress strains vertical downwards on the strain circle construction. Always we need to achieve by simply you know like dropping the vertical on this perpendicular strain construction in the sign convention, you know like as we have already discussed that actually there are some sign conventions in these stresses part.

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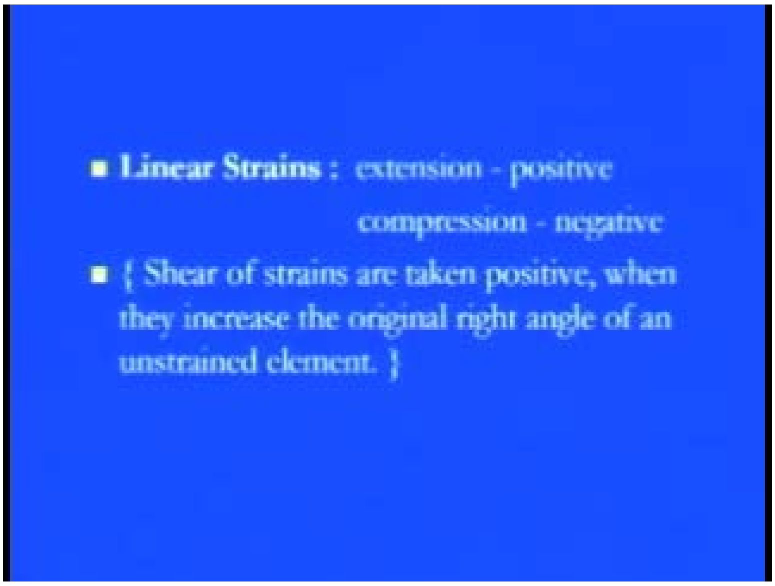

So, again similar kind of sign conventions are there in the strains, and here it is you see the linear strains always as far as the normal strains are concerned. We are going to use if there is an extension. That means, if there is an axial pulling is there, and if it is you like going to verse the extension kind of that always will go for positive because there is an extension and the strain is nothing but the change of length. That is the extension part, positive part divided by the original length, and if there is a compression; that means, you see there is a reduction is there.

So, the overall impact, the change in length is the negative side. So, minus means the change of length in terms of negative divided by the original lengths to compression. It always gives you the negative part extension, always gives you the positive part. So, correspondingly you see we need to take the sign convention as far as the linear strain is concerned. The shear of the strains are always taken positive when increase the original right angle because you see you know like if you want to measure the shear stress, that was nothing if you just remind that the slope of AOB minus slope of A dash O B dash means you see what exactly you know like the change of the shear planes are there. If we have a common you know like the square is there and if we are keeping those shear strains, always you know like the kind of distortion is there from their own planes. So, how you see distortions are taken place and what exactly the final shape is and correspondingly if you know like there is an increase is there in the original right angle of the unstrained element, we are always taken the shear strain positive.

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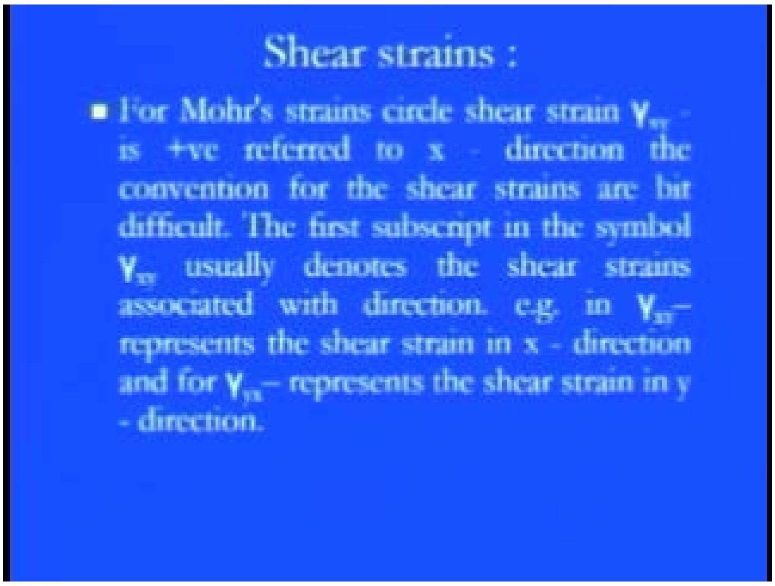

So, again you see if we focus on the shear strain for Mohr's strain, you know like strain circle, the shear strain gamma x y if it is positive, refer to x direction. The conventions for strains are a bit difficult for that actually exactly how they are rotating the first subscript in this gamma x y is always you see you know like the inverse, the shear strain associated with the direction. That means, you see if I am saying that gamma x y is there, so always you see shear strain is going to what is the x direction means. The domain is the x direction and the other one, so that actually what the force direction is there. N is what the extension is there.

So, example in gamma x y represent the shear strain in x direction and gamma y x. If you just you see change the first subscript, it shows the shear strain in the y direction. That means you see what exactly the point you know like the influence is there in the shearing part. Always first subscript will give you that. Actually this is the direction of the strain. So, if gamma x y is there, always it is going to what is the x direction like this distortion is there and if gamma y x is there, the distortion is there into towards the y direction.

So, this kind of you see you know like the notations are there, and that is why you see as we have discussed that actually these are the plane stress or plane strain forms. So, these you see the combinations of these two subscripts are there in that form.

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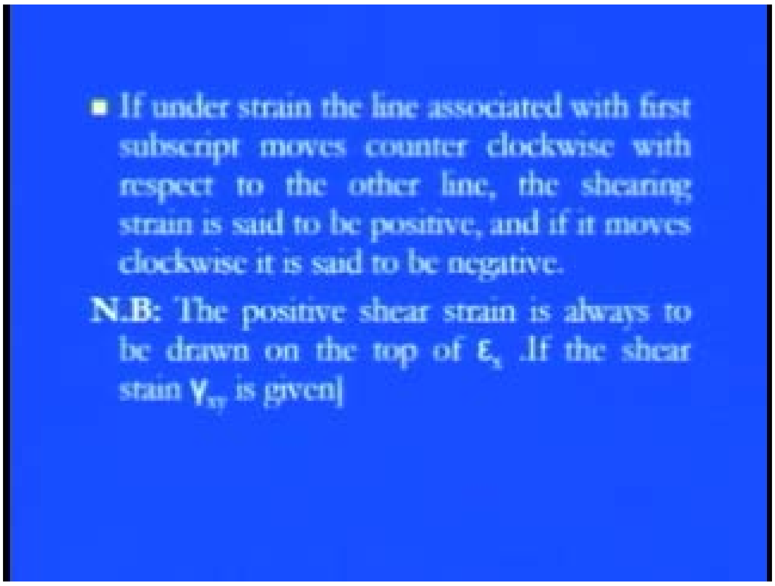

If under strain the line associated you see whatever you see the line associated are there with the shear strain, the first subscript moves counter clockwise always. You see you know like the CCW with respect to the other line, the shearing strain is said to be positive. That means, you know like if the deformation because as we told you that gamma x y is there and it is moving towards the x direction. So, if you are saying that due to that if the line is associated with any of the strain part, and towards that if they are moving towards the anti-clockwise, means the counter clockwise direction always we are taking the shear strain. The deformation due to the shear strain is to be positive, and if it is moving towards the clockwise direction, it should be taken as a negative part.

The positive shear strain is always to be drawn on top of the epsilon, which is normal strain component. If the shear is you know see the gamma x y is to be given with respect to this normal strain component, that means you see if we have the combined part, then how to draw you know like the shear strain with respect to the direct strain or we can say the normal strain. This kind of you know like the relation is fruitful in that sense.

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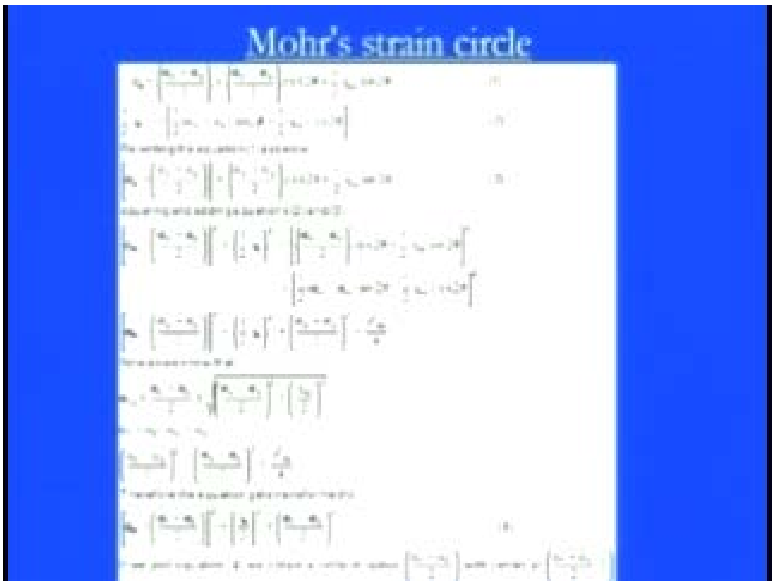

Now, come to the real Mohr's circle strain. So, you see as we discussed the analytical part that you see if you want to calculate this normal strain component of the direct strain component at the theta, then it is nothing but equals to epsilon x plus epsilon y by 2 plus epsilon x minus epsilon y by 2 cos of 2 theta plus gamma x y by 2 into sin of 2 theta. So, this was the first equation which will give the direct strain component at the oblique plane and similarly, we can get you know like the shear strain component to the oblique plane. That is nothing but equals to gamma theta by 2 equals to minus. So, because there is a reduction is there towards the shearing plane, so it equals to you know like the epsilon x minus epsilon y by 2 sin of 2 theta minus gamma x y by 2 cos of 2 theta.

So, you see these are the two key equations which will give you the two strain component at an oblique plane which is one perpendicular to the plane and which is one parallel to the plane. Then, you see what we are doing here if you are writing again by simply you know like the first equation. We are simply you know like this is the independent of theta. Now, if we are taking towards the left direction, then what we will have? We have the epsilon theta minus this epsilon x plus epsilon y by 2 equals to this term which is you see you know like dependent on the theta. So, we have 1 epsilon x minus epsilon y by 2 into cos of theta plus half gamma x y sin of 2 theta.

This means you see we just did rearrangement of these things are there and now, if I am adding both of the term, you see the equation 3 and equation 2 by squaring. Then, what I have? I have epsilon. This component the epsilon theta minus epsilon x plus epsilon y by 2 whole square plus the equation to the left part, that is gamma theta by 2 whole square. So, this square is equal to now this is the term. So, you see if we are taking this term, then it is whole square. You see epsilon x minus epsilon y by 2 cos of 2 theta plus half of gamma x y sin of 2 theta whole square plus this whole. You see if the equation 2 is right side, you see it will be since you see the whole square is there. So, the sign will be positive and you can simply square of those terms.

So, now you see if we expand these things by you know like squaring term, then we can simply find that actually you know like there are some cos square theta term, there are some sin square theta term and there are you see you know like two sign theta cos theta terms are there. So, after you know like generalizing those things in both of the component of this equation, these two, what we will get here? We will get the epsilon theta minus epsilon x plus epsilon y by 2 whole square. This plus gamma theta by 2 whole square equals to epsilon x plus epsilon y by 2 whole square plus gamma square x y by 2. That means you see you know like the pretty simple thing is that you see we can easily get those terms. That means, now all these terms of you see that is independent of theta. Means if we are squaring those terms and you will find those things, then we will get the new terms, that is you see you know like epsilon 1, 2. That is nothing but equals to epsilon 1 plus.

You see if we are taking those terms here, then what we will get here? We will get epsilon x plus epsilon y by 2 plus minus the square root of these things. What I did here? I simply you know like meet the square terms of this and you know like put those things here. So, we have the two main strain terms. The strain terms are there, the new strain terms which are nothing but equals to if I am talking about the epsilon 1. The new term, that is nothing but the epsilon x plus epsilon y by 2 plus square root of epsilon x minus epsilon y by 2 whole square plus gamma x y by 2 whole square, and if I am talking about the minus, then you see you know like this minus will come and if you see here, if we are making those terms. Then we have epsilon x epsilon 1 plus epsilon 2 if we were just adding those terms because you see all the difference in these two terms 1, 2 is just plus minus.

So, if we are you know like keeping 1 and 2 and if we are adding those terms, then this square root terms will simply cancel out because in first we are taking plus and second, we are taking minus. So, this will cancel out. So, we can say that epsilon x plus epsilon 1 plus epsilon 2 will give you the epsilon x plus epsilon y because you see it is some of those terms, or we can say that actually if we are you know like keeping those terms here, then we have epsilon 1 minus epsilon 2 by epsilon 1 minus epsilon 2 by 2 whole square, you know like equals to epsilon 1 minus epsilon 2 whole square plus gamma square x y. That means, you see if you are rearranging those terms just by the first equation and the second equation, we will get you see the final terms formation of those things as epsilon theta just by these equation, this one if we are keeping this one here.

So, we have epsilon theta minus epsilon x plus epsilon 1 plus epsilon 2 by 2 whole square because you see now we can replace this part here. So, epsilon 1 plus epsilon 2 by 2 square root of whole square plus gamma you know like theta whole square equals to epsilon 1 minus epsilon 2 by 2 whole square. So, if you know like by these two equations, this equation and this equation if we are manipulating those things, then we will get these equations or we can say if we want to plot those things. Now, it is pretty simple that actually if there at this particular level we have two main information. One is that we have you see x square plus y square. This is nothing but you see x square plus y square equals to r square. So, we can say the radius of the circle is this r. So, r is nothing but the epsilon 1 minus epsilon 2 by 2.

So, now we have the radius, the fruitful information from that of the Mohr's strain circle and we have the central point because you see x, y is there. So, we can say you know like this the first thing is epsilon 1 plus epsilon 2 by 2. This is one you know like because x minus theta is there 1, 0 in terms of the y. So, you see there is no component with this gamma theta. There is a component with the epsilon theta. So, we can say this is the center point is there of those because x minus x 1 plus y minus y 1 whole square will give you r square. So, x 1, y 1 if the center distances are there or the coordinates of the centre is there. So, similar you see you know like for the circle, this equation we can get and we can extract the information from that.

So, now we have two main important parameters to draw the Mohr's strain circle. One is the radius that is nothing but equals to epsilon 1 minus epsilon 2 by 2, and we have the center coordinate that is epsilon 1 plus epsilon 2 by 2 in the x coordinate and 0 for the y coordinate as far as the Mohr's strain circle is concerned.

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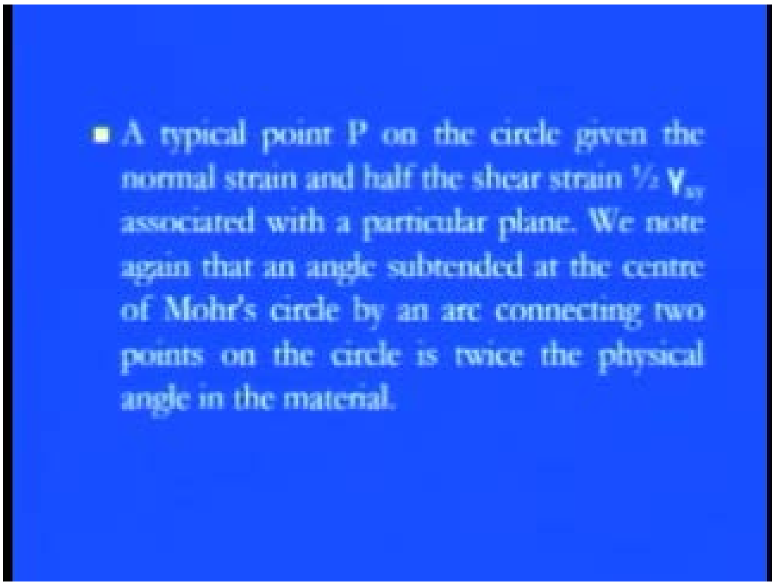

So, you see a typical point P if you want to measure, you see if you just you know like remind that the principle strain, the principle of the stress circle, then you see the similar terms are here. The principle point P is the central point of the circle given, but the normal strain and the half of the shear strain gamma x y associate with the particular plane. Always you see it is just giving by the oblique plane like theta or whatever. We note again that an angle subtended at the center of the Mohr's circle always by connected two main points. Just you see you know two extreme points are there and if you make those points, you see you can simply connect those angle which are subtended of the center of the Mohr's circle by in connecting two main points of the circle twice the physical angle which is measured in the material of the side.

So, that means, you see always you need to take the 2 theta angle that actually what exactly the connecting of the two points are there, and the extreme corner of the strain circle. Since, the transformation equations for the plane like the epsilon theta or gamma theta in terms of the strain, you see plane strain is similar to those for the plane stress like the epsilon in the sigma and tau theta. We can employ similar form of the pictorial representation and is known as the Mohr's strain circle because you see you know like in Mohr's stress circle, again the similar kind of equations came and we discussed about that. Actually you know like on the x axis, you can take the normal stress component and the y axis, you can take the shear stress component.

Similarly, we get the similar equations at the oblique plane also as in terms of you know like the stresses. So, here we can also plot the Mohr's strain circle based on x axis, simple. The normal strain or the direct strain component and you know like the y axis, we can take half of the shear strain component. The main difference between the Mohr's stress and Mohr's strain circle is the factor of half only because you know like we need to maintain the equilibrium side on both the side. So, we are simply multiplying by half. So, that this is the new term. So, in terms of the stress component, you know like the Mohr's circle, we have only the sigma x was there, tau x was there, and here we have the gamma x y by 2.

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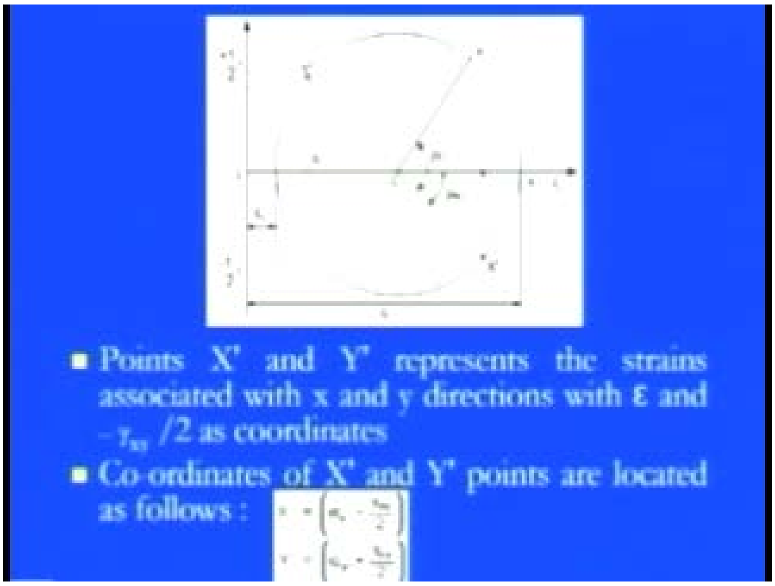

So, now you see you know like just whatever we discussed, it came here. You see we have an x axis, the epsilon we have on y axis that is the gamma y by 2 plus and the minus. So, now, you see we have the center position. So, center position coordinates pretty simple. You see epsilon 1 plus epsilon y by 2, 0 because there is no component on the y direction. So, we have the center position, we have epsilon 1, epsilon 2 is there. This one, epsilon 1 is there. So, you see pretty simple you can calculate these points and you also know the radius was nothing but equals to epsilon 1 plus epsilon 2 by 2.

So, you see you know like you can get this radius and once you know these, you know like the Y dash and this X dash. These you know like the coordinates because it is pretty simple. You see you can get those things, the coordinates of these things. We need to plot those coordinates, make the stress you know like the diagonal, draw the circle. If you drop those things, then we will get the epsilon y and epsilon you see you know like the x and those particular coordinates. So, you have you see all those things and then, you see you need to rotate by 2 theta P, where you see the principle planes are there. So, this is my you see you know like rotate after. So, rotation of those things, we have this X dash and Y dash. The strains are there associated with this epsilon with the gamma x y by 2. So, you have 2 theta and then, you see this is for the optimum plane and this is for the normal planes.

So, again you can calculate this C 2 P coordinate by that. So, point X dash and Y dash simply represent the strain associated with these you know like the x y directions like that in terms of the epsilon, and minus gamma x y in terms of this below point because X dash is there towards the downward directions. So, we have the minus gamma x y by 2 at the coordinate. So, coordinates of X dash and Y dash, we can simply calculate by the gamma. This epsilon x minus gamma x y by 2 here, this is the X dash because you see this is you know epsilon x is there, and this in terms of this minus gamma by gamma x y by 2 or we can say as far as this Y dash point is concerned, we have epsilon you know like the y and plus since it is the positive direction. So, we have positive this gamma x y by 2.

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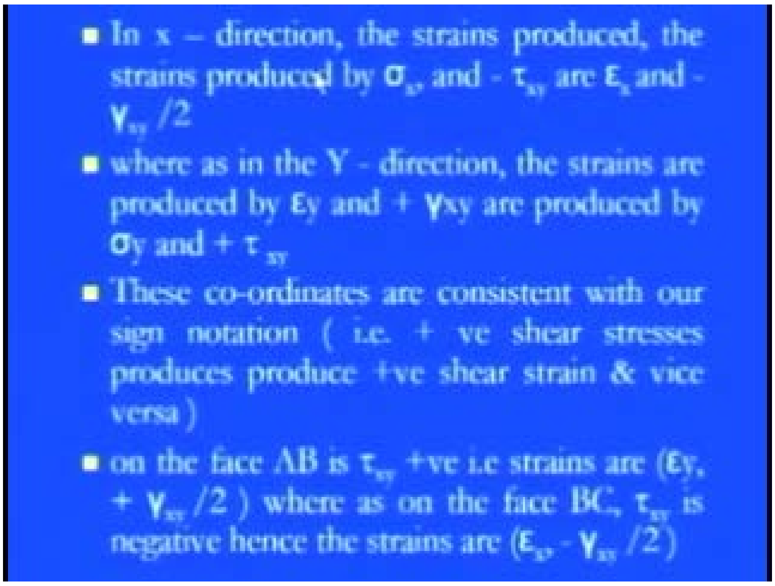

So, these you see you know like we can simply calculate. So, in x direction, the strain produced whatever the strains are there by gamma x and minus tau x y, exactly equals to epsilon x and minus gamma x y by 2. So, you see that is a symmetricity. Only the half is the difference in the stress component, this shear stress and shear strain component, wherein the y direction in the strains are produced by this epsilon y and gamma x y. These are the positive one and the similar terms are there, the sigma y and tau x y.

So, as far as the y direction is concerned, it is pretty symmetricity is there, but as far as the x direction is concerned, we have you see only the half of the you know like this minus sign is there, but they have the symmetric sign conventions. These coordinates are consistent with our sign convention. That means, the positive shear stresses are being produced positive shear strain, and negative shear stresses have always produced the negative shear stress. That means, you see if we are saying that this is due to the shear stresses, element is just going towards the clockwise direction or there is an extension is there, or we can say you know like the distortion is there towards x direction you know like going in the anti-clockwise direction.

So, we can say these are the positive shear stress and the positive shear strain correspondingly, and you see vice versa is there. Always you see when the object is tending to move towards the clock, counter clockwise direction due to the shear effect or we can say this, whatever the contraction or we can say these distortion is there in the angle is towards you see you know like always going in this clockwise direction, then we can say it is a negative shear strains or negative shear stresses are there. So, it is pretty you know like the symmetricity is there in terms of the shear stresses and the shear strains. On the face AB like in the previous figure, the tau x y is always positive that is the strains are nothing but we can simply calculate the epsilon Y, plus gamma x y by 2.

On the top of that you see if you simply note that AB part and as far as the BC which is just below the diagonal part is there, you see we can you know like calculate the coordinates for BC also. That is this tau x y is the negative. Hence, the stresses, all the strains are also coming as the negative part, that is epsilon x is the x coordinate in the y coordinate is minus gamma x y by 2.

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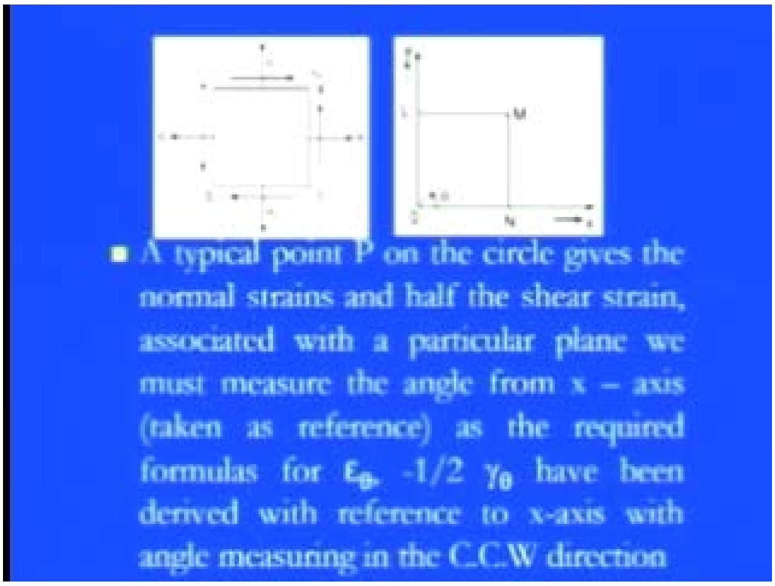

So, you see these are the key features which you know like simply got if the strain components are to be plotted. So, now, come to the main feature. You see these problems says that actually if we have two mutually perpendicular stresses like gamma x is there, you see these are there and gamma y are there, they are mutually perpendicular. Gamma x towards x direction and this sigma x is towards x direction and sigma y in y direction, and if we have tau x y that is the shear strain component, and if it is tending to move towards you see you know like the clockwise direction. So, this is basic element which we discussed a lot in the stress component as well as the strain component also.

We can simply you know like just put the analogy here that you see if we have you know like the LMN and the shear strain. You see there is no distortion, and then we can simply get the ON. This ON ML means actually this particular element is pretty simple, but if you see the distortion is there and if you see this L is simply moving towards L dash or M is moving towards M dash or N is moving towards the N dash, that means there is distortion towards the direction. Always the strain component, the shear strain components can be measured with the distortion in the angle and then, you see we will check it out that whether it is positive or the negative direction.

So, typical point P which you see on the top of that was there in the previous Mohr’s strain circle gives the normal strain, and half of the shear strain associated with the particular plane. It must measure the angle from the x axis because x axis is our reference axis. Always we have taken as the required formula by epsilon, this theta, minus gamma theta of half because you see you know this is simply measured on the oblique plane. So, half of minus half of gamma theta will be the y direction, and epsilon theta is in the x direction has been derived from the reference to x axis with the angle measuring in the counter clockwise direction because you see in that actually we are always taking the strain component as in a positive way.

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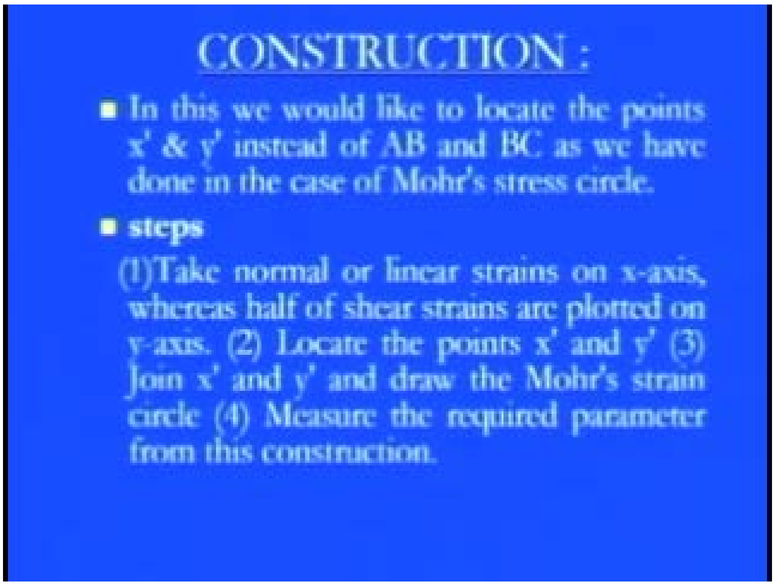

So, now we want to construct the Mohr’s strain circle based on whatever the information which we you know like discussed in the previous slide. So, in this we would like to locate the points of x dash and y dash. You see that what exactly you know x dash y dash point will come of you know instead of this A dash AB or BC which we discussed in that, and these also you know like we have done this kind of analysis or to draw the Mohr’s circle in the stress circle also. So, the following steps are there to draw the Mohr’s strain circle.

The first one you know like just we need to take the normal as well as the linear strain component on x axis as I told you, whereas you see half of the shear strain component will come on the y axis. So, first you see you know like take those point and then, locate the points of x dash and y dash, and you see you know like in the previous slide, you know like we simply found that actually what are the influencing components are there to make the point of x dash and y dash.

Once you make you know like x dash y dash points at these two places, now join those x dash y dash point and then, you will have a diameter of the Mohr's circle, and once you have the two extreme point of the diameter, simply draw the Mohr's circle. So, once you have the Mohr's strain circle on you know like this point, you simply get the center point and the center point will give you know like that what exactly the locations are there of those other points. Once you do that part, then you see you know like what you have. You have horizontal line on the strain part and that is nothing but the direct strain component and you have this x dash y dash joining line. So, now measure those angles. So, measure the required parameter, whatever the parameters are there and also, whatever the angles are there at those things.

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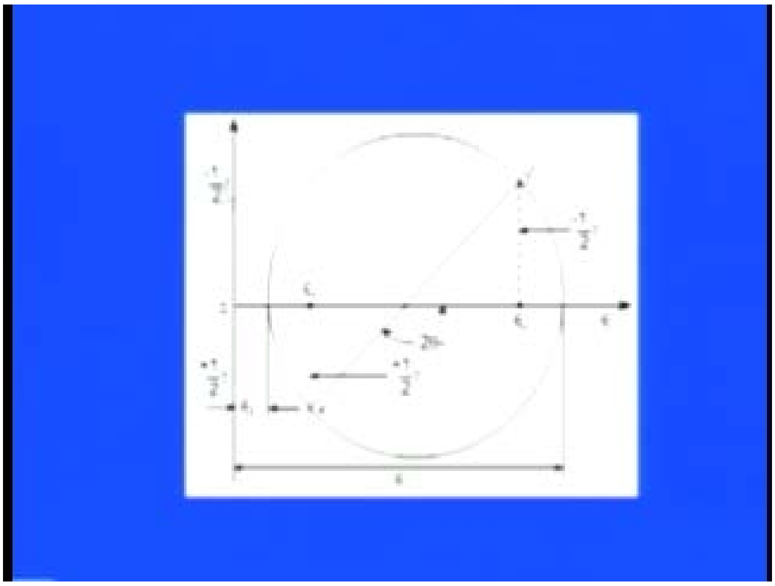

So, here it is you see all these particular things. You have a epsilon part, the normal strain component and y, this x direction you have on y direction minus gamma y by 2 plus gamma by 2. You know like both plus minus axis of y plus minus side of the y axis and then, you see you know like you need to draw the X dash and Y dash components. So, you see this is X dash 1, this is Y dash 1 because you have the coordinates in terms of the strain component.

So, once you plot those things, join this line. Once you join the line, now you have this you know like the circle and once you have the circle, you have you know like the center point. Once you have the center point, you can simply get that what exactly the center point coordinates are there epsilon 1 plus epsilon 2 y by 2. This is the coordinate there. So, once you have these things, then you can straight away plot from the origin. Then, this is you see epsilon 2, this is exactly the epsilon 1 and the coordinate is epsilon 1 plus epsilon 2 by 2, 0. These are the coordinates. Once you have these things, you see now you simply drop those you know like vertical lines and then, you have on Y dash side, you see you have the Y coordinate that is plus gamma y by 2, and on x side you have minus gamma by 2.

So, you see both the things which you have you know like the coordinates or this one. This epsilon x, this plus I should say you know like whatever you see, you can take any of these things, gamma by 2 and this is epsilon y, gamma by 2. Then, you see you know like what the angles are there towards the measure side because we are taking always 2 theta. So, we can say that this is you know like the 2 theta angle is there. So, these are the required parameters which you can easily measure if you simply draw the Mohr's circle with the required whatever the given information is there.

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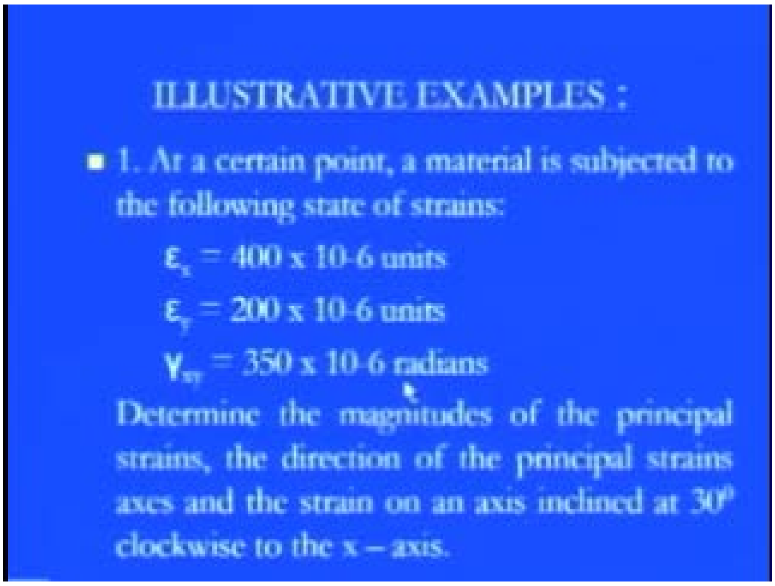

So, now you see you know that actually how to draw the Mohr's strain circle. We have you see the different strain components are there with us and you simply get you see at the oblique plane also as well as the principle planes. So, now you see these examples just gives you a clear cut feeling that actually if we have the numerical data, then how we can get all those things. So, you see here first of all we have your certain point, the material subjected to the following states strains. There is 400 times into 10 to the power minus 6 unit epsilon and y is there 200 into 10 to the power minus 6 units, and gamma x y that is 350 into 10 to the power minus 6 radians, because you see always gamma means the shear strain is always coming in terms of radians while you see there are units you know like because the epsilon, these epsilon things are there. These are you see you know like in the normal straight components. We can straightly measure in without any units are there.

So, determine the magnitudes of principle strains that what exactly the principle strains are there, what the direction of the principle strains is. That means, 2 theta P at where the principle stresses, the strains are occurring and the strain on the axis at 30 degree clockwise to the x axis means you see if you cut the plane at 30 degree with the reference plane towards the clockwise direction, then that means, this is my location of the oblique plane.

Now, we would like to calculate that what will be the normal strain of the direction component and the shear strain. So, this is the main you know like the example is there. So, now, we would like to first draw those you know like the Mohr's strain circle based on these information. We have two strain component, normal strain component and we have the shear strain component. So, based on that, it is pretty easy for us to you know like draw those things. If we can you know like put the x dash, y dash, make the diameter, draw the circle, get the value of center, get the value of this epsilon 1 and epsilon 2, and these are the principle stress and then, measure 2 theta which we can get this angle where it is now. Then, you see you know like we can get by simply rotating the 30 degree towards the clockwise direction of the x axis. So, you see all these things are pretty straight come to the solution.

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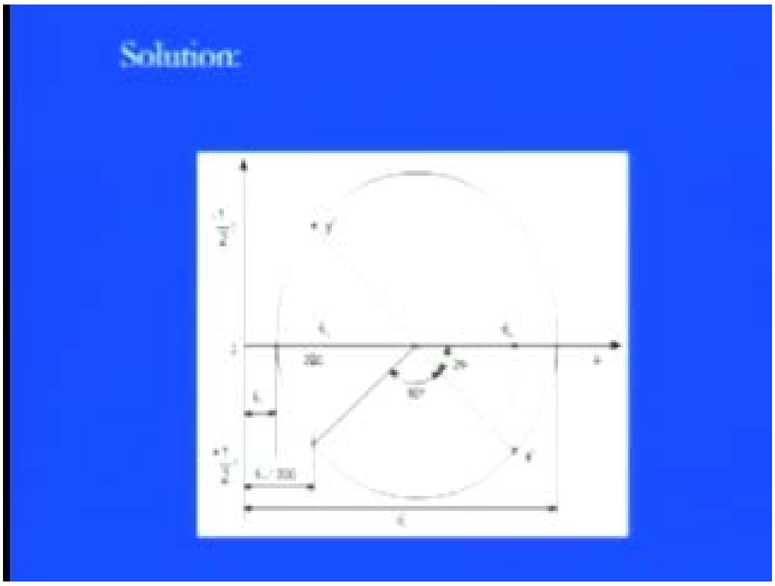

Now, you see here as I told you, first what we need to do is, we have the x dash coordinates, we have the y dash coordinates. So, you see simply plot the epsilon n gamma here, plot those you know like this x and y on mutually perpendicular axis plot. Those x dash and y dash you know like draw the line. Once you draw the line here, now you can make this particular circle. Once you have the circle, since you see you know like at this extreme corner of the circle will give you the epsilon 2 and this extreme corner will give you the epsilon 1. So, just measure those terms and you have the principle stresses.

So, this is you see the required first answer that what will be the principle strains. Then, you see once you get those things, simply rotate, simply measure this angle because you see you know like that what will be our this plane is there. So, now once you have 2 theta 1 because this is the measured angle that at what plane the strains exist. So, now we have 2 theta 1. So, we have the angle 1 and then, you see they were asked that actually what will be the you know like form of the axis. If you know like means what will be the new strain components at the direct as well as the shear strain. Simply rotate 30 degree to the clockwise direction to the x axis.

So, now you see since it is 30, so we need to go with 2 theta as we have discussed previously. So, 60 degree angle we need to make. So, now, you see here the 60 degree angle, draw this drop. So, you see here this is the new coordinate. New coordinate will give you the new values of the x as well as the y direction, and then you see we can say that this is my epsilon theta which is this direction strain component at an oblique plane as well as you see you can get you know like the gamma theta just by keeping that part you see on the vertical direction. So, this vertical direction will give you this one. This vertical direction will give you the strain component in y direction, this on x axis, this projection will give you the strain in the x axis.

So, you see you have both the component, the normal strain component and the shear strain component by just dropping those projection from the x as well as the y direction. The meaning is very simple. You see we do not have to do anything. There is no numerical calculation. Whatever the information is given simply you know like put those information on this Mohr's circle, draw that part, get the value of you know like all those required items.

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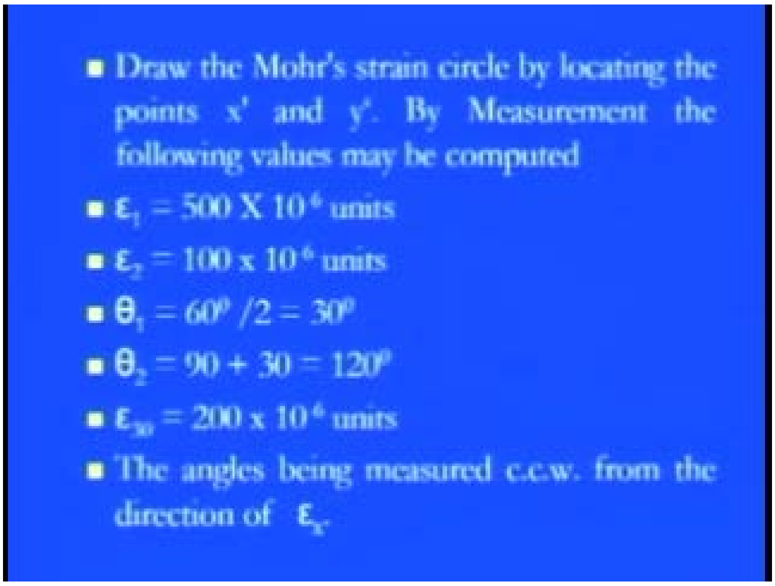

So, you see here you know like once by drawing those things, we need to measure that. So, you see you have epsilon 1 that is 5000 into 10 to the power minus 6 units. You have epsilon to that is 100 into 10 to the power minus 6 units, and you can simply measure the theta 1 and theta 2 because these two planes are always 90 degree to each other as we discussed in the previous lecture. So, once you have 30 degree and then, you see at 90 degree. So, this is theta 1 and theta 2. These are the locations where these principle strains are you know acting mutually perpendicular.

Then, you see we have epsilon 30 degree because we need to measure this kind of thing. So, you see simply put the horizontal projection. This is we have even this 200 into 10 to the power minus 6 units, or even you can also go for tau x y just by you know like putting the projection of the vertical slide. So, we can also measure that part and you can get all this kind of required information which is you see you know like necessary, and which can be only getting by measuring those drops here.

So, the angle is being measured in the clockwise, the counter clockwise direction from the direction of the epsilon x. That is what you see. It is you know like straight way we can get you see the positive values for these required items. So, as you see these are the required parameters for which you see we do not have to calculate straight away. We can measure from the Mohr's strain circle.

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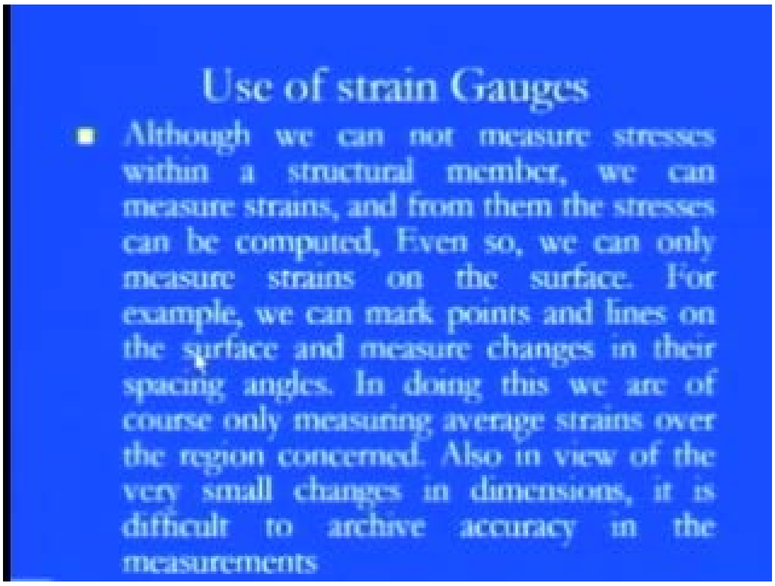

So, this is done now. Now, there is a use of the strain gauge where as you know like the gauges are there to measure the strain. So, although we cannot measure the stresses because stresses are always inducing due to the application of the forces, so we cannot measure that actually. What you know the distortions are there or the deviations are in those object, and what you see the kinds of the resistances are being provided by the material against the application of force.

So, it is not easy or it is not even some times in the possible to base directly. The stresses we can only calculate that part instead of measuring the stresses within the structure number, but we can measure the strain because where is the direct distortion or the deviation is there. We can say the deformation is there in the object. So, we can simply measure those things that what exactly the change of length width or breadth are there in terms of the original one or even the area or even this value. That is why you see you know like the volumetric strains are there or the normal strains are there.

So, we can measure strains. From there you see the stresses can be computed as such even. So, we can only measure the strain on the surface. For an example, we can mark the point on the lines of the surface and measure the change in there you know spacing whatever the angles are there, and in doing that we are you know of course only measuring just average strains, or the reasons concerned. Also, in the view of very small changes in the dimension, it is difficult to achieve the accuracy in the measurement. So, that is what you see you know like we need a perfect because as we know that well application of the load is there. What all deformations, which are coming are very minimal.

So, we cannot exactly go and simply measure that how much is there. Some time it is in the microns only and then, we have to be very careful just in choosing of further strain gauges. That is what exactly the strain gauges are there now which this kind of measurement is suitable and we can get the accurate answer. So, in practice, electrical strain gauges always provide more accurate and convenient method of measuring strains.

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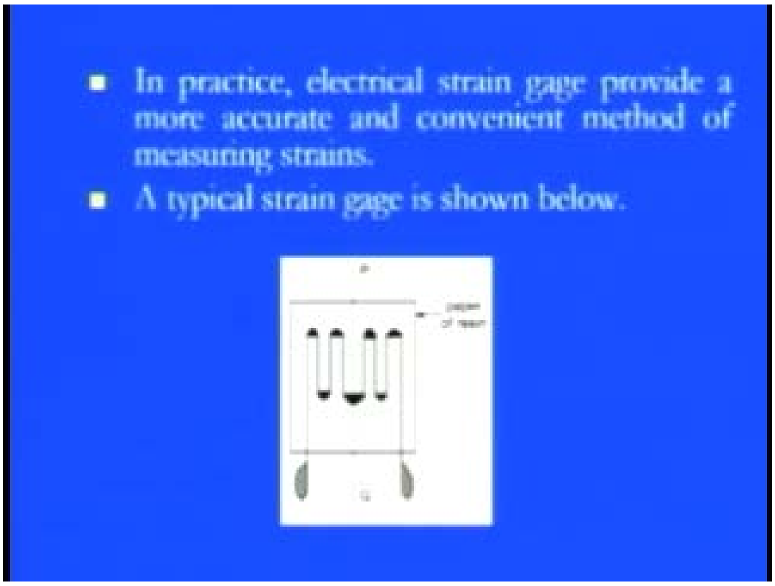

So, you see here we have you know like the electrical strain gauge. So, what is there, it is simply paper or the region are there in which there are two segments. So, what we are doing here is, this is simple you know like block the P of Q. You see this is the center line is there. So, if we apply any kind of you see the distortion or any kind of things are there, there is a change of the potential. That means you see if there is change of the potentiometer that what exactly the potential change is there, and if you know like this modify or amplification is there of that small change, it will give you the clear cut picture that what exactly the distortion or the deviation has happened in the element. So, electrical strains gauges are always based on you see that actually what exactly the change of the potential or these voltages is there within those elements.

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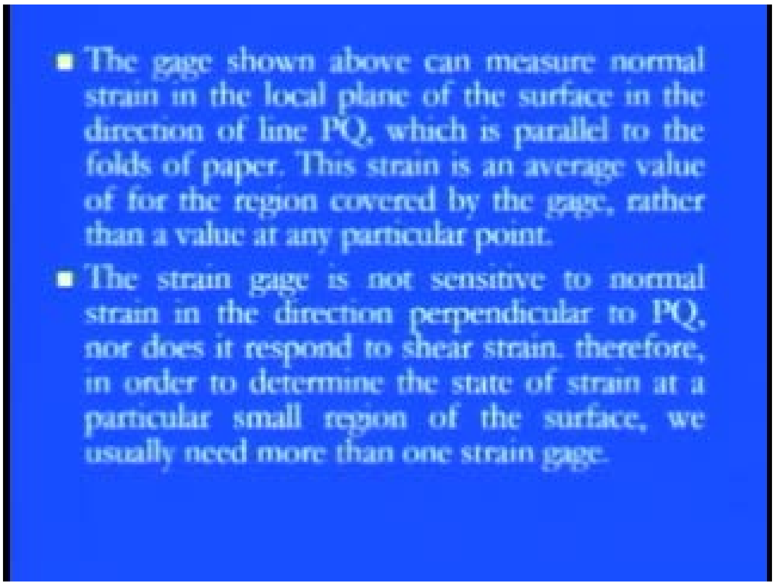

So, the gauge shown and above measure normal strain in the local plane of the surface in direction of PQ, whatever you see, the PQ straight you know horizontal plane was there which is parallel to the folds of a paper. Like you see paper or regions are there, this strain is average value always gives you an average value for a region covered by the gauge, rather than a value at any perpendicular point because there is what are the localized strain are there at a particular point.

The strain gauge is not sensitive to normal strain in the direction of perpendicular to the PQ. So, whatever you see something is going on the normal direction of PQ, it does not matter here because whatever the things are changing in to parallel to those things, it is always concerned with that. It straight way deflect those changes in their final reading, or we can say you know like it simply respond to there is no respond you see to the shear strain. Therefore, in order to determine the state of strain at a particular small region of the surface, we usually need more than one strain gauges for measuring the normal strains as well as the shear strain component.

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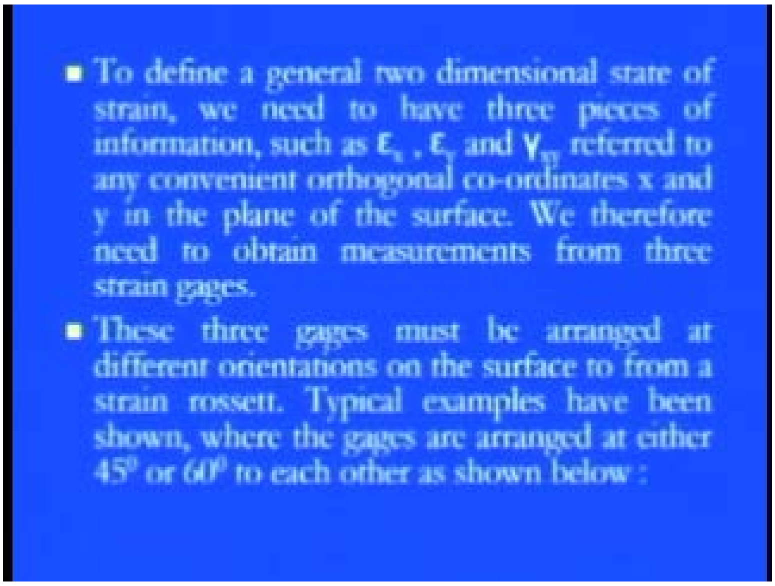

To define a general two-dimensional state of strain because you see we always prefer that actually wherever the application of load or the distortions or deformations are there, we found that there are both the components, the normal strain as well as the shear strain component are there. So, define these general two-dimensional states of strain, we need to have three pieces of the information, such as epsilon x. That means, the strain component in the x direction, strain component in y direction and the shear strain in x y because all three kind of you know like the information is available whenever kind of a load application is there. Refer to any convenient orthogonal coordinates in x and y direction.

So, we therefore need to obtain measurement from these 3, at least 3 strain gauges. So, we can get all those required information in the respective direction in x y direction as well as the plane strain is concerned. These three gauges must be arranged at different orientation because again you see you know like we need to keep those strain gauges in a perfect way, so that we can get exact deformation in a particular direction. So, three gauges, all three gauges must be arranged at different orientation on the surfaces to you know like the formal strain rosette. So, again we need to form a strain rosette to get all three you know respective directions, all the three respective magnitudes of the strains in the respective directions. The typical example has been shown you see in the next figure, where the gauges are arranged either at 45 degree or at 60 degree which is perfect for the strain rosette.

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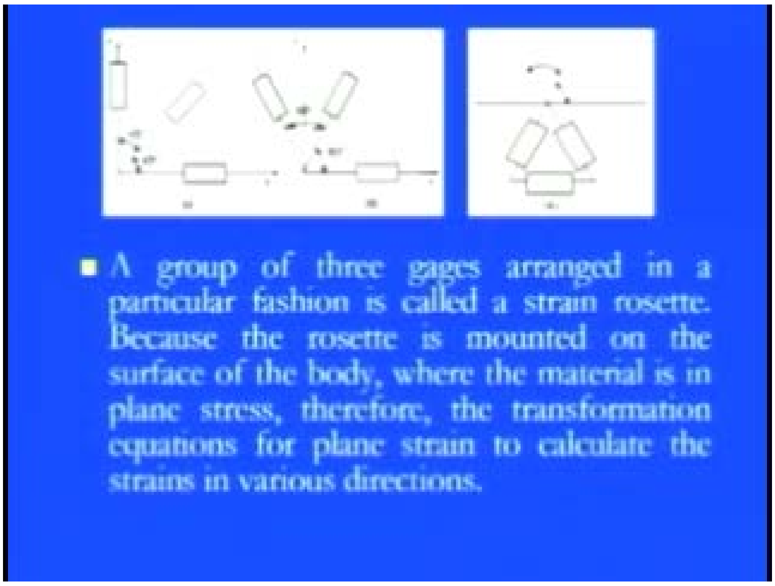

So, you see here in this diagram, we have you know like the 3 rosettes for x for y and for this z which is for gamma x y. That means we can say that for axial strain, this A and B is there and for you see you know the shear strain, we have the middle one at the 45 or we can say at 60 also. This will give you the x and y while the middle one will give you the shear strain component or you can say this third diagram.

So, in the above figure, a group of three gauges can be arranged because in the triangle form, you see you can form because they are again forming in this 60 degree. So, this can be, even this figure can be replaced by this. So, three gauges can be arranged in a particular fashion which is known as the strain rosette. So, all three gauges to measure the respective strains in the respective directions if they are arranging in the particular form irrespective of the triangular form or any form, you see the right angle, they are known as the strain rosette because the rosette is you know like mounted on the surface of the body just to measure actually what exactly the deviation is there due to the application of load, where the material is in plane stress. Therefore, the transformation equation for the plane strain to calculate the stress is always in the various directions.

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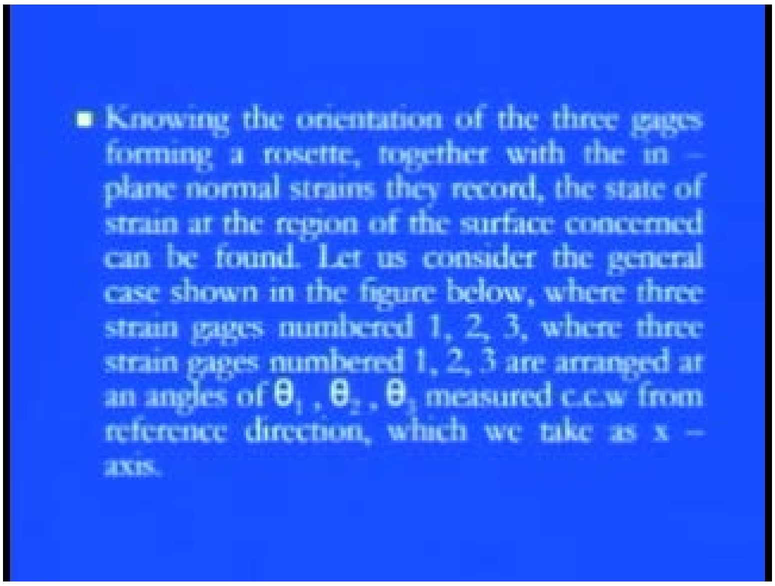

Knowing the orientation in x y or x y of three gauges forming a rosette, together with the in plane normal strain they always record, all three respective state of strain at the reason of the surface concerned to you know like whatever the reasons are there, it can be easily measured and it can be found easily. Let us consider now a general case shown in the next figure which has you know like three gauges. They are mounting at 1, 2, 3 where three gauges, three strain gages number 1, 2, 3 exactly for the corresponding thing, and whatever the angles are there which you know like the distortion angle, I should say here is theta 1, theta 2 and theta 3 which are like measured in the counter clockwise direction referred to the direction of x axis.

So, here you see this is there. So, we have the epsilon you know like this xyz is there, and we have you see whatever the distortion theta 1, theta 2 and theta 3 corresponding is this strain gauges are there, epsilon theta 1, epsilon theta 2 and epsilon theta 3 with those 1, 2, 3 you know the gauges are there.

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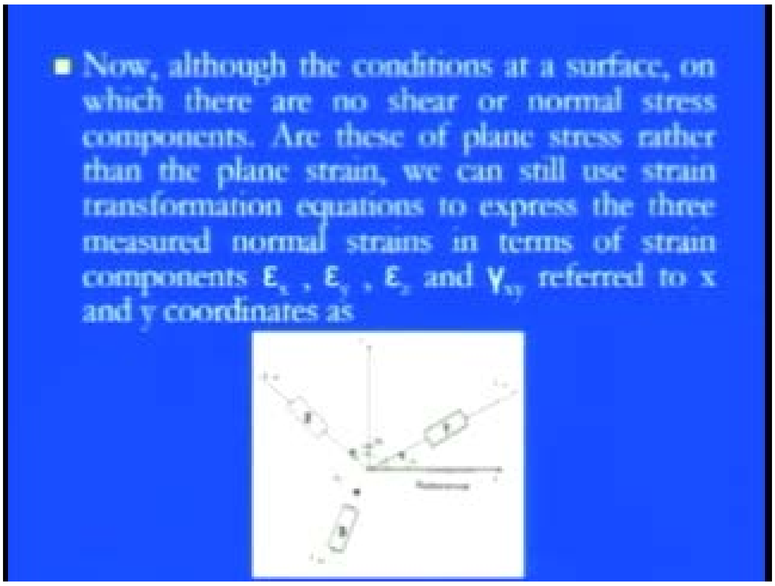

Now, you know like although the condition of whatever the strain gauges at a surface, on which there are no shear or normal stress components are there. So, these are the planes rather you can say the plane strain component, we can extremely use the strain transformation equation, the epsilon theta and gamma theta which we can still use you know like for this general transformation equation to express these three measured normal strain in terms of the strain component epsilon x, epsilon y, epsilon z and this gamma, this gamma x y referred to the x and y coordinates as shown in this particular figure.

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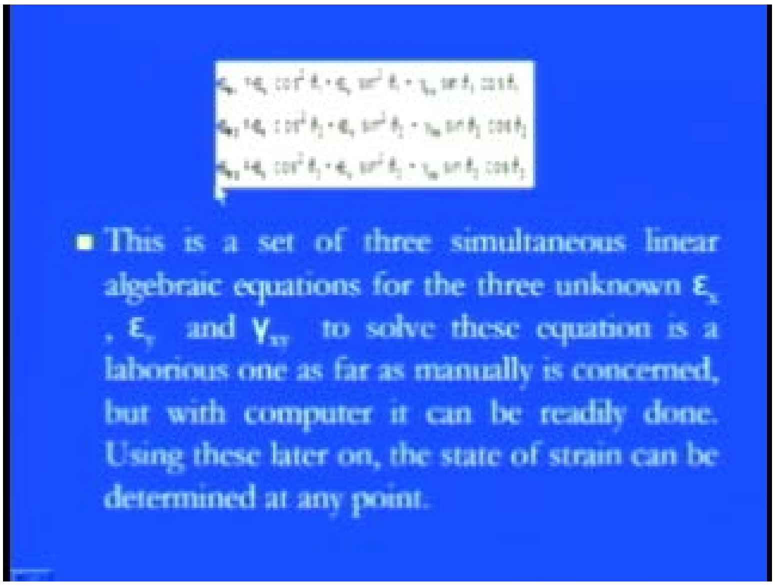

So, what we can do here? Now, we have this epsilon theta 1 epsilon theta 2, epsilon theta 3 and as I told you see we can straight way use those equation rather to measure. So, we are you see you know like epsilon x cos square theta 1, epsilon y sin square theta 1 and gamma x y sin theta 1 because theta 1 will give you the epsilon theta 1. Similarly, you see we can get epsilon theta 2 and epsilon theta 3 just by keeping those values in terms of theta 1, theta 2, and theta 3. So, I mean to say you see that if the theta 1 is there, then we have epsilon x, epsilon y and this gamma x y is there just to measure the strain component at theta 1.

So, similarly at theta 2 and theta 3, whatever the distortion is there due to epsilon x, epsilon y or gamma x y, it can be simply computed in terms of epsilon theta 2 or epsilon theta 3 when you see epsilon x, epsilon y and gamma x y distortions are there. So, this is a set of three simultaneous linear algebraic equations you see in terms of both epsilon x, epsilon y and gamma x y linear like these three unknown to solve this equation you know like is a pretty laborious part is there because we have three unknown equations. Again, you see we need to solve with these equations with all that kind of manipulation, but with the computer, it can be you know like easy and simple. Mat lab programs are there and we can simply keep those things, and we can get the final answer using you know like these thing. The state of strain can also be determined easily at a point.

This is you see pretty informative information’s are there rather once you know the theta 1, theta 2, theta 3, you can get you know like this whatever the strains are there at a particular value which is known as the oblique plane strains. Let us consider now the optimum part which we discussed in the strain rosette at 45 degree.

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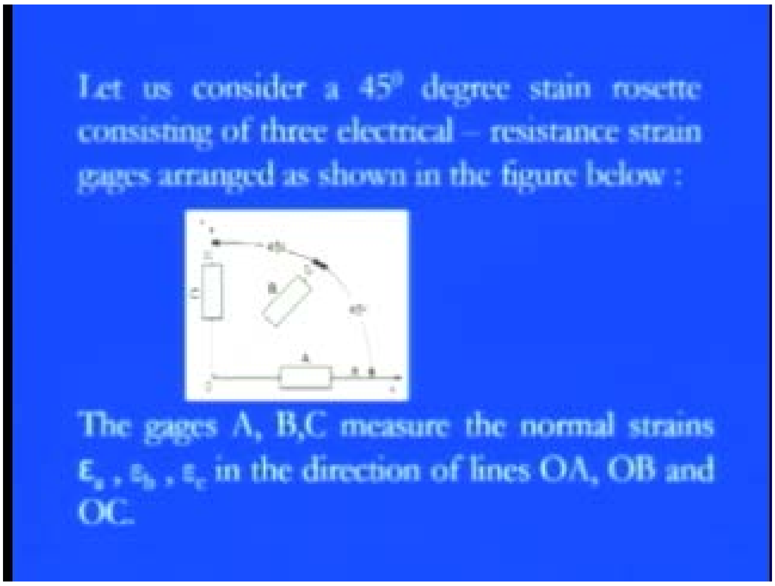

You know considering the three electric resistance strain gauges are there can be arranged exactly in the corresponding direction x y and x y direction like in the middle one. They are simply set up to the 45 degree angles. So, the gauge ABC simply measure this A B C are there. They are simply measuring the normal strain, this epsilon a, epsilon b and epsilon c with their respective directions you see epsilon a, b, c in the directions of lines OA, OB and OC.

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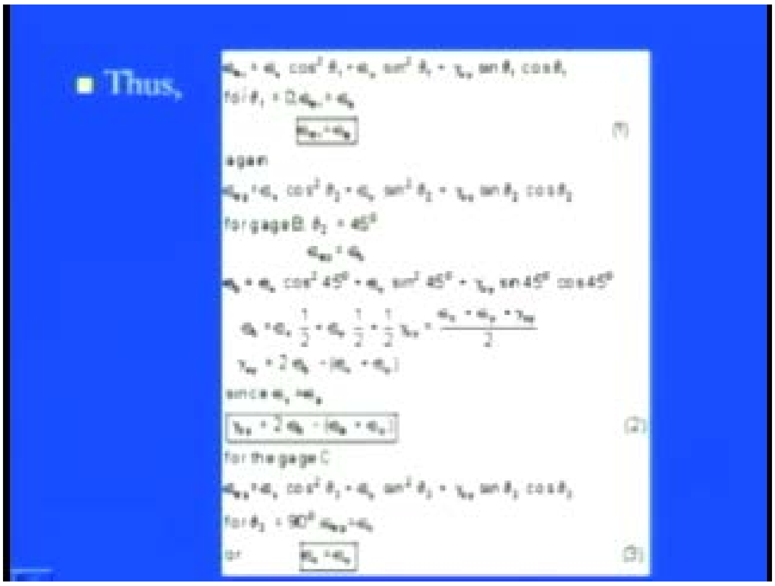

Once you see you know like we can simply put those equations which we found that in the three algebraic equations by keeping those things in the 45 degree at that point. We have epsilon theta was nothing but the epsilon x whole square theta 1 epsilon y sign square theta 1 plus gamma x y sign theta 1 cos theta 1 was there. So, by keeping that theta 1 is exactly you know like parallel to that one. So, theta 1 is 0. So, what we have is epsilon theta 1 is equal to epsilon a.

So, what we have? There is no angle. You are simply rotating 45 degree by theta 2 and theta 3. So, only we have 0 degree at first one. So, we have epsilon theta 1 is epsilon a. Again, you see you know like we can simply epsilon theta 2 which is at 45 degree from the theta 1. So, again epsilon theta 2 is nothing but epsilon x cos square theta 2 plus epsilon y sin square theta 2 gamma x y sign theta 2 and cos theta 2. Keeping 45 degree, what we have? We have epsilon theta 2 is nothing but equals to epsilon b.

So, what we have you see epsilon b is nothing but equals to you know like which we measure that actually the strain gauge b is there at this point just by keeping 45. What we have? We have epsilon b equals to half of epsilon x plus half of epsilon y plus half of the gamma x y, or we can say half of epsilon x plus epsilon y plus gamma x y by 2. So, these you see you know like the meaning is that we have straight relations that actually if you want to get those things, you can get the strain part. This shear strain part gamma x y by just you see if you know the B 2 times of just two times of multiply these two times here.

So, times of epsilon B minus epsilon x plus epsilon y. That means, if you know the normal strain component, the middle portion at the 45 degree from this one, you can easily get the shear strain component from that or we can say gamma x y is nothing but equals to twice of these terms. Now, you see go back to the third one which is exactly at the 90 degree of this one, the third c 1. So, now you see by keeping these theta at 90, what we have? We have epsilon theta 3 is equal to this ec means epsilon c which is there. So, epsilon c equals to epsilon y. So, we have epsilon a equals to epsilon theta 1, epsilon c is equals to epsilon y.

So, we can easily get either a and you know like b from the x axis and then, you see what we have means epsilon a is epsilon x, epsilon c is epsilon y. So, you see these are the normal strain component. The shear strain component is nothing but equals to twice of epsilon b minus of the summation as epsilon a epsilon x plus epsilon y. So, these three terms can be easily computed in for strain rosette if it is you know like shifted at 45 degree angle.

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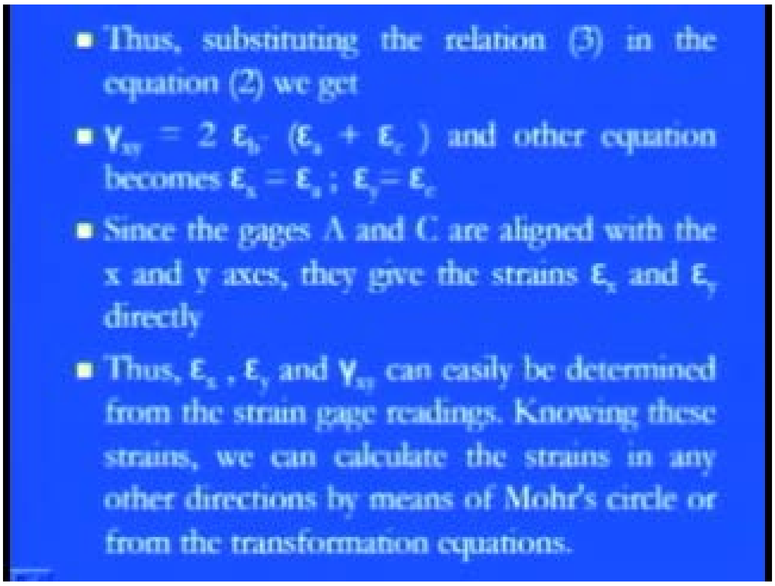

Thus, you see you know like all these things just by substituting all these things, we have gamma x y. So, this is the shear strain component and the normal strain component. So, if the gauges A and C are aligned with x and y, so obviously they will give you the direct or normal strain component while you see this middle one is settle with the shearing plane. So, obviously, it will give you the shear strain component. Thus, you see all these you know can be easily determined from the strain gauges. Knowing these strain gauges, we can simply calculate the strain in any direction by means of Mohr's circle or we can say from the transformation equations.

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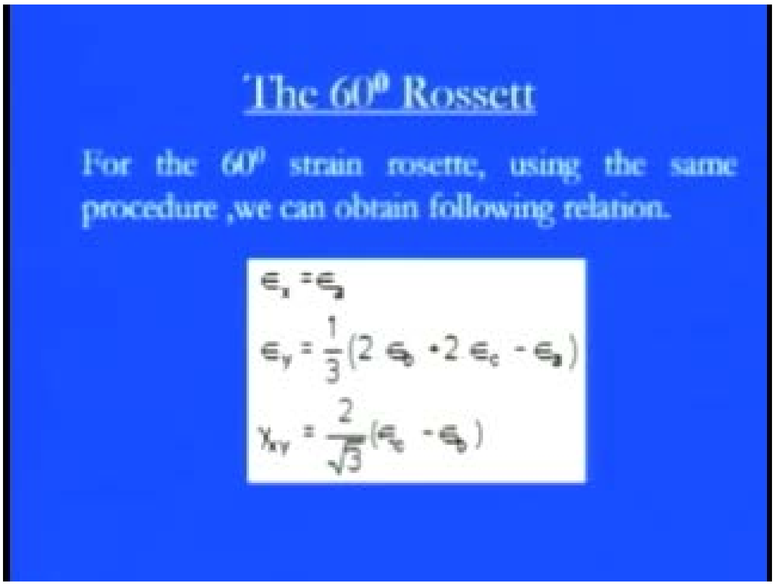

Then, you come to the next portion that is the 60 degree. So, previous case was 45degree. Now, if you put the 60 degree rosette, then what we have? We have epsilon axis epsilon a straight way, similarly epsilon y which is just going beyond these things. If you make the triangle or if you just do this thing, then it is one-third of 2 times of epsilon b plus 2 times of epsilon c minus of epsilon a. So, that means, you see it is not exactly you know like vertical to the part a. So, it is not you see we can say that these are the normal strain component. They are you know like since it is going certain beyond thing, so it has an impact of both b as well as c, and this gamma x y is nothing but 2 by square root of 3, that is epsilon c minus epsilon b.

So, now you see in this lecture we discussed about you see that if we have the different strain component, then how to calculate by using Mohr's circle. This you know like the strains at the oblique plane as well as if you see if you want to measure the strain because to measure parameter of strain, so you see which strain rosette will give you the perfect answer. So, we need to arrange at 45 degree to get the optimum or the perfect value of the shear strain or you see this 60 degree, it is also usable in that sense.

So, in the lecture now we are going to discuss about what the exact applications are there in the real future that you see. If we have a real bar, if you are applying the loads, how we can exactly measure those you know strains. And once you measure the strains, then what exactly the relations are there in between you know like those stress and strain component for different types of material that if we have you know like the ductile material, perfect elastic material, not perfectly elastic material, but there is you see some sort of distortions are there. Then, what exactly the relations are then between them, and what the properties are there of these material which will influence those relation between the stress and strains.

Thank you.