Strength of Materials

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Lecture - 8

Hi, this is Dr. S. P. Harsha from Mechanical and Industrial Department, IIT, Roorkee. I am going to present my lecture 8, here of that subject of Strength of Material, which is ((Refer Time: 00:37)) like development of this national program and technology enhanced learning. As far as my you know like the last lecture is concerned, we discussed about you know like most of the time the stress components that actually what are the normal stress components are there and what the shear stress components are there.

And what are that impact is there of these the stresses on the material and within the material, if we want to calculate that where you see the maximum stress components is there, like if a material is subjected by a pure normal stress, that does not means that there is no shear stress is there. We can also get the value of the shear stress, if we simply cut the plane and if you want to calculate the sigma theta or the tau theta, that is what we discussed.

And even if you see a material is subjected by a pure, the shear stress you see means the parallel stress component, also you see we can get the normal stress component with in this the object with the shear stress component. And if a material is subjected by both, the mutual perpendicular stresses as well as the shear stress component, then what the impact is there or what the interaction is there of this stress components and what the resultant values are there. We can also you see get those things by the two different types of solutions, which we discussed.

One is the analytical solution, we calculated at the inclined plane or we can say the oblique plane, the sigma theta as well as tau theta values by interaction of the sigma x sigma y, in the two mutually perpendicular stresses and the tau x, y which is the shear stresses across the plane. Then you see another solution which we discussed in the previous lecture was the Mohr's circle means, you see the graphical solution that rather to calculate all those sigma x, sigma y, sigma theta and the interaction of our sigma theta and tau theta.

Respective you see if you have the values of these two mutually perpendicular stresses and if we have the value of the shear stress. We can straight away you see plot those sigma x tau x and sigma y tau x y and get the value of you see a circle and you see just get the values of within those things what is the coordinate of the circle. What is the central point coordinate is there, what is the outer periphery is there, what is the radius is there and with those things you see you can pretty easy calculate that what the angle is there at which the principle stresses are there.

What the values of the principle stresses are there just by measuring no need to calculate by analytical way simply measure sigma 1, sigma 2 that what is the minimum point is there, what is the maximum point is there. What is the planes are there the two ((Refer Time: 03:03)) at what angle they are putting and what is the first principle plane a theta 1 and what is the next principle plane theta 1 plus 90. Meaning is pretty simple, if we have this kind of a interaction it is pretty easy you see to calculate first or to measure those things and also we discussed that actually what are the drawbacks of these things of the Mohr's circle.

If you have some sort of the values like 25.25 or 25.47 like this kind of things, then it is very hard to draw those things and then, we need to round up a last values. And then, it will give you some sort of error in a plotting those things even or in measuring those things, so this kind of the truncation error is always considerable, while solving stress values. So, every either to get the exact value by analytical way by calculating sigma theta, tau theta, sigma 1, tau 1, theta 1, theta 2 all those things theta p, theta x pretty easy, but it is the lengthy process.

For simplicity, but it has some sort of containing either if you have a thicker pencil, then again a we cannot measure exact angle and all those things the kind of truncation errors are there, so both features are unique as well as the stress is concerned. So, you see in the previous lecture, come to the main conclusion that in the previous lectures all you see, we were discussing about all these stresses, stresses, stresses that what exactly the internal intensity of the resistances are by application of the force.

So, now you see in the present lecture of this, we are going to discuss about that if the load application is there, then what kind of extension is there. Because, a stresses are there, but what the impact on the stresses there is a deviation is there in the shape, that means there is kind of deformation is there. So, how to measure the deformation and actually how, what exactly the technical term is there to measure the deformation and then, if you have the deformation in x direction, y direction or z direction, then what exactly the impact is and how we can measure. There this deformation in x as well as y and z direction, if we have the combined loading means, the loading is there or the stress components are there in two mutually or three mutually perpendicular direction. Or if all in the normal stress component is there or if pure shear stress is there or if the combination of both at the normal stress as well as the shear stress is there. Then, how we can major the strains, we can say the deformation that we are going to discuss in the next lecture and then, also we are going to discuss about that.

Actually if we are applying the load and the deformation is there, then what is the limiting the value of the load is there through, which we can say that it is the elastic deformation or if you go beyond certain thing, then it is a plast deformation. So, if you are working in the elastic deformation, then what will be the property which are applicable only within this region and what are the different property, which are applicable to the another region. So, this kind of all those properties, which we are going to discuss, in this particular lecture.

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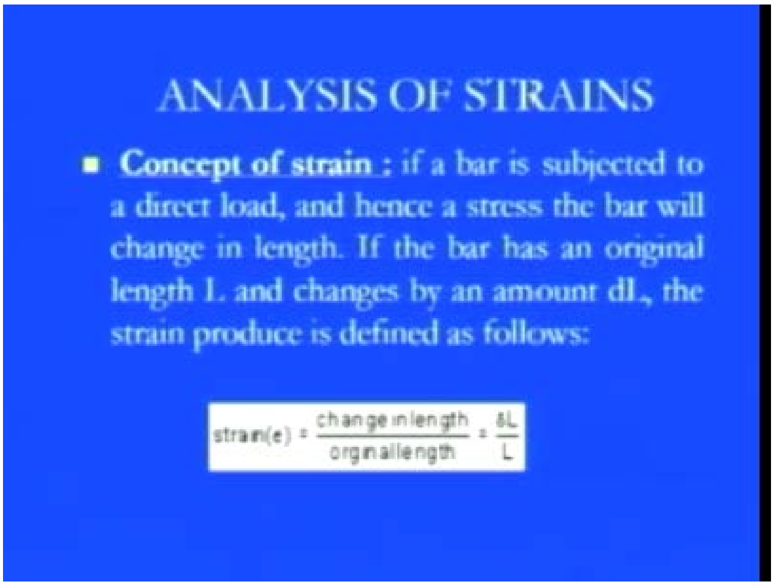

So, here it is the first analysis of the strains, so you see here the concept the strain coming from the same thing, which we discussed that if a bar is subjected by a direct load, that means you see if you have a tensile pulling. You see at the extreme corner of any bar rectangular bar, circular bar like that, hence a stress of a bar will change the length, because of the stress, stress formations are there all across the structure of the this bar.

Then this will just tend to change the length of the bar, if the bar has an original length L and the change of this bar is just by an amount of d L, then the strain which is a measure of the deformation. If you measure the deformation respective of whether it is a tensile load, compressive load, there is a sort of deformation and if you want to measure the deformation the technical term for that is the strain.

So, strain can be defined by the change of length divided by the original length or here, if I am saying that the delta L is the change of length and the L is the original length, then the strain which is the size there which is equals to del L by L. So, this is you see the technical term for measurement of any kind of deformation.

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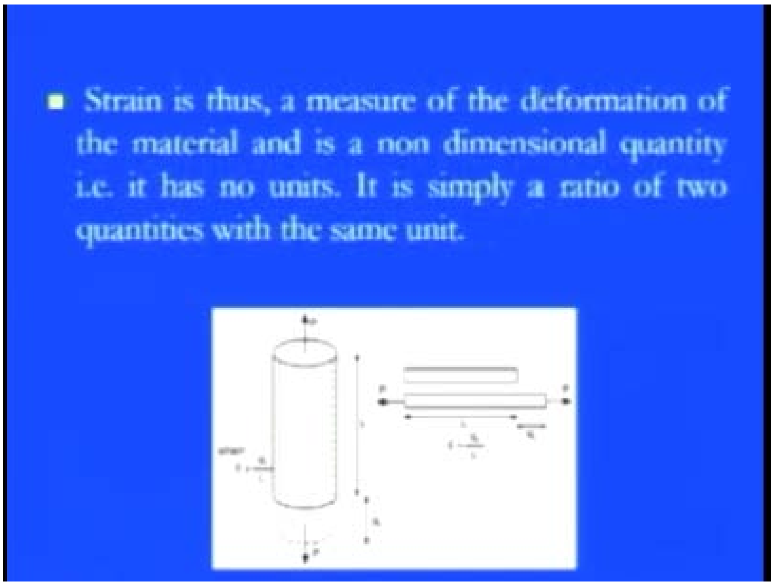

Since you see strain is thus the measure of deformation of material and it is nothing but, equals to delta L by L, so obviously it is a non dimensional quantity. Because, you see delta L is also in somewhere in millimeter, meter, centimeter, micrometer like that, and L is also having the same unit of the length, so you see both are cancel out, so it has no unit. So, strain is the non dimensional parameter which has no unit and it is simply ratio of two quantities of the same unit that is why it is like that.

So, come to this particular figure, where we found that actually if there is the rectangular circle bar, if you have a circular bar, simply keeping these forces at the extreme end of those things like P, extension is there and with this extension, the strain is nothing but equals to delta L by L. So, if it is a vertical part this is the original length is there, this is a kind of extension is there or else if you have this one extension is there, means if you have a rectangular bar which is in the horizontal direction.

Again if you are keeping the two different, two same amount of load at the two extreme corner and by keeping those things, if we have an extension of delta L with the original length L, we can simply get the strain value that is nothing but, equals to delta L by L. So, this is the basic concept of the measurement of the deformation and for that we define the technical term known as the strain.

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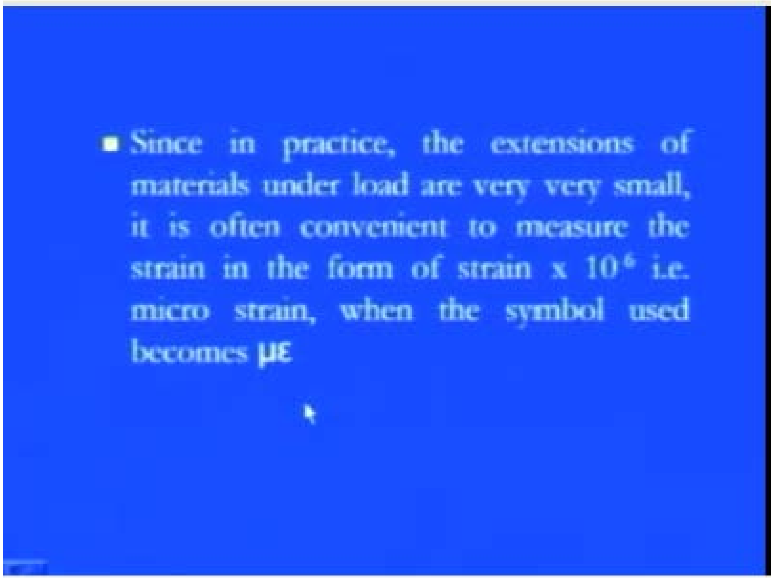

Since in practice the extension of material under any load are very, very small always, because we are putting, putting, putting and there is a internal extensions are there of the microstructure, but we cannot see by naked eye. So, whatever the extension or we can say sort of the deformation is very, very small, it is often convenient to measure the strain or the whatever the measurement of the deformation in form of the strain into 10 to power 6.

Because, you need to magnify those things, just to see that what exactly going on within the structure if you apply the load, that is always that is why we are always going with the micro strain or we can say the microns of the strains, when the symbol is mu times of strain symbol. So, here mu times of epsilon will give you the micro strain here, then the next point will come that actually respective of whatever the loading is there, what is the sign convention of the strain is.

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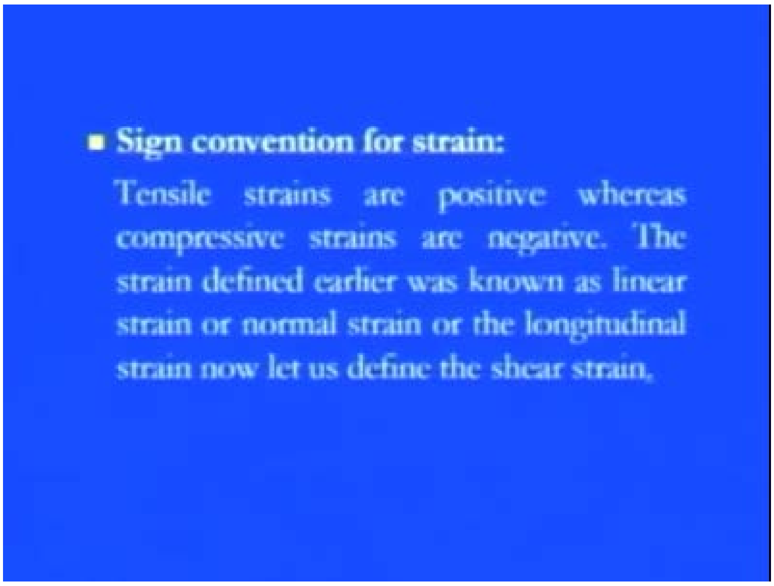

So, as we discussed again I just want to refresh those things that actually, whenever there is a tensile pulling is there the tensile, load is there, tensile stresses are there and the tensile stresses are always termed as the positive stresses. Compressive stresses are always termed as the negative stresses in the normal stress form, and if you have the shear stress, then if you due to the shear stress, if object is tending to move towards the clockwise direction.

You always use the positive sign and if the object is tending to move towards the anticlockwise direction like that, we are always tending to put the negative direction. So, similarly you see it is a kind of analogy here that, if we apply the load and there is an expansion towards the same direction, because you see the normal stresses are nothing but, the axial stresses. So, when we apply the load and there is an expansion in that bar, these are the tensile strains are there delta L, because the change of the length divided by the original length.

So, tensile strains are always positive, just like the tensile stresses, whereas the compressive strains are always negative just like the compressive stresses. So, it is pretty analogy kind of that and the strains defined earlier was we say the linear strain or the normal strain is there, just like the normal stress component. Because, it is axial strain always towards the same axial direction either in a x this extension part is there or the compression is there or we can say it is the longitudinal strain, because it is all along the same longitudinal way.

Because, the same axis is there is no eccentricity is there, wherever the load application is there, it is passing through the neutral axis of a bar and that is why we can say either irrespective, it is a axial strain, normal strain, linear strain or we can say it is a longitudinal strain. And now you see we can another strain which we can define as the shear strain, shear strain is nothing but, as the shear stress is there it is always along the plane. Means you see it is in the normal stress the stresses were there perpendicular to the plane, these stresses the shear stresses were parallel to the those planes are there.

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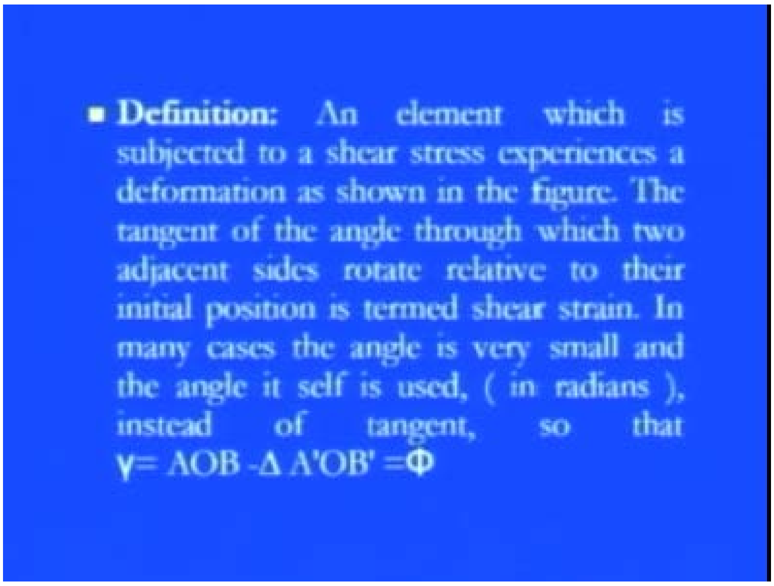

So, similarly you see if you want to define the shear strain, an element which is subjected to a shear stress, means parallel to the layers of the plane experiences a deformation. Because, if you want to twist that element from right from the across the plane you see here, towards the clockwise or anticlockwise direction, always you see the whatever the segments are there or layers are there or even the microstructures are there. They are always trying to experience a kind of deformation and the deformation it is different than the normal stress component. Then you see we need to get the tangent of that and once you get the tangent of the angle through, which two adjacent sides are just tending to rotate relative to their initial position always termed as the shear strain, means you it will just to try to tend towards that movement.

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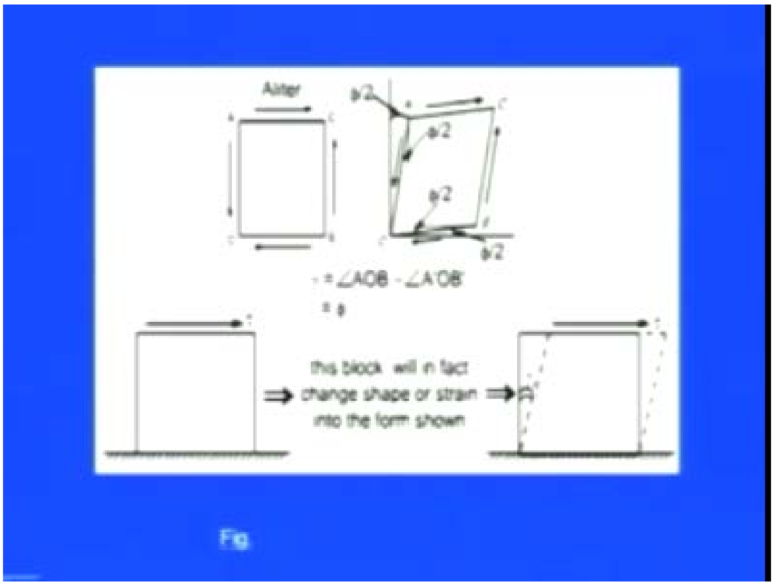

So, I just want figure out those things that you see here, if this is my object and here initially we were just want to this is the shear stress, shear stresses are there the direction and these are the complimentary shear stresses are there. And these due to this shear stresses this object is standing to move towards the clockwise direction, here this will tend to move this direction, this will tend to move in this direction.

So, you see we have a kind of extension at this, so this will, this object will move towards this direction, this object will move towards the lower direction. So, you see after equilibrium side if we have this kind of extension that, it is going towards the direction, this is going in towards this direction and then, you see the respective directions are just to balance that part. If we have this phi by 2 or we can say this gamma is there, the gamma is nothing but, equals to the angle of AOB initially, the angle of AOB this and minus the angle of A dash, this A dash OB.

So, whatever the change is there right from first two, the initial part to final part it will give a phi means, what is the change of the overall change is there and this can be denoted here by the phi. Or else you can say this block in fact, there is a change of the shape or the strain into the different form is there like you see here, if you have the this rigid the term is there and we are applying the stresses across this plane, this is across this layer of plane.

So, this is very much parallel to these things, so it will tend to move this object, in this direction, so you see if we are saying that after stabilizing those things, if you have this kind of the deformation. Like this one is shifted to this direction, this one is shifted to this direction and if you want to measure this kind of deformation due to the parallel forces all across the plane, this kind of deformation is known as the shear strain and shear strain can be calculated by whatever the change of length means you see here.

The angle of AOB minus angle of A dash OB divided by what was the original one, so you see come go back to the main position here, we were discussing that actually whatever the tangent of the angle, which is passing through that two adjacent side. Just rotate relative to their initial position as we shown that, actually they are simply trying to rotate in that way, always termed at the shear strain which is known as by, which is we can symbol by the gamma.

In many cases an angle is very, very small, because always the layers are tending, they have the tendency to make a rigid base, so that is what you see it has a less tendency to make the rotation kind of that. So, they are very, very small as compared to even the normal strains component and the angle itself is we can say that, we can use these things in terms of the radiations or the the degree form instead of making the tangent. So, we can say that the gamma, which is equals to either AOB minus whatever the change of these things or we can say it is a phi.

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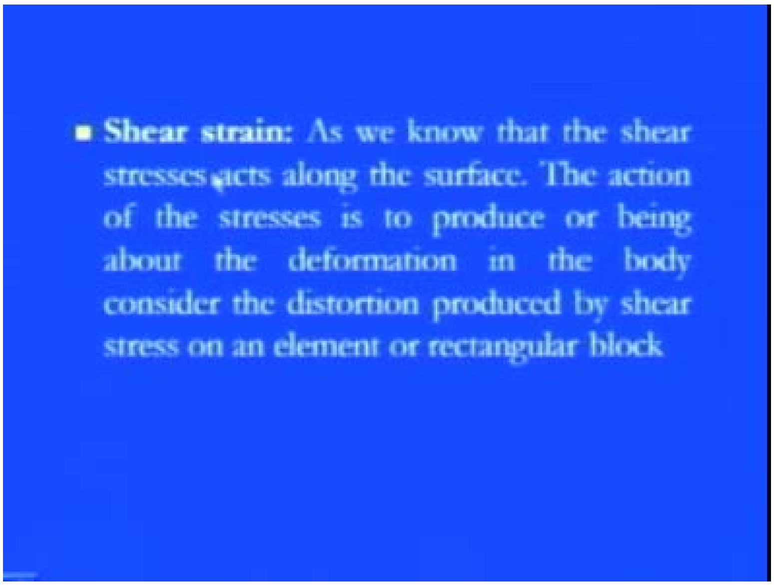

So, come to the real phenomena of the shear strain which we discussed in the previous slide, that as we know that the shear stress is acts along the surfaces, which the parallel to the surfaces or the parallel forces are acting in that. The action of the stress is to produce or being about the deformation, in any body consider the distortion produced by the shear stresses on an element or on rectangular the block is to be considerable to calculate the shear strain.

Means we need to see that actually how the distortion is there, means if we put the block as you can see in this particular figure, that if you put the block here ((Refer Time: 15:47)) the rigid base and the how what kind of distortions are there on the top of layer. So, this distortion can be easily computed with the using of concert of the shear strain, so this is the gamma is the shear strain which can easily find it out that, actually how this upper layers are to be distorted with respect to the lower layer.

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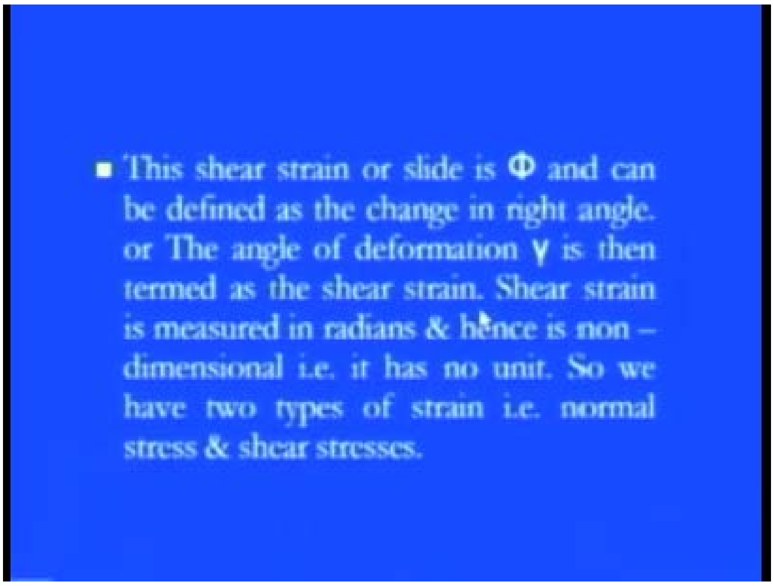

So, this is which we discussed about these things, so this shear strain or we can say the sliding of those layers from the original one is phi can also be defined as the change in the right angle. Means whatever the changes are there of the right angle, earlier right angle was the shape and how these changes are there, you see this basis is just changing like that. Or the angle of the deformation gamma is then, termed as the shear strain and shear strain is measured, in terms of radians as I told you.

And hence, you see it is a non dimensional parameter, because the change of angle divided by the original angle is there, or we can say the right angle is there, hence the non dimensional parameter always gives you no unit, because the unit can be cancel out from denominator to nominator. So, we have two types of strains as similar to the stress, the normal strain component or the shear strain component, just like similar to the normal stress and the shear stress.

You need the normal strains also we have the different formations like, we have the tensile strains, compressive strains are there or even you see we have the bearing strains, bearing strains are similar when the two surfaces are meeting to each other and there is the compression is there. So, what exactly the deformations are there in the different points, if we are measuring those things that is nothing but the bearing strains or even we can say that when the bending stresses are applying means, if there is a combination of the tensile as well as the compress stresses are there.

Similarly, we have at some if we are talking about bending of beam at some point there is an extension, so we can say the tensile strains are there at some point there is compression, so we can say the compressive strains are there. Means it is, if we are talking about bending strains, it is the combination of tensile as well as the compressive strains. And the third one is derived stress is the this torsional strains, means whenever rotating shaft is there and the shear strains are there that means, what are the moment which are applying at the extreme corner of these rotating shaft.

We will get the shear strains at the end or we can say it will termed as the torsional strains, and the similar the last form of the strains which we can also, we can get those due to the temperature change is the thermal strains. If because of the expansion, if high temperature region is there and any material is which is being subjected by this kind of temperature, that always there is an expansion of the material. Or there is an contraction if the less temperature is there, lower than the melting point of that temperature, contraction is there.

So, whatever the change of the size of these material due to the temperature effect, these the terms the strains which are measuring this kind of deformation is termed as the thermal strains. So, here these kind of strains are there which are pretty similar to the stresses, but you see they are simply measuring the deformation due to the load application or we can say due to the, whatever the point of application of these applied loads are.

So, now we have the two different terms, we have the stress because of the load application, we have the strains because of the load application. So, both are simultaneously forming the object when there is a load application, means you see when we apply the load in irrespective of whether it is a tensile load, compressive load, shear load like that. There is always the stresses are being formed, across the layer or perpendicular the layer and also the same time there is a change of the deformation, means there is a change of the size of the element.

Means kind of deformations are there or the displacements are there, so means the strains are there, so we just want to set up the relations that actually if the stresses are being formed, strains are being formed the same time. Then what exactly the relations are there in between the stress and the strains, so for that there is a basic law of in the solid mechanics that is the Hooke's law.

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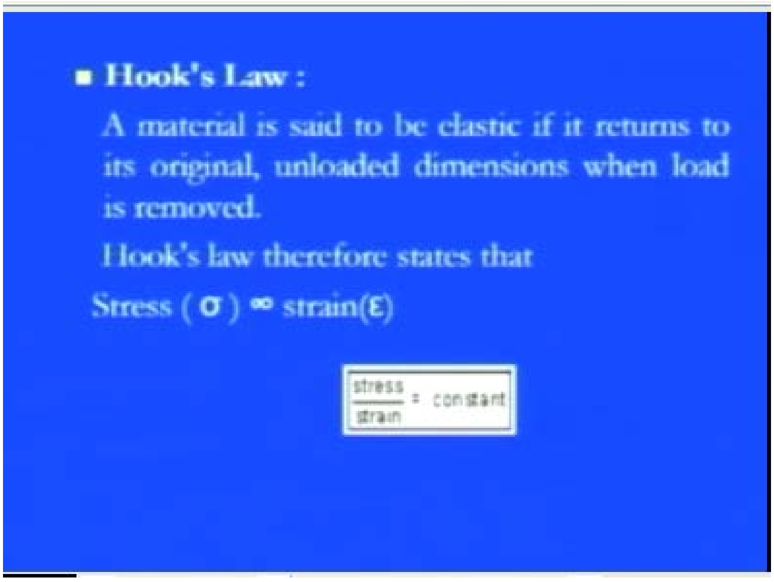

Hooke's law says that, if the material is said to be an elastic, elastic is nothing but, elastic deformation, because an elastic deformation we can say that there is a kind of deformation, if you apply the load and deformation is there once you release the load. And if body comes to it is original shape without any kind of permanent set up deformation, this deformation is known as the elastic deformation.

So, material set up in elastic if it returns its original shape, unloaded dimensions when load is removed, means when load is removed we cannot realize that or we cannot visual those things that the kind of deformation was there, when it was loading; we can say that this is the elastic deformation. And Hooke's law says, Hooke's law again this is the must condition that Hooke's law is only valid when there is a elastic deformation under any kind of loading, tensile compressive, shear, whatever the kind of loading is there.

Hooke's law is only valid when material is subjected under the elastic deformation, then we can say that, then what is the statement of Hooke's law. Hooke's law says that under the elastic deformation stresses when the load acting is there, stress formation is perpendicular to the deformation measure. Means a stress proportional to the strain and once we equalize those things we have a constant, so stress by strain is will give you the constant and this constant has a real meaning.

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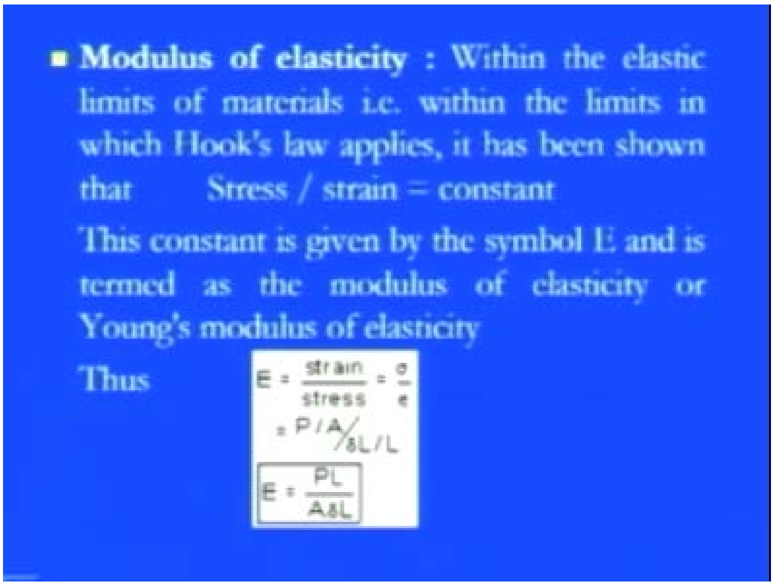

So, this kind of a constant which we are going to discuss as the modulus of elasticity, means we have a stress, we have a strain and if you are saying that actually you are applying the load, the stresses are being forming within the object. And the same time there is a change of the length or the diameter any dimension, and if they are proportional means at one point of time, the stress is increasing, strain is increasing and they are proportional.

And once you want to set up the relation in between the stress and the strains at this particular point, it will give you a new constant, the constant is known as the modulus of elasticity, because we are considering only the elastic deformation here. So, within the elastic limit a material that, within the limits of this Hooke's law applying, it is shown that actually stress divided by strain is constant and is constant nothing but, equals to the modulus of elasticity.

So, this a constant is given by a symbol E, so modulus of Young's, this is known as a Young's modulus of elasticity, because Young's was first defined this that, if you are taken this kind of material, then what is the limiting part of the elastic region is. So, that is what for putting the limitation of the elastic region, the Young since, Young's produces those kind of calculation that is way it termed as the Young's modulus of elasticity and it is always denoting by E.

So, if you see this the E is nothing but equals to, E is nothing but the constant, E is nothing but, equals to the strain divided by stress, these kind of thing or we can say it is kind of a sigma divided by the epsilon value. Or we can say these P by A divided del L by L or we can say that actually if you put all those values together, we have the Young's modulus of elasticity which is nothing but, equals to PL by AE or we can say this delta L.

Delta L is nothing but, equals to whatever the change of the length, this is the original length, this is the applied load and this is the effective area of matter of concern. So, this is a kind of we can say the main property of the material is there that, what the limiting values there, so this is absolutely valid when we are talking about a material ductile. Means if a material is having ductility property, then you see it can be easily expanded, does not have to, we can say it did not so those kind of thing, the rupture or the fracture or this kind of property when the load application is there. And for which it is pretty easy for us to define the elastic region and the plastic region within these things. And for this kind of thing the value of E, because it is defining the region, the value of E is having a significant importance for this kind of region dividing.

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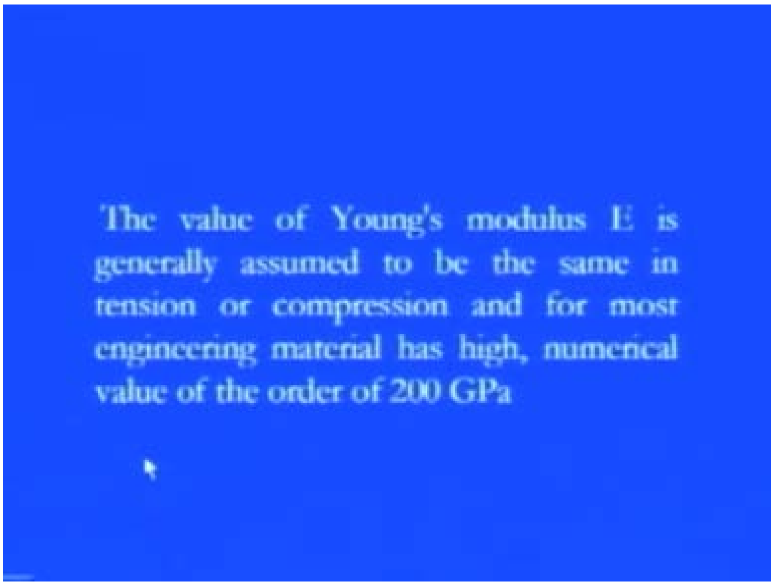

So, the value of E the Young's modulus as I told you is generally assumed to be same in the tension or the compression whatever, because it is nothing but, the stress and the strain which are there in both of the loading, the tensile as well as the compression. And for most engineering material it has a high value and it is almost equal to 200 Giga Pascal and this 200 Giga Pascal generally we are using for common mild steel.

But, if we are using the speed steel or high carbon steel, which has high carbon percentage, so for that actually it is a different value more than this values there. So, this was the one property which was only valid for the elastic region, then we have the another property which is also equally important, if we are talking about the stress and strain simultaneously which is known as the Poisson's ratio.

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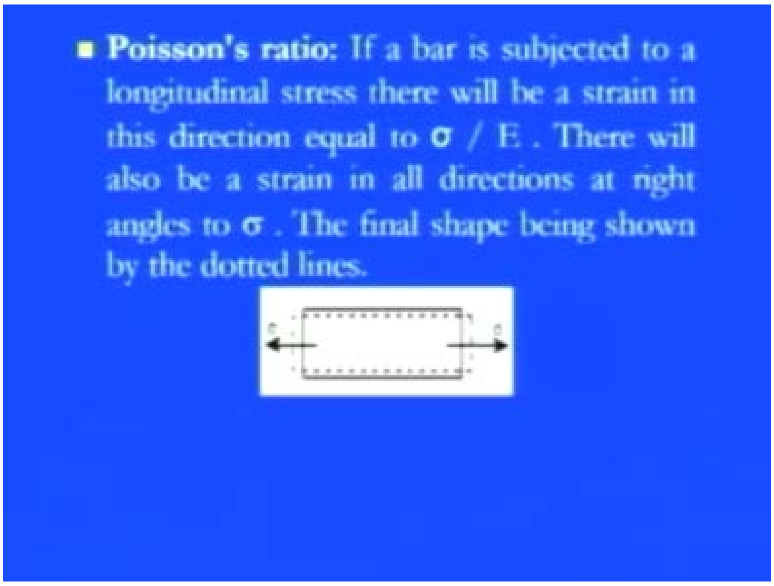

If a bar just like if we are considering those things, we have a bar here and it is being subjected by a tensile loading and due to tensile loading, we can see the dotted region, dotted region is nothing but, the extension. So, the initial length though we can say of this bar was the firm line in this figure and the dotted lines simply shows that what the deformation or I should say it is the extension. So, if a bar is subjected to a longitudinal stress, longitudinal stress as we termed out that it is a tensile stress or we can say the normal stress.

There will be a strain in this direction in the longitudinal direction only, so this is the longitudinal direction, will you just keep this things in your mind it is a longitudinal direction which is equal to sigma by E. Obviously, the strain is nothing but, equals to the stress by strain E is so strain is nothing but, equals to a stress by modulus of elasticity, because within this elastic region only. There will also be a strain in other direction, because what we are considering here that there is a change of the length.

So, the change of length is ((Refer Time: 25:44)) this is the extension, this is the extension, but if you go to the another direction means, if you are going for this direction and if what will happen to the mutually perpendicular action. So, if you are just looking on the top of that, we found that this one was the earlier length and then, if you look at these thing after the deformation, we found that there is the deformation along these things.

So, this is lateral direction, so this was earlier the extension was there in longitudinal direction, but what has happened to the circumferential or we can say the lateral direction. So, there is a contraction here that means, if you pull the things it will extended, but across the circumference there is a reduction is there and this reduction we need to measure. So, this Poisson's ratio will define the relation between that towards the longitudinal as well as the lateral direction. So, there will be also strain is there in other direction, right angle to the sigma and the final shape is being shown here by the dotted lines.

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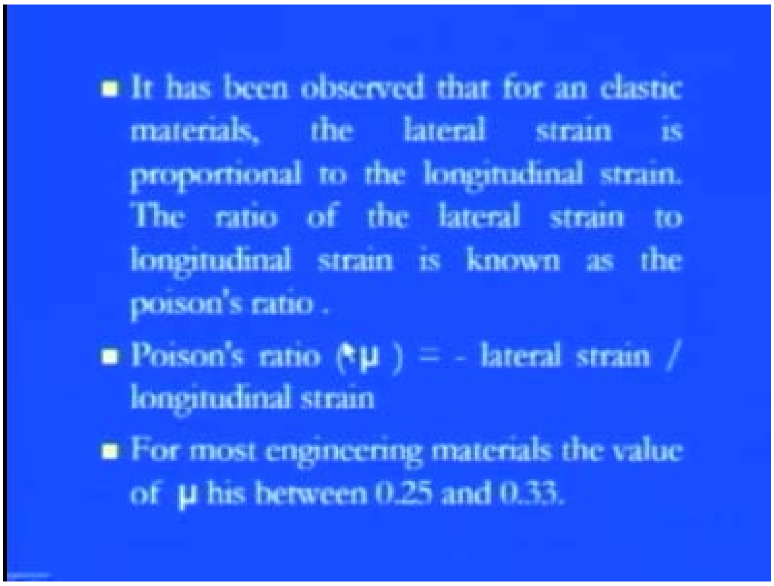

It has been observed like, as we discussed that for an elastic material, the lateral strain is the lateral means the circumferential part is proportional to the longitudinal strain. And the ratio of the lateral strain to the longitudinal strain, again the lateral means towards the circumference of that divided by the longitudinal where the load application is there. So, ratio of the lateral strain divided by longitudinal strain will give you a new term, again you see it is a strain by strain, so it is a non dimensional parameter no value, no unit is there, no dimension is there.

Because, it is a strain by strain, so but it will give you a direct relation that once you apply a compressive load or extension, there is how the changes are there or we can say what kind of deformations are there in all the directions. So, here the Poisson ratio that is also known as the mu is equals to minus, because it is a kind of contraction is there on the lateral part, so minus lateral strain divided by the longitudinal strain.

So, again keep this thing in your mind that lateral strain is nothing but, the circumferential part where the contraction is going on here in the previous figure, longitudinal strain is that part where the extension is there or where the kind of load is applied here, so this is the direction of the lateral divided by the longitudinal strain. And for most of the engineering material, the value of mu is always falling in between 0.25 to 0.33, but keep this thing in your mind that wherever we are applying the Poisson ratio, this is only applicable within the elastic region only.

So, now we discussed about the two main parameter, one was you see the Young's modulus of elasticity and one is the Poisson ratio; so when we are discussing about the stress and strain simultaneously, then these two are the real good parameter to be introduced. Now, we see we are going to discuss about as we discussed in the previous cases also that, if we have a stress and then what is the three dimensional state of stress.

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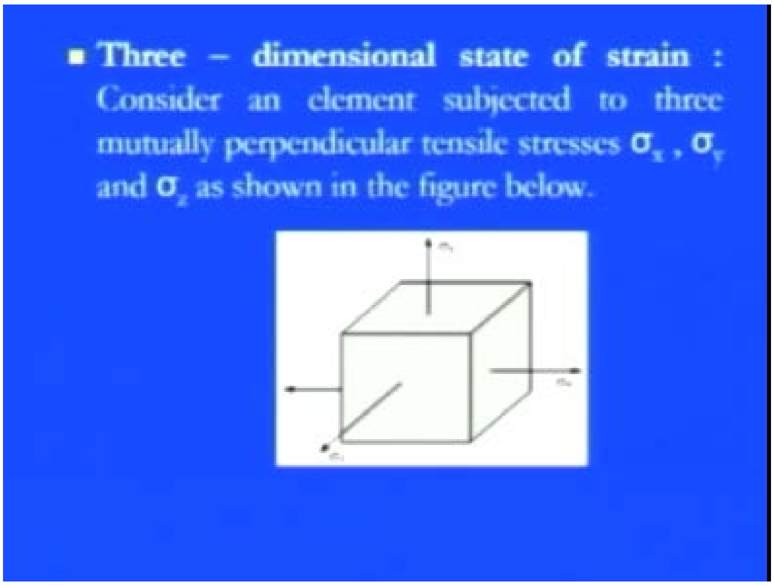

So, here similarly we have the three dimensional state of strain, so again for that we need to consider a symmetric element, so the symmetric element is this here the unit cube is there and unit cube has, unit depth, unit width and unit height is there. So, all these in these three directions, we have the unit structures and in that now we are considering the three mutually perpendicular this tensile stresses are there. So, here in the x axis, y axis and the z axis all the three mutually perpendicular stresses and they are in the form of the tensile, so they are always trying to pull this symmetric element in mutually perpendicular directions.

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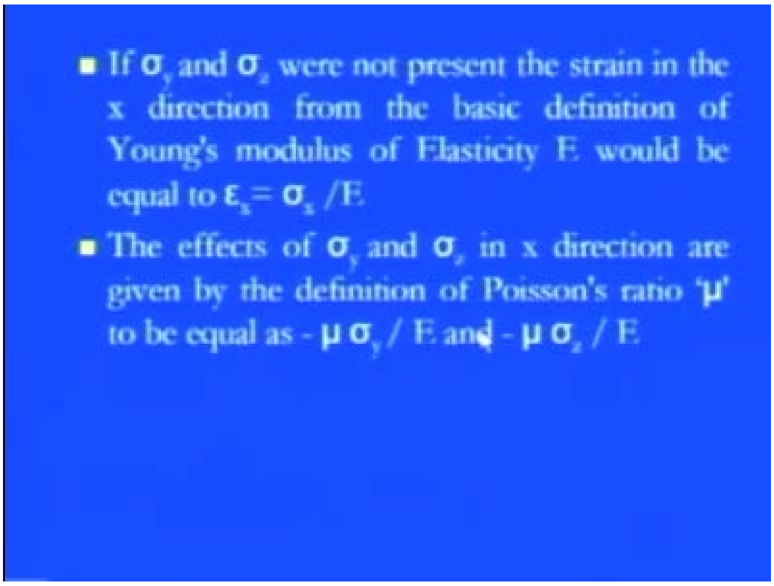

So, if you are saying that if sigma, sigma y and sigma z were not present the strain in the x direction from the basic definition, we can also get simply by the strain in x direction this xi x is nothing but, equals to sigma x by E. Meaning that if there is if we have only one force is there in the x direction axial, we have a three mutual direction which are presenting, but if you are saying that these two loadings are not there, only the loading is there in the x direction. So, in the x direction there is an extension and due to the extension there is a deformation, which is in the positive direction.

So, the strain in x direction, xi x is given you equals to sigma x by E, but the effect of this because, if you are pulling the thing, then other direction will be contracting. So, there is the deformation is there in other two direction, so the effect of sigma y and sigma z in x direction will be given by the Poisson ratio mu, which is equals to minus mu times, because if you look at those things this Poisson ratio is nothing but, equals to minus lateral strain by longitudinal strain.

So, that means there is a contraction is there in other part, so if you want to calculate the strain component in that, then we have this xi y is nothing but, equals to minus mu times of this whatever xi y is there and xi z is minus mu times of this xi z or we can say minus mu times of this xi z value is sigma y by E and xi z value is sigma z by E. So, I mean to say that if we have only one loading we can simply get those other values by this xi x, xi y, xi z and we need to keep the things that actually there is only in extension in one direction, but there is any contraction is there in the other two directions.

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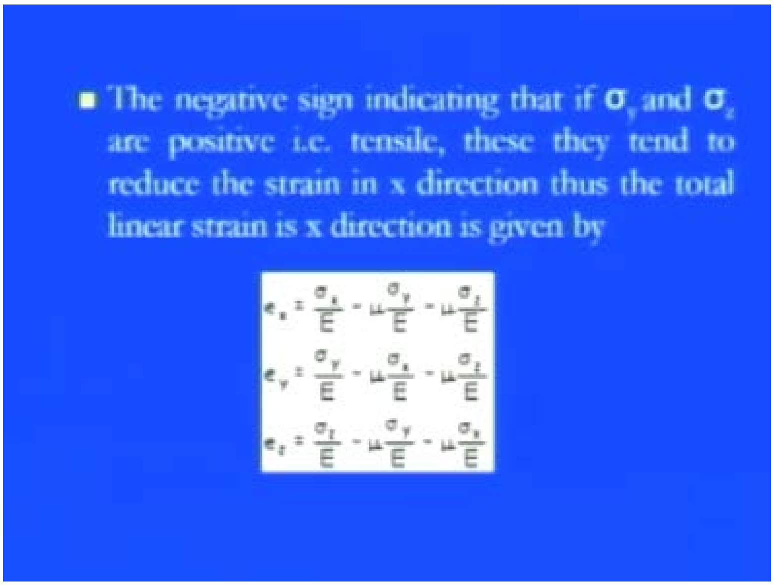

The negative sign indicating that, actually if you are applying the load in the x direction, there was an compression in there, in other two directions even if you are taking the sigma y and sigma z. That means the stresses are positive, that is tensile they always tend to reduce the strain in the x direction whatever the strains are there in other the other two directions, thus the total linear strain in the x direction.

If you want to calculate, then we always go for the epsilon axis is equals to sigma x by E due to the longitudinal strain and the other two strains are the lateral strains, so that is what you need compute in the proper way with respect to the x axis. So, we have the total strain the combination of three axes are this sigma x by E minus mu times of sigma y by E minus mu times of sigma z by E, so this is this will give you the total strain in x direction by interaction, in interactive effect of x y and z direction of the stresses.

Similarly, we can compute in other two directions also, like if there is an axial pulling is there in the y direction, so the strain in y direction will give you the sigma y by E that is the main the extension, but in the extension in y direction means the there is in a contraction in other two direction, which can be computed using the Poisson ratio. As we have seen in the previous case, so that is equals to minus mu times of sigma x by E and minus mu times of sigma z by E.

And similarly we have the strain component the z direction, while there is a loading in z direction means the pulling is there toward the z direction, sigma z by E minus mu times of sigma y by E minus mu times of sigma x by E. So, we have all the three kind of the strain formations are there, the strain in x direction, strain in y direction, strain in z direction. But, to compute the strain in all three direction there is interactive fact is there of the stress in a mutually through the perpendicular directions like sigma x, sigma y, sigma z corresponding to what type of loading is.

So, now come to the principle strains in terms of the stresses, because as we discussed already that actually there are some principle stresses are there and the principle planes are there. And if you want to compute the principle stresses and principle planes there is no form of the shear stresses are, so now we just want to check that actually what about the principle strains.

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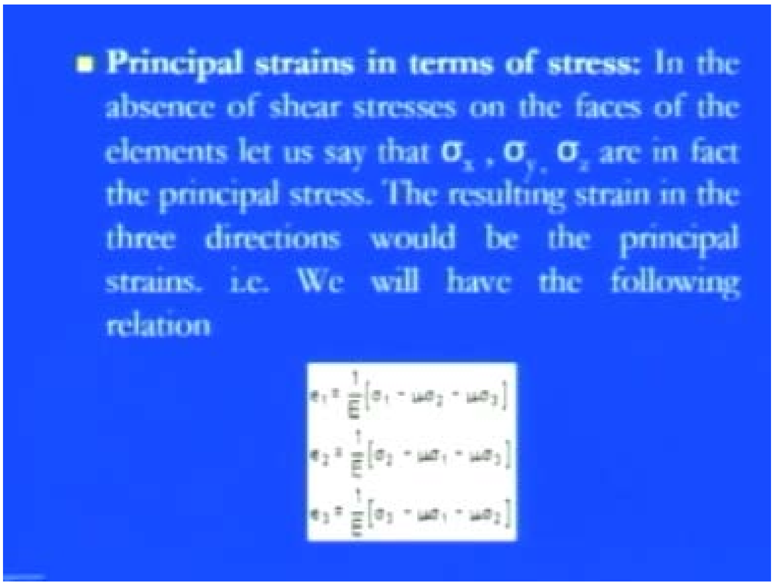

So, in the absence of shear stresses, means there is no element of the shear stress on the faces of the elements, let us say we have sigma x, sigma y, sigma z and there are in fact if you saying that these are the principle stresses, the resulting strain in these three directions, that would always gives you the principle stains. Because, there is no element of the shear stresses are there to in computing of sigma x sigma y sigma z, so this will give you the indication of the principle strains.

And we can say that actually the principle strain this epsilon 1 which is nothing but, equals 1 by E times sigma 1 minus mu times of sigma 2 minus mu times of sigma 3, that means that actually we have three different components of principle stress sigma 1, sigma 2, sigma 3. And if you want to compute with the induction if I talk with the inducing of inclusion of all these a three combination of sigma 1, sigma 2, sigma 3, we can easily compute the principle strain in one direction.

Means in one principle plane direction that will give you the epsilon is equal which is equals to 1 by E sigma 1 minus mu times of sigma 2 minus mu times of sigma 3, similarly we can also compute like the principle strains in other two directions, other two planes I should say rather that is a principle. Stain on the second plane, which is equals to the epsilon 2 is equals to 1 by E mu sigma 2 minus mu times of sigma 1 minus mu times of sigma 3.

Or we can say in the third plane, if we are talking and if you want to measure the deformation in the third plane which will give you the principle strain in the third plane, means third principle plane, that is epsilon 3 is nothing but, equals to 1 by E into the sigma 3 minus mu times of sigma 1 minus mu times of sigma 2. So, these the analog is there that if we are talking about normal stress and shear stress components, then similarly we can go for the normal strain, the normal stress as well as the shear strain part.

But, if you are talking about the principle strains where there is no element of the shear strains are there, we can also compute this principle strain component with the using of all three mutually perpendicular principle stresses are. So, now if we are talking about plane stress or we can say the plane strain, so for that the third direction is gone automatically, and if you are saying that only we are working in the x y plane. Where the sigma x and sigma y is there or we say the epsilon x and epsilon y is there, as well as the stress and a strain components are, so sigma z is gone so sigma z is equals to 0.

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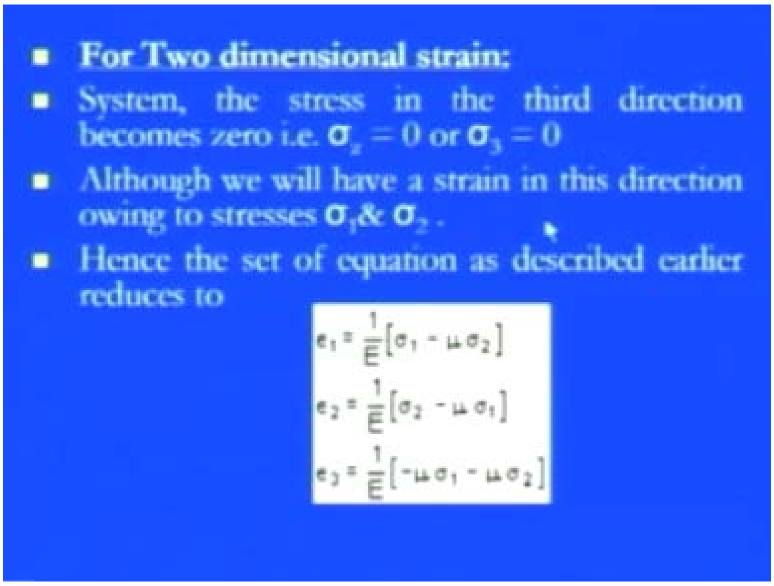

Or we can say rather, if we are talking about the principle stress, the principle plane component sigma 3 is equals to 0, then only we have sigma 1, sigma 2 or we can say sigma x and sigma y. So, for that we can calculate the principle strain formulations apart from those things, we have epsilon 1, which is equals to 1 by E sigma 1 minus mu times of sigma 2. Similarly, in other two directions also we can calculate like sigma, the epsilon 2 the principle strain in other direction is nothing but, equals to 1 by E.

This sigma 2 minus mu times of sigma 1, but now the third one, because it is not the there is, the strain is there in other direction also which we need to compute, because of the induction effect of these two loading sigma 1 and sigma 2. So, the principle the principle strain in the third direction or the third principle plane, which will give you 1 by E into minus mu times of sigma 1 minus mu times of sigma 2, because mu 3 or we can say the sigma 3 is we are not considering or we can say that is equals to 0.

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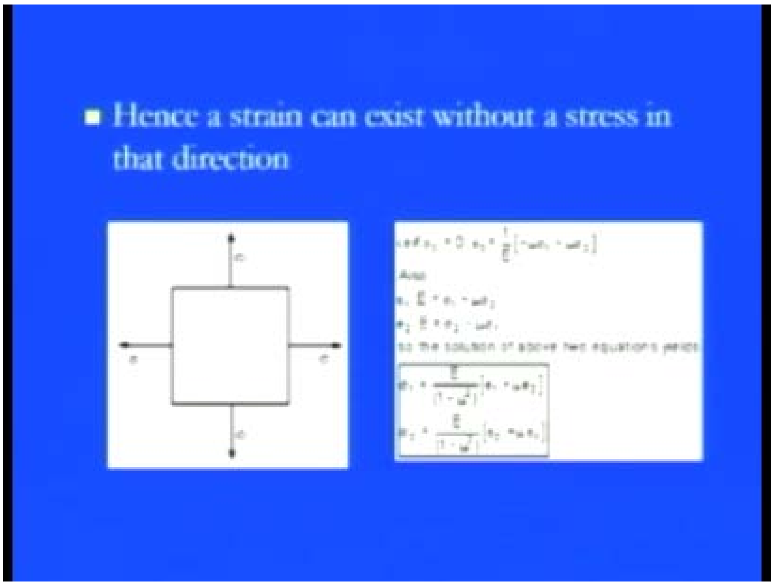

So, hence we just want to see that actually how these formations are there and how we can calculate those things, so it is now we have a plane stress component, this block is there in which only the stresses are there in the x as well as the y direction. So, because of all with two directions are there sigma 1 and sigma 2 or I can say sigma x and sigma y mutually perpendicular one, so we can go straight way to this part that sigma 3 is gone there is no consideration of those three.

But, we can calculate the strain in third principle plane which is equals to 1 by E into minus mu times of sigma 1 and minus mu times sigma 2 or we can say the E is nothing but, equals to sigma 1 which is epsilon 1 times E. From the first formula sigma 1 minus mu times of sigma 2 or we can say epsilon 2 times E which will give you sigma 2 minus mu times of sigma 1. So, if revolve these things, because the two different formations and two relations are there we can simply calculate that what the principle stress formulas.

Also for that actually we can get sigma 1, which is equals to E divided by 1 minus mu square into this epsilon 1 plus mu times of epsilon 2, or we can say sigma 2 is equals to we put these values in this particular equation, we can get this epsilon 2 from this equation. So, epsilon 2 is nothing but, equals to E over 1 minus mu square into epsilon 2 plus mu times of epsilon, means only there is a change of that actually what where the this Poisson ratio will multiply.

So, if we are talking about the first region, then obviously the Poisson ratio will come into the another direction, so here if I talking about the first plane sigma 1, then obviously the mu times of epsilon 2 will give you the deformation measure. And if you are talking about the another plane, that means sigma 2 obviously this mu times of sigma 1 give you that what that measure of deformation is.

So, it is pretty simple that actually, if we are talking about principle stresses or principle strains, we have to be very careful that actually what kind of loading is there and due to that what the interactive effect is there on the other two directions, so this was the principle strains. Now, we are coming towards the hydrostatic means, hydro means the liquid part, so what is the meaning of the hydrostatic stress?

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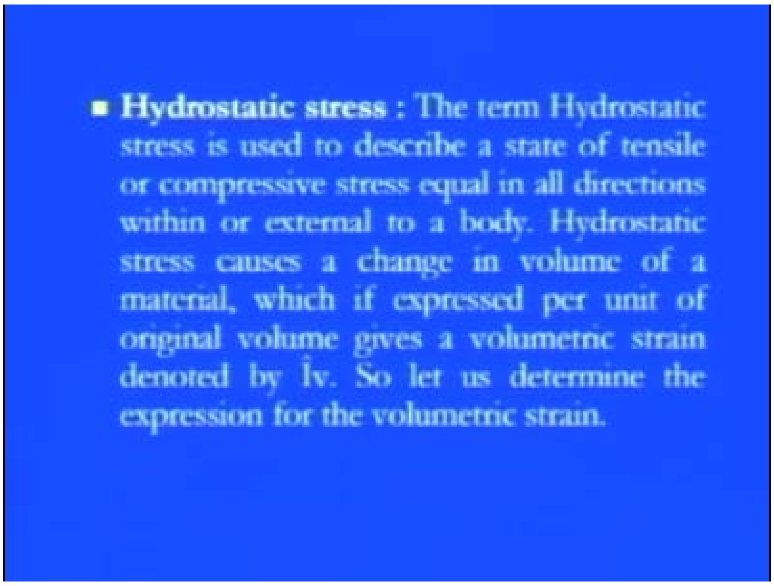

The term hydrostatic stress is used to describe the state of the tensile or the compressive stresses equal in all directions within or we can say external to the body. Means actually if this particular I should say, if this state of stresses are being forming tensile or compressive and they are behaving exactly equal isometric way, I should say in all three directions, then we can say these whatever the stresses are there these are the hydrostatic stresses.

And hydrostatic stresses causes the change in the volume, because since they are going equally in the all directions, so whatever the change will come, the change will come in this volume volumetric way. That means d x, d y, d z they are there, but the if they are presenting which is uniform in other directions we always focused on what the net change of these things are there, and net change will come in the volume, not in area and not in length only the volume will take an important place to measure the hydrostatic stresses.

So, this will cause a change of volume in a material which if expressed per unit original volume will give you the volumetric strain. So, what the net change of the volume is there divided by original volume will give you the volumetric strain and it can be shown by this i times of v.

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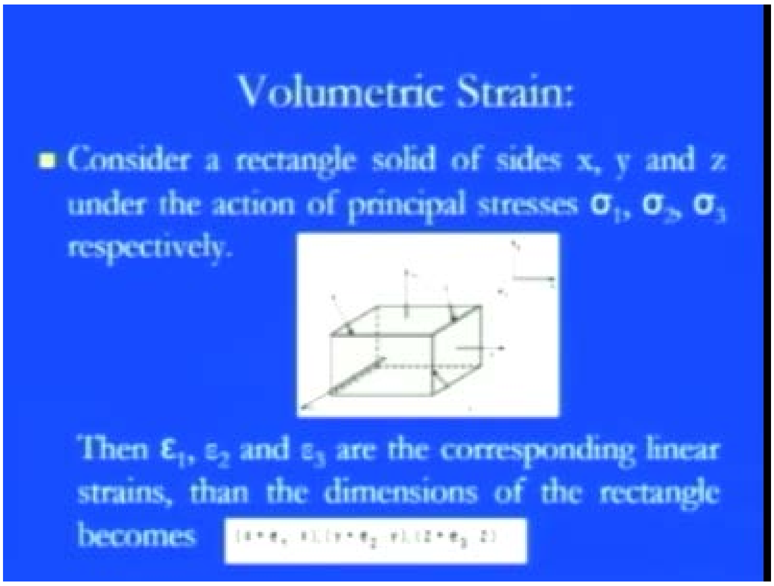

Or we can say, so if you want to express those things, then here we can consider again the same element which we considered in the previous cases that we have the uniform, this unit bar is there and all in, all three directions we have the stress components sigma 1, sigma 2 and sigma 3 is there, in these three mutually perpendicular directions. And if we are say saying that these three stress formations are equally distributed among this element, then we can say the sigma 1 or sigma 2 or sigma 3, they are absolutely apply on the principle planes.

Or we can say these are the principle stresses which are forming these things and due to these, due to the application of the principle stresses we have the principle strains the epsilon 1, epsilon 2 and epsilon 3, they are the corresponding the linear strain, because of the application of those things. And we can say that the what exactly the change is there in the x direction we have the plane, x plus epsilon into x, in the y direction y plus epsilon 2 into the y and in the z direction we have the z plus epsilon 3 into z. So, now what the net change is there in the volume divided by the original volume will give you the volumetric strain.

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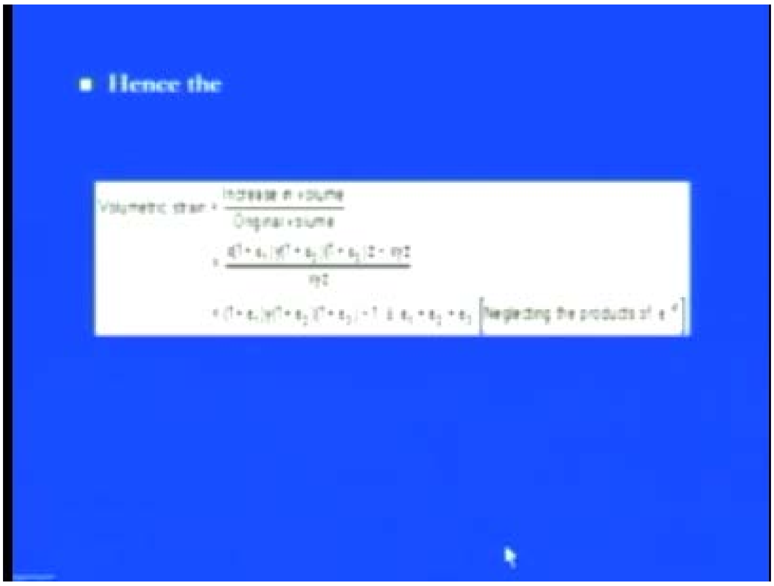

So, come to the main definition volumetric strain is nothing but, equals to the increasing volume divided by this original volume or we can say whatever the change in the volume I should say, it is a rather defined definition is there, the change in volume divided by original volume will give you this volumetric strain. So, now come put those values, which we got in the previous figure that x times 1 plus epsilon. y times 1 plus epsilon 2. z times 1 plus epsilon 3 divided by x y z this is nothing but, the original volume which will give you the volumetric strain.

So, we can get 1 plus epsilon 1 into 1 plus epsilon 2 into 1 plus epsilon 3 will give you the volumetric strain or since. these epsilon 1, epsilon 2, epsilon 3 they are measuring in very very small amount. So, if you are going in the higher order of these things means, epsilon 1 into epsilon 2 or in the square term always it will give a very minimal effect on that kind of thing, so we can neglect that part of the square or as well as the multiple term, so the product term. So, we have the final term as far as the volumetric strain is concerned is epsilon 1 plus epsilon 2 plus epsilon 3. Means the individual measure of the strain or we can say the deformation in the these three mutual direction will give you the volumetric strain of the object.

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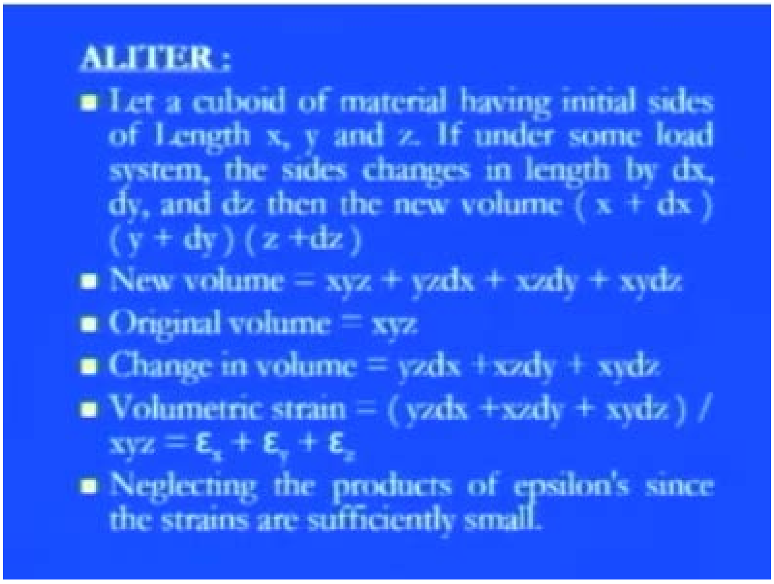

Or we can say let us take a cuboid which we have taken in the previous case, I mean the initial the length is x, y, z under some load, whatever the loads are there we have the changes in the length is d x, d y, d z. So, the new volume will be x plus d x, because the x plus what are the change is there, y plus d y what are the change in y direction and z plus d z because the change is there in the z direction.

So, now the new volume will nothing but, equals to x, y, z the original volume into in change in x direction, y direction and z direction, so x, y, z is the first volume. Second volume if there is a change in the x direction, then y z into d x if it there is a change in the y direction, then it is x z into d y and if there is a change in the z direction, then it is x y into d z. So, original volume is d a this x, y, z and the change in volume if you replace those things then, we have this d x into y z d y into x z and d z into x y.

So, the volumetric strain is nothing but, the change whatever the change in volume divided by the original volume which will give epsilon x plus epsilon y plus epsilon z, if you divide those things. Meaning is very simple, that even if we are talking about a principle plane or principle strain then we have epsilon 1, epsilon 2, epsilon 3. Or if you are talking about these and the normal strain and this normal plane is concerned we have epsilon x, epsilon y, epsilon z which will give you the volumetric strain, because again we are neglecting the higher term of the strains here.

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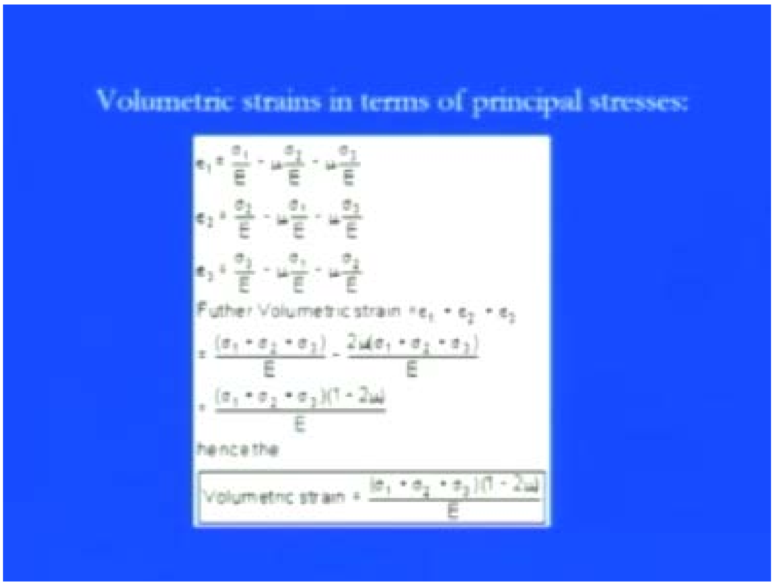

Volumetric strain in terms of the principle stresses also we can computed, because we can easily express the volumetric strain in terms of sigma 1. sigma 2. sigma 3. So, further if we keep all these value which we are calculated this epsilon 1, epsilon 2, epsilon 3 in this volumetric strain we will get the sigma 1 plus sigma 2 plus sigma 3 divided by E minus this 2 times of mu, because mu 1 and mu 2 is there here.

So, the 2 times of mu into sigma 1, sigma plus sigma 2 sigma 3 divided by E or we can say that the volumetric strain, which is also equal to in terms of the principle stresses is sigma 1 plus sigma 2 plus sigma 3 into 1 plus 2 times of mu divided by E. The meaning is very simple that actually if we know the principle stresses sigma 1, sigma 2, sigma 3, we can easily get that what the net change of the volume is due to the effect of these principle stresses, which are mutually perpendicular to each other and what the interactive effect is there of these things we can easily find it out.

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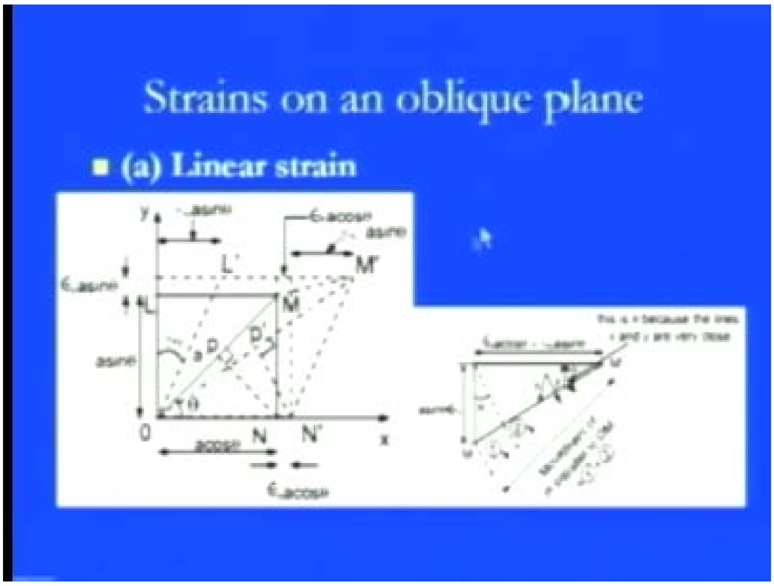

Now, again we similar to the stresses on the oblique plane, we just want to calculate the stresses the strain on the oblique plane, so what we have earlier we have a simple this O L M N, so O L M N is the original our the unit this cube was there, in the x, y planes, so this x and y plane is there, x y planes like that. And after applying those strains, now we want to calculate that actually if this M is shifted to M dash, if N is shifted to N dash and the corresponding this L is shifted to L dash, means we have the new element under the application of the shear strain is O, L dash, M dash, N dash.

So, now we see we can simply compute that how this will come, this here the shear strain is there and this gamma x y, so the shear strain is always coming in the picture whenever there is any deformation or we can say the distortion is there. So, this distortion coming from L to L dash, M to M dash or N to N dash and we can simply calculate this part also that what is the axial strains are there and what is the plane strains are there. And simply for this kind of formation where these are forming, we can again take these things that actually what M into M dash.

So, this nothing but, M M dash and this another part is there that means, we are calculating about this particular this triangle and if you are imposing this triangle here, we can also says that actually the deviation is there in the theta in the strain term.

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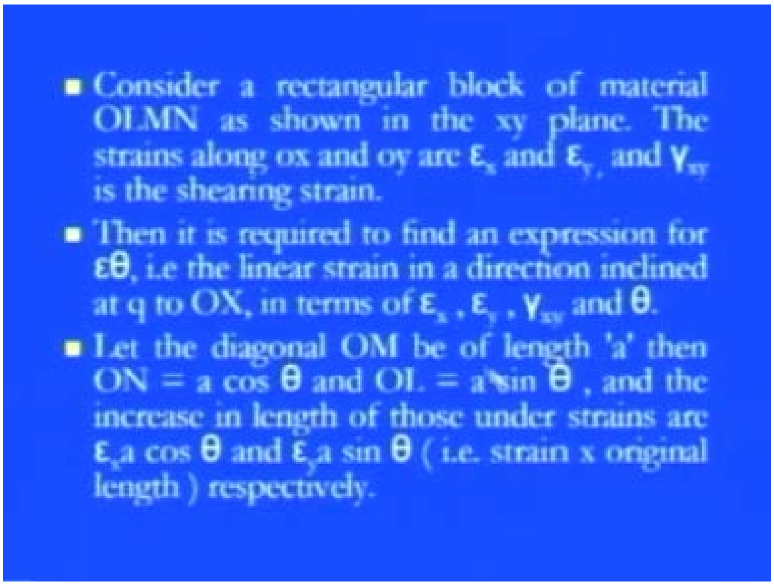

So, the linear strain consider the rectangular block which we are considering O L M N as I discussed in the x y plane the strain along O x and O y simple x and y direction is epsilon x epsilon y and gamma x y is the distortion in the shearing form, so this gamma x y is the shearing strain. Then it is required to find the expression for this epsilon theta, that means I just want to find it out that actually what will be the strain value at the oblique plane.

And for that again I am going to analyze the similar manner as we have discussed in the previous cases for the stress formation, the normal stress as well as the shear stress form in the oblique plane. So, here the linear strain in the direction inclined to whatever the q part is there of the O x in terms of epsilon x, epsilon y, gamma x y and the theta, so let us now we just make the diagonal you see O M of let us say length a.

And now if you resolving this part, then we have the two main component in x direction O N which is a cos theta, O L which is a sin theta and then, if you are saying that there is an increase in a length. So, it will give you inducing the strain component, so the strain is in those a directions we have this epsilon x into a cos theta, because there is an x direction and in the y direction there is O L is there, so epsilon y into a sin theta, so this will give you the strain in respect to x and y directions.

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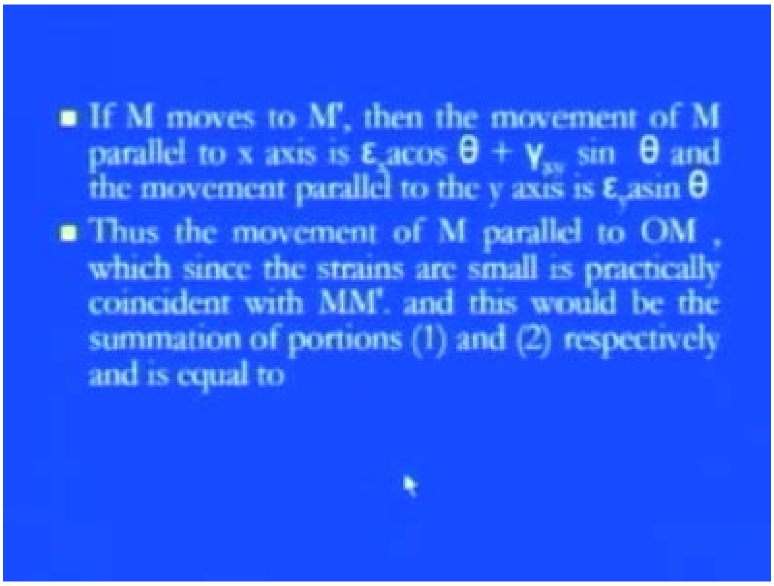

So, now if we are saying that due to the shear if M is moving to M dash, then the movement of M which is parallel to x axis is this epsilon x a cos theta plus gamma x y sin theta, because it has both the component it is moving towards the kind of that, the structure is there. So, for that it has a variation in x direction this direction and this shearing direction, so we have the component of the both epsilon x a cos theta and gamma x y sin theta and the movement parallel to y axis will epsilon y a sin theta. Thus the total movement which we want a calculate parallel to O M, since the strains are very, very small as practically, and we are considering always that.

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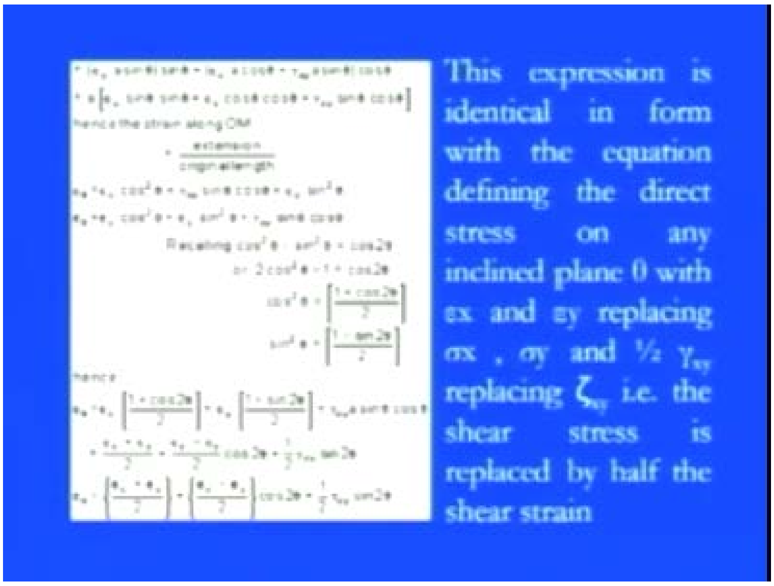

Actually if it is coincident to the M M dash this would summation of portion (1) and (2), which is equal to now here pretty simple epsilon y a sin theta into sin theta plus epsilon a epsilon x into a cos theta plus gamma x y a sin theta multiplied by the this component, the resolution component a cos theta. So, if you resolve this part what we have, we have a strain along this particular O to M which is equals to the extension, whatever the extension is there due to the combined effect of axial, as well as the shear strain divided by the original at the original length.

So, what we can do here we can simply calculate this epsilon theta along this oblique plane is equals to epsilon x cos square theta plus gamma x y sin theta cos theta plus epsilon y sin square theta. So, now if you resolve this means, if you are keeping this cos square theta minus sin square theta cos 2 theta or we are saying that they this cos square theta which is nothing but, equals to 1 plus cos 2 theta by 2 are all trigonometric relations are there.

Sin square theta which is nothing but, 1 plus sin theta by 2 if we are keeping those things, then what we have the formula for calculating the strain in normal direction, this epsilon theta is equals to epsilon x plus epsilon y by 2 plus epsilon x minus epsilon y by 2 into cos 2 theta plus half of gamma x y sin 2 theta. Means we have both the component, we have one component due to this epsilon as this epsilon x and epsilon y which is independent of theta and other two components are very much dependent or we can say governed by the theta.

If you remind just in the stress case, that if you have the component which is under the effect of two mutually perpendicular stresses and the shear stress is there, the sigma theta, the theta at the inclined plane is nothing but, equals to sigma x plus sigma y by 2 plus sigma x minus sigma y by 2 into cos of 2 theta plus half of this tau x y cos of the sin of 2 theta. So, it is exactly similar, this expression is very much identical in the form of equation which we define for the direct stress at any inclined plane theta with this epsilon x and epsilon y replacing sigma x sigma y and half of gamma x y, in terms of that.

So, here it has a clear and analogy that irrespective of whether if we are talking about the stress or if we are talking about the strain, always if you want to calculate the stress and strains at the inclined plane, in terms of the normal or direct stress or direct strain, the formula is very simple and it has a clear analogy.

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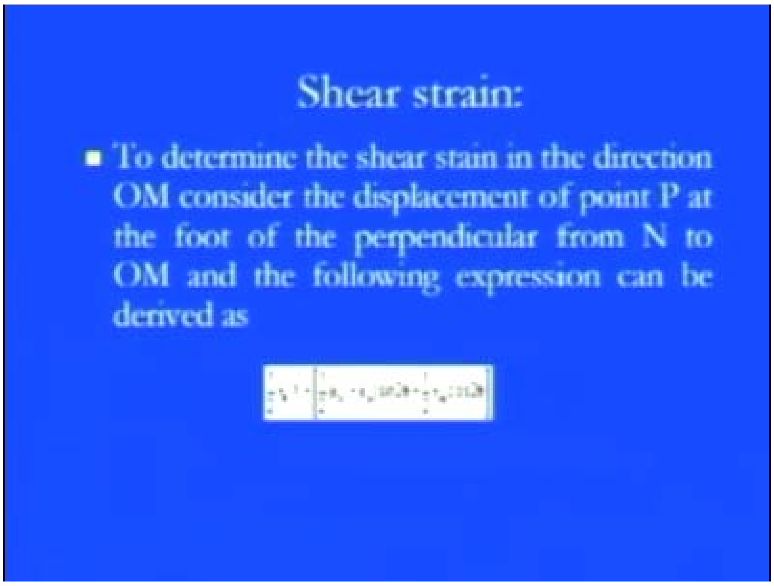

And if we are going for the shear strain component that means, we discussed about the shear stress, then how to calculate the shear stress, so now we would like to see that what the shear strain is there to determine the shear strain in the direction of O M. Because, due to the shear strain it punctuate right from the middle section, the direction O M consider, the displacement of P at the foot of the perpendicular where the N point was there.

So, N to O M if you simply put the perpendicular, then you will find that this sigma theta is nothing but, equals to half of or we can say just put half; because of the symmetricity half of that the sigma theta is nothing but, equals to half of epsilon x minus epsilon y sin 2 theta minus half of tau x y cos of 2 theta, because you see we just want to put the symmetricity.

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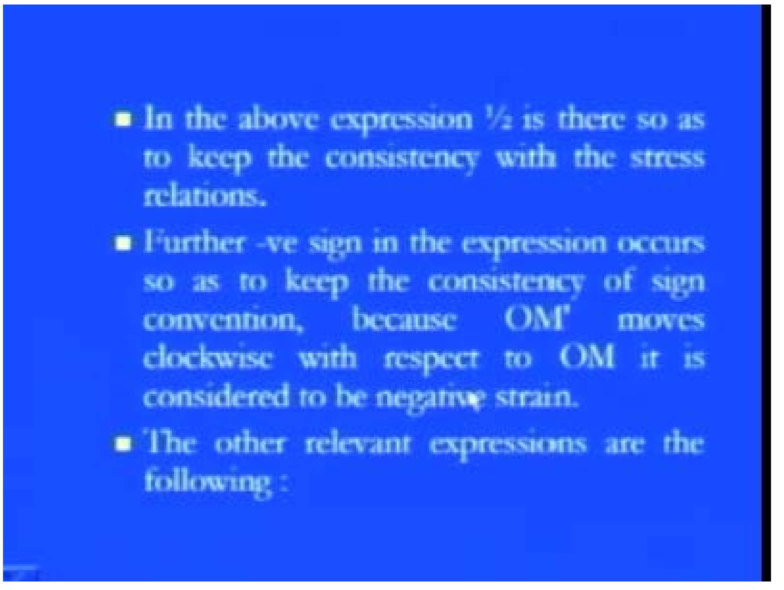

So, in the above expression half is there, so as to keep the consistency with the a stress relation, because we know that actually this tau theta was nothing but, equals to sigma x minus sigma y by 2 plus half of this tau x y into sin of 2 theta. So, it is pretty similar to those things we just wanted to keep the consistency, so it has the same relation further the negative sign in the expression occurs, so as to keep the consistency of the convention.

And because of O M dash moving towards the clockwise direction with respect to those thing, so we are always considering that whenever the shear stress or strain is there. And if the object is tending to move towards the clockwise direction always the positive sign is there, while O M is always considered to be negative, because it is tending to move the counter clockwise direction.

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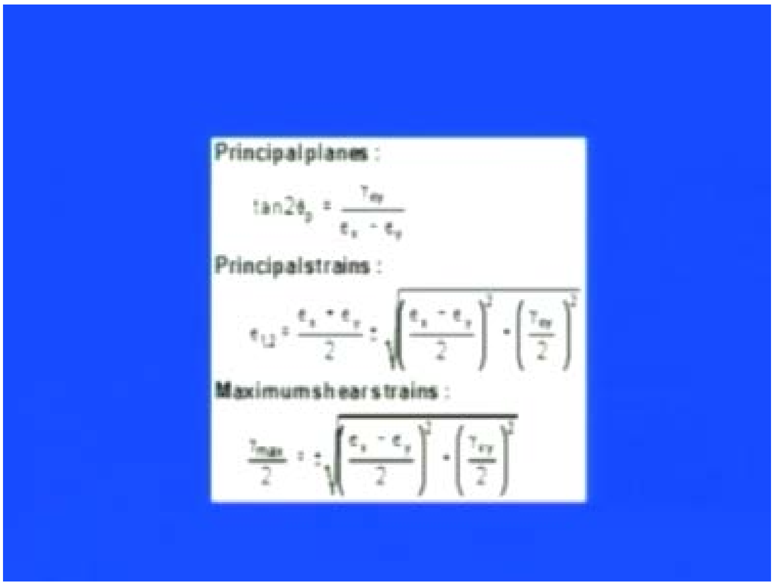

So, here now the principle planes once we know that it is a pretty analogy to the previous thing, so we can get the similar things here that this principle plane we can get by the strain ways tan to theta p is nothing but, equals to gamma x y divided by epsilon x minus epsilon y. And with the principle strains is also we can get this epsilon 1 comma 2 is nothing but, equals to the epsilon x plus epsilon y by 2 plus minus, so these are the two principle strains.

One and two the maximum minimum just by with a taking of plus minus sign square root of epsilon x minus epsilon y by 2 whole square plus gamma x y by 2 into square. Or we can say the maximum shear strain also can be obtained by this gamma x y by 2 is equals to plus minus square root of gamma this epsilon x minus epsilon y divided by 2 whole square plus gamma x y by 2 whole square. So, it is pretty simple, because as we have already derived for the stress formulation, so there is nothing to be discussed here, because it has already being discussed that it is pretty analogy those kind of thing.

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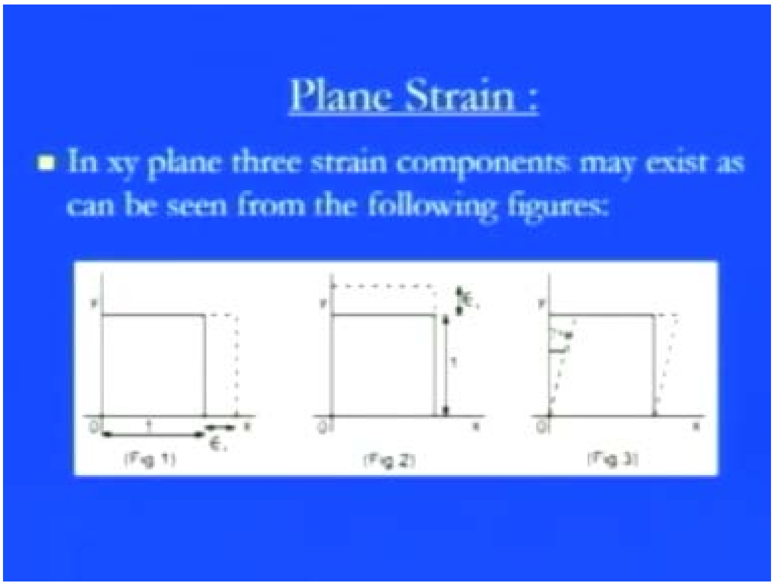

So, now if you are coming towards the plane strain, because in the shearing we found that actually it has combined effect and distortion is there. So, in a x y plane three stress components are there, which can be easily viewed, that if you are talking about the x part the normal strain component towards the x direction. We have the epsilon x even in the vertical direction, if the normal stress component the strain component is there, then epsilon y is there and this is the shear strain, because of the plane strain part is there. So, these three figures clearly shows that actually what kind of the displacement or the distortion are there in corresponding relation.

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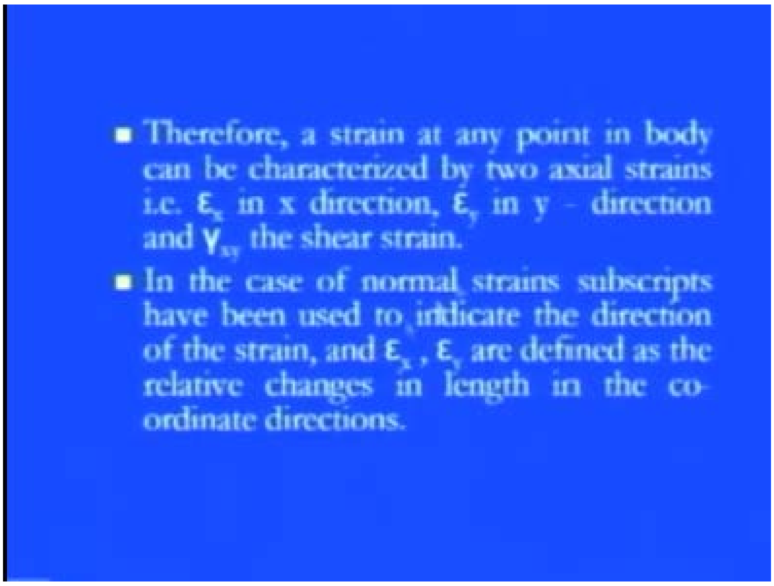

Therefore, a strain at any point in the body can be characterized by two axial strain, this epsilon x in x direction and epsilon y and a plane strain that is the gamma x y as the shear strain. In case of the normal strains, the subscript you have used to indicate the direction of the strain, that means the epsilon x or epsilon y means, the direction of the change in the x and y direction respectively.

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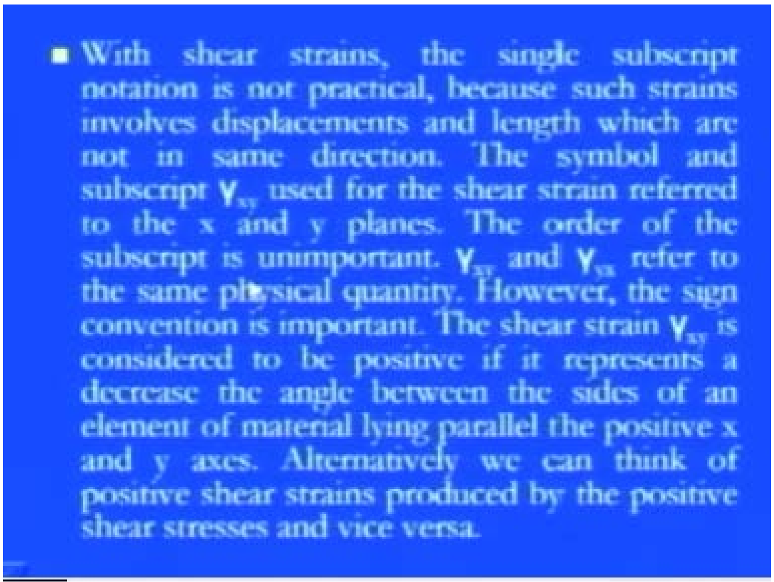

While gamma x y the shear strain it has two subscripts, one is the single, means the first subscript single subscript notation is not practical as we shown, because it is a plane strain or we can say it is always there towards the x y side. So, we need to concern both the things the symbol this gamma x y gives you the two main formation, one is the x and y or we can say the y of x, if the element is pretty symmetric, we can say it is pretty easy to describe the symmetricity. Because, of the this either the nature of the force, as well as the if the nature of the what the geometry of shape, or we can say geometry of the objectives, these are the shape of the objectives.

The shear strain gamma x y is always considered to be positive, if it is represent the decrease the angle between the sides of the element of material, which is lying in the parallel to the positive x or y axis. That means, if it is decreasing in the angle towards that, we always concerned the positive side, while we can think of the positive shear strain produced by positive shear stress, and the...

Again if we are talking about the negative shear strains, that means which there is any increase the angle between sides of any element material, which is lying a parallel to the x or y direction.

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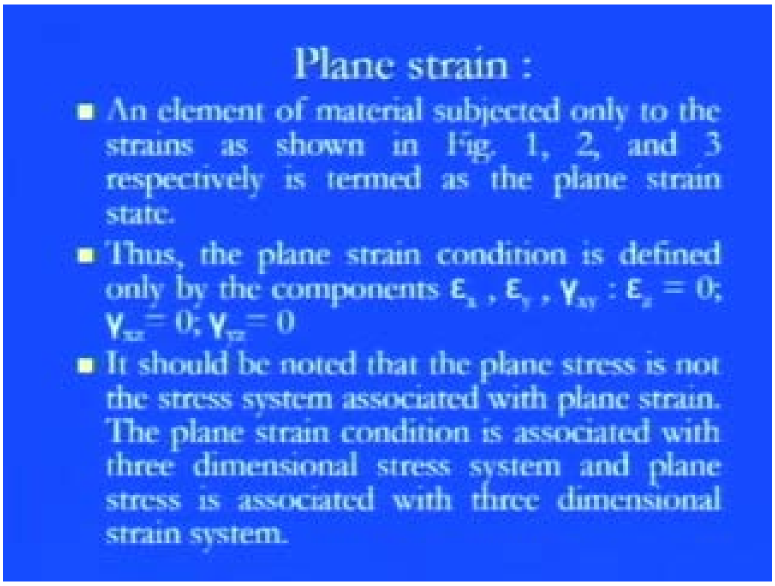

That means we can simply conclude those things that actually, if we are talking about a plane strain, the plane strain is not associated with the plane stresses, plane strain is basically associated with the three dimensional stress conditions; because in the plane stress we have simply concerned with the two directions while in the plane strain. We can go for all three directions like that, and that is why it can be defined by the components of the epsilon x, epsilon y and this gamma x y, because it is in the all three directions like that.

So, we can say the plane strain condition associated with the three dimensional stress system, in the plane stress is associated with the three dimensional strain system. So, in this chapter we discussed about that what the stress strains are there, because of the application of load is and if the strain, if we are pulling in the one direction, then what are the corresponding changes are there in the other direction.

And if you want to relate those things then the Poisson ratio is an important factor and also the Young's modulus elasticity is defined, for the elastic region where the stress and stress the strains are proportional to each other. And then last part we found that, if you want to calculate if a material is under subjected to all three kinds of strains, then the strain at inclined plane is exactly similar to the stresses at the inclined plane and it is pretty symmetricity. So, now in the next lecture would like to discuss about that how to resolve this issues, because these analytical part is pretty simple, but what about the Mohr's circle, so the next lecture we are going to discuss about the Mohr's circle for the strain component.

Thank you.