Strength of Materials

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Lecture - 7

Hi, this is Dr. S. P. Harsha from mechanical and industrial engineering department, IIT Roorkee. I am going to deliver today my lecture seven of this title of this strength of materials in which you know like we are going to discuss about some of the problems. If you refresh the previous lectures, you know like the last two lectures we have discussed about the various stress components and what the distribution is there of these stresses when you know like the load applications are there of either the single or the mutual kind. As you see actually in my lecture five, we discussed about the analytical solution when a material is being subjected by a pure normal stress at the normal forces or a material is subjected by pure – this shear loads are there or we can say when it is under the normal stress components are there or pure shear stresses are there or when a material is subjected by the two mutually perpendicular like the stress components like sigma x and sigma y.

And, at the same time simultaneously, we can say that, if the shear stress is there, then what exactly the interaction is there of these stresses and what exactly is the impact of the stresses on the material. And through the analysis, we can also find it out that actually how to get the maximum value of the normal stress component or how to get the maximum value of the shear stress component by analytically, we discussed all those things. So, if you just remind, then actually… Just I want to refresh those things that, actually if we have a material, which is being subjected under a two mutually perpendicular stresses like sigma x and sigma y; and also the same time there is a shear stress is there along the plane, because it is a parallel to the layers of the surfaces.

So, if the shear stress is there, which is standing to move towards the clockwise direction; and simultaneously, to balance this material, we always need the complementary shear stresses queued as… That means we can say that, if a material is subjected under the combination of these three components: sigma x, sigma y and the tau xy, then we can also find it out that, actually what exactly the stress component is there at any angle – we can say at oblique plane, which is at angle of theta. We can calculate like… let us say if it is at the angle of theta, we can say the normal stress component as well as the shear stress component at the theta; we can easily calculate by analytical technique like sigma theta is nothing but equals to tau x plus tau y by 2 plus sigma x minus sigma y by 2 cos of 2 theta plus tau xy by 2 sin of 2 theta. So, this is the normal stress component if all combination is there – these all three combination is there of this stress component.

Or, also, we can calculate the tau theta – that is nothing but equals to sigma x minus sigma y by 2 plus half of square root of sigma x minus sigma y whole square plus four times of tau square xy; means we can easily calculate that, what the stress distribution is there. And, if you want to calculate the maximum normal stress component within this object if it is under the application of these three mutually perpendicular stresses, we can also calculate by simply differentiating the sigma theta by D theta; putting equals to 0, we know that, what is the location, that is, 2 tan 2 theta P, which is nothing but equals to 2 times of tau xy divided by sigma x minus sigma y. And also, if you keep those things there in the mean value, you can get the maximum value or minimum value of the normal stress component, that is, sigma 1 comma sigma 2; which is nothing but equals to sigma x plus sigma y by 2 plus minus square root of sigma x minus sigma y whole square plus 4 times of tau square xy; that means you have two values of these things. One is the maximum value when you are choosing plus sign. When you are using minus sign; that means it has a minimum value. The meaning is that, these two values: sigma 1 and sigma 2 are nothing but the principal stresses. And also, we can say that, these stresses are always occurring at the planes; and these planes are known as the principal planes.

Then, we conclude that, actually if you want to calculate what is the value of the shear stress if we have the maximum normal stress is there, we found that, the value of the shear stress is 0. So, we concluded that, actually always the maximum normal stresses are occurring in a material always, where the shear stress is 0; that means the principal stresses are always one can get if the shear stress value is 0. Or, we can say wherever the shear stress – these principal stresses are occurring in the object, these planes are known as the principal planes. And, if you want to locate those principal planes, one can easily put the shear stress is 0, get the value; and you can simply locate the theta P.

And also, we relate that, if the maximum shear stress is there, then what is the tan 2 theta is there. So, we calculate the tan 2 theta as is nothing but equals to sigma x minus times of sigma x minus sigma y divided by 2 times of this tau xy. So, we have a mutually perpendicular… Or, I should say that, we have reciprocal relations between the tan 2 theta P and tan 2 theta S. So, that is what we discussed about the analytical solution of any problem, which is under the subjected of variety of these stresses. Then other solution, which we discussed about the Mohr's circle; that is the graphical solution. So, in that also, we are going to use all those analytical parts, which we have used for solving those things. That means if a material is subjected under the same pure normal stress, pure shear stress or two mutually perpendicular normal stress with the shear stress; then how to draw the Mohr's circle; what will be the coordinate of the centre point; what will be the radius of that, and on x-axis…

As you discuss that, on x-axis, we are taking the normal stress component; on y-axis, we are taking the shear stress component. And then how to plot that part; what will be the centre distance is there right from the origin to this part or what will be the maximum diameter is there; how to put the… If you want to calculate the shear stress or the normal stress at the oblique plane, then how to put the 2 theta. Or, we can say what will be the relation between the beta and theta. So, all those sorts of thing we discussed in the lecture six. So, today is lecture seven. So, whatever we discussed in lecture five about the analytical solution; in lecture six about the graphical solution; today, we are going to merge those things. And, we just want to apply in a real practical phenomena, that is, the numerical problems. And today, we are going to discuss about both the solutions for a specific problem.

So, the first problem… We are going to discuss about the two problems: the first problem is pretty simply problem; and the second problem – somewhat more complex problem is there. So, we just want to generalize that problem; we just want to find it out the state – this general state of stress; what will be the value of these things of the general strength of the stress if it is subjected under these kind of loadings. So, this kind of the phenomena, which we are going to discuss in this lecture; so, here it is we have the first problem. So, this chapter itself – the name is illustrative problems. And, we are going to discuss a few representative problem dealing with the complex state of stress, which has to be solved either analytically or graphically. So, here we are going to apply whatever we discussed in the lecture five and lecture six.

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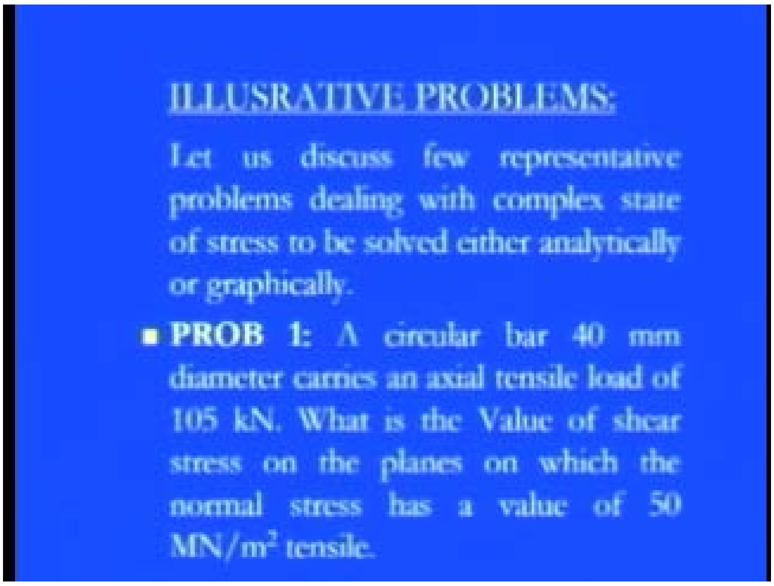

So, here the statement of the problem 1, which is a circular bar. We have a circular bar are of the diameter is 40 mm carries an axial tensile load of 105 kilo newton – means we have a kind of bar, which has a diameter of 40 mm or we can say the 4 centimeter like a pencil. And, there is Newton, which is… There is a load, which is applying on just two extreme ends, which is the nature of the load is axial pulling. So, when you are pulling, there is a tendency of a material to resist that thing as per the Newton’s third law. So, what will happen; once the load application is there, the internal load distribution is like that. And, if you are saying that, the action whatever – by 105 kilo newton; and the reaction – just to resist that force, if they are well-maintained, we can say that, the system is under equilibrium position. So, whatever the stresses, which are being set up within the structure – these stresses are the tensile stresses because of the axial pulling.

So, now, we have this load; we have the diameter of this in the circular bar. So, what we can do now, we can simply calculate the tensile stress. But, here the other information is given the – what is the value of the shear stress. We need to find it out the shear stress value just by applying the load of tensile on a planes on which the normal stress has a value of 50 mega newton per meter square tensile – means that, we just want to cut the plane at a certain angle. We do not know the angle; but, at this particular plane, we have a normal stress component, that is, the sigma theta, which is 50 mega newton per meter square. And, we just want to find it out that, what will be the value of tau theta at this particular plane. So, now, the solution – just as we discussed, it is a tensile loading. So, we can simply calculate the tensile stress.

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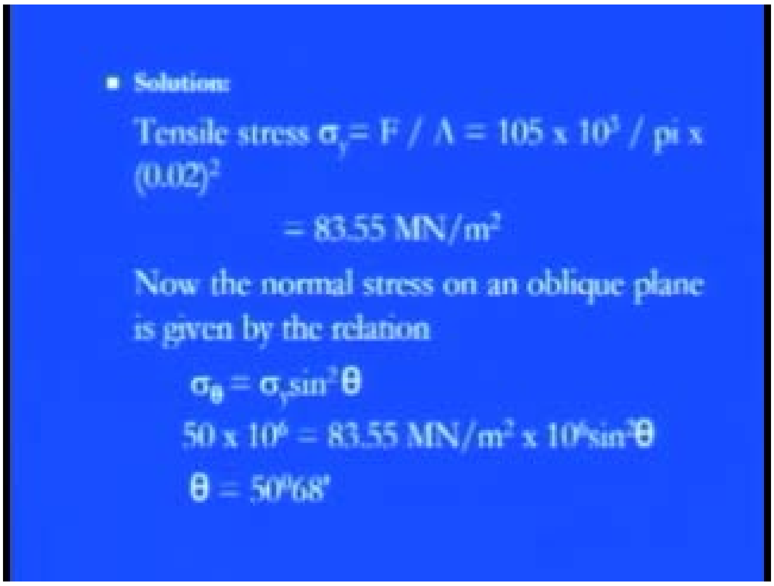

And, tensile stress is nothing but equals to the applied load – the applied tensile load divided by the effective area. And, since it is a circular bar; so, effective area is pi by 4 D square or pi R square. And, here we have the diameter is 40 mm. We can say it is a 20 mm radius is there. So, 105 into 10 to power cube – that is the applied load divided by pi into 0.02 square. So, you can calculate the tensile stress, which is nothing but equals to the 83.55 mega newton per meter square.

Now, the other information was given that, we have a normal stress at an oblique plane, which is equals to 50 mega newton per meter square. But, by the formula, when a material or any object – if it is subjected by a pure tensile load, then the normal stress component is nothing but equals to the applied stress into sin square theta. So, here if we apply the concept, then we have the sigma theta, which is the stress at an oblique plane. Normal stress component is equal to the applied stress at the extreme and that is sigma y into sin square theta. But, we do not know what is the angle of the normal plane is; means actually where the oblique plane is cutting at the particular point. So, we would like to know these things. So, if we put all those values; means the sigma theta is nothing but equals to 50 into 10 to the power 6, which is equals to sigma y; sigma y is nothing but equal to 83.55 mega newton per meter square into 10 to the power 6, because we just want to equate those things into sin square theta.

So, once we put all those values, we can simply calculate the sin square theta has some value, because if you put all those things at this particular side of the equation, we have sin square theta equals to some value. And, once you square root those things and put the inverse sign, you have the angle – means at what angle, this oblique plane is there. And, at the oblique plane, at this particular 50 degree – in 50.68 degree, we have the value of the stress is 50 mega newton per meter square. So, now, we know the location. Once you know the location, you can easily get what is the value of the shear stress is. So, if you move further, then you will find that, the shear stress at this particular plane, where the theta is 50.68 is… Now, you see this theta is for the normal stress component, because it is there. So, if you want to calculate the shear stress part, then what we need to do? We need to simply keep that particular theta in a real formula.

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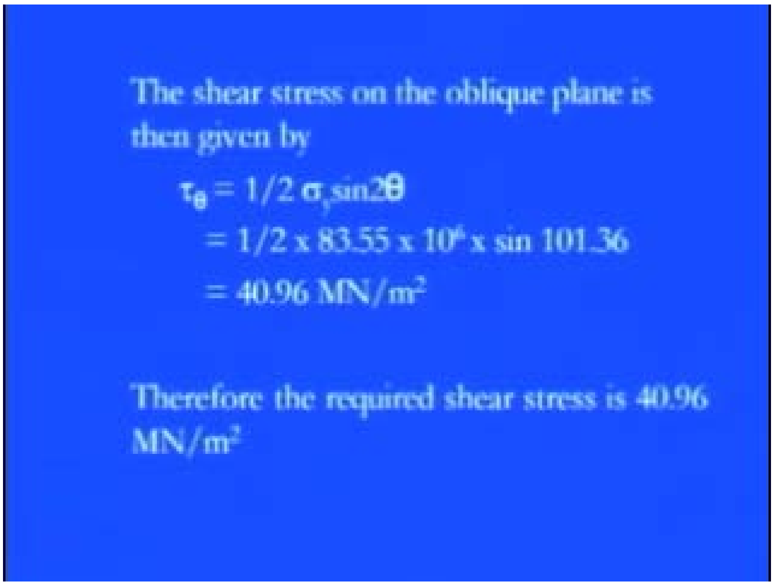

So, for tau theta, the extended formula when a materially is subjected by a pure tensile load, the formula says that, the tau theta should be calculated by sigma y, which is the applied stress into sin 2 theta divided by 2. So, once we apply this formula, now, we can get this value of tau theta, which is half of sigma y, which is 83.55 mega newton per meter square – so, into 10 to the power 6 into sin 2 theta. And, the theta was 50… If you see the previous slide, then we have 50.68. So, if you multiply by 2, then we have this value. So, we can simply put sin of 101.36. So, we can calculate the tau theta, which is equals to 40.96 mega newton per meter square. So, here these stress components – either the sigma theta or tau theta – one can easily calculate if you know the formula or if you know what the stress distribution is there under the application of any load.

Also, here you see this – the example was a pretty simple example. And, this example just gives you the information that, actually even if a material is subjected by a pure normal stress, in this particular micro structure or the structural part of the element, is always carrying the normal stress component as well as the shear stress component. So, you can simply compare that, we have the magnitude of the tensile stress is 83.55. But, also apart from that, we have both the component at an oblique plane. We have the sigma theta, which has a value of the sigma theta, which has the value of 50 mega newton per meter square. And, we have the value of the shear stress at the oblique plane; that is equals to 40.96 mega newton per meter square. So, actually, how to calculate what is the physical significance? That is what we calculate here.

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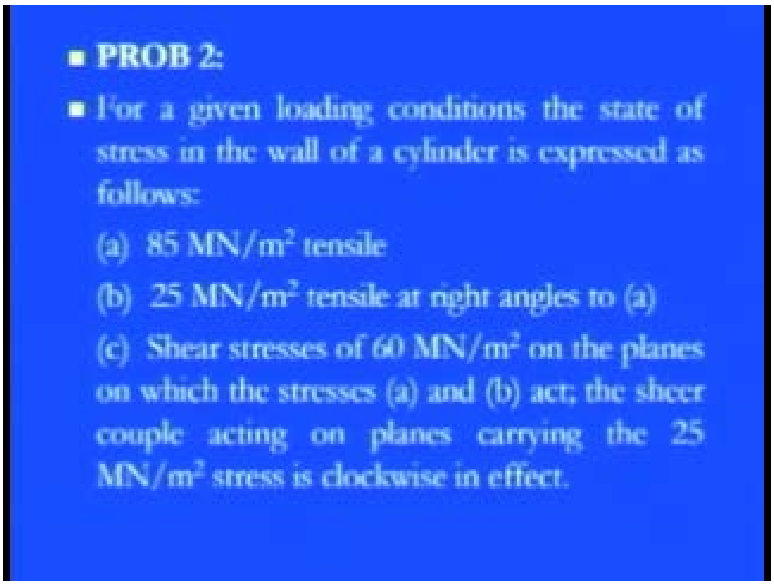

Now, come to the second problem. Second problem is somewhat like I should say the mixed nature, because here not only the tensile loading is there, but also apart from the tensile loading, we have the shear loading here. So, here the statement of the problem is for a given loading conditions, the state of stress in a wall of cylindrical things; means now, our object is cylinder. And, in the cylinder, we would like to see that, actually what exactly the coordinate system is and how we can correlate with the stress component is. So, here the wall of a cylinder is expressed as follows. Here first of all, the first loading is tensile loading. So, first part – 85 – this mega newton per meter square tensile loading is there. Then b part says that, 25 mega newton per meter square tensile at right angle to a – means wherever this tensile loading is there; if you consider that, this tensile loading is in x direction, because it is a normal stress component; tensile is there.

So, if we are considering that, this is in the x direction, then the other component, which is of 25 mega Newton per meter square is also having the tensile nature, but it is exactly right angle to this means say that if I am saying that, this – here on an element, if this is the direction, which gives you the 85 mega newton per meter square in this direction means this tensile direction. Then, the 25 mega newton per meter square – like the other loading is there, is always in the right angle to this particular – the previous one. And, it is also in the tensile loading. So, here this is the correct physical description about these two: a and b. And, the third thing is that, the shear stress is there, which is a parallel plane stress; that means it has a parallel concept to these layers of the object or we can say the cylinder. So, shear stress of 60 mega newton per meter square on the planes on which the stress a and b acts here; that means this shear plane – it is in the… As I told you, these normal stress components or the normal stresses are the axial stresses while the shear stress is a plane stress.

So, now, it is along a plane, where these to above stresses are acting; means at one point of time, I can say that, here 85 mega newton per meter square is acting; at this point, this 25 mega newton per meter square is acting. And, along this plane, means the shearing part – here the 60 mega newton per meter square is acting. And then the shear couple – whatever the shear couple is there – acting on planes carrying 25 mega newton stress in a clockwise direction; meaning that, wherever if I am considering the 25 of mega newton per meter square, the b part on the top of side. And, the top of side, I have the shear stress means actually the shear stress is occurring on a plane, where the vertical direction I should say the 25 mega newton per meter square is stress is acting. So, both are acting on a same plane: one is a normal; one is a parallel. So, the shear stress component is always combining with the 25 mega newton per meter square. But, the direction is always the shear stress is directed towards a clockwise direction as it is.

So, here as far as if you go to the conveyance and real convention; and we say always we said that, if it is a tensile stress component, then it is the positive direction. You need to take the sign convention as positive And also, if the shear stress – because of the induction, in fact, of the shear stress – if object is tending to move towards a clockwise direction, always take positive. So, here you see here I have now the complete configuration of this problem is we have two stresses perpendicular to each other 85, 25 and we have this 60 mega newton shear stress, which just try to tend this cylindrical object into the clockwise direction. So, this is the clockwise direction. So, the anti-clockwise – the complimentary shear stress will come just to balance the condition.

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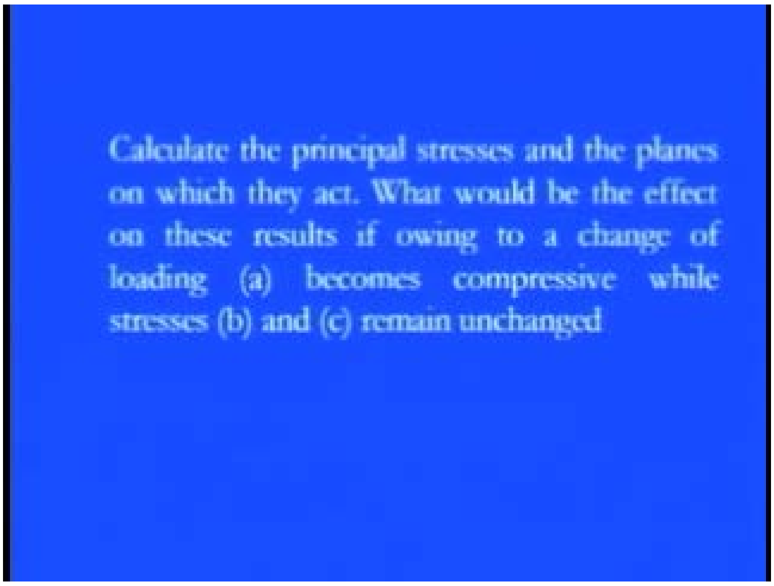

So, now, if we now, the calculative part is – calculate the principal stresses, because now all three combinations of or we can say all three types of the stresses are there. So, we need to calculate what will be the sigma 1 value and sigma 2 value – the minimum and maximum principal stresses. Also, the plane that, what is the principal planes are there – means what is the location of the principal plane at which they are acting all three components. And, what would be the effect on these results if owing to a change of loading a becomes compressive means just now in the first case, what we are taking; we are taking that, the first case means 85 mega newton per meter square; the load is a tensile load.

Now, if I reverse this; means now, I have a compressive load of 80. The same magnitude – that means the 85 mega newton per meter square, then what will happen while the stress b and c remains same – means the b is always going to the tensile 25 mega newton per meter square; and the c means the shear stress, is just tending to move this object towards the clockwise direction. So, it has a value of 60 mega newton per meter square. So, now, this is the complete problem. Also, we can simply calculate either by analytical solution or by graphical solution.

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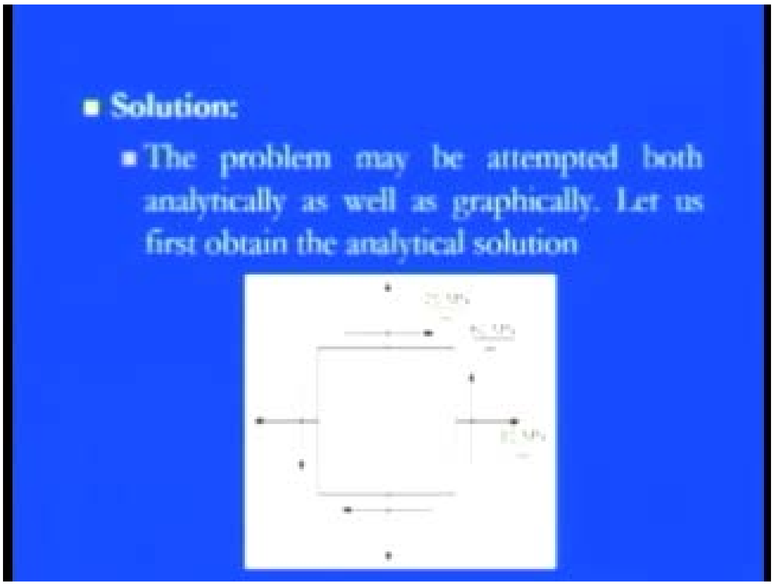

So, now, here the solution is. So, as I discussed already that, actually what is the physical phenomena of this problem is the problem may be attempted both analytical as well as a graphical. So, here first, we will go with the analytical solution. So, here the problem is we have a cylindrical part; we just taken a small element, which is symmetrical in all the six directions. So, it is a kind of a unit cube, which has unit – the width, unit depth and unit breath is there. So, now, on these, the axial stresses are the sigma x, which is the tensile nature. So, it is of 85 mega newton per meter square perpendicular to that; that is, this one sigma y, that is, 25 mega newton per meter square. And, the shear stress – these are the shear stress component, which always try to turn this object in the clockwise direction and it has a value of 60 mega newton per meter square. And, it is only acting, where this 25 mega newton per meter square is. So, this is the active plane, where the perpendicular stresses are there; where the parallel stresses are there. So, the meaning is this is the complete configuration of the problem, which has written in the previous slides. So, this is the graphical description of a problem in a perfect way.

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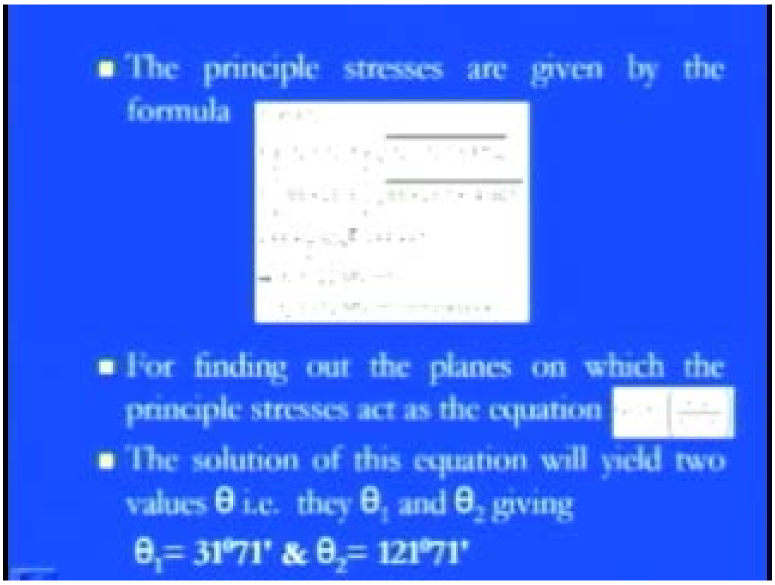

So, here now come to the real formula, which we discussed in the sixth chapter that, the analytical solution always gives you the value of the principal stresses; where, the shear stress is 0 or we can say we can simply calculate the sigma 1 and sigma 2 value if you know the value of sigma x, sigma y and tau xy. So, all the values are given to us; sigma x – 85 mega newton per meter square; sigma y – that is, 25 mega newton per meter square. And, both are tensile nature. So, we have to take the positive sign for that. And, the tau xy, which is 60 mega newton per meter square; and it is also tending to move this particular object in the clockwise direction. So, it is also positive; that means all three values are given to us; they are positive. So, we need to put the positive value and we can simply calculate if you know the formula: sigma 1 and sigma 2. These are the two principal stresses – maximum and minimum just by keeping plus minus signs.

So, here it is the formula is sigma x plus sigma y by 2 plus minus half of sigma x minus sigma y whole square plus 4 times of tau square xy. So, by keeping those things, we have the 85, that is, sigma x plus 25, that is, sigma y plus minus half square root of the sigma x values 85 plus sigma y, that is, 25 whole square plus four times of 60; that is, the tau xy square – 60 square. So, if you calculate those things, then we have this 85 plus 25 divided by 2; then we have the 55 plus minus half of this 60 into square root of 5. So, once you calculate those things, you have 2 values 55 plus minus 67 – means we have the maximum value of the principal stress is 122 mega newton per meter square; we have the minimum value of principal stress; that is, minus 12 mega newton per meter square and since it is a minus. So, it has a compressive nature – the first principal stress, which is a positive. So, it has a tensile nature; meaning is that, you can pretty simply calculate those things once you know the values of sigma x, sigma y and tau xy – plus if you know the sign convention that, actually what is the point of application of the force is; means how these forces are acting.

And, due to these forces, what types of stresses are inducing – means the tensile nature, compressive nature – shear stress is there; then what is the complementary shear stress is there; whether it is tending towards the clockwise direction or counter clockwise direction. So, these two important points are there that, what the information is given to us – means what kind of stress components are there or the material is under what types of stresses. And, the second part, which is more important than actually that what the sign convention is; means actually, how the stresses are being set up within the object. So, for finding out the planes on which the principal stresses are acting, it is pretty simple. We can get the tan 2 theta is nothing but equals to 2 times of tau xy divided by sigma x minus sigma y. So, we know the sigma x value; we know the sigma y value; we know the tau xy value; and all are positive. So, we can simply keep those things.

Here the 2 times of 60 divided by 85 minus 25. So, once you put all three values, the solution to the equation – you can get the two values, because the plus minus sign is there here – the tan inverse of that So, if you take the plus sign, then you have the theta 1, that is, 31.71 degree – means if you tilt – like right now, the cylindrical object is there; the previously, which I shown you if you tilt by 31 degree Then, whatever the planes are coming, these planes are the principal planes. And whatever the stresses are there like the sigma 1, that is, in the tensile nature and the magnitude is 122 mega newton per meter square; and the compressive stresses – that is, like the principal stresses too, which is in this direction – the compressive one – means directed towards the element, which has the value of this 12 mega newton per meter square. You can also calculate the theta 2 value; that is, just by adding 90 degree because both are perpendicular to each other, because the mutually perpendicular stresses are there. So, the theta 1 is 31.71 degree. And, theta 2 is just 31.71 degree plus 90 degree; that means the 121.71 degree.

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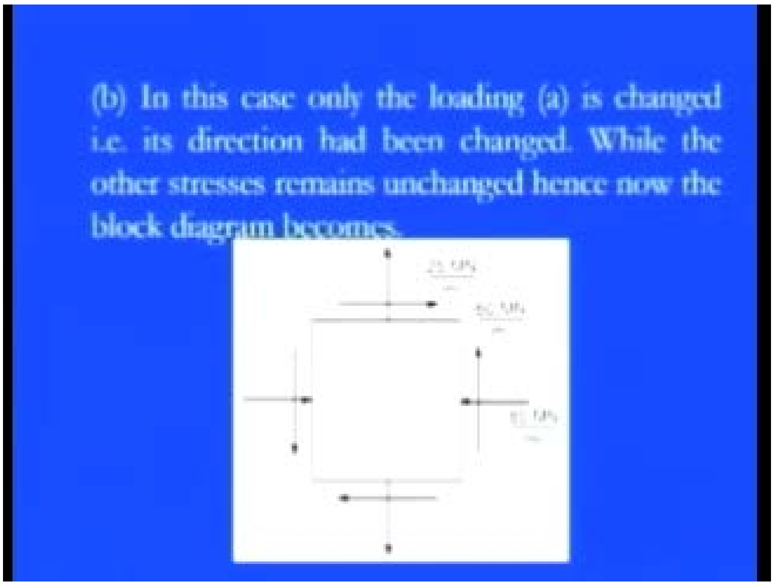

So, you see here first, in this case, now, the second part, which we discussed that, actually if we reverse the loading of part a, part a has the x direction. Earlier we had taken the 85 mega newton per meter square in the tensile part. So, now, in this part, we can found that, now, it is a compressive one, because earlier it was a tensile. So, compressive stresses are there. So, you can see here 85 mega Newton per meter square in the compressive way. And, the other stress components are pretty similar to the part a like here we have the tensile stresses in the vertical direction, that is, 25 mega newton per meter square. And, we have the shear stress component, that is, 60 mega newton per meter square, which is going towards the clockwise direction. So, while you see the other stress like…

As we discussed those things, now, we want to analyze those things by analytical solution. So, again I would like to go with the sigma 1 and sigma 2 by the same formula. Just the difference is sigma x is 85 minus, because it is a compressive one. So, it will contain the negative sign; otherwise other two stress components, because on y-direction, we have the 25 mega newton per meter square, that is, in the tensile nature and the shear stress, that is, tau xy, that is… which has a value of 60 mega newton per meter square is the clockwise direction.

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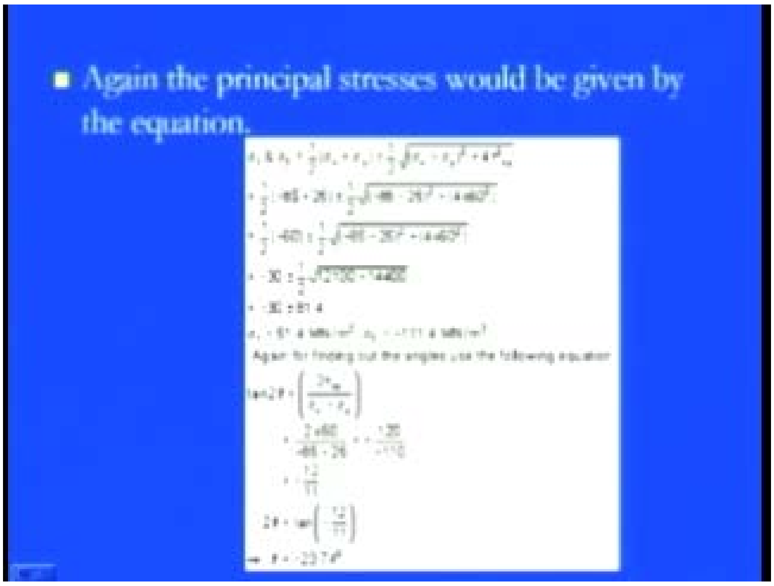

So, keeping those values here – sigma 1 and sigma 2, we have half of sigma x plus sigma y plus minus half of square root of sigma x minus y whole square plus 4 times of tau square xy. So, by keeping these things, we have the value minus 85 plus 25. These values – sigma x and sigma y divided by 2 plus minus this half comes out and square root of minus 85 for sigma x; 25 for sigma y plus 4 times of 64 tau square xy. So, 60 square is there. So, if we are keeping these values, what we have? We have the two equations minus 30 because of this; minus 85 plus 25 plus minus – the 81.4. So, now, you have two values: the first value, which is sigma 1, that is, the 51.4 mega newton per meter square; and you have the sigma 2, that is, the minus 111.4 mega newton per meter square; that means again the two principal stresses – one is in tensile nature with the magnitude of a 51.4 mega newton per meter square; another principal stress, which has the magnitude is 111.4 mega newton per meter square, but in the compressive nature. So, these two we can simply put those values to get what exactly the angle is there. So, again for finding out that, actually what the angle is there to get the oblique plane simply; or, we can say the principal plane that where exactly the principal planes are there or where the shear stress value is 0.

We can again go for the same formula, which we applied in the previous case – tan 2 theta is equals to 2 times of tau xy divided by sigma x minus sigma y. So, just we need to keep the values of 2 times of 60 divided by the sigma x, which is minus 85 minus 25. So, we have the value of minus 12 by 11 or if we are keeping that value; 2 theta is tan inverse of minus of 12 by 11 or we can say we have the value of theta is minus 23.74 degree – means, now, we have the location of just minus; minus means if we are going towards the right direction, we have these kind of values.

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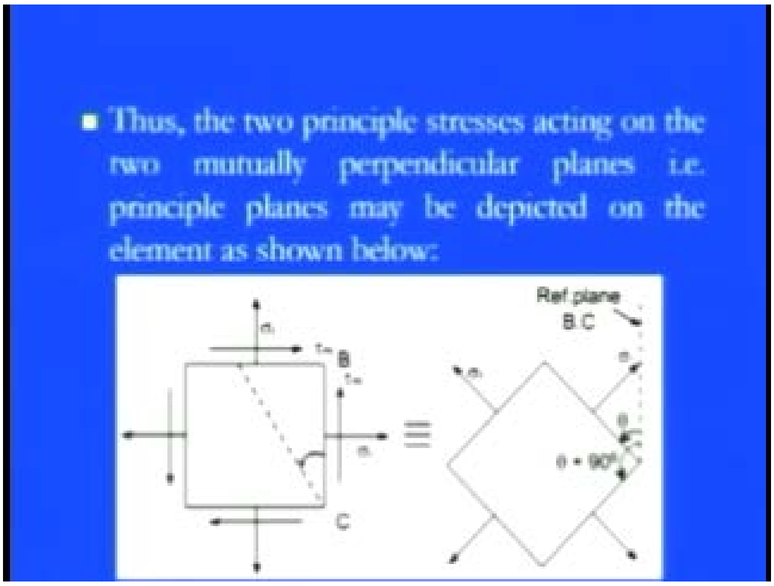

So, just we want to plot those things. The two principal stresses acting on two mutually perpendicular planes; always sigma x and sigma y is always acting at two mutually perpendicular planes. And, those planes are the principal planes. Here you see… And, we can easily depicted on the element as like here So, we have the pretty simple elements, which has all those three types of stresses: sigma x in this direction, sigma y in this direction, tau xy in this direction. It is equivalent to this direction. This is the reference plane, that is, the BC plane – if you rotate this by the theta, as we calculate in the first case – theta by this, you have sigma 1. So, this is the sigma 1. In this direction, sigma 1 is there; this direction gives you the sigma 2. And, this is the first plane – theta. Another plane – that was if you just remind that, the previous case; we discussed about the theta 1 and then we added the theta. For theta 2, we added the 90 degree. So, this is the location of another plane. So, this is the principal plane 1; this is the principal plane 2. On these, we have both the components: sigma 1 and sigma 2, which is reference to this particular plane.

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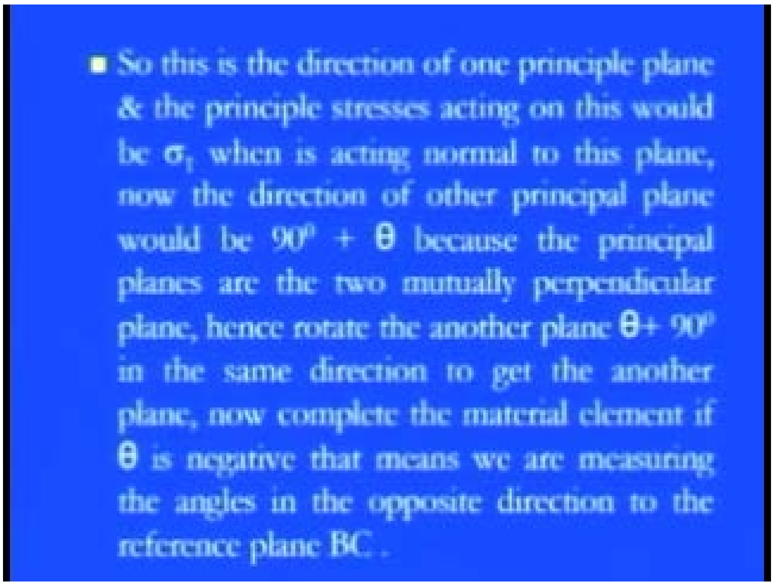

So, this is the reference plane BC or which we can simply rotate that part. So, the location of the first principal stress, that is, sigma theta 1; and corresponding sigma 1, which is pretty easily calculated. Once you can plot the sigma 1 and the theta 1 on the oblique plane, one can easily get the other direction; that is, other direction is nothing but the perpendicular to the previous one, that is, sigma to a is always acting at the 90 degree of the previous one – theta 1 plus 90 degree will give another location of the principal plane. So, here we have the two mutually perpendicular planes. Hence, we can simply rotate the first plane by 90 degree to get another plane direction.

So, now, we can simply complete all those things with… if the positive angles are there. But, if theta is negative; that means, we are measuring the angle in the opposite direction – means we are going towards another direction of the reference plane BC – means right now you see the theta was positive. So, what we are doing here; we are simply going towards this direction; this is BC. And, this was the first principal plane is there; the location of this theta. Now, if I am saying that, in the second case, where we found that, when this tensile stresses was negative, we found that, the theta 1 was negative – means what we need to do; we need to locate this theta now by this reference plane to that direction. So, theta 1 will go in that direction; then you need to add plus 90 to get another location. So, here it is…

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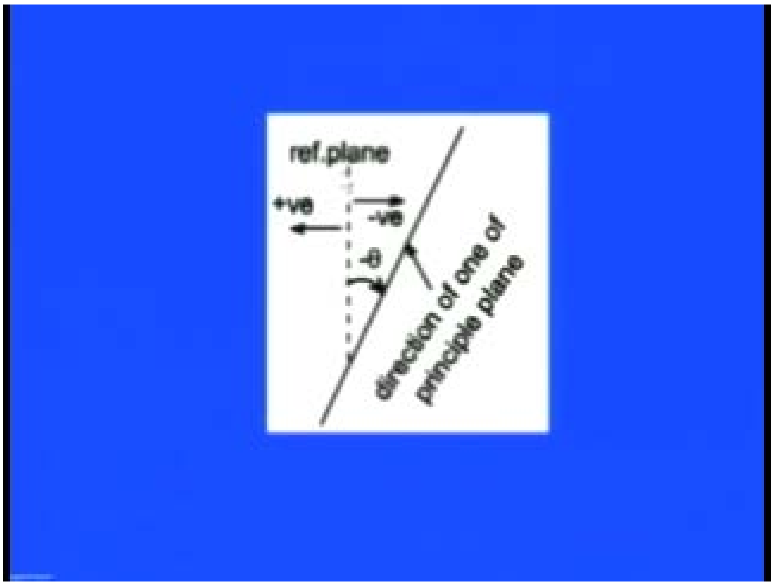

This is the reference plane as I discussed; BC kind of that. If theta is positive, then you need to go to the positive direction here; if it is negative, then you need to go by minus theta. So, this is the negative direction and this is… Once you have this; so, this is my direction of the principal plane 1. And, once you have the theta 1; to get another value of theta 2, you need to add 90 degree. So, here it is…

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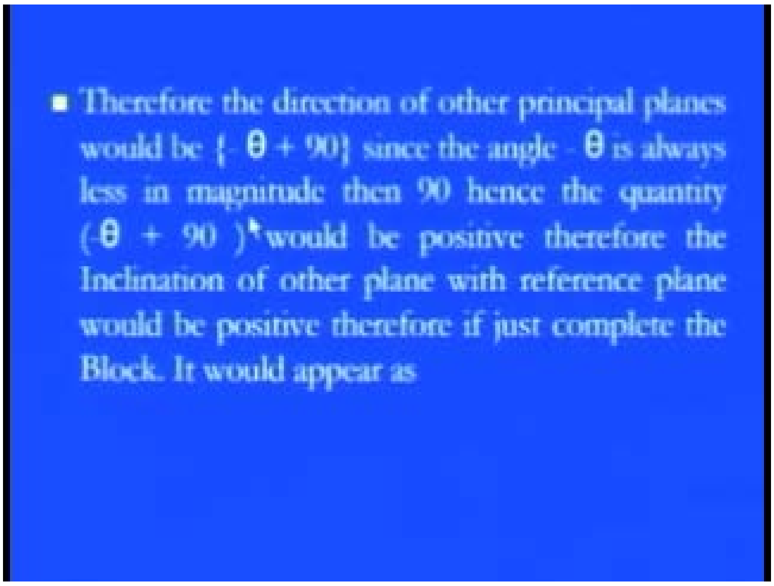

Therefore, the direction of another principal plane would be minus theta plus 90 – means what you need to do; just you put the minus theta is always less than whatever the magnitude is there and you need to go in the other direction by adding 90 degree. Hence, the quantity is minus theta plus 90 would be positive. Therefore, the inclination of other plane with the reference plane would be positive, because we are adding the 90 degree here. Therefore, it will just complete the block, because we just want to complete the total block, but just by rotating by adding the 90 degree in the minus theta. And, it would appear as exactly like that.

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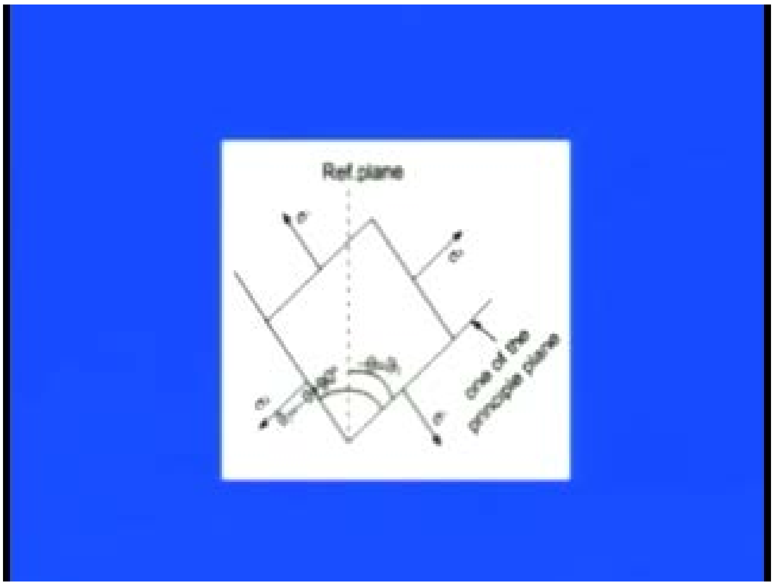

You see here; if it is minus theta; so, this is the reference plane. It would be here – minus theta; this is one. So, this is the first reference plane. Now, what we need to do; we need to add those things. So, this is another reference plane, which is coming by minus theta plus 90 degree. So, this is the sigma 2; sigma 1 is this. So, this is… Just to complete the block, what we need to do; we need to first put minus theta here and minus theta plus 90 here. So, this one just gives you a clear picture about that, if we have theta is minus or theta is positive, we can simply rather go in positive direction or in negative direction here on the just two sides of the reference plane.

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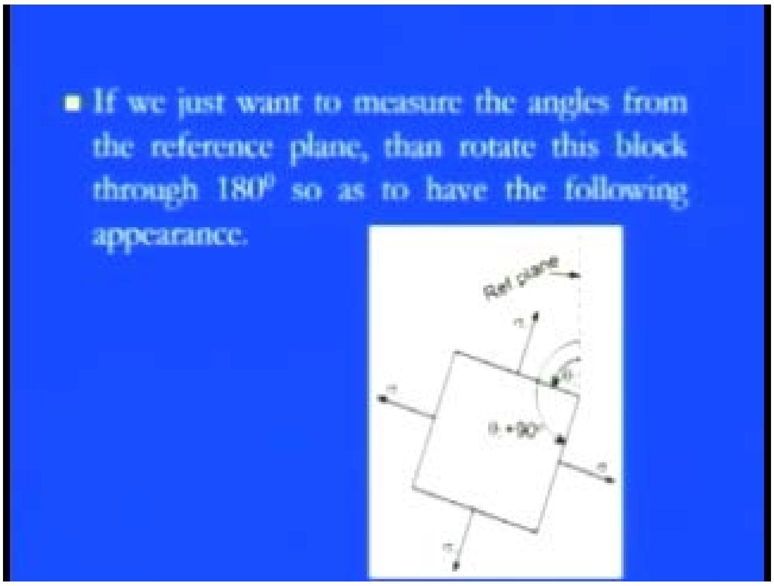

If we just want to measure the angle from the reference plane, then you need to rotate the just block, because what we need to do here; we just want to measure those angle; that actually if it is in the minus sign or in the positive sign, and then how it appears. So, what we need to do here; just rotate the block by 180 degree, so as to have the following appearance. So, what we did here; this is the reference plane. Now, this one is the theta and this is – whatever the theta angle is there; at which this sigma 2 is acting. And, this is the plane, where just 90 degrees there; theta 2 plus 90… So, this is sigma 1; meaning is pretty simple. If you want to appear those stresses; if it is minus theta or positive theta; and if you feel that, the things are pretty complex; then what we need to do? You need to simply make the block in such a way that, either by rotating 90 or 180 degree just to make the simplest way to represent those principal planes – this theta 1 and theta 2 at both sigma 1 and sigma 2 stresses are there.

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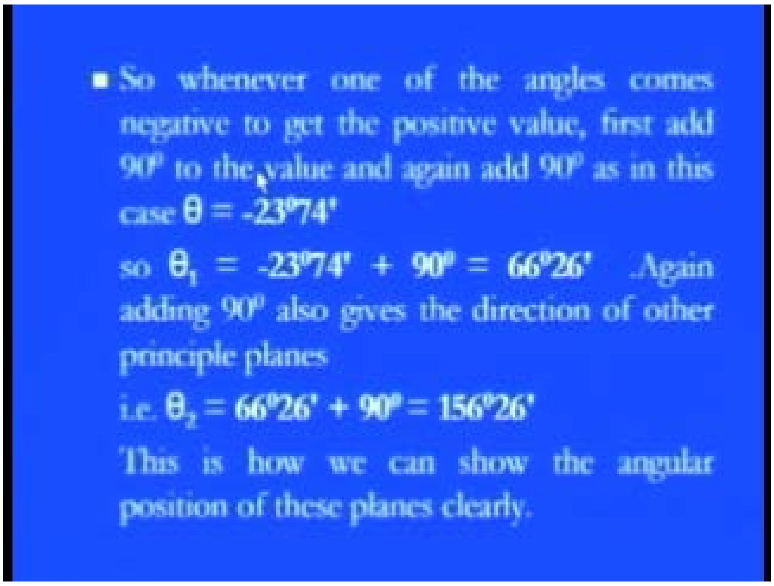

Whenever one of the angle comes negative like minus theta as we discussed in the previous case to get the positive value, first you need to add the 90 degree just to get the positive value and then another 90 degree to rotate just to locate the another principal plane; means pretty simple – if we have let us say minus like this one; in this case, we have the first thing – theta is minus 23.74 degree. So, either to rotate this slide rather to go that direction, what we need to do? You need to simply add to locate the first principal plane just adding 90… So, here minus 23.74 plus 90 is 66.26 degree. So, this is the location for first principal plane. Again by adding 90 degree will give you the direction of another principal plane – means again you need to add. So, that is what in the previous slide, we discussed that, actually if the things are pretty complex; what you need to do; you need to add 180 degree in total. 90 degree to just give the positive value and 90 degree to rotate the first principal plane to go to the second principal plane.

So, here we have the theta 2, that is, the 66.26 degree plus 90, that is, 156.26 degree. This is how we can show the angular position of these planes clearly, because if there is no ambiguity in that; if minus there; you need to go like that. If it is plus theta 1, is plus. So, just by adding 90 degree, you can get the location of the another principal plane. Now, this was all about the analytical solution. So, now, come to the graphical solution. The graphical solution just gives you… You do not have to calculate all these plus minus theta, sigma theta, tau theta – all those what the maximum and minimum values;

straightaway you have the problem. Plot those problems; get the… Just by measuring the other terms in the Mohr's circle, you can get exactly the solution is.

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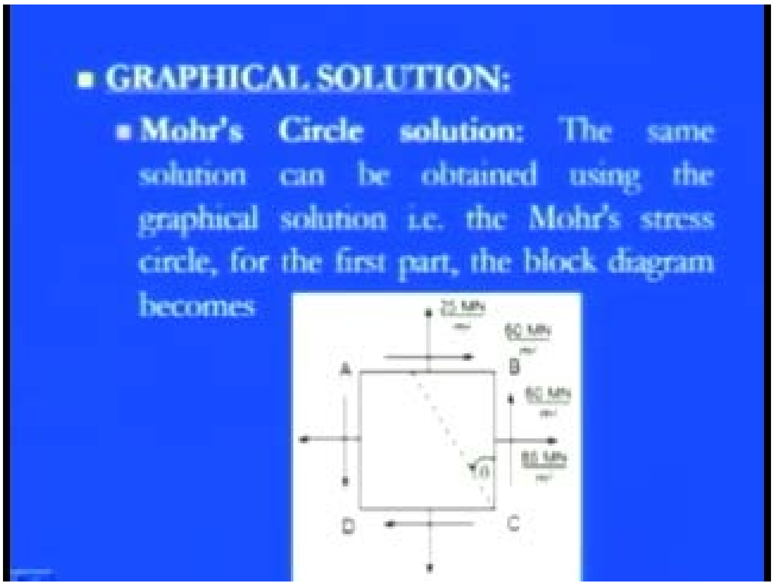

So, here comes to the main problem again. The same problem is there. Only the solution – this variety of the solution is there. So, solution way is different – Mohr's circle solution; the same problem, which we have taken. The two tensile stresses are there: the first is 85 mega newton per meter square; second one is 25 mega newton per meter square. Both are mutually perpendicular: one is along the x-axis; one is along the y-axis. And, this material is under the clockwise direction rotating under the effect of the shear stress. And, the value of the shear stress is 60 mega newton per meter square. So, this is the problem.

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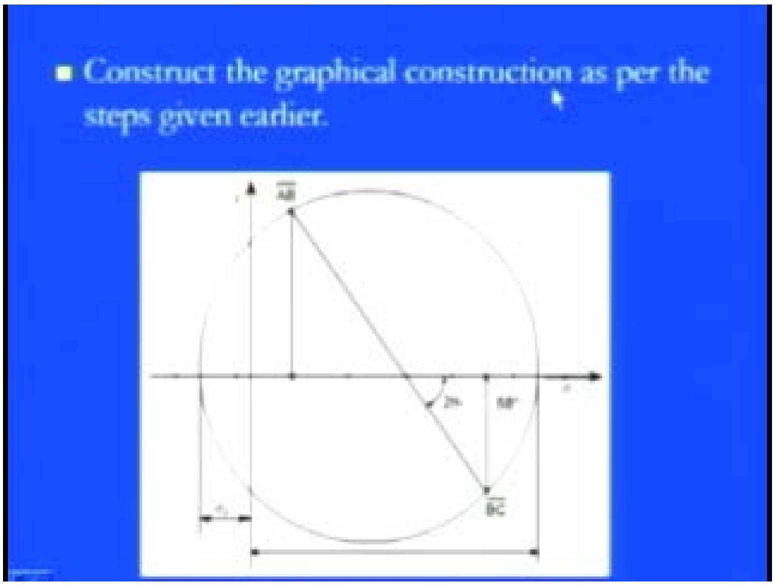

Then, construct the graphical constructions. Just we need to construct the graphical just by putting all those formulae. So, here what we have; we have sigma 1; that is, right from this to this, we have the sigma 1. So, all these things can be easily calculated. This is the O point; from that we can get the sigma 2; that is the sigma 2. So, O to this part is sigma 2; O to this part, is sigma 1; this one is sigma 1. So, once you have sigma 1 and sigma 2, we can simply get those values; then this is the centre point. We can get also the centre point coordinates. This is the diameter. So, through this, if you know that, 2 theta; then we simply put this particular… Simply rotate this by 2 theta; get this AB bar to BC bar as a diameter.

And then once you have these things, then we can simply get this radius. Or, by simply giving the interception from these two: AB 2 or BC 2 on the x-axis, what we have? We have the tau xy. So, this tau xy; tau xy is there; the sigma 1 and sigma 2 – we can simply get from the sigma x and sigma y. So, these – the Mohr’s circle – as we discuss that, actually it is always in between the sigma – x-axis, tau y – axis; you can draw these things and by measuring.

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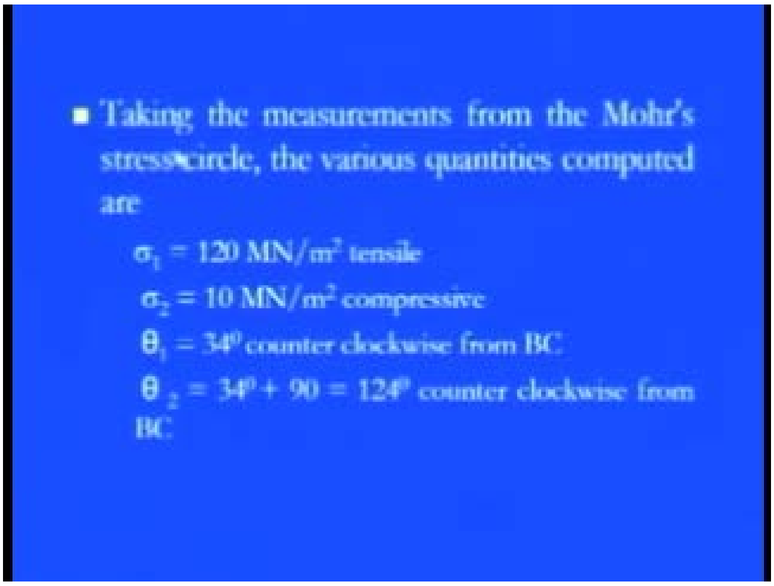

Now, we can simply measure the required values. By measuring – by taking the measurement of the Mohr's stress circle, the various quantities can be computed easily; like sigma 1 is 120 mega newton per meter square, that is, in the tensile nature, because it is in the positive direction. Just go back; it is towards origin to this direction. So, it is the positive direction. And, the sigma 2, which is origin towards the left direction. So, it is in the compressive nature and it has a value of 10, because very closest to the origin. So, 10 mega newton per meter square. And, again because the 2 theta is there, for AB bar to BC bar, you can again calculate the theta 1, that is, the 34 degree is the positive direction is there, because of the clockwise from the BC. And then theta 2 is always coming by adding the 90 degree for the next principal plane location. So, 34 degree plus 90 degree is 124 counter clockwise from the BC. So, these values can easily… You do not have to calculate those things that, what is the value of the sigma x, sigma y, tau xy, sigma theta, tau theta – all those things. Straightaway plot those things – sigma x, sigma y, tau xy. With the using of this information, plot the curve Mohr's circle and get those values by measuring those things.

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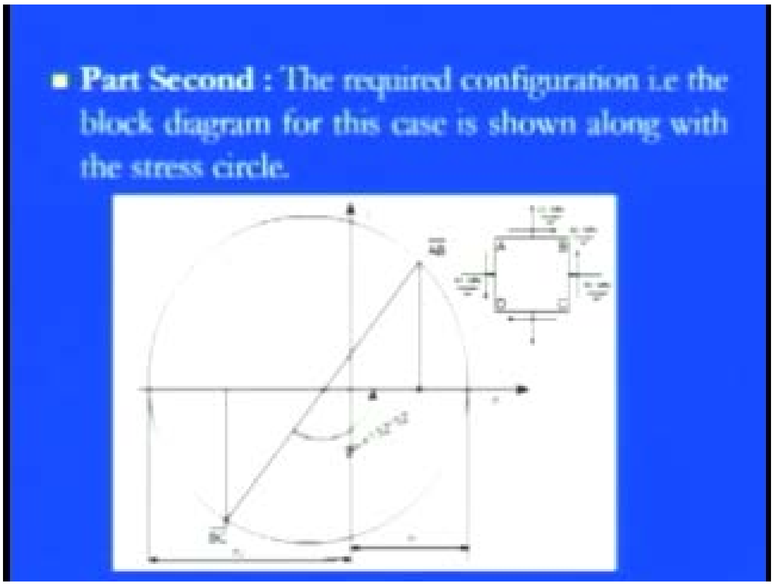

So, the part 2 – in which the tau xy that, sigma x was the compressive one. The problem here in this slide; this is like the square element; this sigma x is the compressive one. Sigma y is the tensile – means that is 25, this is 85, and this is 60 in the clockwise direction. So, now, keeping by these things, put those values – sigma x, sigma y and get these… This is the centre coordinate through which this is passing. So, what we need to do here; this is sigma tau. So, we have 2 theta. We simply calculate the theta. So, multiply those things. So, we have 132.32 degree and we have… This is the sigma 1 towards the positive direction; sigma 2 because it has a direct impact of this minus 1. So, we can also calculate this sigma 2 here like plotting these things. So, this is the Mohr's circle, which has a radius of right from… So, if I am saying that, this is P. So, P to BC or P to AB; we have a Mohr's circle radius. And, by dropping those things, this is nothing but the tau xy; this is nothing but the tau xy. And, you can simply get those things by sigma x plus sigma y by 2 here. So, here I mean to say that, if you have these information – sigma x, sigma y and tau xy, simply get the value of tau 1 and tau 2 here, and the theta.

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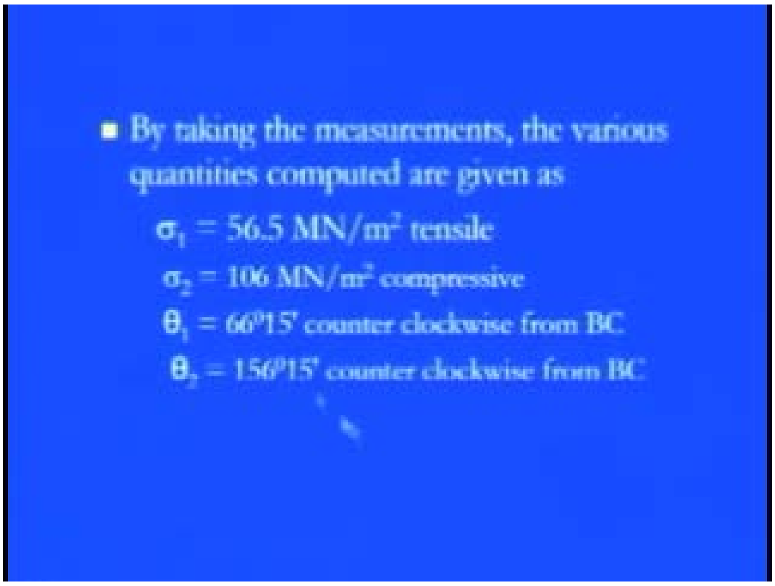

So, again by measuring those things, we can get these things, because the sigma 1 was towards the positive direction. So, we have 56.5 mega newton per meter square, that is, in the tensile nature. And, the sigma 2, which is towards the left direction of the origin. So, it is 106 mega newton per meter square compressive. And, theta 1 was pretty… because it was towards the clockwise direction. So, we have from the BC. So, it has a positive value – 66.15 degree. And, by adding 90, we have the theta 2 – means another location of another principal plane. So, 156.15 degree counter clockwise from the BC. So, here just we need a paper on which you can simply plot the curve. Just by measuring those things, we can get those values.

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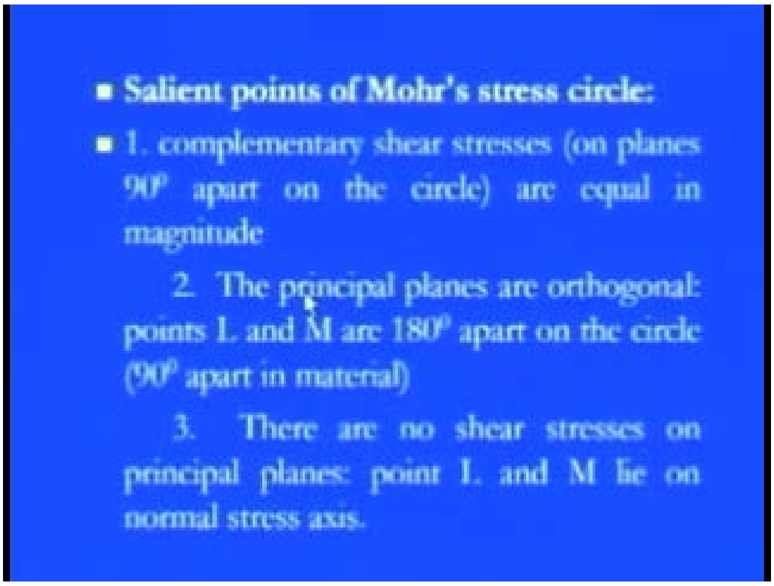

So, what is the salient features or the points of the Mohr's circle is the compressive shear stress on the plane – 90 degree apart from the circle are equal in the magnitude; that means it does not matter. If we have a shear stress of complementary shear stress, either apart at the 90 degree or another plane of the 90 degree plus minus of the circle, they are equal. Second feature is the principal planes are always orthogonal; like either sigma 1 and sigma 2 – they are always at the 90 degree to each other; they are orthogonal. So, point L and M – whatever – that two points are there, which we located, are always 180 degree apart from the circle or we can say 90-90 degree apart from the material side. There are no shear stress on principal planes as we discussed. So, point L and M lying on the normal stress axis always, because we are talking about the principal planes only. So, the principal planes does not contain any shear component is there; that means, whatever the points are there – L and M – they are always lying on the normal stress axis.

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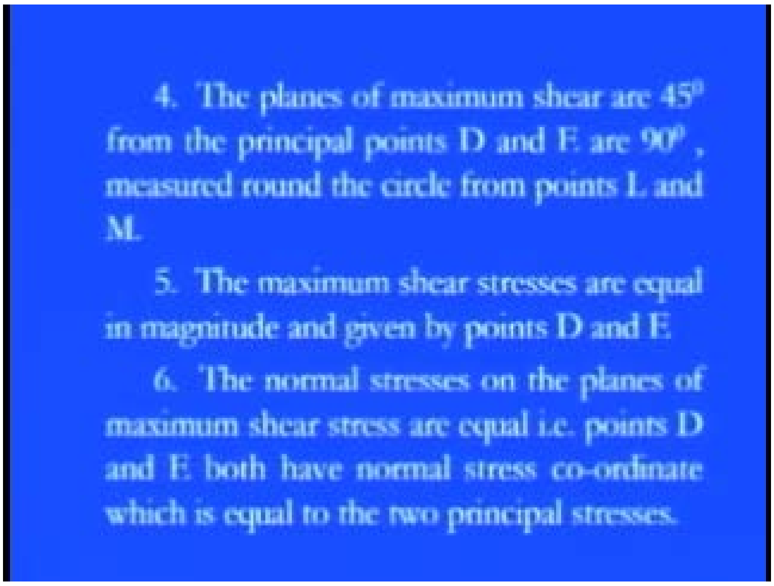

The planes of maximum shear are always at the 45 degree from the principal point D and E, which are 90 degree; that means they are always exactly at 45 degree. And, these two planes – the principal planes are always containing the maximum shear stresses. And, if you count both the parts, then it is apart from the 90 degree; means that round from the circle from the points of L and M. The fifth point the maximum shear stress are always equal in magnitude and given by D and E point as we discussed in the previous graph. The normal stress on the plane of maximum shear stress; whatever the normal stress component is there; if you are talking about the maximum shear stress plane – 45 degree are always equal. That is the point D and E, which we discussed in this point 5; both have the normal stress coordinates, which are equal to the principal stresses; that means if you are talking about the maximum shear stresses and if you are talking about the plane of that, where these stresses are occurring; always we need to be very careful that, actually what about the normal stress components are there about these planes. And, they are always equal to two principal stresses.

And, if you are talking about the principal planes or principal stresses; always we have to be very careful that, actually what there is… There should not be any normal stress component is there amongst those principal stresses or the principal planes. So, now, as we know that, this particular circle – the Mohr’s circle was represented all the possible state of normal as well as the shear stress on any plane through a stress point in a material. Further, we have to see that, the coordinates of point Q are seen to be same as those derived from the equilibrium elements in the analytical solution, that is, the normal as well as the shear components on any plane passing through the point can be found using Mohr's circle. If it is passing from the centre point, where the coordinates are pretty symmetry, we can easily get any value of the normal as well as the shear stress from the Mohr's circle itself.

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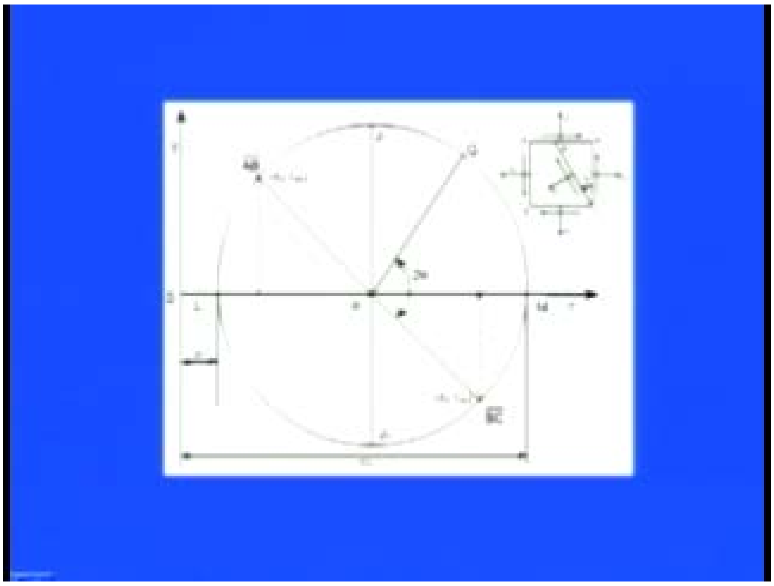

So, now, come to whatever we discussed these things; these points are there. We have… This is tau; this is sigma; this is tau. And, these whatever we discussed – all these salient features… Here this AB bar and BC bar – this is the P coordinate. And, this is the first sigma 1, which is always coming from the first principal stresses; sigma 2 is the circle principal stresses. So, sigma 1 plus sigma 2 divided by 2 will give you this coordinate of the centre point. So, this is the centre point.

Then, if you rotate this point – this Q, then you will get these points, where this sigma x, this tau xy and tau xy here is there. So, here if you want to check the coordinate of AB bar, you have the two values of sigma y and tau xy. If you want to calculate the coordinate of BC bar or if you have those things; you can simply plot those things – the sigma x and tau xy. So, once you have these two points, put those points here with those coordinates from these; you have sigma x, sigma y, tau xy. So, put these two values here; fill up those things; get the value of sigma 1 and sigma 2; get the value of P. Once you have the P, then rotate these things by 2 theta, which will give the principal plane. So, now, the principal plane is like that; earlier the straight one. Now, the principal plane is like that.

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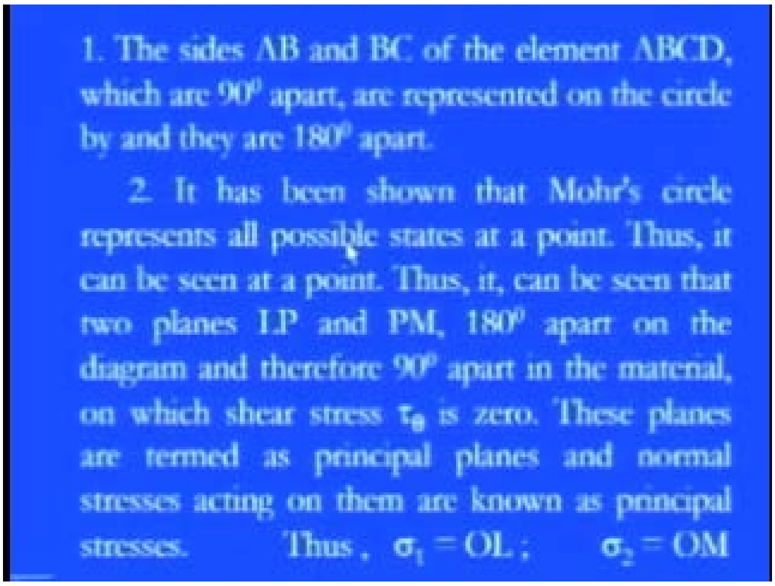

So, all the sides – AB and BC of the element ABC, which are 90 degree apart from each other, are always representing the circle and they are always… Since 90-90 degrees, they are the total 180 degree apart from these things. So, it has been shown that, the Mohr's circle represent all possible states of stress as we discussed. Thus, it can be seen that… If you want to calculate the stress at a point, can be easily calculated without any sort of calculation just by measurement. Thus, it can be seen the two planes: IP and PM in the previous figure – they are always 180 degree apart on the diagram. And therefore, 90 degree apart from the material side on which the shear stress – tau theta is 0; that means the principal planes are there. And, these planes are termed as the principal planes. And, the normal stress acting on these are the principal stresses. Thus, we can say that, the sigma 1 is the first OL; sigma 2 is the total – the OM. And, the centre point is sigma 1 plus sigma y by 2; sigma 1 plus sigma 2 by 2.

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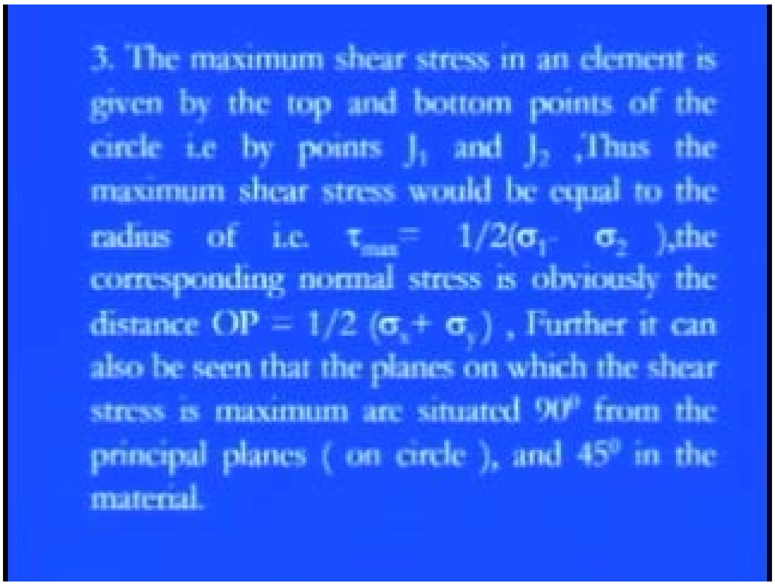

The maximum shear stress in an element is given by the top. Always it is on the top of the circle, which is equal to the radius. So, bottom of the points; like top and bottom – J 1 and J 2. Thus, the maximum shear stress would be sigma 1 minus sigma y by 2 equal to the radius of the… As I told you the Mohr's circle, which is equal to sigma 1 minus sigma y by 2. And, the corresponding normal stress is obviously distance OP, that is, sigma 1 plus sigma y by 2; that is the coordinate of the P – the centre point. Further, it can be seen that, the planes on which the shear stress is maximum are situated 90 degree from the principal plane always; that means you can say the 45 degree from the material. If you want to calculate what is the maximum value and where it is exerting; it is always 90 degree from the principal plane, where the maximum shear stresses are there. And, you know that, the principal planes are having 0 shear degree. So, you can simply make those points just to make easy of the calculation.

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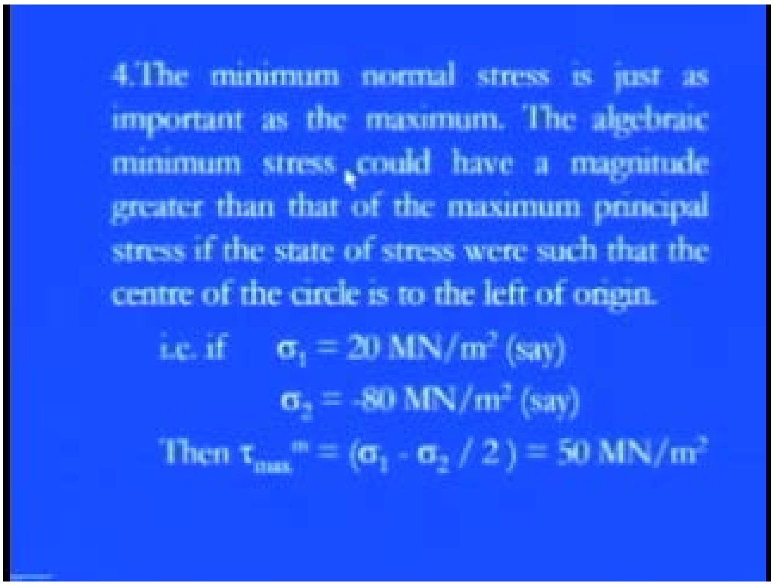

And then the minimum normal stress component – just the important. As you see, it is simply the maximum part is there from the algebraic way. So, the algebraic minimum stress could have a magnitude greater than that of the maximum principal stresses, if the state of stress were such that the centre of the circle is exactly left towards the origin; means if it is going beyond certain things, then always we have sigma 1, which is the positive value; sigma 2 is always compressive as we discussed in the previous case. So, that is here if we are saying that, the sigma 1 is 20 mega newton per meter square, and if it is the sigma 2, which is the compressive nature – the principal stress another is minus 80 mega newton per meter square; if these two values are there; let us assume then tau maximum – means the maximum shear stress is always left towards the origin; that means it has a value of 50 just by putting minus here. So, 50 mega newton per meter square.

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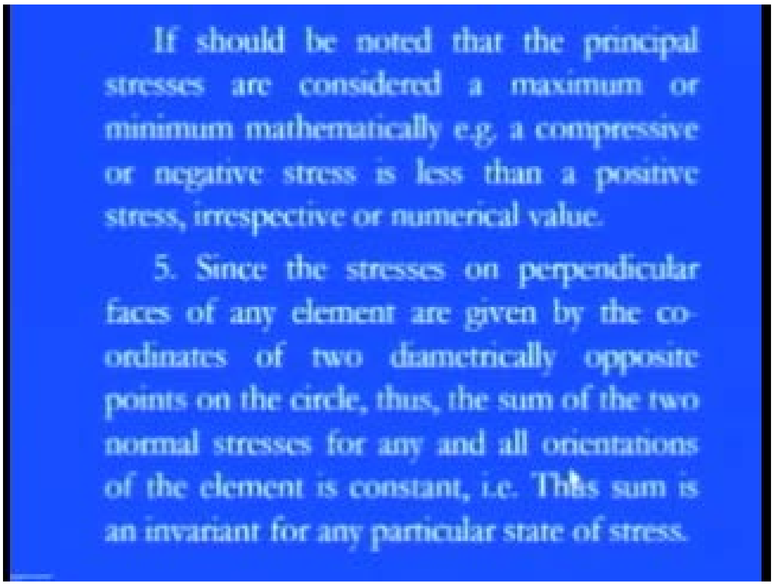

It should be noted that, the principal stresses are considered maximum or the minimum mathematically. Just it is not that, whatever the compressive or the negative stresses are there or the positive stresses are there; they are just giving you the feeling about whether it is a tensile or whether it is a compressive nature. Since the stresses are always perpendicular faces of any element, which is given by the coordinates of two diametrically opposite points on the circle, they are always sum of the two normal stresses for any – always sum of the two normal stresses: sigma x and sigma y; or, we can say sigma 1 plus sigma 2. And, the orientation of the element is always constant; that means the sum of the invariant, whatever, the things are therefore, any particular state of stress. They are always giving you the feeling about the sigma 1, sigma 2, sigma x and sigma y.

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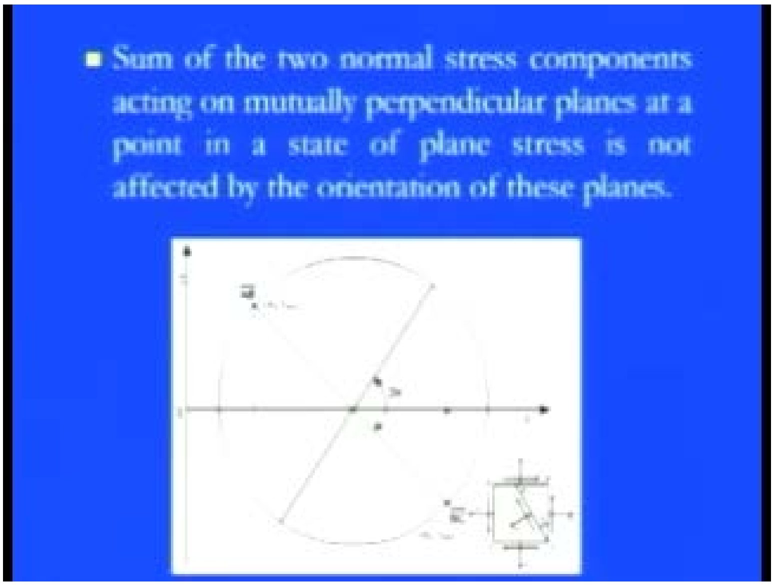

So, here, which we can simply feeling about the two perpendicular normal stress components are there; here like these sigma y, tau xy, sigma x, tau xy, and the rotation of those things. And, this is the sigma 1; this is the sigma 2. And, these components are like that.

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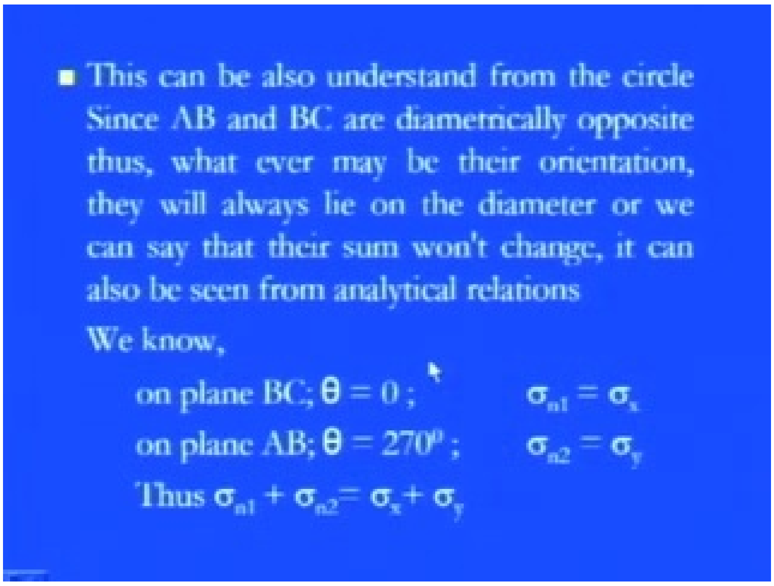

So, all those things, which we discussed; which can be easily understood from either the sigma x or sigma y are positive or negative. We can simply get.

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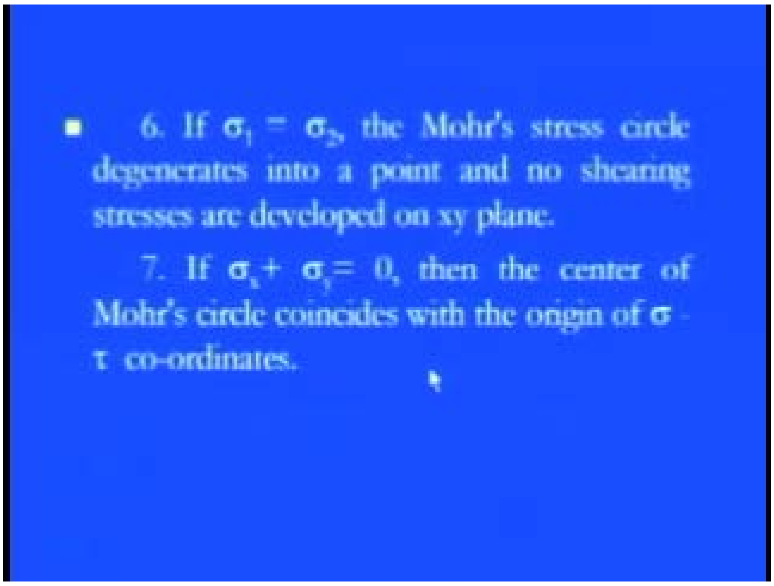

And, if sigma 1 and sigma 2 are equal, the Mohr's circle degenerates into a point. Just it squeezes into the point and there is no shear stress is there, which can be developed on xy plane; means if we can say that, there is the both principal stresses are equal, then what is the point is there; there is no shearing is there in that. And, always whatever the things will come; they are coming in the xy plane. And, the second case – if the sigma x plus sigma y is 0 – means the total summation of both – one is tensile; one is compressive; and the effect of the normal stress component – 0; then the centre of Mohr's circle is coinciding exactly on the origin of this sigma and tau coordinate. So, we found that, actually if both typical cases are there; then how to get the value of the extreme points – sigma 1 sigma 2, or how to get the coordinates of those values about the circumference of the stress circle.

If the last case, which we discussed about that, if both – if we have the tensile nature of stress; if we have the compressive nature of stress; and if the summation is exactly… and if both are equal in the magnitude; then definitely the summation means the combined effect is 0. Always whatever this centre is there, because the centre is nothing but the sigma x plus sigma y by 2. So, centre is always coming towards the point O. So, P is coming towards the point O. So, these are some of the special cases of the Mohr's circle. And, once we have the values of sigma x, sigma y, tau xy, we can easily plot this Mohr's circle radius by keeping one value on the circumference, that is, the sigma x and tau xy; another value is sigma x and tau xy. And, once you have those values on the circle, you can simply draw the circle, because both are nothing but the diameter on the extreme corner of those things.

And, once you know the theta, where it is they are exerting, you can simply measure the 2 theta. And, this 2 theta – once you rotate that part from the sigma or tau, these things are simply… By measuring those things, we have sigma 1; we have sigma 2; we have theta 1; we have theta 2. And, theta 2 is nothing but equals to theta 1 plus 90. So, these kind of exercises are pretty simple by measuring. There is no need to calculate. But, there are certain drawbacks of this Mohr's circle as I told in my previous lecture also that, once if we have the value of sigma 1 or sigma 2 or tau xy are the absolute – as we have discussed like 10, 20, 30, 40, 50 – all those things, then it is pretty easy to plot those curves here.

But, if we have the value of sigma 1 like 15.25 or 15.29, then again what we need to do; you need to again plot those things, which is pretty… Again, it is in the 2 or 3 digits; and then to make it out, you need to round up those values. And, once you round up those values, then whatever the angle is coming, this is not exactly accurate. So, some sort of like trial and error method is usable to plot those things. And, always some sort of we can say – a kind of errors are there in measuring those things. Or, sometimes the thickness – whatever the pencil thickness is not proper, then always… Or, the scale is not correct, then always some sort of magnification error is there. Or, we can say this is not the correct way to resolve those stress components if we have this kind of stress nature. But, if it is absolute value, there is no problem; you can straightaway put the scale; get those values by measuring these components in the Mohr's stress circle.

So, you see here in this lecture, we discussed about the two different approaches, which we discussed in the previous lectures: analytical approaches and this graphical approach. In the graphical approach, again we discussed about that actually. If we have the location of these theta 1 or theta 2; if theta 1 is negative, then how to go in another way, and how to part by 180 degree by just giving positive value by adding 90 degree; and then by adding 90 degree to locate another principal plane. Or, if the theta 1 – like the principal plane location is positive, then there is no need to go in the positive direction. It is already in the feasible region. And then by adding 90 degree, will give you another location of the principal plane. So, this is all about the stress components, which we discussed in this lecture or in the previous lecture.

So, actually if we have the different stresses are there; if they are inducing in a component either in the axial way, in the normal stress components, shear stress component. And, if the combined effect is there; the interaction is there of these kind of stresses; then how to resolve the matter and how to get the maximum value of normal stress, minimum value of normal stress, maximum value of the location; and what is the principal stress; what is the different stresses. And what is these kind of maximum shear stresses are there; and what exactly the relation is there in between those normal as well as the shear stress components. This is all about the stresses.

In the next lecture, now, we are going to discuss about the strains, because whenever the load application is there, we know that, though the internal intensity of the resistances are there and due to this the stresses are forming in the object; but, there is a deformation. And, when there is a deformation, there is a change of the shape of the material. And, once we are saying that, there is a change of the shape of material, we need to measure that. And, for measuring, there is another technical term is there, that is, the strain; that whether if you are pulling those things, then there is a pulling in one direction; but, the compression is there on the other direction – means the tensile is there in one direction – x direction; but, compression is there in another direction. Then, what exactly the relation is there in between this extension and the compression? So, again we need to define some different parameters.

And then in the next lecture, we are going to discuss about that up to what extent we need to pull the chain or we can say the bar or any kind of material, so that we can say that, if you release those things, it is in the elastic nature. And, once you are just pulling those things and once you go beyond certain limit, you are in the plastic region. And then if you are working in the elastic region, then what the different coefficients are there. And, once you go beyond certain region, then what the coefficients are there. So, these main terms are coming in the next lecture that, what is the strain – means how to measure the deformation – whether it is a tensile or compressive or in the shear formations. So, all these types of strains, which we are going to discuss; and then the other properties of the material.

Thank you.