Strength of Materials
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Lecture - 06

Good morning. This is Dr. S.P. Harsha from mechanical and industrial engineering Department, IIT Roorkee. I am going to present the lecture six on the topic of the stress formation and the analysis of the stresses and the topic of the strength of material subject, which was developed in the national program on technological enhanced learning. If you see that, like the previous lectures, we discussed about that, what are the like the resultant stresses are there at the oblique plane if there is like the material is subjected by the pure normal stress or what will be the stress components are there at the theta as well as the normal stress component or the shear stress component if the material is subjected by pure shear.

Or, if we have two combined stress – means two mutually perpendicular stresses like the normal stress components: sigma x and sigma y; if the material is subjected by that, then what will be the stress component at the oblique plane? Sigma theta means the normal stress component and the shear stress component. So, these like the components – the sigma theta and the tau theta means this normal stress component as well as this shear stress component; we have already discussed and calculated that actually what the impacts are there. And also, we just put the different conditions to get the value of the maximum as well as the minimum shear stresses if a component is subjected by both.

So, in the last… there is a discussion there, which was pretty fruitful that, if a material is subjected by both; if we have the mutually perpendicular stresses as well as the shear stress component is there; if there is a combination of these like stress components like sigma theta or the sigma x, sigma y and tau xy, then what will be the resultant like the stress components are there – like the normal stress component and the shear stress component at the oblique plane. Because if we cut the plane at theta, that is the oblique plane and which is going to formulate those things.

So, that is why we say we discussed that if these three components are applied at the same time means that the sigma x, sigma y and tau xy at the plane; then, we had the sigma theta, which is equal to sigma x plus sigma y by 2, which is independent of the theta plus sigma x minus sigma y by 2 into cos of 2 theta plus tau xy sin of 2 theta. So, this was the first – the normal stress component at the oblique plane if these three stress components are there.

And also, we formulate that, actually if and if the other component was there – the shear stress component – the tau theta, which is pretty like the combinational effect is there by sigma x minus sigma y by 2 into sin 2 theta or plus tau xy – this cos of 2 theta. So, that means, like if you see either the sigma theta or tau theta, we found that, there is the combined effect is there of sigma x, sigma y and tau xy. And also, we calculated that, actually if you want to find out that, in the component, where the maximum or minimum values of these normal stress components are there; then, we observe that, there are some of the places, where this maximum or minimum normal stress component is there. And those planes are known as the principal planes. And whatever the stresses values are there like sigma 1 and sigma 2; these stresses are known as the principal stresses. So, you see we calculate that, sigma 1 is nothing but equal to sigma x plus sigma y by 2 plus the square root of sigma x minus sigma y whole square plus 4 times of tau square xy. So, this was the first principal of stress.

And, second principal of stress was sigma x plus sigma y by 2 minus square root of that was the hypotenuse was there – sigma x minus sigma y whole square plus 4 times of tau square xy. So, these were the two stresses, which we found that, these are the principal stresses and these are always occurs at the principal planes. And we can find the principal planes by differentiating the sigma theta with the theta and keeping zero. And we found that, there is a location; and that location we can calculate by 2 times of theta P, which is equals to tan inverse of 2 times of tau xy divided by sigma x minus sigma y. So, this kind of discussion, which we made and also we found that, which is a very important observation was there; that the principal planes are always occurs where there is no shear stress; means wherever the shear stress is zero, we had like the principal stresses, which are sigma 1 and sigma 2.

Also, we discussed that, actually we can get the maximum shear stresses like the location was tan of 2 theta as… which is equals to this sigma x minus sigma y divided by 2 times of tau xy. So, we had a relation – the last part, which we discussed in the previous lecture was – we had a relation with the theta P as well as the theta S – that means, where the maximum shear stress is there; where the maximum normal stress component is there.

So, the relation was tan of 2 times theta P into tan of 2 times theta S equals to 1; that means, they had a reciprocal relation to each other. So, these kind of relations, which we made in the previous lecture. And then, also, we discussed that, we had like… We can solve by two methods: one is the analytical solution; that means, the numerical solution is there that, what… like the angles are there. And by putting all the trignometric relations, we can observed that, now, these are the places if a component is there, which is subjected by many of the forces; we can easily resolve those forces if we know and if we… We can also find out that actually where these stress values means the internal intensity of the resistances are the maximum or what is the stress concentration is there in this particular component just by knowing those trigonometric relations about this particular object. So, this is one method.

But, the popular method is the graphical solution. So, in this particular lecture, we are going to discuss in detail that, what is the graphical solution is there; and generally, it is termed as it is a Mohr's circle – means with the individual effect of the two mutually perpendicular stresses, the normal stress is sigma x and sigma y with the third stress, that is, the parallel stress. If we have a component and there the parallel stresses are there, these are tau xy.

And at the same time, to balance this component under the effect of the shear stress, we had a counter balance shear stress; that is known as the complementary shear stress tau dash. So, under all those stress components, if the material is well-balanced, we can say that, yes, all the stresses are well setup within those objects; or, we can say under the influence of these forces – of the external forces, we are in the equilibrium position. And that is what all those principal stresses, principal planes or the location of the principal stresses like that or the location of the maximum shear stresses within those objects are valid if this material or we can say the object is equilibrium under influence of these stresses.

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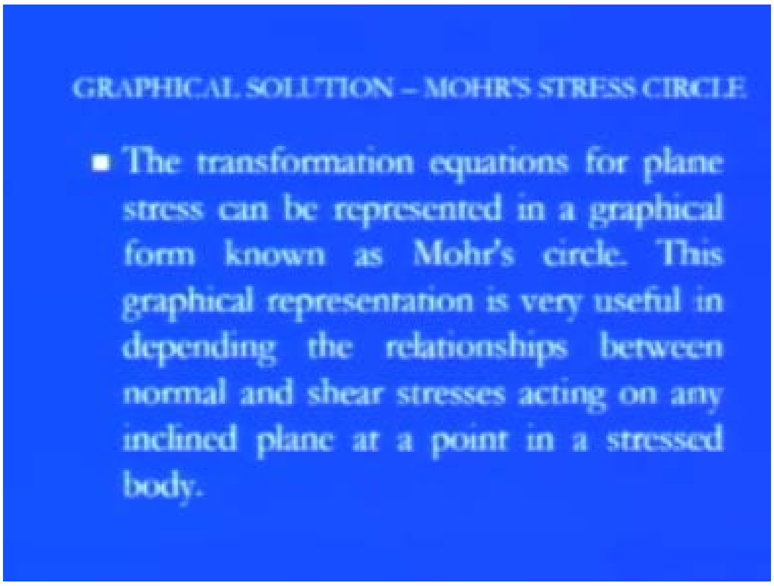

So, now, you see the lecture six here. As I told you, now, in this lecture, we are going to start with the graphical solution, that is, the Mohr's stress circle, because we are only dealing with the stresses right now. So, here the transformation equations like the sigma theta, tau theta – all those things for the plane stress, the plane stress means the two-dimensional stress in which these like the plane is there either the xy, yz or zx can be represented in a graphical form known as the Mohr's, because Mohr is a well-known… A scientist was there, who generated this kind like this graphical solution. So, based on that, actually we simply named this particular graphical solution as the Mohr's circle. This graphical representation is a very useful in depending upon the relationship between the normal as well as the shear stress.

As I told you, now, in this particular case, which we discussed the last thing was we had two mutually perpendicular normal stresses, are there and there is only one shear stress component; that means we have three mutually perpendicular stresses: one is parallel to axis; one is two mutually perpendicular axes. So, all three – two normal stress component and one shear stress component – if they are acting, then how we can formulize the stress component; and then, what will be the resultant like the stress component is there at the inclined plane, that is, the oblique plane you can say at a point in the stressed body? So, this kind of the transformation as well as the relation we just want to describe on a graphical solution.

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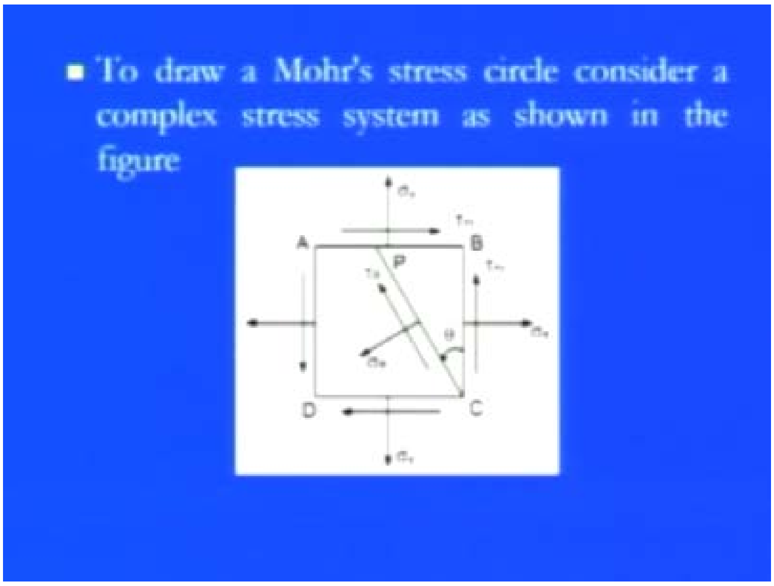

So, now, come to the main point that, to draw a Mohr's stress circle, consider a complex stress system as we discussed that, if we have… First, we see this figure; then, we have this sigma x on both of the sides. So, this stress component is the normal stress component; component in the x direction; we have the y direction means the normal stress component in the y direction, that is, the sigma y, which is perpendicular to the y section. And we have the same time, the shear stress, which is parallel – the AB as well as the CD. We have the shear stress component tau xy, which is parallel to the plane. So, to counter balance those, the shear stresses, what we have the tau xy; this is the counter balance. So, it is known as the complementary shear stress. So, we have both the component at AD as well as the CB.

So, this material means whatever the material, which we have; it is subjected by three stresses like one or the two stresses or the normal stress component – sigma x, which is in this direction. As we can see in this figure, sigma y, which is in this vertical direction and the tau xy, which is in this figure; then, we would like to see the resultant of the combination of these stresses. So, that is what; what we did here – we simply cut the plane at an angle theta. So, this PC is the cutting part, which is we simply cut at the theta angle. So, this is the angle of a measurement. So, we just want to measure those things and then at this particular plane, we had this PC plane. So, normal to this plane, we have a normal stress component, that is, the sigma theta and we have a parallel stress component, that is, the tau theta means those normal stresses as well as the shear stress components are there. And we have already stabilized the relation this analytically.

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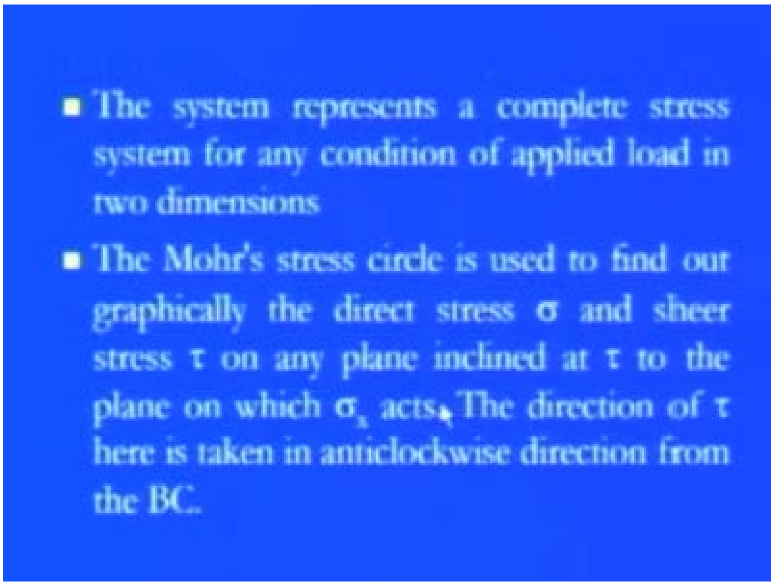

So, now, here the system represents a complete stress system for any condition of the applied load in these two dimensions like the x and y. The Mohr's circle stress wherever these is used to find out graphically the direct stress sigma as well as this shear stress tau on this particular plane that, theta plane, which is inclined at tau to the plane on which the sigma x is acting. And the direction of tau here is taken as the anticlockwise direction from the BC. So, here whatever the tau is there, it is just… because it just tries to oppose that thing. So, we have the shear stress, which is exactly opposing to the concerned shear stresses. So, now, in order to achieve whatever the desired objective is there; because we just want to calculate what the resultant tau is the… which is going to represent also the same time. What is the resultant tau means the shear stress? What is the resultant sigma, that is, the normal stress component? So, there is a ((Refer Time: 11:24)) for that.

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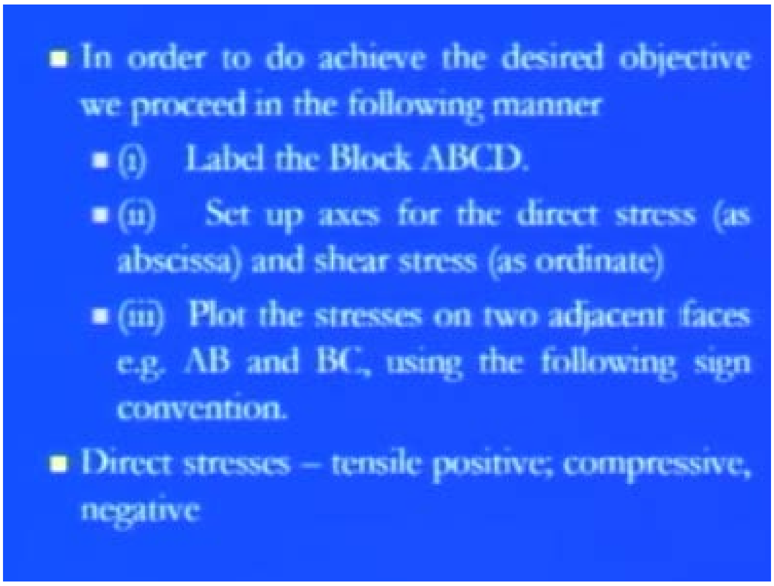

So, first of all, one should know that, you have a material, which is a kind of regular shape simply noted… you just simply noted or simply labeled those corner points. Here we have a simple – the square part is there. So, just label that block ABCD. So, now, we have ABCD. So, the AB at AB plane or this particular like the CD plane, we have the shear stress component, that is, tau xy. And at the these two corners, either you can say the CD or AB; what we have? We have just these planes normal to those planes. We have the normal stress component, that is, sigma x in x direction, sigma y in y direction. So, first, label those ABCD – this is the first step; then, second step is the – set up the axes for the direct stress; that means, you see whatever the direct stresses are there, now, set those axes as abscissa; means… because if you want to make the circle, we need to have the two, because it is a plane stress. So, both the stresses are there. So, just for abscissa, you need to put the direct normal stresses; that means, the sigma. And on this ordinate side, means, the vertical side, what you need to put? You need to put the shear stresses; that means now, whatever the stresses are there – these complex stresses are there in the real manner, now, we are transferring those stresses in the Mohr's circle by keeping x axis with the direct stress component, that means, the sigma only, and vertical direction – it is the shear stress only – tau.

Now, plot these stresses on these two adjacent faces; that means, either AB or BC using the following sign convention. So, here if we want to plot whatever the resultant stresses are there; first of all, we have to be very careful that, actually what exactly the direction of the stresses are there. Because whatever the direction is there, we have to be very careful that, actually what exactly the sign is – whether it is a summation of them, means, the resultant part – the summation of them or they are cancelling to each other or what exactly the interaction is there in between these stress components. So, first of all, if we are looking at the sign convention of these stress components, then first of all, we found that, there is a direct stress component; and the direct stress component we have if there is a pulling in the normal stress; that means, if there is an extension is there, we are always taking the positive. And that is known as the tensile stresses or tensile loading you can say.

And, if there is a compression or which the internal intensity of these forces – they are always just tried to go towards outward direction; always there is a compression is there inside the object or the outside external surrounding always take the negative direction. So, as well as the normal stress components are there – just pretty standard thing is there; tensile is always taking positive, compressive is always taking negative sign. And if we are looking at the shear stress component; as we discussed that, actually… because they are always across the layers of the object; we always see that, what is the tendency of this shear stress component, because if it is just tending to turn the block, whatever the ABCD block is there in the clockwise direction, means like this thing. They are acting in this direction and object is moving towards this direction always take the positive side.

And then, we have the complementary shear stress component, which will just try to resist this clockwise motion. So, if we have this kind of the shear stress component, which we will just try to… means actually this one; if these shear stress directions are there and if they are tending this particular block to move in the anticlockwise direction, always take the negative direction. That means shearing stresses are positive when the movement or the tendency of the block is towards the clockwise direction or the element is just tending to move towards the clockwise direction. Or if it is moving in counter clockwise direction; that means, it is the negative direction. So, these are the sign conventions.

And, we are very much strictly, we need to follow those; that in the normal stress component, the tensile is there – always take positive; compressive is there – always take negative. Shear stress – if it is moving towards clockwise, take positive; if it is moving anticlockwise, take negative. So, just keep this thing in your mind. And by keeping those things that, we have like the abscissa with this normal stress component; the ordinate – the y axis with the shear stress component, and these sign conventions. Now, we can easily find it out that, what is the Mohr's circle is. So, in the next slide, it is going to present you that, what exactly those combinations are there; and with inclusion of those combinations, how we can draw this Mohr's circle diagram.

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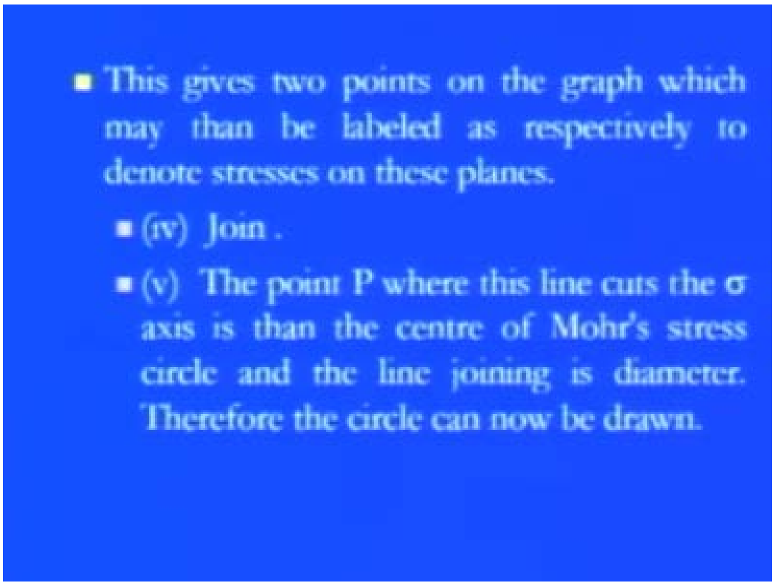

So, now, here this gives the two points on a graph, which may labeled or represented by the stresses on this particular planes. First of all, you have sigma x and sigma y on x-axis. So, you need to label, you need to join those things that, what is the value of the sigma x is there; what is the value of the sigma y is there; and what is the combination is there, because we have the two normal stress components. So, just join those two components on this particular plane. So, we have one plane, that is, a sigma plane. And if we have the tau plane, again, we have to be very careful that, actually whether this tau xy is positive means above 0 in the first quadrant; or, it is negative means it is in the fourth quadrant like that in the below side.

So, the point P, which is the cutting plane is there; so wherever the cutting plane was there at the theta, this point P; where, this line cuts the sigma axis is now like the center of the Mohr circle; and line joining is the diameter. So, here now, the two main features have come in this particular line. First, whatever the cutting part is there – the extreme end point on top of the surface gives you the center of the Mohr circle. Whatever the location of the point P is there, this gives you the center point of the Mohr's circle and the line joining of whatever the diameter; means if it is just passing from these things, it will give you the diameter. Therefore, the circle can now be easily drawn with the using of all the information, which we discussed here.

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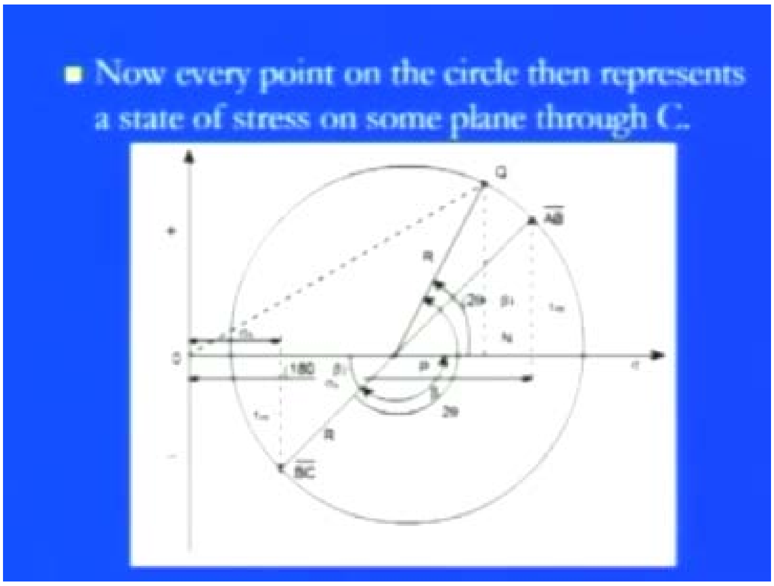

So, now, this is the Mohr's circle. Now, every point on the circle then represents a state of stress on some plane through center or we can say the C. So, now, we can look this particular circle; you found that, as we discussed, this is the x-axis, where the normal stress component is there; this is y-axis, where the shear stress component is there. So, now, by taking all the information of that, we have a material, which is subjected by two normal stress components and the shear stress – the shear stress is passing. Under the combination of all three components of the stress, now, we see how we can formulate. So, now, what we have? Sigma theta and tau theta. What is the sigma theta? Sigma theta is sigma x plus sigma y by 2. So, sigma x – now, we can straight the plot that, this sigma x. So, sigma x is nothing but this BC is there. So, if you plot these things, now, this is the sigma x; and whatever this diameter, which is passing from these things – we have – if you draw this line, project this line on the x-axis, because whatever the sigma x and sigma y is there – it is nothing but the interception of these points – this BC bar or AB bar on an x-axis.

So, if you see, we have the point P, which is the center point; take the radius and draw the circle. So, wherever you found that, by simply clicking of this particular… now, this – we have the x-axis and we have the beta angle. So, once you rotate this beta angle, you found that from P to BC bar, we have the radius – radius is nothing but equal to square root of sigma x minus sigma y whole square plus 4 times of tau square xy. So, once you calculate these things – this P to BC bar, that is, the radius; simply intercept this project – this particular part on the x-axis; you will get the sigma x.

Similarly, once it is passed through the P and it will give – this is the whole diameter of that. So, we have the AB disk simply intercept this part – the tau xy; and on the x-axis, whatever the distance is there from O to this part, this will give you the sigma y. So, this is sigma y. So, now, we have sigma x; we have sigma y. So, we can calculate the total sigma theta, which is nothing but equals to sigma x plus sigma y by 2 plus sigma x minus sigma y by 2 into cos of 2 theta.

So, now, if we want to calculate the cos of 2 theta, now, you look at this point that, what is 2 theta. So, 2 theta is nothing but equals to… right from this forced the projection. So, this was the first projection from P to BC bar. So, from here you can simply plot the 2 theta. So, now, here this one is projected; that means, now, this is simply projected by that; earlier this was there; now, the 2 theta is this and if you simply rotate by beta degree, that means the beta 1 – this one. So, we have 180 minus beta of this part or else you can say from this part, you can get the exact value of 2 theta minus beta means actually what exactly the difference is there in between the projected angle and the measured angle. So, this is there. So, the meaning is that, whatever the information, which we are carrying in the numerical way – that means, irrespective of the sigma theta or tau theta, you can simply plot irrespective whatever the relation is there from the Mohr's circle. So, you do not have to calculate if the complex stresses are there and the complex formation is there and the material concerned. So, we are…

Again starting from that, this is sigma tau; simply plot this… simply get the value of the radius. Once you have the radius, plot the diameter. Once you have the diameter, make the projection. So, once you make the projection of the theoretical axis on the horizontal axis, then you have both the components: sigma x and sigma y, which is pretty easy or you can say in vice versa – you have sigma x; you have sigma y; you have tau xy. So, it is pretty easy for you, because once you know the sigma x, straightway take from the origin; draw the sigma x and make the vertical projection wherever it meets to this one point; just make those points.

And this is one point. Take another point sigma y. So, straightway on the x-axis, this is sigma y and then take tau xy. So, this is the tau xy. So, once you have this point from the sigma x and tau xy, you have from another point from sigma y and tau xy. So, now, make this diameter. And once you have the whole diameter, take the point at the center on the x-axis and draw the circle. And once you draw the circle, you have all the information required – means you can easily draw irrespective whatever the information, which you can simply plot the Mohr's circle.

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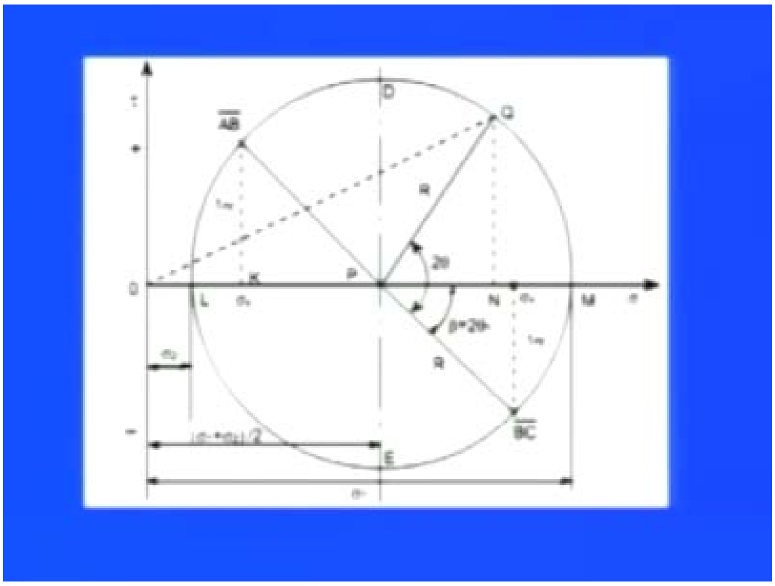

So, now, this is the well-generated form of the Mohr's circle; means what we did here; we simply convert whatever the information, which we have – means we have the sigma x, sigma y and tau xy So, based on these three information in the previous figure, we are showing that, how to plot all those curves means the diameter and the Mohr's circle. Now, you see we converted that, actually, wherever the minimum and the maximum value of the normal stress component is. And those values are known as the principal stresses. So, now, this is the conversion of the simplest form of the stresses, because if we have both this mutually perpendicular stresses: sigma x and sigma y and if you have the tau xy, this is sort of the complex form of the stresses, because if you want to calculate the resultant of these stresses, then you found that it is very complex.

So, what we need to do here; we just want to make the general state of stress. So, that is what in the previous lecture also, we converted all those combined – these complex stresses into the simplest stress and those stresses are known as the principal stresses. So, here starting from that sigma 1; sigma 1 is nothing but because we know sigma x, sigma y and tau xy; sigma 1 is easily calculated like we have… Sigma 1 is nothing but sigma x plus sigma y by 2 plus square root of sigma x minus sigma y whole square plus 4 times of tau square xy.

So, now you have the sigma 1, which is the maximum value – the summation is there; sigma 2, which is sigma x plus sigma y by 2 minus square root of sigma x minus sigma y whole square plus 4 times of tau square xy. So, here you have both the maximum value of normal stress, because the normal stress component is there on this plane. So, this plane is the principal stress 1. So, this is principal plane or which the principal stresses are there. So, we can simply load both maximum value of the principal stress, that is, the sigma 1. So, once you know the maximum value, simply take on the x-axis; and if you know the sigma 2, that is, the minimum value of that principal stress, then put the sigma 2.

So, now, you have sigma 1, sigma 2. So, wherever it interacts, you have the full diameter of these things. So, you see here you can simply calculate the coordinates of P, that is, nothing but the sigma 1 plus sigma y by 2. So, that is this one. So, you can straightway first locate that, what is this coordinate points are there of the center point of the Mohr's circle; that is nothing but if we are talking about these principal stresses, then it is sigma 1 plus sigma y by 2. Once you plot these things, now, what we need to do here; we are simply… First of all, we need to rotate these things by 2 theta, because we know that, tan 2 theta P is nothing but equals to sigma x minus sigma y divided by 2 tau xy. So, here the 2 theta P can be easily rotated. So, you need to rotate these things. So, now, it is rotated. So, we have the BC bar. Or, we can say this BC bar is nothing but equals to either you can take this sigma x and tau xy is there. So, here pretty simple – the sigma x, which is nothing but this distance and the tau xy of this distance.

Similarly, you can plot also the AB bar, which is nothing but equals to this, is the sigma y. So, once have the sigma y, and you can also put the tau xy. So, you can plot this thing either one way or another way is you have this particular B… Just simply rotate this main axis, where the sigma 2 and sigma 1 is there; rotate this part by beta or we can say the 2 two theta P, where the principal planes are locating; and then, we can get those on this particular circumference of this Mohr's circle; you can get both the points of the principal planes. So, here one – we have AB bar. Once we have the BC bar; you have sigma 1 sigma 2; you have sigma x; you have sigma y and you have tau xy. So, exactly, the relation is there; it is pretty easy to find it out. And another if we are saying that, now, it is rotating, where the maximum shear stresses are there. So, again you can rotate by 2 theta S and you can get the value of the P to Q.

So, you see here; now, if you rotate these things by 2 theta; now, you have these things. So, pretty simple; you can simply did the circular part here – this, this, this – either one and another is this one circle. So, both the triangles are pretty straight; only you are simply making the relation between the sigma x, sigma y and tau xy, if the combination is there; or, the simplest form is sigma 1 and sigma 2, if the principal stresses are there or the principal planes are there. So, this is all about the Mohr's circle in the previous diagram; it was with the complex form is there. And in this form, it was a simple principal stress formation.

So, here consider any point cube, whichever angle by 2 theta on the circumference of the circle – outer periphery of the circle, we just found that, the PQ makes an angle of 2 theta as we discussed with the BC; whatever the BC on this particular plane is there – drop a perpendicular from Q as we drop that with the sigma x; that means, the x-axis and it is meeting at the end point.

Then, we have the OQ; OQ – the O is origin of the point; Q is the point, where we are meeting. So, OQ represents the resultant stress on a plane at an angle of theta to the BC. So, here pretty simple – we are assuming that, the sigma x whatever the x component is there, because it is in the abscissa; that means x-axis. So, always it is sigma x is greater than to the sigma y; that means this is a simplest assumption is there. With that assumptions, we can simply say that, sigma x minus sigma y will be positive; and accordingly, the direction of the rotation is there of the shear stresses. That is why we are assuming this sigma x is greater than to the sigma y.

Now, let us find out the coordinate of the cube, because it is rotated by 2 theta from the main angle. So, what exactly… This main angle means AB to BC bar; you simply rotate it in the previous figure, if you see previous figure. So, that is what you see; if we rotated them about the new coordinates of this Q is. So, what we did here; simply we have chosen the two things: one is the ON and one is the QN. So, here from the figure – from this figure, we found that, this figure – from the figure, this is the Q point.

So, what we did here – simply we want to calculate that; actually, if we draw these things, then what is the Q to N part is there; means what the coordinates are there and what the coordinates of the PN. So, now, what we have; we have a triangle PQ and we just want to find it out, because all other coordinates like this tau xy, sigma x or tau y, tau xy – all those coordinates are very established; pretty well known things. But, as far as this thing is concerned, which is because it is simply rotated by 2 theta, then what is the new formation of the Q is we want to find out the coordinates and we want also want to setup the relation between the P to N, P to Q, and Q to N.

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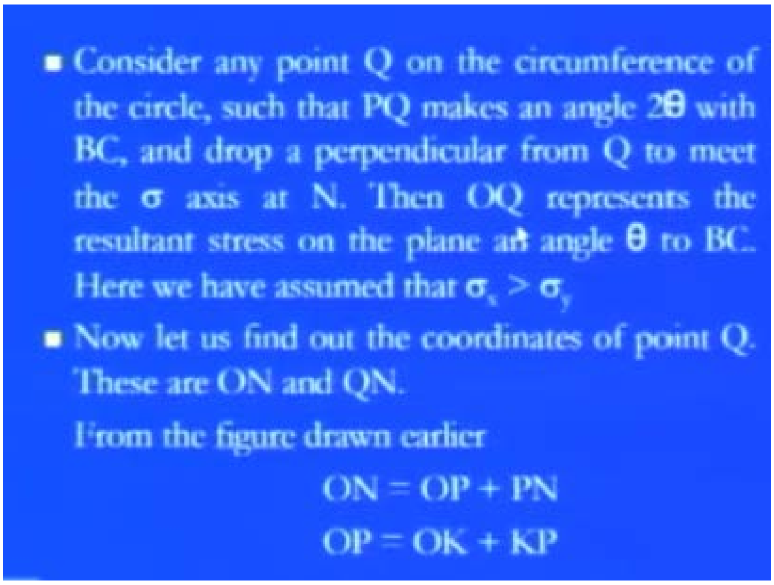

So, that is what you see here from that figure now; you simply say that, the ON is nothing but equals to OP plus PN; or, we can say that, this OP, which is coming the horizontally; OP is nothing but there is a middle point that is OK to KP. We can simply confirm these things. This is O to P; O to P is nothing but equals to O to K plus K to P. So, this OP can be easily divided and this was the OP, was nothing but the OK plus KP.

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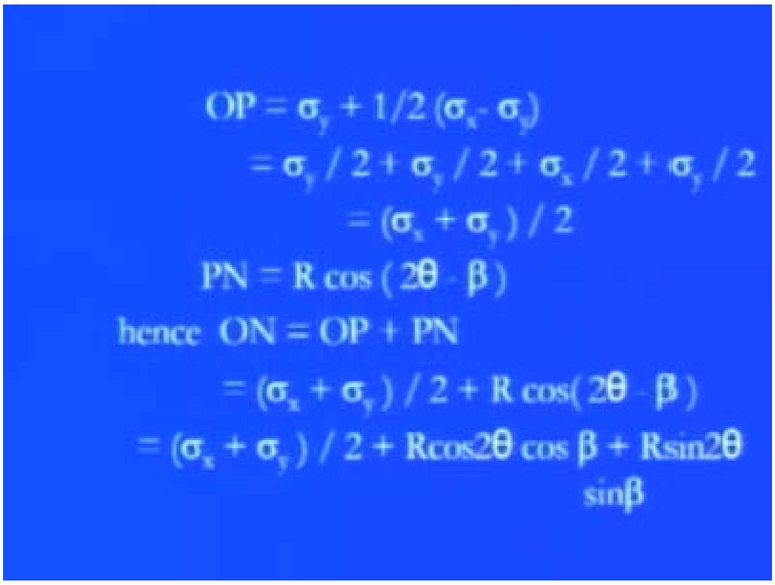

So, we have discussed in the previous slide that, actually there is a straight relation with the OP to ON. So, we would like to see that, actually from the Mohr’s circle, how we can get the value of the OP. So, here in the OP, OP is nothing but equals to the sigma y plus sigma x minus sigma y by 2. Again, here the biggest assumption, which is made here is sigma x is greater than sigma y. So, that is what you see – sigma x – if you deduct sigma x minus sigma y, it gives you a positive value. So, here by taking the consideration of these assumptions, we have OP; OP is right from origin to the center point; we have what sigma y, which is the O to K and K to P was sigma x minus sigma y by 2.

So, if you compare those things, then what we have sigma y by 2 plus sigma y by 2; simply you may divided into these 2 form plus we have sigma x by 2 plus sigma y by 2. So, all and all if you relate those things, then what we have – we have the sigma x plus sigma y by 2; that means OP; OP also – if we are going for this principal stresses, then we can simply locate the P position by sigma 1 plus sigma y by 2, if the principal stresses are there.

But, if we are talking about like that, if we have the normal stress component like sigma x, sigma y and tau xy; we can also find it out that, the P location is nothing but equals to sigma x plus sigma y to… which is proved here. And then, you see the P to N; P to N is the nothing but – the N is the point, where the projection was there from the Q. So, P to N is nothing but R of cos R into – what is the result? R is the radius – the radius into cos of 2 theta minus B. So, 2 theta minus B was the angle if you just remember that figure. So, 2 theta minus was the B and the R was the radius. So, if you made the positive resolution of this force, then you find that, PN is nothing but equals to R into cos of 2 theta minus B.

So, here it is pretty simple – ON is nothing but equals to… Again go back to that – OP plus PN; OP is nothing but equals to sigma x plus sigma y by 2. So, this is sigma x plus sigma y by 2 and PN, which is the cos resolution of this R. So, R times of cos of 2 theta minus B. Or, we can also use this; if we expand, this cos A minus B is nothing but equals to cos A cos B plus sign A sign B. So, by resolving these things, we have sigma x plus sigma y by 2 plus R times of cos of 2 theta cos of beta by this plus R times of sin 2 theta sin beta. So, what we have; we have now O to N. O to N is nothing but equals… The O is the origin – the starting point on the x-axis; N is the point, where the Q is dropping. So, we have the total dimension of ON by summing up the OP to PN.

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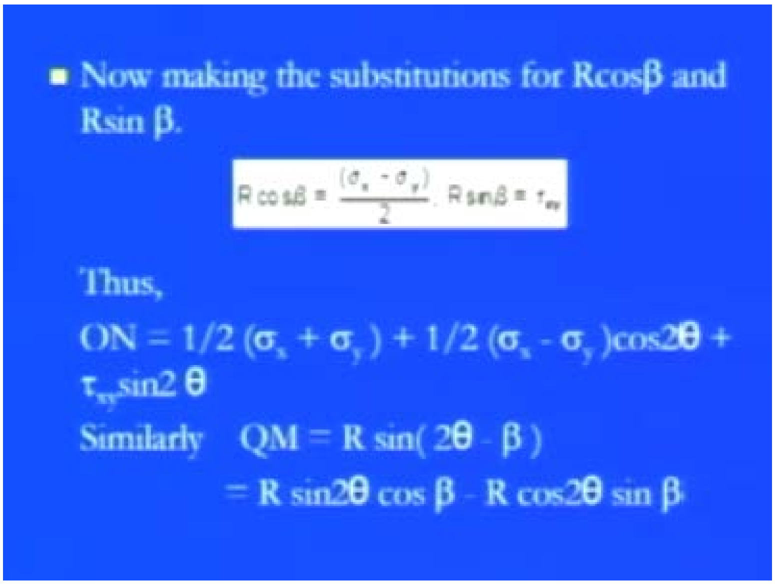

So, now, making the substitution of R cos beta and R sin beta, we can simply… that, the value R cos beta is nothing but equals to… Here these are the substitutions, which we need to make and we can get these things by the geometry itself. So, what we have? We have R cos beta is nothing but equals to sigma x minus sigma y by 2 and R sin beta, which is nothing but the tau xy. Again, this is a very good assumption. If you go back to like those figure, which is Mohr’s circle; you will find that, the beta is that angle and the first circle I should say. In the first circle, you can find that, the beta is the angle and the beta you can easily calculate, because it is just sloping with the R. So, R of the positive slope; R cos beta will give you sigma x and sigma y by 2; in between that, sigma x is this; sigma y is this. So, sigma x minus sigma y by 2. And the tau xy – the vertical component will give you the sin of beta. So, sin of beta is nothing but the tau x by y tau xy divided by R. So, from that, we can easily get these values – cos beta and sin beta.

So, if we are keeping these values here, we have O to N – means origin to the last point O to N is nothing but equals to sigma x plus sigma y by 2 plus half of sigma x minus sigma y by 2. And now, we can simply keep these things. So, cos of 2 theta plus the tau xy, which is R sin theta was there. So… And this sigma x minus sigma y by 2 was the R cos theta. So, sigma x minus sigma y by 2 cos of 2 theta plus tau xy sin of 2 theta. Similarly, we can also calculate Q to M. So, here because this is the end part was there, in that particular way, you can also calculate the Q to M; and Q to M is nothing but equals to another resolution of the force. The first formation of the force was there that, R cos of 2 theta minus B.

Now, if we go back to 90 degree, then you have Q to M, that is, R sin 2 theta minus B or we have… If we can simply keep the formula of sin A minus B is nothing but equal to sign A cos B minus cos A sign B. So, by keeping that formula here, we have Q to M, is nothing but equals to R times sin of 2 theta into cos beta minus R times sin of cos 2 theta into sin beta. So, here what we have; we have in the previously, if you see then, we have ON; ON we always get by PN, which is R cos theta B. And you see this is the PN and then we have the Q to M, which is R sin 2 theta means if you resolve those things, we have both OM as well as the QM.

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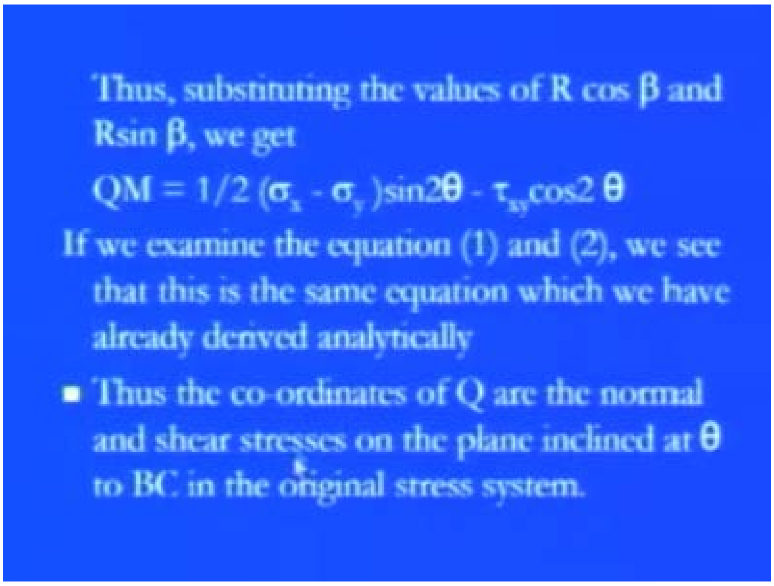

So, now by keeping those values R cos beta, which is nothing but equals to sigma x minus sigma y by 2 or R sin beta, which was the tau xy; by keeping those values in the previous formula, this QM; so now, what we have – the QM is the new formulation of the QM is sigma x minus sigma y by 2, which was the value of R cos beta into sin theta minus tau xy, which is coming because of the R sin beta into cos of 2 theta. So, if you see both the formation irrespective of the QM or PM or OM; so OM is nothing but equals to we have sigma x plus sigma y by 2 plus this sigma x minus sigma y by 2 – cos of 2 theta plus tau xy sin 2 theta, or we have this QM. So, here these two equations are pretty close to calculate those things. But, if we examine the equations 1 and 2 the previously, we found that, these equations – the physical science of these equations are pretty same, which we have already derived analytically from the previous equations – means that, what we have if we want to calculate the theta – the sigma theta… What was the value of the sigma theta? Sigma theta was nothing but equals to sigma x plus sigma y by 2 plus sigma x minus sigma y by 2 cos of 2 theta plus tau xy cos sin of 2 theta. So, same equation – sigma theta means ON; ON means it is the normal stress component at the inclined plane.

Here we are considering inclined plane is at beta. So, you see here we have ON. ON is nothing but the sigma theta. So, this is the sigma theta formula, which is exactly similar to ON. And then, the tau theta – tau theta was the shear stress component at the oblique plane, where the theta angle is there. So, similarly, if you compare these things, then you find that, the tau theta was sigma x minus sigma y by 2 sin of 2 theta plus tau xy cos theta. So, here all of the differences, what the rotation is there; so here the QM is the tau theta; that means the shear stress at the theta angle, which is again the similar thing is there – tau xy minus tau; tau x minus tau y divided by 2 into sin of 2 theta minus tau xy cos of 2 theta; that means you have… If you want to calculate either this ON, which is the sigma theta or QM, which is the tau theta, you have all those combined effect of these real form of the stresses; that means the sigma x, sigma y, tau xy. And because of those things, there is a rotation that is the end. That is why this theta angle is there.

So, if you want to calculate either this one – the normal stress component at the oblique plane or the shear stress component at the oblique plane is pretty easy. And these equations are well-established; that means Mohr’s circle is perfectly okay and there is no ambiguity in that to say that, the Mohr’s circle is the perfect graphical solution for analyzing the stresses, because the analytical equation is giving the same result; the graphical results are giving the same results.

And that is why they are perfect matching. Thus, the coordinates of Q are the normal and the shear stress of the plane inclined at the theta to BC in the original stress system; that means we can say that, whatever… Even after rotating by 2 theta, the coordinates of these things are nothing but equals to sigma x and sigma y in the x direction if the normal stress is there and the tau xy – means the same. It is a combination of normal as well as the shear stress in a plane stress composition; and theta is always just making the angle with the BC of this particular plane. So, this is… From that now, we can found some of the important observation.

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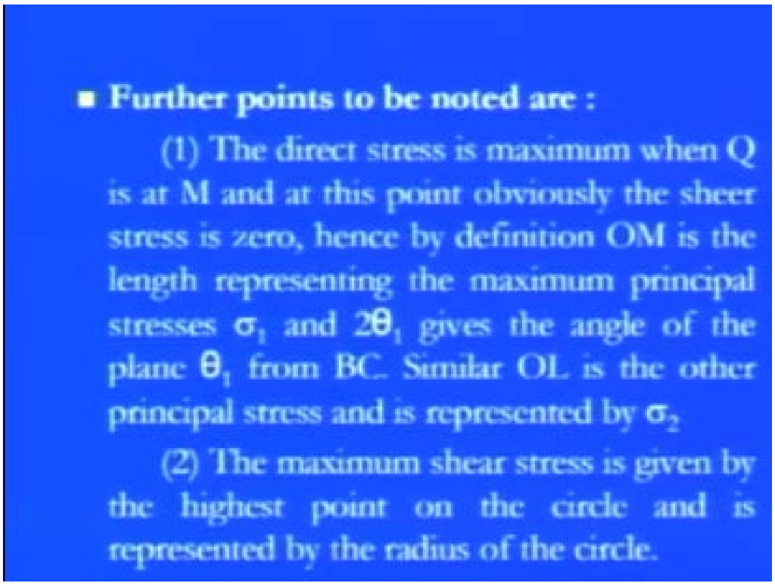

And, these points are to be noted that, the direct stress is nothing but the maximum… Direct stress is the sigma theta is maximum Q is at M. And this point obviously, shear… This point is the point, where there is no shear stress is there – meaning is that, if you just go to the analytical solution, we found that, if we differentiate the sigma theta by D sigma theta divided by D theta. And if we equate to 0, we found that, the plane is there, that is, the 2 tan times of 2 theta P equals to sigma x minus sigma y divided by 2 xy; that is the location, where there is no shear stress; the principal stresses are there. And this was the analytical part.

But, if you compare that analytical part here to the graphical solution, we found that, the direct stress is maximum; that means the theta… sigma 1 and sigma 2 is there; and sigma theta is maximum when Q is at M means when the Q – the top Q is meeting to the M; if you go back to the figure, then you find that, if the Q and M is meeting, then at this point, there is no shear stress is there because the vertical component is 0 – means there is no shear stress and the normal stress component is maximum. Hence, by definition, OM is the length representing the maximum principal stresses sigma 1 and 2 theta 1 gives you the angle for the plane – the 2 theta P, which we observe, where the theta 1 will be always from the BC.

Similarly, OL is the another principal stresses and representing by sigma 2 – means that, if Q is meeting to M, we have a coincidence that, actually whatever the stress formations are coming in the second Mohr's circle figure, it gives you the maximum value of normal stress, that is, the sigma 1; it also gives you the minimum value of the principal stress, that is, the sigma 2; and it always coming by graphically. So, we can easily plot the graph – this principal stresses on the principal plane by calculating this Q to M and by calculating the 2 theta.

So, it gives you graphical solution, no need to calculate those things; just by replacing those things, you can have all the solutions in your hand. So, this is one for the maximum or minimum direct stress component. But, the second point – if we go to the maximum shear stress, always it is given by the higher point of the circle, because on the vertical part – on the abscissa, you found that, actually we have the shear stress component. So, on the vertical axis, you are making a circle – the highest point of the circle, which gives you the maximum shear stresses.

Or, you can say that, the ((Refer Time: 41:56)) is represented by the radius of circle; that means, if we want to calculate; if you remember the previous lecture; we discussed that, if you want to calculate analytically that, what is the location, where the maximum shear stress is there; then, it is tan 2 theta P, is nothing but equals to 2 times of tau xy divided by sigma x minus sigma y. So, this is the location. Now, if you want to calculate the maximum value of these things, then it is nothing but equals to square root of sigma x minus sigma y whole square plus 4 times of tau square xy divided by 2. So, this is nothing but the radius of the circle. So, here by graphical solution, also, you can noted that, the highest point of the circle, which gives you the maximum shear stress. And you can also calculate from P origin to the highest point; and that is nothing but the radius of that and you can also calculate from the circle as well as from the analytical solution.

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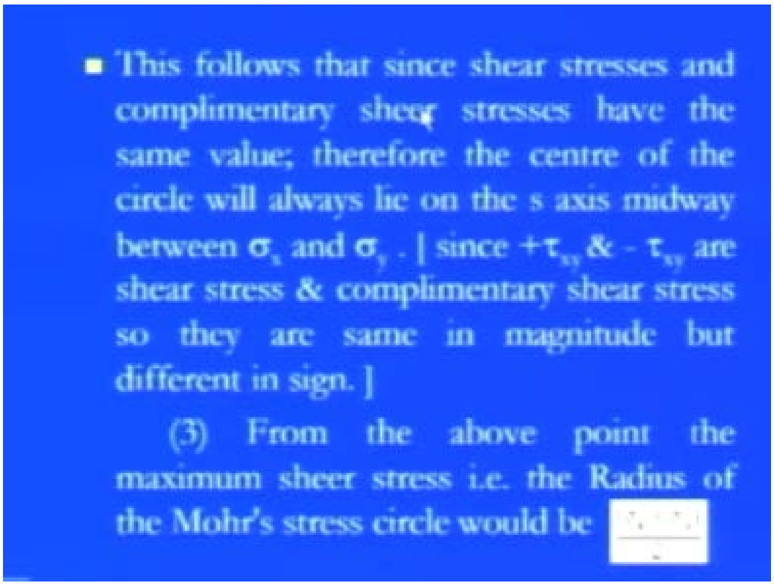

This follows that the shear stress and the complementary shear stress – that means the rotation and the anti-rotational part of the shear stress have the same value; obviously, because we have the symmetric in the geometry as well as the applied conditions are pretty symmetric in the unit cube or parallel pipe. Therefore, the center of the circle will always lie on the x-axis. So, we have one axis, which we are saying that, center is always lying; there is no eccentricity is there towards the axis midway between the sigma x or sigma y – means the plus tau xy minus tau xy are the shear stresses; complementary shear stresses are there and they have the same magnitude; only they have the difference in the sign; means that always if you want to calculate the location of P, it is given by two ways. One is the summation of sigma x plus sigma y by 2. So, this is P. So, you can simply locate the center; or else, if you are rotating these stress values – the normal stress value by sigma 1, sigma 2; then, easily we can put that, what is the center position or we can say the location of the center C is sigma 1 plus sigma y by 2.

So, here irrespective of what is the symmetricity is there, it is simply lying on the axis – the x axis, and it is always in between, or we can say the midway of sigma x and sigma y or sigma 1 and sigma 2. And in that case also, we found that, there is a symmetricity. And that is why we always concerned about tau xy, never tau yx. Or, because of the symmetricity, this is always equal; tau xy equals tau yx; tau xz equals to tau zx; or, tau yz equals to tau zy. So, this is due to the symmetricity. So, here also we are considering those things; only plus and minus signs are just the shear stress and the complementary shear stress and they are counter balanced to each other. And that is why we can say that, the object is under ((Refer Time: 44:40)) because of these two components of the shear stresses.

Now, from the above point, the maximum shear stress, that is, the radius of the Mohr's circle would be at sigma x minus sigma y by 2; that means we can say that, if we want to calculate the maximum shear stress, it is pretty simple, because if you go back to that, then you found that, this tau theta is nothing but equals to sigma x minus sigma y divided by 2 into sin of 2 theta minus tau xy cos of 2 theta. So, now, if you put those things; if you want to get the maximum value, always the theta is at either 45 – means theta is at 45 degree angle; that means what we have – we have the maximum value of the shear stress, that is, the tau xy sigma x minus sigma y by 2, because the tau xy will go on into the 0 part.

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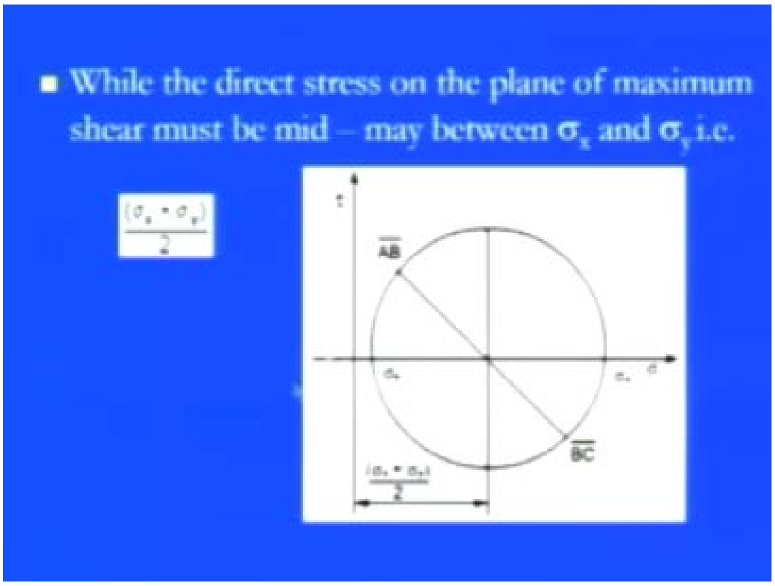

So, now, come back to this point. As we discussed, while the direct stresses are there, the P point – the center point is always coming in between the sigma x and y. So, sigma x plus sigma y by 2 is nothing but this is the location; and it is the radius. So, if you want to calculate the maximum value of the shear stress, it is always at this top point of the circle, which is nothing but this is a radius from this P to this point; and this is nothing but equals to sigma x minus sigma y. So, here if you want to calculate the direct stress on the plane, where the maximum shear stress is the midpoint is there, you can easily calculate by sigma x plus sigma y by 2. So, this is pretty simple. Once you have all the information, you can play with the graph – this diagram and you can get those required values.

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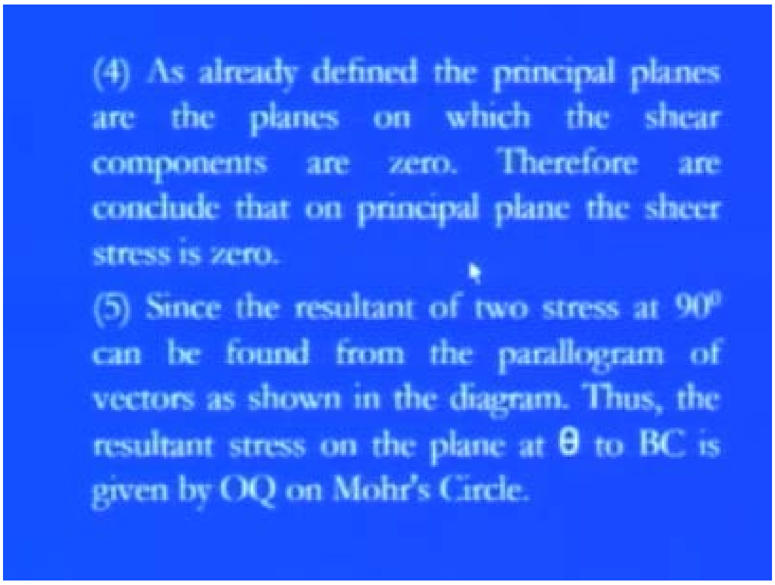

So, now, as already defined the principal planes, which are the planes, where there is no shear component is there – the planes on which there is no shear stresses; that is why these principal stresses are there on this plane. Therefore, we can conclude that, the principal planes have no shear; only normal stress components are there – means all those parallel forces are not at all concerning on this principal planes; only the perpendicular stresses are there on this particular planes. Since the resultant of these two stresses at 90 degree can be formed from the parallelogram, which we have shown in this – of the vectors; so we can say that, actually the parallelogram is a perfect background, because it has unit width, unit depth as well as the unit thickness. So, the parallelogram is a perfect thing, because both – the mutually perpendicular stresses can be easily shown then.

And also, we can say that, the resultant of these two can also be at the 90 degree, which can be easily founded by those things. Thus, the resultant stress on the plane at theta to BC is given by OQ on the Mohr's circle. So, you see here whatever the O to Q was there; O to Q means O is that point and Q was that point. It simply gives you that, actually what exactly for this resultant is there of these two stresses, which are mutually perpendicular to those things. So, it pretty simple to check all those points if you have the circular positions.

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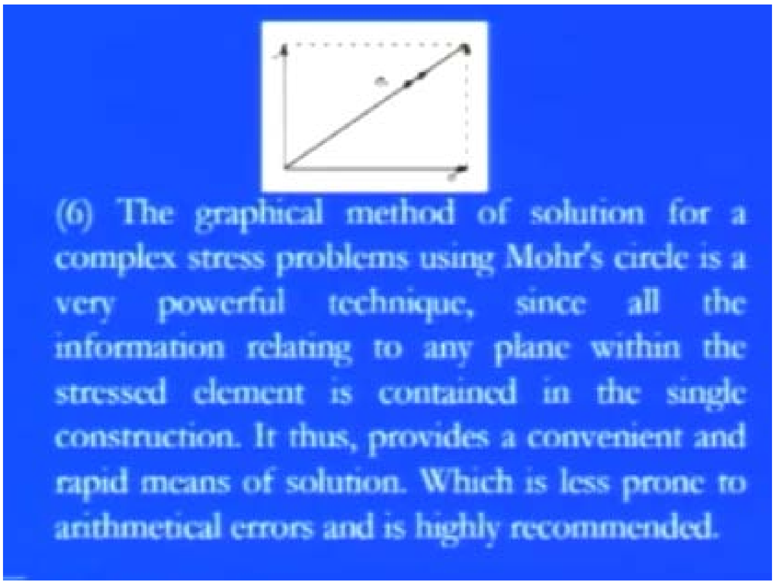

So, now, go back to this particular thing. You have the sigma; you have the tau – this is the resultant part. And if you know these values, it is pretty easy to get those other components of the stresses. The graphical method of the solution for a complete stress problem – Mohr's circle is a very powerful technique, because it gives you all kinds of flexibility that, just rotate this angle and get those values; rotate this angle in the counter clockwise or the clockwise. You can get all those values by simply locating the points, rather than you need to do all the analytical analysis here. Since all the information relating to any plane within the stressed element is contained in the single construction. So, once you… That is what I told you that, a lots of flexibility is containing that, That is what we can say that, it provides a convenient and the rapid means of the solutions if we have the complex shear stresses or if we have the complex normal stress component or if we have the combination of all these stresses, which is less prone to this arithmetical errors and is highly recommended. Only…

Now, this is the beauty of the analysis that, we have all kinds of flexibility to put the points that, actually if you want to calculate the sigma x or sigma y or tau xy, or if you have these values; you can easily go for the principal planes – sigma 1 and sigma 2. You can also go for that; where, is the maximum value of the sigma x is there where, is the minimum value of the sigma x is there; where, is the maximum value and minimum value of the tau xy is there just by rotating the elements. So, we can say that, actually this method is highly recommended, where you simply want to put those points and want to check those with the scaling value. But, the problem is that, actually if we have some higher values; that again what we need to do here – the basic drawback of this Mohr's circle is that, if we have some higher value of the stresses or we can say that, actually if we have some real values; means let us say if we have sigma x is 50.20 or if we have the sigma y is 20.25 or if we have the tau xy is somewhere 15.45; then, what we need to do here – we need to plot the graph, which is again…

And, if you are calculating those angles, then it contains some sort of the truncation error – means you are calculating… When you are plotting those – the 2 theta or theta; and then, if you are calculating where is the tan theta of that or cos theta or sin theta; then, it contains some types of error. And we cannot go beyond certain 2 to 3 digit. And then, what we are doing here? We are summing up all the required digits of that. And that is why, sometimes these assumptions gives not exactly the answer, because analytical results always gives you the perfect answer.

But, from graphical solution, as it is pretty standard thing is there, the graphical solution will not give you the exact solution. So, in that case, what we need to do? We need to restrict ourselves that, actually if we have limited values of sigma x, sigma y and tau xy; or, if we have the direct values like 50 Newton, this 50 kilo Newton per millimeter square; or, if we have the 20 kilo Newton per millimeter square or whatever like that. If this kind of the exact values are there of any sigma x, sigma y or tau xy, it is pretty easy for us to get the required values and to draw the Mohr's circle. So, this is the basic – I should say the requirement of the Mohr's circle.

So, in this lecture, we found that, the Mohr's circle is also the perfect solution for the stress calculation if we have the sigma x, sigma y or tau xy or in the separate way or in this combination way. So, in this chapter, we discussed about those things. And in the next lecture, we just want to do some problems that, actually if you have these values, then how to first draw the Mohr's circle. And if we have the required values – whatever, how to calculate the desired value – first; and second – if we have the Mohr's circle, then how to extract the information about the shear stress or the normal stress component.

So, this kind of the discussion, which we are going to discuss in the next class that, the numerical problems – some 4-5 problems are there of the different values of the sigma x, sigma y and tau xy as well as the different rotation; that actually if it is rotating by this angle, then in the clockwise or counter clockwise of the sigma x, sigma y or tau xy – shear stress. If it is rotating like that, then what is the resultant is there and what is the combined effect of those things are there.

Thank you.