Strength of Materials

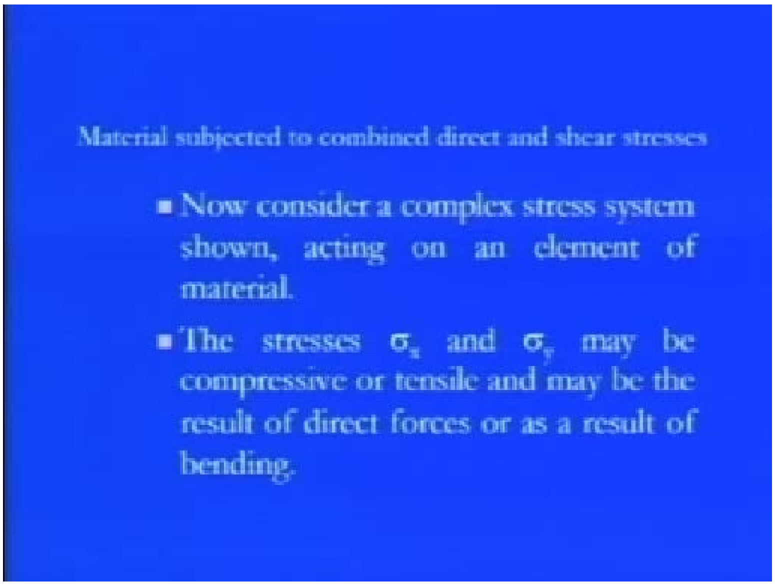
Prof. Dr. Suraj Prakash Harsha
Department of Mechanical and Industrial Engineering
Indian Institute of Technology, Roorkee

Lecture – 5

Hi, this is Dr. S. P. Harsha from Mechanical and Industrial Engineering Department, IIT Roorkee. I am going to present the lecture 5 which is basically you know like of the strength of materials subject which is basically on how the stresses are being set up if the different kind of stresses are being induced within the system, or the different variety of forces are being applied because in the previous lecture, we found that if the material is under the influence of pure normal stress or if it is under the influence of pure shear stress or if two mutually perpendicular stresses are there of the normal stress, then how we can get stresses at the oblique plane like the sigma theta or tau theta.

So, how we can get those things and what kind of formulations are there, we can easily find it out those things. In that you see like in the previous case as we found that only the pure shear stress or the pure normal stress cases were there which we discussed, and because of that you see the sigma theta or tau theta were there, but now you see we are going to discuss that if these mutually perpendicular planes or the axes if you see the combination of both shear as well as the normal stress components are there, and we just want to check out the sigma theta as well as the tau theta. If both means the tau x as well as the sigma x is there.

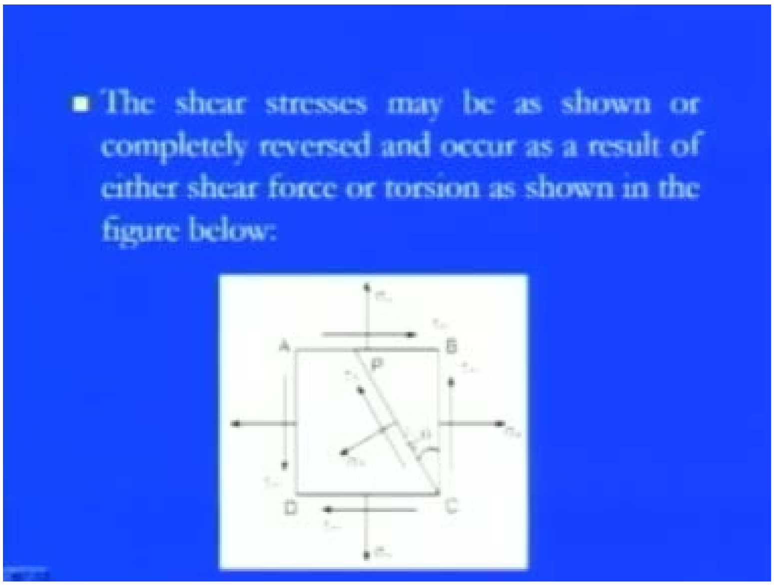
(Refer Slide Time: 01:56)



Now, you see you know like this slide if the material is subjected by the combined direct and stress as the shear stress is there, so we are going to consider again the same unit cube which is like all three mutually perpendicular stresses are there, and on these axis stresses, we have sigma x, sigma y, sigma z is there tau xy, tau yz and tau zx is there, but to make the simplicity in that case, the system is more complex and it is pretty difficult to analyze those stress. The stress you know like the values if we have all those nine components because of the tensile stress. So, to make the simplicity, again we are considering that we have the same element of the unit length, unit depth and unit width of the unit cube and you see this xy plane is there. That means, we are going to concern about the plane stresses in normal as well as the shear stress.

So, if we have the stresses sigma x and the sigma y in the x axis, let us say if it is either may be you see the compressive nature or tensile nature and may be of the result of those stresses if we have the bending because in the general bending case, if we have this beam and the forces are applying, then we found that you see at certain places, we have the tensile and at certain places, we have the compressive nature. So, you see in the bending, always we have combination of both tensile as well as these compressive stresses. So, we would like to analyze those cases here. So, now here you see we have the same element which we have discussed.

(Refer Slide Time: 03:31)

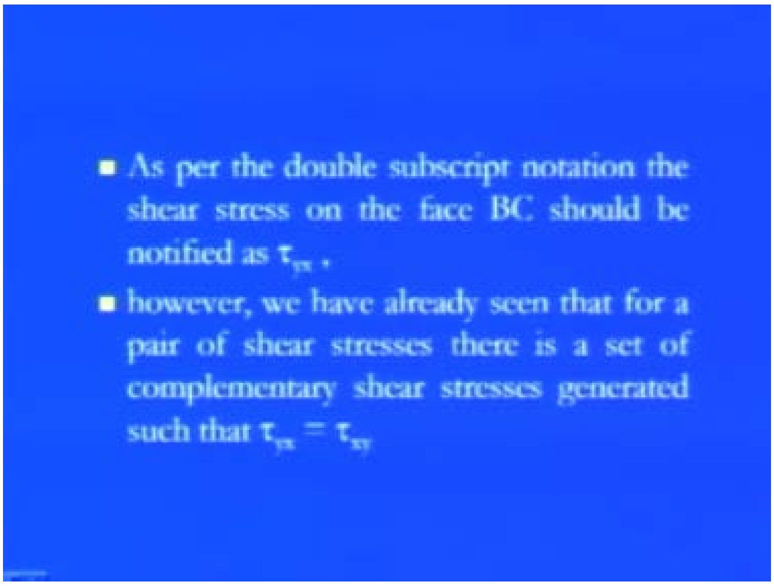


In the plane, xy plane you see this x and this is y on the AB AD side and the BC side, you know like we found that we have the sigma x component which is exactly perpendicular to the plane. We have parallel forces and because of those things we have the shear stresses. So, tau yx in this direction and tau yx, tau xy in this direction, we have both. As you know like the complementary shear stresses and if you are concerning about the AB and CD, we found that the both stresses are there. Both are mutually perpendicular. This is perpendicular to this AB plane and that is the normal stress component. That is known as sigma y and parallel to this AB plane, we have shear stress component that is tau xy. So, in the opposite direction means you see under the influence of these two types of stresses which are mutually perpendicular, this system is under equilibrium position.

So, now you see here we just want to concern that if the metal is being subjected by both, shear as well as the normal stress component, then what will be the stresses at the oblique plane. Now, we are cutting the plane at an angle theta and the plane is rotating here in this diagram by PC. So, at this plane now we would like to calculate that sigma theta and tau theta is if this plane is there, this is my PC. So, what will be perpendicular you know like the forces or the stress component is there. That is the normal stress component denoting by sigma theta what the parallel system is there. That means the parallel forces are there, parallel stresses are there that is known as the tau theta. So, we would like to calculate those things.

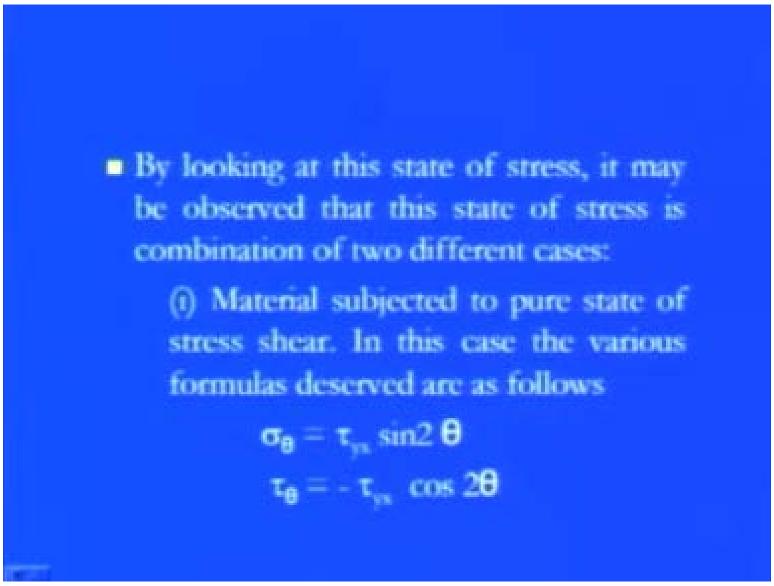
So, for that as per you know like double the subscription, what I said notation is there, the shear stress on the phase BC always B is tau xy or tau yx. Because if you are applying the tau xy, always there is complimentary shear stresses which has to be applied on the element, so that we can counter balance those stresses, and we can say that now under the influence of either tau xy or tau yx, if you see in the previous figure these either the tau xy or tau yx, they have to be counter balanced to each other and then only we can say that this object or this element is well equilibrium under the influence of the shear stresses. There is no movement, there is no rotation is there in these kinds of elements.

(Refer Slide Time: 05:46)



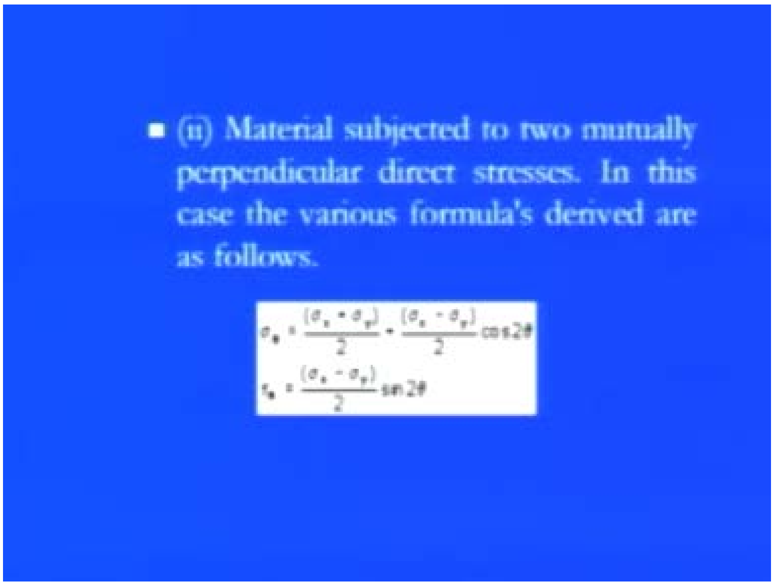
So, here you see you know like for simplicity or we can say for symmetricity because of the nature of the geometry or we can say the nature of the point of force, we have tau xy equal to tau yx. That means always we are notating normal stress by sigma x, sigma y, sigma z and always we are notating tau xy, tau yz and tau zx if we have plane stresses or if we have you know like the symmetricity in the geometry.

(Refer Slide Time: 06:25)



So, by looking at these states of stresses in the previous case, it may be observed that the state of stress is the combination of two different cases as we have discussed already. If the material is subjected to the pure state of stress shear means if the shear stresses are there, then at the oblique plane, we have sigma theta which is equal to tau xy or tau yx whatever it is because both are equal in symmetricity into sin of 2 theta, and this tau theta which is the shear stress at an oblique plane is equal to minus of tau xy or tau yx, both are equal into cos of 2 theta. That means you see here we have both the stress component which is influenced by the shear stress into the component of the cos or sin into the different category of the normal as well as the shear stress, if it is subjected to the pure state of shear stress.

(Refer Slide Time: 07:15)

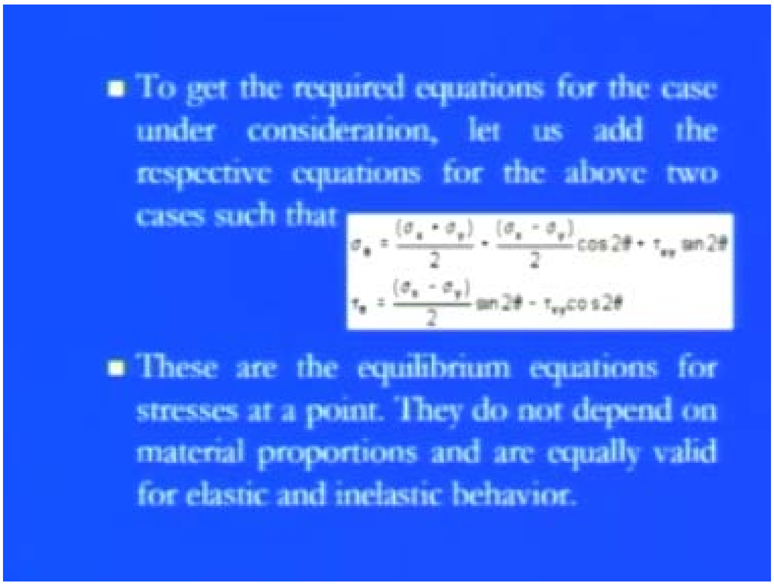


The materials subjected to two mutually perpendicular direct stresses which we discussed in the third case. So, you see the previous case was discussed in the second case of the previous lecture, and this was the third case of the previous lecture when two mutually perpendicular direct stresses are there. And if they are acting on the different sides of the element, then how we can calculate the sigma theta and tau theta at the oblique plane that has already been discussed.

So, we can say that sigma theta, if two mutually perpendicular stresses are there at sigma x in the x direction, sigma y in the y direction, then the combined effect of these two normal stress components on the normal stress components at the oblique plane equals to sigma x plus sigma y by 2 which is not influenced by any angle theta plus sigma x minus sigma y by 2 into cos of 2 theta. So, that will give you the normal stress component at the theta, and the shear stress component at the oblique plane can also be calculated by sigma x minus sigma y by 2 into sin of 2 theta. So, you see in both of the cases, we have already discussed that actually at which point we have the maximum value of a normal stress or maximum value of shear stress or minimum value of normal stress, and the minimum value of shear stresses.

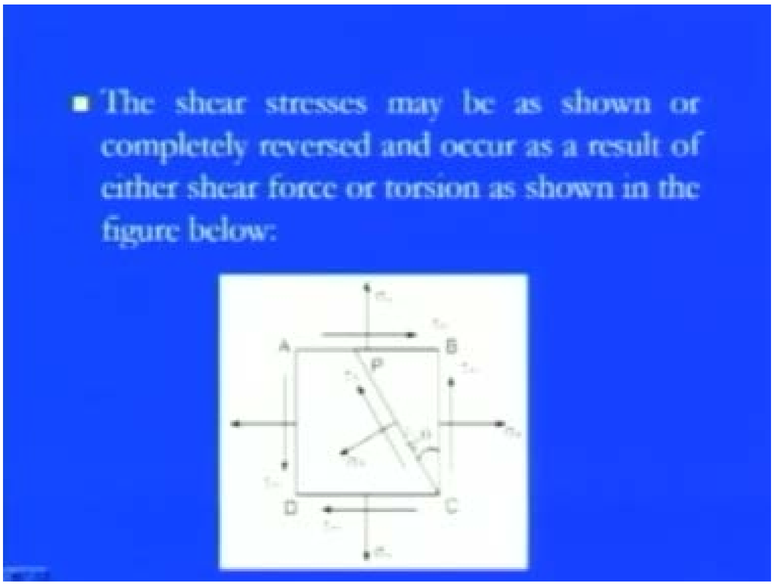
So, these are the real cases through which we can get that actually if a material is subjected by pure shear or pure normal stress, or two mutually perpendicular of the normal stresses. Then, what are those locations where we can get the maximum stresses under the influence of these forces at the particular area. So, that is the cases, those cases has been discussed, but here you see this case which we are discussing here that if we have an object which is under the influence of two mutually perpendicular stresses, sigma x and sigma y with the shear stress, so the combination of all these stresses are there.

(Refer Slide Time: 09:06)



So, we can add both the cases and we can get the value means you see to get the required equation for the case under consideration in which we have an element.

(Refer Slide Time: 09:10)



If you back to the figure. If you go back to the figure, you found that this case where you see the sigma x is there in the x direction, sigma y in the y direction, so we have two mutually perpendicular normal stresses and on the same side, we have the shear stresses. So, you see only we need to compile the previous work which we have already being derived. So, if you compare those two, get the required equations for this kind of case which we discussed recently under consideration.

Let us add both the respective equations, which we discussed. That means, if we have a pure shear, then this equation is there of the sigma theta and tau theta. If we have two mutually perpendicular direct stresses, then we have sigma theta and tau theta of the values. So, if we combine those things, then we have the sigma theta and tau theta value when the material is subjected by two mutually perpendicular direct stresses with the shear stress component which is parallel to the active plane. So, you see sigma theta is nothing but equals to sigma x plus sigma y by 2 plus sigma x minus sigma y by 2 into cos of 2 theta. This component is coming because of the two mutually perpendicular stresses of direct stress.

Sigma x and sigma y and plus tau xy sin of 2 theta which this component is coming because of the direct stress, this pure direct, pure shear stress component. So, we have the combination of these things. So, sigma theta is equal to sum of these two components which is sigma x plus sigma y by 2 plus sigma x minus sigma y by 2 into cos theta plus tau xy sin of 2 theta. Similarly, you see we can also configure this shear stress at oblique plane, that is tau of theta is equal to sigma x minus sigma y by 2 into sin of 2 theta. This is being influenced by two mutually perpendicular normal stress components. So, sigma x and sigma y is there.

So, this is the contribution of that minus tau xy cos of 2 theta. The minus is always there because as we have discussed that there is a complimentary shear stress which always exist or occurs in an object because to counter balance, this object to make in a equilibrium position. So, we have you know like this is the contribution due to the pure shear stress. So, tau xy cos of 2 theta is that component. So, we have the combined effect. So, we can get sigma theta and tau theta at the oblique plane. If this object is being you know like subjected by pure shear as well as two mutually perpendicular, these normal stress components are there.

So, these are the equilibrium equations for the stresses at a particular point Q let us say, and they do not depend on the material portions. That is the key feature and they are equally valid for elastic as well as the plastic behavior. So, you see this is a great phenomena which we got that actually that this is more of the realistic situation that if we have a material and if material is being subjected by three normal forces, or we can say three main components like sigma x, sigma y and tau xy. Then what are the real combinations of the that resultant stresses are there in this sigma theta as well as the tau theta.

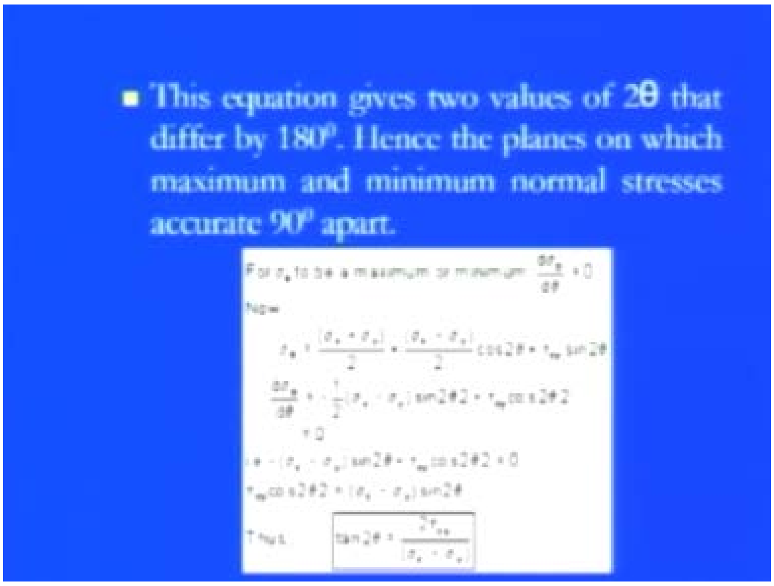
So, we can simply compute those things, and we can get the final results in terms of sigma theta and tau theta which is highly immaterial to you know like that what material which we are using whether it is this high steel, high carbon steel you know the mild steel because it is a ductile material. They have the ductility properties. So, if you are pulling those things, they will exhibit all those things and they will show this kind of relation. Again if we are pulling those things, then whether it is going in the elastic region or plastic region, it does not matter.

So, this is a key feature here in these kind of you know like the stress components that actually whether we are considering the pure stress, normal stress, shear stress or two mutually perpendicular. This normal stresses are there, or the combined effect is there. We have all those components at the oblique plane which is immaterial to the material proportion as well as it is valid for elastic as well as the inelastic behavior. That means elastic and inelastic means you see if we are talking about the elastic material, whatever the force is applying to an object, they have you see you know like the stresses are there and the stresses if whenever we apply the force, there is a kind of deformation.

So, stress is proportional to the applied deformation or I should say because we are measuring the deformation by one technical term which we are going to discuss is strain. So, stress is proportional to strain and you see we can define another you see you know like this Young's modulus of elasticity which is a material of property which is absolutely based on which material is there. So, for elastic region or inelastic means if you are going to the plastic region, where the permanent set of deformation is there and in that also, if these you know like the forces are there and these forces are well setup within this structure, we can get the stress component sigma theta and tau theta at the oblique plane even in that region also.

So, you see here if you are talking about the plastic region or elastic region, it does not matter if it is in high steel, high carbon steel, mild steel, it does not matter. These equations are valid in all these kind of stresses.

(Refer Slide Time: 14:17)



Then, you see the above equations you know like gives the true value of theta 2 theta differ by 180 degree because at 180 degrees, these signs, these all are containing sin of 2 theta or cos of 2 theta. So, they always vary by 180 degrees. Hence, the plane on which maximum and minimum normal stresses are accurate, just they are existing. They are accurately 90 degree apart from that. So far you see know if you see this one.

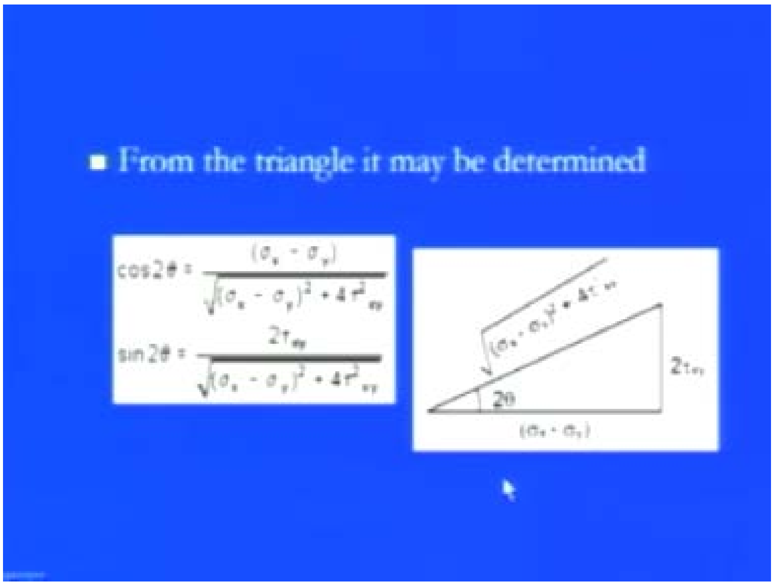
So, to calculate the sigma theta for the maximum or minimum value, a pretty standard technique is there to get the maximum or minimum value of any function. We need to differentiate those things and equate to 0 to get the value of those things that at particular what location we are getting this sigma theta is minimum or maximum by plus minus, and put those values into the main function and you can get the exact value of those things. So, this is pretty standard you know like the phenomena which we can also apply here to get the value, the minimum or maximum value of sigma theta.

So, here you see the same process which we are following here differentiate those things, sigma theta by d theta equate to 0. So, if we have the sigma theta which is sigma x plus sigma y by 2 plus sigma x minus sigma y by 2 into cos of 2 theta plus tau x tau xy cos of 2 theta, so if we are you know like computing all these things, you are differentiating.

So, d sigma theta by d theta if you differentiate, then you have minus half because this is gone and there is no influence by theta. So, this has gone. Now, this is you see minus of 2 will come. So, minus half sigma x minus sigma y sin of 2 theta into 2 theta is 2 plus you see here tau xy cos of 2 theta into 2. So, you see if you equate to 0, then we have minus of tau x minus tau y sin of 2 theta plus tau. This tau xy cos of 2 theta into 2 equates to 0, or we have you see if you divide sin theta divided by cos theta, then you have finally tan of 2 theta is two times of tau xy divided by sigma x minus sigma y. That means here 2 theta is equal to 10 inverse of two times tau xy divided by sigma x minus sigma y.

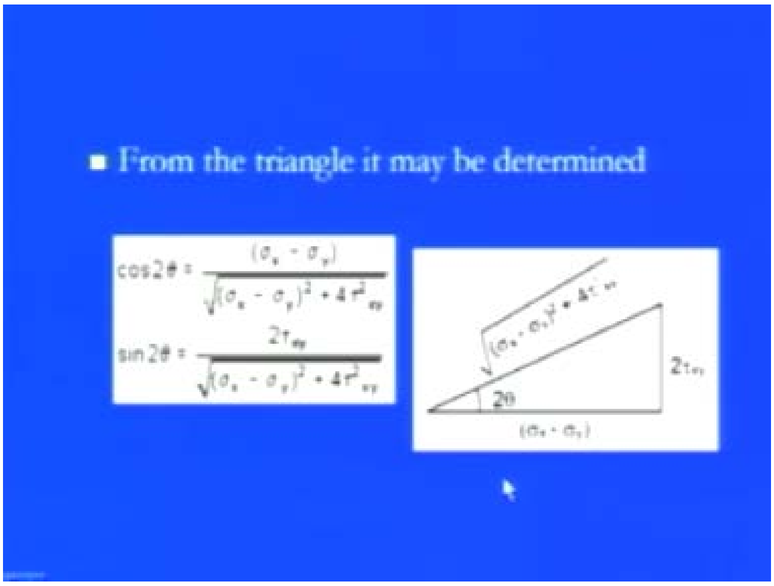
2 theta is a plane where you can get the maximum value of this normal stress component. That means you see here now this is the location 2 theta, where actually within this structure, we have some values, maximum values of this normal stress component. So, this is you see the pretty standard theory through which we can calculate that what is the location of the theta. 2 theta is there through which at particular we can get the maximum value of sigma theta.

(Refer Slide Time: 16:55)



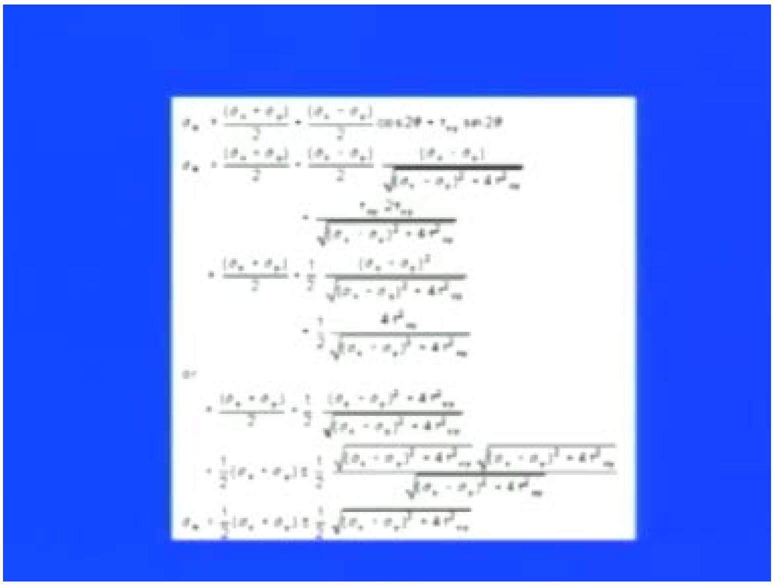
Now, you see from the triangle, we can again you know like determine those things that this is 2 theta, where we can get those things. So, it is the combination of sigma x minus sigma y. This is the combination of 2 xy, because you see if you go to that, this tan theta we have you know like this perpendicular divided by this particular you know like the base.

(Refer Slide Time: 17:13)



So, you simply put this. This is my base, this is perpendicular. So, hypotenuse is nothing but applying the Pythagoras theorem and we can get those things. So, it is nothing but equals to the square root of sigma x minus sigma y square plus 4 into tau xy, this one. So, now you see here we have the full triangle, we have all the values. So, we can get cos, this cos of 2 theta sin of 2 theta where we have the maximum value of direct stress component sigma theta. So, these are the values of cos of 2 theta sin of 2 theta. So, we can simply put those values in the main frame of the equation to get the maximum value because now we know the location, we know the value at this particular location. You just put in the main equation and get the final form of those things. So, you see here there is a pretty standard you know like this process is there.

(Refer Slide Time: 18:01)



So, sigma theta is nothing but equals to what we have discussed is sigma x plus sigma y by 2 plus sigma x minus sigma y by 2 into cos of 2 theta plus tau xy sin of 2 theta. Now, you see in the previous case, we have cos of 2 theta which is sigma x minus sigma y divided by this hypotenuse sin of 2 theta is two times tau xy divided by this hypotenuse. So, through this we can get these values here.

So, now apply these things here. Sigma x is nothing but equals to this. Sigma theta is nothing but equal to sigma x plus sigma y by 2 plus sigma x minus sigma y by 2 into cos of 2 theta is this sigma x minus sigma y divided by this hypotenuse. Ok plus you see here we have tau xy into this like the tau xy because of this cos theta. So, tau xy is as it is this. We need to replace this sin of 2 theta. So, sin 2 theta will give you two times of tau xy divided by square root of sigma x minus sigma whole square plus four times of tau x square tau square xy. So, now you see we have the total resolution of those things because we are applying sin of 2 theta cos of 2 theta here.

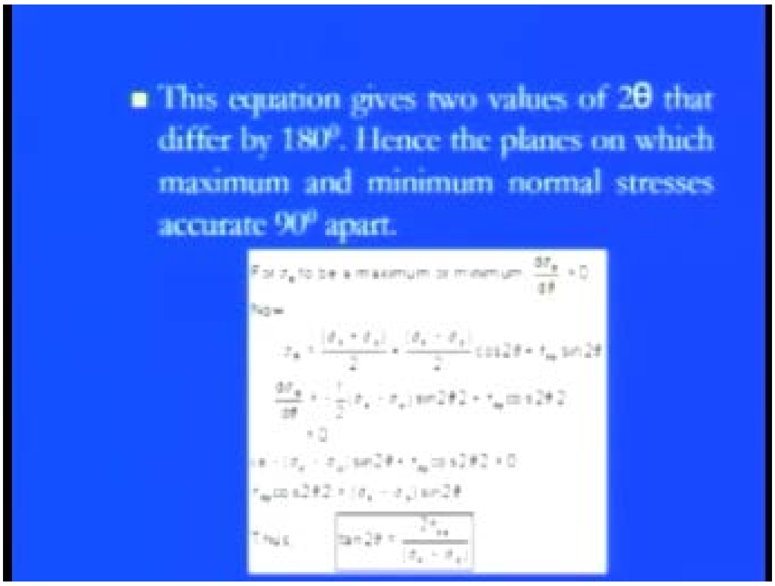
So, at the end you see what we have just by equating those things. We have sigma x plus sigma y by 2 which is highly independent of the cos theta or sin theta plus half of these things. So, now you see if you resolve these things, then we have this sigma x minus sigma y by into square. So, multiply those things and we have this in denominator, we have this hypotenuse form plus half of four times of tau square xy divided by this hypotenuse form.

So, even if you resolve these things, then we have sigma x plus sigma y by 2 plus these if you sum up because hypotenuse is same. So, we have sigma x minus sigma y whole square plus four times of tau xy and divided by square root of this. So, you see like the square form is there, we can simply you know like cancel one form. So, at the end you see we have the sigma theta which is the combination of those values, where sigma theta means sigma theta differentiation and divided by d theta equals to 0, and we have this location tan of 2 theta which was two times of tau xy divided by sigma x minus sigma y.

So, if you locate those things, you will find that we have the final value of sigma theta. This is the maximum value of sigma theta is sigma x plus sigma y by 2 plus minus half of square root of sigma x minus sigma y sigma x minus sigma y whole square plus four times of tau square xy. That means you see if you want to calculate the maximum value of sigma theta, you need to consider the plus sin and if you want to calculate the minimum value of sin, this sigma theta means the normal stress component. You have to be chosen of the minus value.

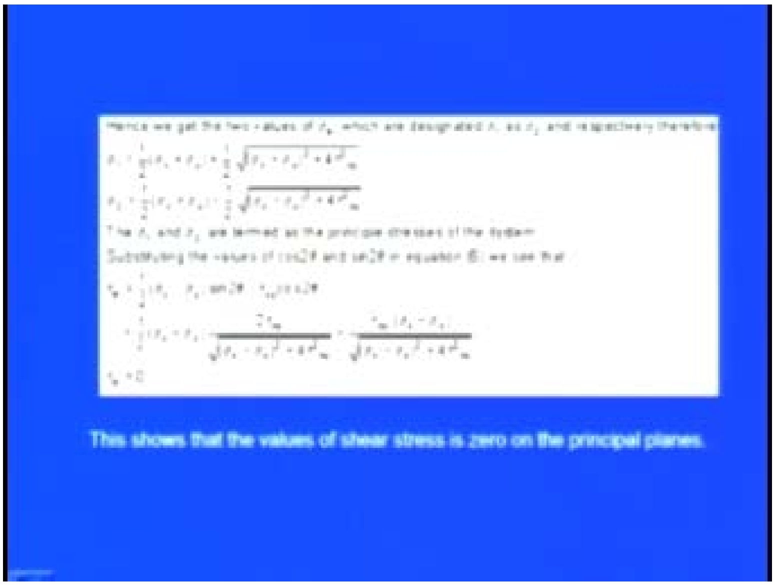
So, these you see this formula is very important while you see when a material is subjected by two mutually perpendicular stresses, the normal stresses sigma x and sigma y, and the same time the shear stresses also being influenced there. So, we have all three combinations and what is sigma theta is there at the oblique plane. This is the perfect formula that sigma theta is maximum value of sigma theta has maximum value you see and the location is also there. This is the value is there, the amplitude is there. So, we can get that at four particular locations. This value will occur the maximum if you consider sigma x plus sigma y by 2 plus square root of sigma x minus sigma y by 2 square plus four times of tau square xy divided by 2.

(Refer Slide Time: 21:20)



So, where we will locate? So, you need to again go back, and you can see that actually that if this 2 theta is plus half tan inverse two times of tau xy divided by sigma x minus sigma y, so it will give you the plus value. That is what you see you know like in the previous slide, we have already discussed that actually the equation will give you two values. So, at one value you see we have the maximum. The value of sigma theta at another value, we have the minimum value of these and it is apart in by 90 degrees. So, this is a part which we have discussed.

(Refer Slide Time: 21:48)

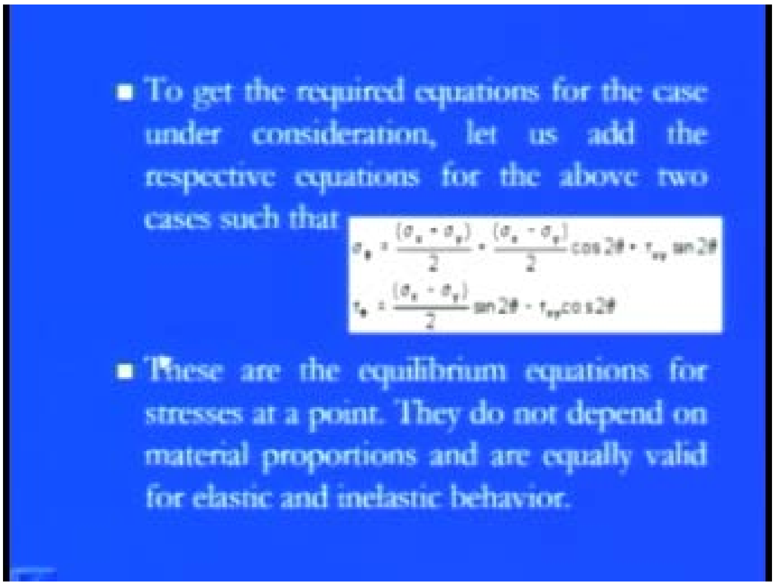


So, now you see if you want to get the two values of sigma theta as we discussed, this is being designated by the special term that is sigma 1 and sigma 2. So, sigma 1 is the maximum value of sigma theta, and sigma 2 is the minimum value of sigma theta as we discussed. So, sigma 1 is equal to as you know like we discussed that here also. Sigma x plus sigma y by 2 plus half of square root of sigma x minus sigma y by sigma x minus sigma y square whole bracket square plus four times of tau square xy, or sigma 2 which is the minimum value which is nothing but equals to sigma x plus sigma y by 2 minus half of square root of sigma x minus sigma y whole square plus four times of tau square xy.

So, these two you know like the different values are there, though you see they are occurring at the different locations that you see another difference is these vector you see. We have you see you know like in those things, we have the influence of sigma x, we have the influence of sigma y, we have the influence of tau xy. So, through this you see we can simply calculate sigma n, sigma 1 and sigma 2, and these sigma 1 and sigma 2 terms are known as the principle stresses of the system. That means you see now we have the new term, the principle stresses. Till now you see we discussed about the stresses that is the normal stress. One is the shear stress and another is the normal stress.

Again you see there were derived stresses are there, the tensile stress, compressive stress, the bending stress, the bearing stress or even you see the torsional stresses are there, the shear stress or even the thermal stresses are there, but now you see this is the new term of the stress that is the principle stresses, and they are occurring at a particular location that is 2 theta was there tan inverse of 2 sigma tau times of xy divided by sigma x minus sigma y. So, these principle stresses are always having two values sigma, this sigma 1 and sigma 2. If you see we have all three different sigma x, sigma y and tau xy stress components are being influenced.

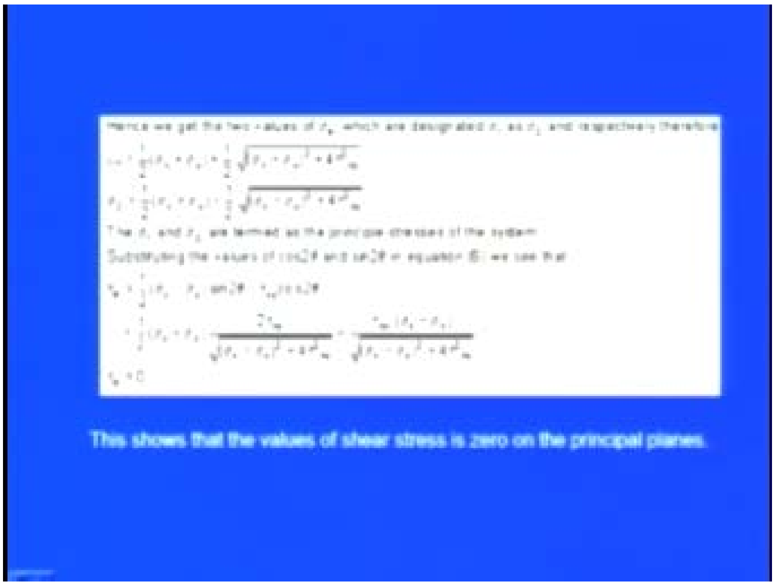
(Refer Slide Time: 24:00)



So, if you substitute those cos 2 theta and sin 2 theta into the equation, then we have tau 2 theta again. So, again you see whatever we discussed here cos 2 theta and sin 2 theta, now if I would like to replace in the tau theta means here in this tau theta. Now, you see here we have sigma x minus sigma y by 2 if i want to replace this sin 2 theta minus tau xy into cos of 2 theta.

Now, if I just want to replace here because you see here this 2 theta will give you the location, where the maximum direct stress is there or the minimum direct normal stress is there. So, now I just would like to check that actually the location where the maximum or minimum normal stress is occurring. What is the value of the shear stress? So, now this is beauty thing you see here that we have different things here. That means, all those stresses are mutually coordinating to each other, mutually interacting to each other and we would like to find out that actually what the principle stresses are there. So, principle stresses we have already calculated by putting those values.

(Refer Slide Time: 24:42)

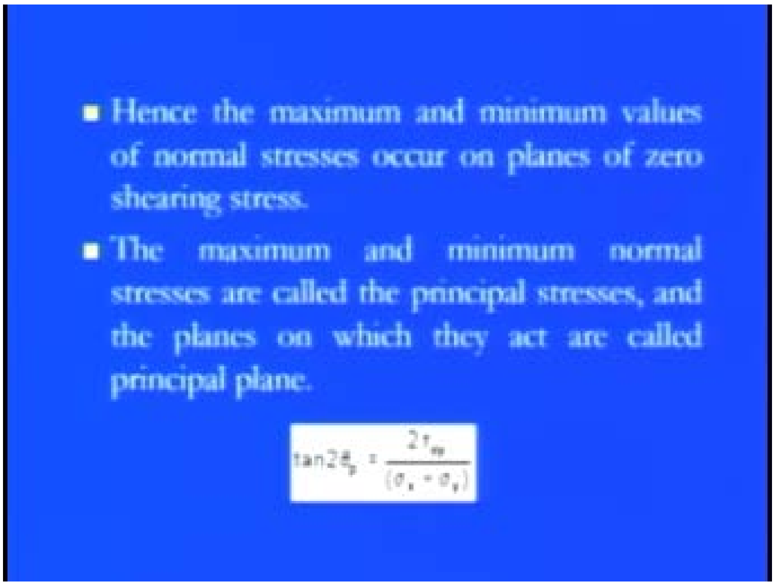


Now, I would like to check out that what the shear stress is there at that point because principle stresses are the form of the normal stress component. Sigma x maximum was there, sigma x minimum was there. So, now here you see if you put those values there in tau theta, you have half of sigma x minus sigma y sin of 2 theta which was the previous formula minus tau xy cos of 2 theta. So, if you put those values here, sin 2 theta or cos 2 theta. Now, you see that actually they are exactly equal. They will simply you know like two times. 2, 2 will cancel out. So, tau xy sigma x minus sigma y divided by this hypotenuse minus tau xy sigma x minus sigma y divided by this hypotenuse or we can say tau theta is 0.

What is the physical significance? Physical significance is that there are some of the locations, or we can say the planes are there. Some of the planes are there where the principle stresses are occurring means where you see the maximum or minimum normal stresses are there. At these locations, there is no shear stress means you see wherever the principle stresses are occurring because these principle stresses are there. The minimum or maximum, this normal stress component, the principle stresses at these particular location, there is no shear stress and this was that actually you know like the real you know like a meaningful output that always if we are concerning about the principle planes, there is no shearing. Only direct stress components are there.

So, if we are discussing even in the future also that this principle stresses are there, sigma 1 sigma 2 or the principle planes are there, you have to put the shear stress 0 or by putting shear stress 0, we can calculate easily the principle plane locations. So, this is you see the beauty of this analysis.

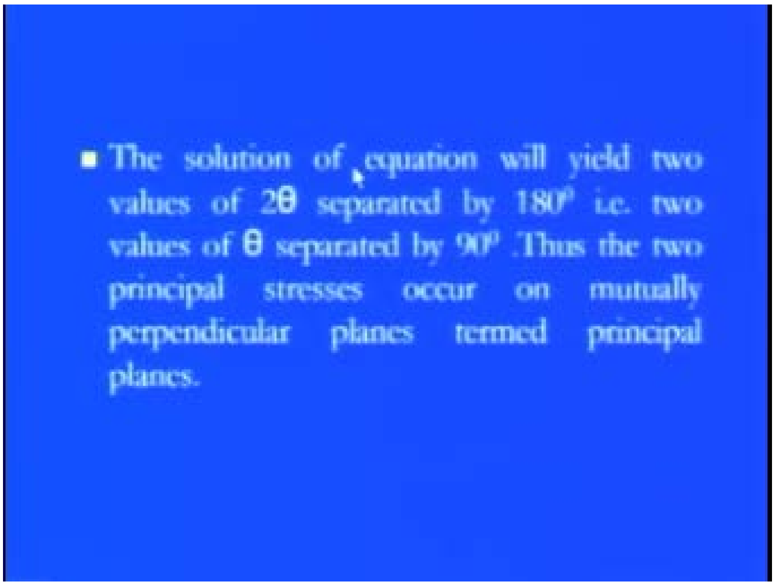
(Refer Slide Time: 26:38)



Hence the maximum or minimum values of normal stresses occur only on those planes where there is no shear stress, and these planes are known as the principle plane. So, the maximum and minimum normal stresses are called the principle stresses sigma 1 and sigma 2, and these can be calculate easily by you know like sigma x plus sigma y by 2 plus minus square root of sigma x minus sigma y whole square plus four times of square you know tau square xy divided by 2. So, this can be calculated, and these are known as the principle stresses and the planes on which these stresses are acting, these planes are known as the principle plane and these principle plane locations we can easily calculate that is the two times of theta p equals to tan inverse of two times tau xy divided by sigma x minus sigma y.

So, this is from this particular formulae, we can simply get that what are the principle location of the principle planes within those object. If the object is subjected by two mutually perpendicular normal stresses sigma x and sigma y at the same time, the shear stress influences there by tau xy. So, this is you see even to locate this thing, the location 2 theta p, we have all the influencing quantity sigma x, sigma y, tau x or even if you want to calculate the principle stresses, the principle stress components even if the maximum or minimum you have, all those you see sigma x, sigma y and tau is there in these slides.

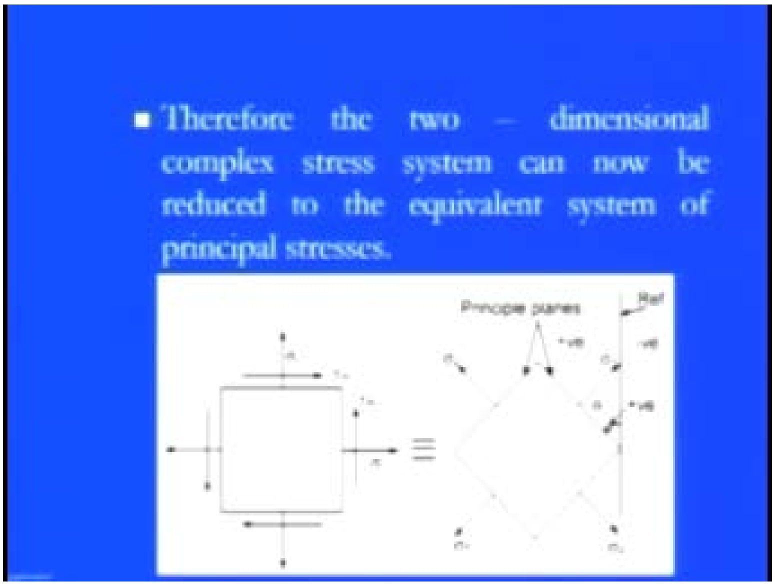
(Refer Slide Time: 28:05)



So, we can say that actually you know like to calculate the minimum or maximum, these values always influence the principle stresses are there and at those things, we have shear stress. There is no shear stress component is there. That means, the shear stresses are always 0 at the principle planes. The solution of the equation will yield two values of theta, 2 theta as we discussed separated by 180 degree, that is the two values of theta is separated by 90 degrees. Thus, the two principle stresses occurs at mutually perpendicular planes termed as the principle planes.

Again the important significance of this analysis says that we have two different planes, and both planes where the sigma 1 and sigma 2 are occurring, they are always mutually perpendicular to each other, and you see even if like we have discussed that actually these sigma x and sigma y was like they were acting at two mutually perpendicular axes. Similar things are there. Principle planes are always mutually perpendicular and you see corresponding principle stresses are always mutually perpendicular. Stresses are there which are termed as the principle stresses and the principle planes. So, this is you see which we have discussed, and we just want to configure into the real figure.

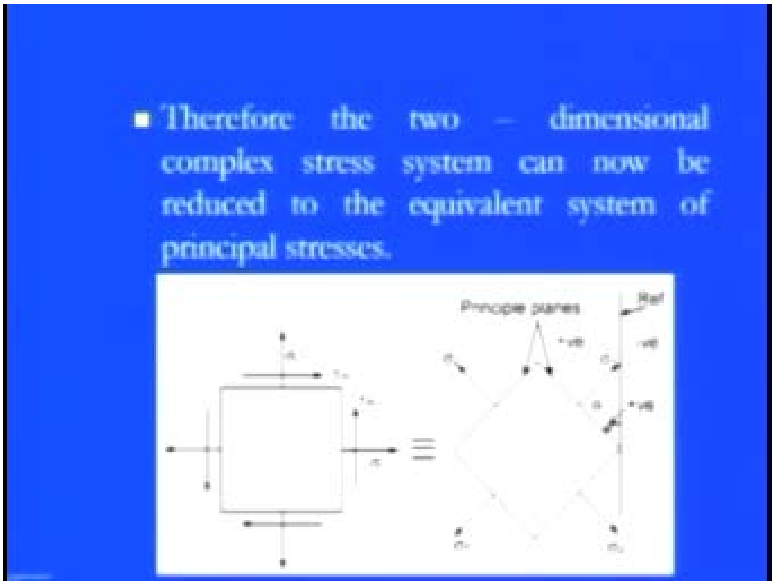
(Refer Slide Time: 29:16)



Therefore, the two-dimensional, the plane stress components where you see the complex stress systems are there now can be reduced to the equivalent systems like in the principle stresses. Now, you see pretty simple formula is there. We have the principle, the plane stresses are there on this component where xy is there. So, in the x and y, we have sigma x, sigma y is there, tau xy is there. This is the real combination of two mutually perpendicular stresses and the shear stress component. These are the complimentary shear stresses are there just to counter balance the system.

So, this is a real complex system in which the stresses are being well set up. So, now we can simply by rotating at you know like these 45 means the 2 theta that is by rotating at the 90 degrees. Just you see you know like we have two principles planes are there. So, these are the principle planes, where sigma 1 and sigma 2 are acting, and as I told you these both principle stresses are mutually perpendicular. So, you see here they are exactly at 90 degree of angle. So, mutually perpendicular stresses are there and these planes are principle planes. The key feature is that at these planes, there is no shear stress while here you see these shear stresses are there, the counter complimentary shear stresses are there, but these are the planes where only this direct stresses are there, and we can calculate the sigma 1 and sigma 2 by this formulae. This sigma 1 and sigma 2 are there.

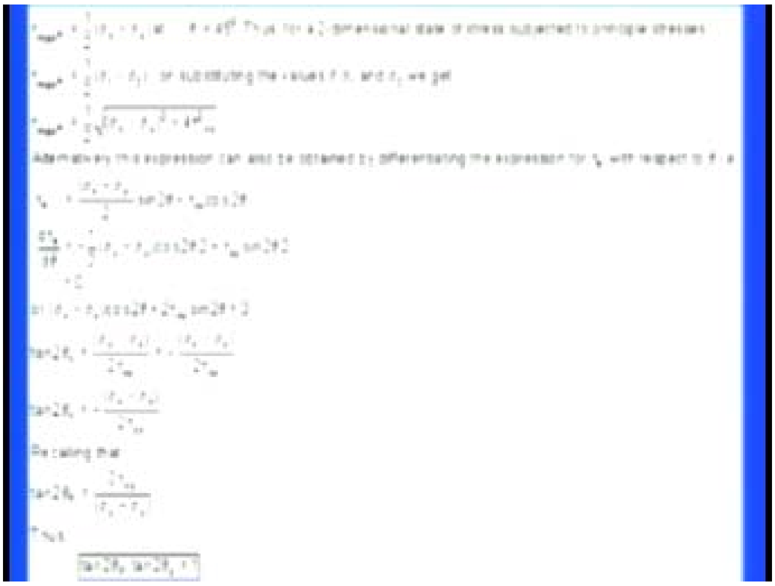
(Refer Slide Time: 30:45)



So, it can be easily you know like find the magnitude of these things, and if you want to calculate the theta, this theta you see where you know like what is the location, then we just want to go you see this 2 theta p is 2 tan inverse of 2 tau xy minus divided by sigma x minus sigma y. So, we have seen that for calculating the location, where the maximum normal stress components exists, or we can say the principle stresses are existing. Always all three mutually perpendicular stresses or we can say all three forms of the stresses, sigma x, sigma y or tau xy are equally responsible. That is why we have seen that you know like if you want to calculate theta p that was you see this one is theta p, where this principle stresses are occurring like sigma 1 and sigma 2.

So, they are you know like they are coming out from the responsibility of sigma x, sigma y and tau xy as we have discussed in the previous formula, and these are you see the positive principle planes because they are exerting in the positive rotation of theta from the reference plane. If you are going towards the right side of the reference plane, then we found that actually the principle planes are there, but they are in the negative directions. So, you see this is the simplest representation of the principle stresses which is equivalent to the complex stress form of two mutually stress sigma x and sigma y, and this shear stress is tau xy.

(Refer Slide Time: 32:17)



So, you know like if you want to calculate the two-dimensional state of stress which is subjected by the principle stresses, we can simply choose the theta which is exactly at the 45 degree as we have seen the previous figure that actually if it is exactly at the 45 degree theta p. That means you see like both the sigma 1 and sigma 2 are parting from the 90 degrees which is two theta part, where you see like this tau maximum is equal to sigma x minus sigma y by 2 because the cos will be equal to 0, cos of two theta equals to 1.

So, it will give the maximum value of the shear stress or we can say tau maximum is equal to sigma 1 minus sigma 2. By 2 we are talking about the principle plane. That means it is pretty configured that if we are talking about two mutually perpendicular principle stresses, we can get the maximum shear stress by subtracting sigma 1 minus sigma 2 divided by 2. We can get easily those value, and we can say that actually tau maximum which is also equal to half of this hypotenuse form was there. That means, square root of sigma x minus sigma y whole square plus four times of tau xy square means these are the different forms of maximum shear stresses, and they are occurring at the different locations. Only the key feature is that actually who is responsible for what.

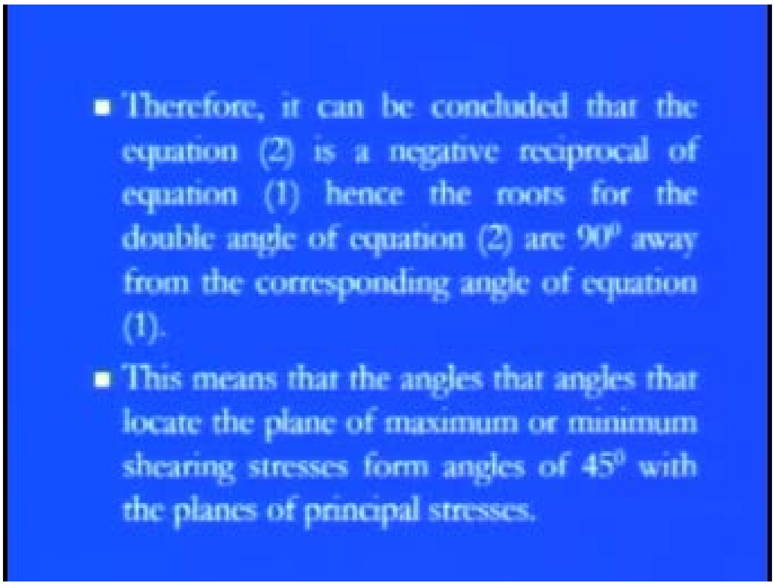
That means you see if I am saying that shear stress is there because of only two mutually perpendicular stresses, then you see tau maximum is sigma x minus sigma y by 2, but if I am saying that because of the principle stresses, we have the shear stress form you know like, then we can say that tau maximum is sigma 1 minus sigma y by 2, and if I am saying that both mutually perpendicular stresses are there, simultaneously the shear stresses also being acted on the parallel axis. Then, the tau maximum at a particular oblique plane we can say that this is half of that square root of sigma x minus sigma y whole square plus four times of tau xy square. You know like alternately if this expression can also be obtained by differentiating the expression tau theta with respect to theta as we have seen. In the previous case, we have sigma theta was there. We simply differentiated with d theta equate to 0 to get the value of different planes, where you see you know like the principle planes were there, your shear stresses were 0.

So, here you see you know like again we just want to calculate that part. So, here the tau theta which we have chosen here that sigma x minus sigma y by 2 into sin of 2 theta minus tau xy cos of 2 theta, so if I just want to differentiate, then after differentiating, we have minus of half you know like these because this cos of 2 theta is there. So, it will come minus out of that. So, minus half sigma x minus sigma y cos of 2 theta into 2 because the 2 theta plus tau xy sin of 2 theta into 2 because of the 2 theta. So, if I equate to 0, then I have the tan of 2 theta x because you see here the shear stress is 0. Earlier you see you know like the normal stress component was 0, so 2 theta p was there.

So, it is here tan 2 theta x which is equal to sigma x minus sigma y divided by two times of tau xy, or I should say that actually sigma x because if I want to take the minus sign, the minus sign was there because of the cos of 2 theta. So, we have minus sign sigma x minus sigma y divided by two times of tau xy. We can say that in the previous case which we have discussed tan of 2 theta p, where you see the normal stress component was maximum tan of 2 theta p was two times of tau xy divided by sigma x minus sigma y, where the shear stress by differentiating shear stress if we want to get what is the plane is there, where we can get the maximum shear stress, that is tan of 2 theta x equals to you know like either you can say minus sigma x minus sigma y divided by two times of tau xy.

So, if we multiply those things, then we have the final equation tan of 2 theta p into tan of 2 theta x always give you 1. That means, here tan of 2 theta x is equals to the 1 divided by tan of 2 theta p means both are mutually perpendicular. That means they are in the reciprocal relationship. That means if you are saying that the location where the maximum normal stresses is occurring, that is the principle stresses. So, you see the shear stress is always 0 while you see if you are saying that the shear stresses are 0, you know like by differentiating those things, the location always gives you the reciprocal of that. That means, you see they have the perpendicular relation to each other. So, tan 2, this is a very good relation, important relation is there. Tan 2 tan 2 times of theta p into tan two times of theta x always gives you 1 because they are mutually perpendicular to each other at the different location.

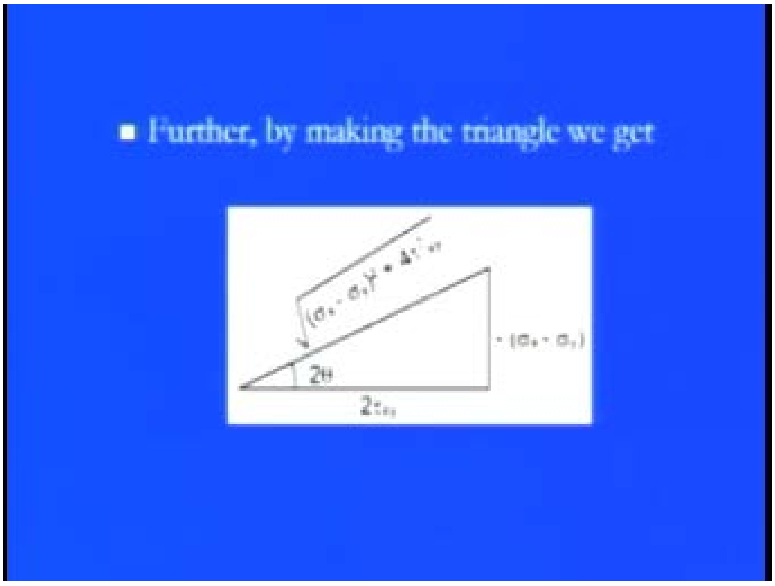
(Refer Slide Time: 37:23)



So, therefore, you see it can be concluded that the equation 2 is the negative reciprocal of the equation 1 because minus 1 by tan of 2 theta p is there, and hence the roots for these double angle equations of that a are always 90 degree away from the corresponding angle of equation. That means as I told you, they are mutually perpendicular to each other because they have the reciprocal relations and since, the minus sign is there. So, they are always going out towards the outer direction. This means that the angles that you know like whatever the angles are there, they are locating at the plane where the maximum or minimum shearing stresses are there at an angle of 45 degree with the planes of the principle stresses.

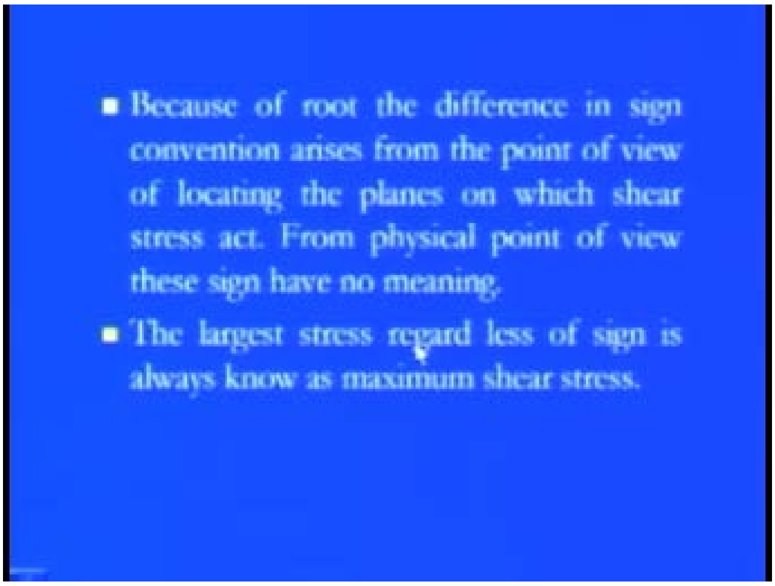
That means the clear picture says that actually if the principle planes are there and if we want to locate the maximum or minimum shearing stresses, they are always 45 degree away from the principle planes or we can say the principle stresses where they are exerting on the planes. So, this is the three locations we know that actually if it is subjected by purely the maximum or we can say you know like the minimum normal stress component, we have the principle stresses. There is no shear stress. So, once you locate that point where there is no shear stress after just rotating those things by 45 degree, we have the shearing stresses irrespective whether it is maximum or minimum values.

(Refer Slide Time: 38:44)



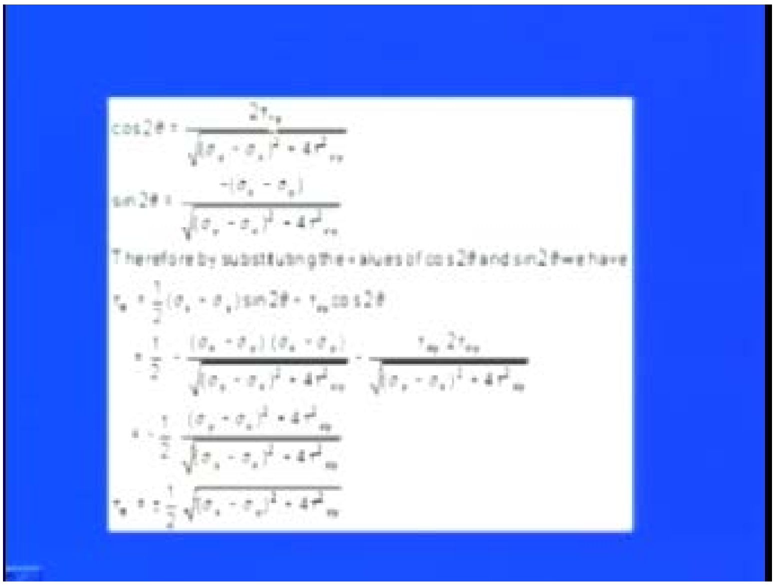
Then, you see further by making triangle as we have discussed previously because we have the tan 2 theta x. So, in that the base is two times of tau xy, and you see you know like the perpendicular is there minus sigma x minus sigma y here in this case. So, we can get you know like the sigma x minus sigma y whole square plus four times of this. This is a hypotenuse, the similar you see for 2 theta x is there.

(Refer Slide Time: 39:05)



Again once you know that these things you can easily calculate cos of 2 theta as an sin of 2 theta and you can put those value because of the root, the difference in sign convention arises from the point of view of locating the planes on which the shear stress is occurring. If they are acting from the physical point of view, these signs have no meaning because you see they simply give you the maximum value and the minimum value of the shear stresses, and they are always locating the 45 degree angle. Apart from you see the principle planes and the larger stresses regard the less of sign. Obviously, it is always known as the maximum shear stresses because of the counter, this shear part or we can say the complementary shear stresses tau dash.

(Refer Slide Time: 39:54)



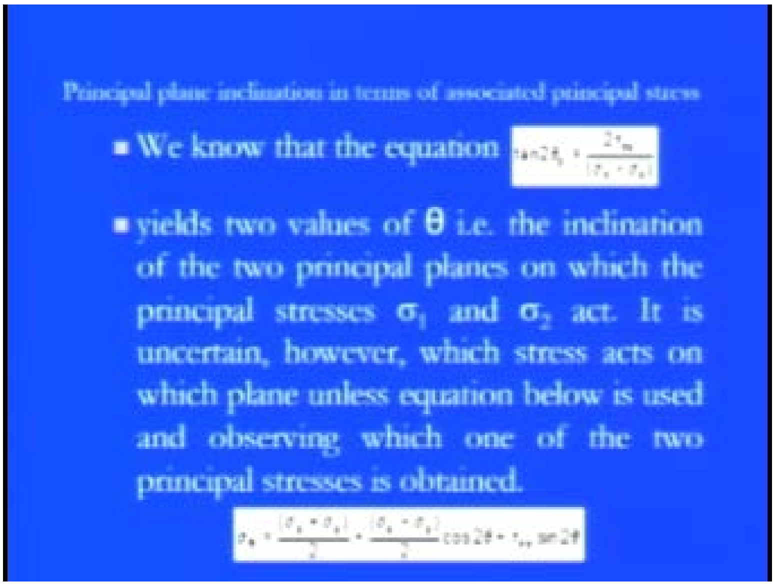
So, now you see you know like from the previous triangle of 2 theta x, we can get cos of 2 theta, that is you know like the 2 tau xy divided by the square root of this like tau x minus tau y minus tau x whole square plus this one. That is hypotenuse sin of 2 theta. We can also get minus sigma x minus sigma y over this. So, now you see if you want to substitute those things in the sigma, this tau theta formula where you see means once you have the location that where you see the maximum, this shear stresses are occurring in this particular plane, or we can say in this object we can easily get it by 2 theta x which is tan inverse of two times of this sigma y minus sigma x divided by two times of tau xy.

Once you know the location, you can get the maximum value by putting that value here in this. So, see this sigma theta as tau theta is half of sigma x minus sigma y sin 2 theta minus tau xy cos of 2 theta. Put those values here, the cos theta here and the sign theta here, you have half which is you see you know like into this sigma x minus sigma y. You know after multiplying those values here, you have divided by hypotenuse minus this tau xy into 2 xy. This is cos of 2 theta divided by this. Once you multiply those things, you have tau x minus tau y whole square plus 4 you know like tau xy square because if you sum up those things, both have the same notation minus.

So, it will come out divided by this. So, now, we have tau theta is plus equals to plus minus like half of square root of sigma x minus sigma y whole square plus four times of tau square xy. That means, you see here if we have in a material which is subjected under mutually perpendicular to the stress is sigma x and sigma y at the same time. If the shear stress is also occurring at the planes, parallel planes, we have the location of the shearing location at which the shear stresses is maximum. 2 theta x equals to minus times of you see you know like sigma x means we can say sigma x maximum minus sigma y divided by two times of tau xy.

After you know like getting that location, we can get the maximum or minimum value of that shear stresses tau theta equals to plus. If maximum value is there plus half of you know like sigma x minus sigma y whole square plus four times of tau xy square, or if you put minus, then it is minimum. So, you have both the maximum and minimum value and the location as well.

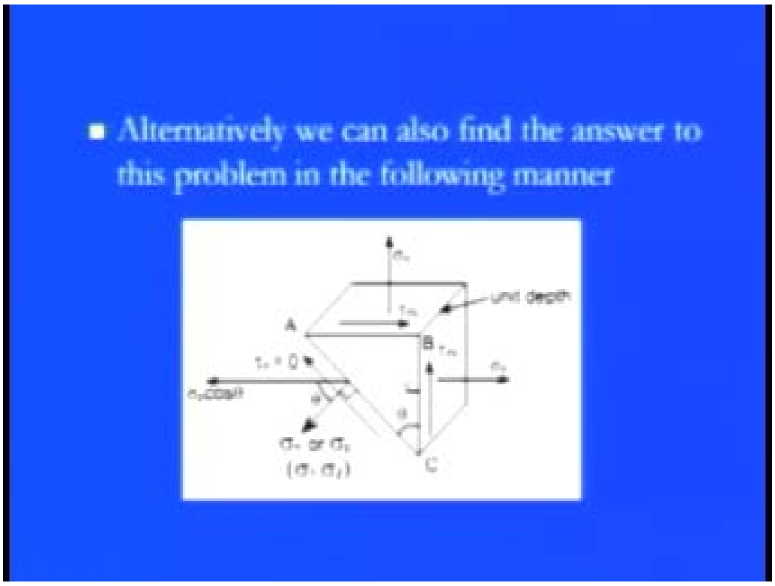
(Refer Slide Time: 42:19)



So, now you see we know the equation tan 2 theta p which is you see two times of tau xy minus sigma x minus sigma y, which yields two values of theta. As we discussed that is the inclination of two principle planes which are mutually perpendicular or means the principle stresses sigma 1 and sigma 2 are acting. It is uncertain because you see the stresses acts on which plane unless equation below is used to observe which one of two principle stresses are obtained may sigma theta because this is coming from the principle stresses. So, principle plane formula is given on the top of that you see here 2 theta.

So, here we have the maximum stresses, here sigma theta is sigma x plus sigma y by 2 plus sigma x minus sigma y by 2 cos of 2 theta plus tau xy sin of 2 theta. So, we can easily you know like get the principle plane inclination in terms of the associated principle stresses.

(Refer Slide Time: 43:12)

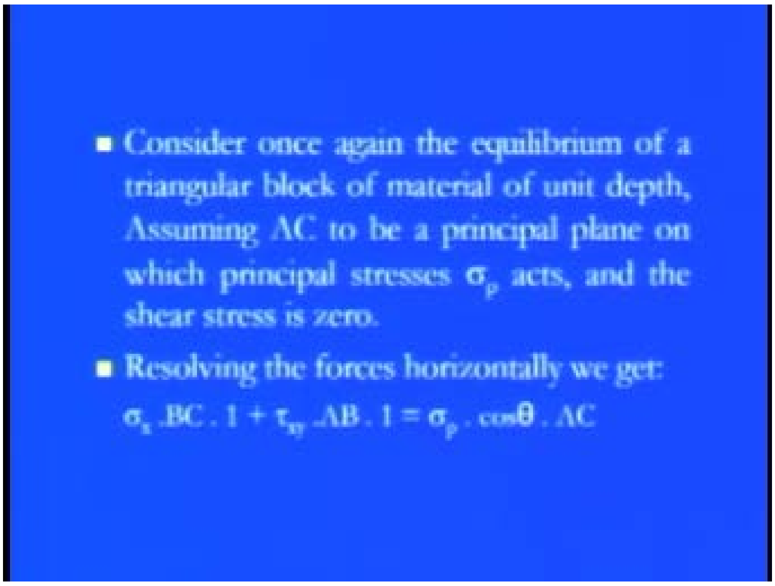


Hence, we have seen that in the previous case the uncertain values are there of those sigma 1 and sigma 2, the principle stresses until and unless you do not know the formula of sigma theta or if you do not know the value of tan 2 theta. So, we need to you know like we need to go for the alternative path that until unless we do not know that sigma theta or those values, then how could you go means like if you do not know the sigma 1 and sigma 2, then you cannot locate those parts, especially the principle planes. So, this alternative way just gives you that we have the same ABC, this triangle is there of the unit depth, this unit path is there.

So, in that we found that the sigma theta or sigma p are nothing but these are the normal stress components on the principle plane or we can say I should say it is on the oblique plane. And since, principle plane we can say because this is tau theta is 0, but if I am saying that if I do not know that what the location is, there I can straight away go that this is sigma theta or sigma p. Some principle plane is there instead of saying sigma 1 sigma 2. So, now you see I just want to resolve. So, I have you see the sigma x. So, this is the straight part of the sigma x. So, now if I just want to resolve the sigma theta, so sigma theta is nothing but if we are resolving this axis, so sigma theta cos you know like the cos of sigma theta sigma p or sigma theta cos theta or we can say if I resolve in the downward direction which is just parallel and opposite to the shear stress, that is sigma theta or sigma p sin of theta. So, you see here we have in this domain, where the AB is there. We have the shear stresses tau xy, here the normal stress component sigma x.

Similarly, you see for this BC plane where you see the unit depth is there, and in this domain we have the sigma x and tau xy. These are the responsible stresses. So, now if I want to resolve these things that if I do not know that you see means if it is uncertain to locate the sigma 1 sigma 2 means if I do not know the formula, then how could I go.

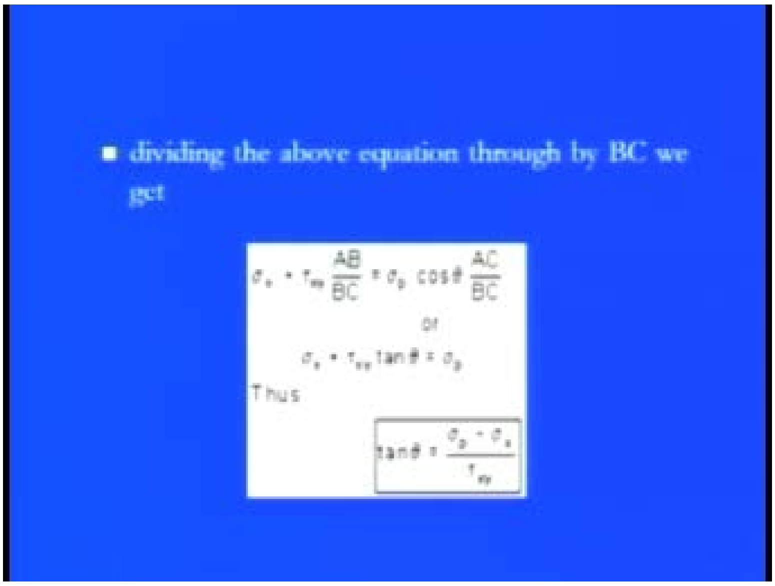
(Refer Slide Time: 45:10)



So, this is you see consider once again the equilibrium because you see again if we want to analyze the stress formation, then you see we need to consider that actually whatever the element is there under the action of all those forces and the stresses, this is well equilibrium. So, under the equilibrium position of the triangular block ABC, where you see the oblique plane is there of the AC, the principle planes are there and the principle stresses at this is sigma p because we consider that there is no shear stress sigma tau theta was 0.

So, you know like since the shear stress was 0, there is a principle stress sigma p, and now you see I just want to resolve those forces and we know that actually since sigma x was going parallely, so sigma x into BC into 1 plus tau xy which was perpendicular part was there. So, tau xy into AB into 1 will give you sigma p and sigma p, since sigma p is acting at the oblique plane. So, we need to resolve into horizontal as well as the vertical part. So, sigma p cos theta is nothing but it is the horizontal part as you see we resolved the previous one. So, sigma p cos theta into AC because it is acting on the AC path.

(Refer Slide Time: 46:25)



So, now you see if I am dividing the above equation by the component BC, then we are getting sigma x because sigma x BC was there. So, sigma x plus tau xy AB divided by BC equals to sigma p cos of theta into AC by BC. So, now you see you know like going back to these components where ABC triangle is there, so if I just go back and form those equations, then I found that sigma x plus tau xy AB by BC will give you this kind of tan theta into AC by BC. BC by AC is cos of theta. So, it will get cancelled out. So, it will be sigma p.

So, now you see we have the new location of this principle plane where the principle plane will form. So, principle plane, the location of the principle plane, this tan theta is sigma p minus sigma x divided by tau xy. That means is that if you know sigma p, if you know the value of sigma p that now this is the sigma p, if you know sigma x which is you know like more of the influencing part. And if you know the value of tau xy, you can easily get the location of principle stress at the principle plane, and you can also go for the principle stresses for the minimum and the maximum value.

So, here you see in this chapter we have discussed about if the material is subjected by a pure shear stress and then, what is the stress formation is there at the oblique plane, that is at the sigma theta and tau theta and what are the minimum and the maximum values are there at that different locations. The second case which we discussed that actually if we have the combination means if we have two mutually perpendicular direct stresses are there, normal stresses, there is no shear stress, then you see what will be the value of sigma theta and tau theta is there at the oblique plane. That means you see what the value of the normal stress is and the shear stress stresses are there at the oblique plane location. We can also get that actually at what the locations we can get the maximum or minimum values of these stress component, shear stress as well as the normal stress component.

Now, the third case which was more realistic case I should say or the complex case is if we have the combination of two mutually perpendicular direct stresses, and the third one is the shear stress is there, then how we can figure out these problems. So, you see you know like we found that if we have these combinations, then sigma theta which is the normal stress component is nothing but equals to sigma x plus sigma y by 2 plus sigma x minus sigma y by 2 influenced by cos of 2 theta plus tau xy sin of 2 theta. That means you see here we have the two different kinds of components. One which is independent of the theta that is sigma x plus sigma y by 2, one purely depends on the influenced by the theta that is sigma x minus sigma y by 2 into cos of 2 theta plus tau xy sin of 2 theta.

So, this is you see the real combination of both mutually perpendicular shear stresses and the tau xy, that is the two mutually perpendicular normal stresses and the shear stress which will give you the normal stress at theta. While tau theta will also give you that, what is the location you see you know like where we can get the shear stresses, and after that if we want to calculate because you see here we have all three parts of the stress sigma x, sigma y and tau xy. We can easily get that actually with the influence of the combination of these three stresses, where the minimum and the maximum values of these stresses are.

So, by making the differentiation, we got that tau, sigma 1 and sigma 2 are there which was nothing but equals to sigma x plus sigma y by 2 minus of those properties square root of tau x sigma x minus sigma y sigma x minus sigma y whole square plus four times of tau square xy divided by 2. So, these was the combination of sigma 1 and sigma 2 which gives you the principle stresses, and the principle stresses are always forming at the principle planes. To locate the principle planes, there is one formula that is the tan to theta which was again influencing by three parameters like two times of tau xy divided by sigma x minus sigma y.

If we conclude those things, then we found that there are planes, the principle plane where the principle shear stresses occurring, they are always exerting where the shear stresses are not there. So, this was the one conclusion and another conclusion was there that we have the different location, where the shear stresses are also there at the maximum or minimum, and these locations we can get by tilting principle planes by 45 degree at minimum or maximum. So, you see here we found that the both tan 2 theta x and as tan 2 theta p where the maximum shear stresses occurring in the different locations, and they are having mutually perpendicular locations. That is why you see they have the reciprocal relations with each other.

Also, you see we found that if we do not know formally to calculate those you know like the locations tan to theta p or tan 2 theta x, we can easily configure if we know the value of sigma p or if we know the values of other components by tan theta was sigma p minus sigma x divided by tau xy. So, these you see the discussions were there. So, here we found that you know like these are the formulations of the stresses. If the stresses, how the stresses are being set up within those objects if they are influenced by different forces.

Now, you see in the next chapter, consequently we will discuss that actually how to resolve these things by means how to solve those stress components. So, we have both the numerical solutions, and we have the graphical solutions. So, as far as the numerical solutions are concerned, we always use the trigonometric relations. Now, you see if these forces are you know like configured, and once they configure those things, how to resolve those forces and the another solution was the graphical solution.

And this solution is also known as Mohr’s circle which we need to plot that actually the circle is there, what is the radius of circle if you are going for the principle planes or principle stresses and then, we know that there is no shear stress component is there. So, how to locate those planes, how to put those stress values in x as well as the y directions, and how to draw the principle, the circle of Mohr’s circle on the principle as well as the normal stress planes. So, this you see this part which we are going to discuss in the next lecture.

Thank you.