Strength of Materials
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Lecture – 40

Hi, this is Dr. S. P. Harsha from Mechanical and Industrial Department, IIT Roorkee. I am going to deliver my lecture, last you see, the number 40. And you see, now our journey is going to be end of the course of the Strength of Materials, and this course is developed under the National Programme on Technology Enhanced Learning (NPTEL).

So, you see, in this lecture we would definitely brief it out about the Castiglione's theorems, which we discussed, you see, know like in the last term. Also we were trying to solve some of the numerical problems and then, you see, we would like to conclude whole, you know like, the course, that what we have discussed entire in entire 39 lectures and, you see, what exactly the importance and the applications of these, you know like, different segments of the strength of materials in our future courses.

So, prior to start the next lecture, you see, this lecture we would like to discuss about, you know like, briefly that because, you see, in this lecture basically we are going to solve the numerical problems. So, our main intention is first to discuss about, you know like, what we have discussed about the Castiglione's theorems and all that part strain energy part. So, we started the last lecture with one of the basic form of the strain energy, that the strain energy due to the bending action.

And we found that, you see, if we want to calculate the strain energy under the bending action, then always, you see, we need to segregate those segments where the load conditions are there. So, you see, here we have taken you see, you know like, the simply supported beam and point load is there somewhere distance, you see, from A from point this left end and we found that, you see, we need to segregate as I told you that there are two regions - one is for A one is for B.

And within that A region if we are saying that the M 1 is, you know like, the bending moment, the bending moment is there due to the action of load. Then the strain energy can be easily formulated as U is equal to this within, you know like, the volumetric domain integration of M square divided by 2 EI into dv. And for another section also we can calculate the same strain energy and then, you see, we can simply sum up algebraically those strain energy components U 1 plus U 2. So, you see, this is the one of the basic form, you know like, through which we can simply calculate that if, you see, the deflections are there under the bending action, then what will be the strain energy; that means, how much energy can be absorbed under the action of bending moment; so, that can be calculated.

And also we can calculate, you see that, within the elastic region what exactly the value of those things through which we can say that, yes, this is under the bending action, we have this much strain energy, or this much energy can be absorbed by the material when it is subjected by a bending action. So, that is what you see, we discussed in the first segment of our last lecture.

And then, you see, we discussed about, you know like, then if we have the number of different loads, then you see, there are two different forms of, you know like, the energy is there, it depends on which domain which we are talking. So, if we are talking about the normal domain like, you see, here in the stress-strain diagram if domain is there, that means, you see, the basis is there on the strain.

Then, you see, the area under the elastic curve within that particular domain of strain will give this elastic strain energy. But if we change the domain, then, that means, you see, if we are talking about, you know like, the area under the curve in which the base is there, where the stress is there, then this energy is known as the complementary strain energy. So, complementary strain energy can be easily calculated, that is, you see, generally we have termed as the U star is being calculated by half of integration of epsilon x into the differentiate term of d into sigma x; that means, you see here, we are simply checking the variation of the stress components and the corresponding strains that are there. So, whatever this multiplication divided by half is there will give you the complementary strain energy.

So, now, you see, with the changing of the domain, we could easily figure out that what the exact things are there with the complementary strain energy and the elastic strain energy. And we also found that, when we are talking about the linear region, then you see, we have you know like, the complementary strain energy and the elastic energy have the same value and have the same significance also, but if we go beyond that, that means, you see, after the yield point we have the non-linear region where the stress is not directly propositional to the strain.

Then we will find that, you see, that the complementary strain energy and the elastic this strain energy is somewhat different. Because, you see, here in that which domain is providing what kind of deformation corresponding is the changes are there in the elastic strain energy and the complementary strain energy.

And then in the last segment of that we discussed about the Castiglione's theorem that, you see, when the number of externally applied loads are there, then how we could figure out because, you see, when there are many points where you see, the load application is there, then the deflection at those point are nothing but equal to the change of strain energy per unit, you see, divided by the change of load is there. So, del U divided by del P i will give you the deflection at those points.

So, this is the first theorem of the Castiglione's or we can say in other terms also, that the load at different points can be easily calculated by del U divided by del Di. That means, you see here, the change in the strain energy within the domain of the change of displacement will give you the load at these points, when the load application, when the deformation is there towards the direction of the load. So, this is, you see, in any of the way, you see, we can simply define the Castiglione's first theorem.

And then you see, we could also figure out that if the bending action is there, that means, you see, the moment being applied on an object, then also we could simply figure out that what the angular rotation is there based on the strain energy.

So, Castiglione's setup one more relation is there that del U divided by del MI at various location - MI means the higher point is there. So, del U divided by del MI will give you the theta; that means, the angular rotations at this point. So, also we can say that whatever the angular rotations are there within the, you know like, this domain of any part, then strain energy within the domain of this angular rotation will give you the moments at those point when the angular rotation of this object is there within the direction of the bending moment.

So, these relations are being setup in the previous part and that is what, you see, now with those relations in this lecture particular we are going to, you know like, use those relations and trying to, you know like, solve the numerical problems. So, we are going to take two numerical problem based on that. So, that we can clearly see that the difference of this Castiglione's theorem applications with the other term. So, here, you see, in this the last lecture, we have some of the numerical problems based on the Castiglione's theorem.

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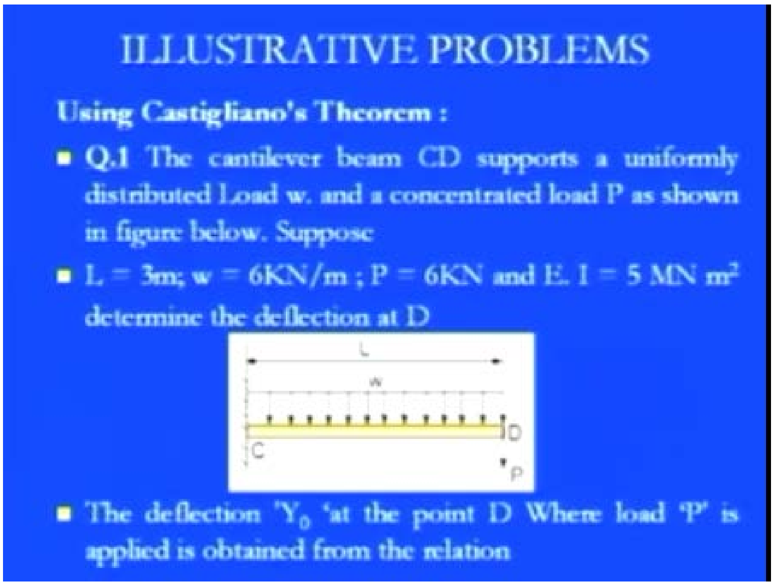

So, the question number one: we have, you see, the cantilever beam in which, you see, the one end is rigidly fixed up and other end is free. And the, you know like, in this cantilever beams CD which has a, you know like, supports uniformly distributed load UDL is there with the intensity of w and a concentrated load P is there at the free end.

So, you see here, this simply cantilever beam is, you know like, acted by two main loads one is the UDL with the intensity of - load intensity - of w and the P is there that is the point load at the extreme end. Now, you see, there are the numerical problems are given: the total length of the beam is L, which is 3 meter; the intensity of the UDL is given as 6 kilo Newton per meter; and the load application - the point load at the free end - which is there at, you know like, the 3 meter is 6 kilo Newton. And, you see, we have the flexural rigidity EI is also given as 5 mega Newton meter square.

We need to find it out the deflection at point D; that means, you see, you know like, at the extreme corner where, you see, the combined load action is there, the UDL as well as the point load what will be the deflection is there at point D? So, let us say, if we are assuming that the deflection is there Y zero at point D when the load P is applied. So obviously, you see, when only P load is there the direct load is there.

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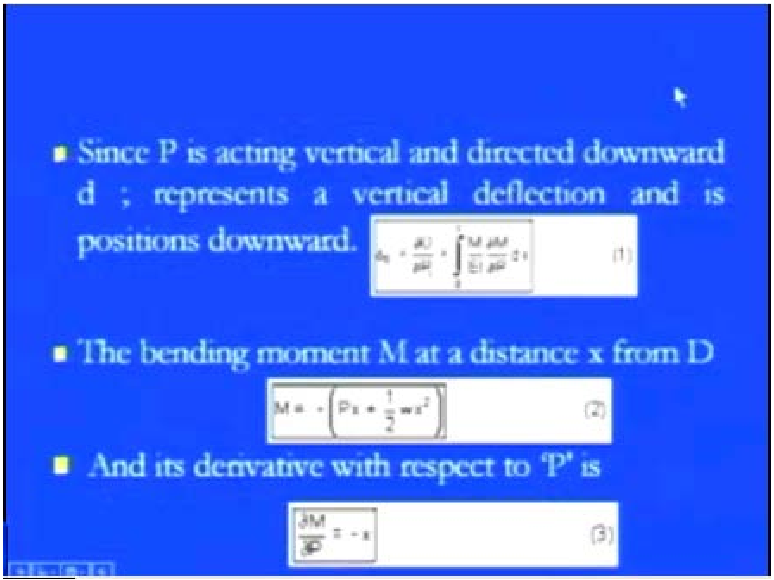

We can simply get, you know like, the deflection at this point since P is acting vertically, you know like, a vertical and directed downward directions D. Then it can be simply, you know like, represented by a vertical deflection and a position towards downward. So, we have, you see, that the delta zero means, you see, dU by d Pi.

So, we know that actually the Castiglione's first theorem says that you can simply get the deflection in the load direction when it is equal to dU by d Pi. So, the deflection and the load direction has to be in the same direction. So, in our case also, you see, the deflection is there in the downward direction and the load is also applied towards the downward direction. So, we could simply figure out this thing. So, this theorem is well applicable.

So, you see, delta zero which is equal to dU by d Pi which is equal to zero to l the total – the entire length of the beam - into now, you see, since the UDL is there, so we need to apply the bending moment. So, M divided by EI into dM by dP into dX. So, now you see, the bending moment as I told you, you know like, it is there and it is due to the presence of this UDL is there. So, that is what and this point load is there because, you see, one end is fixed and one end is, you know like, loaded by point load. So obviously, you see, the moment will be there towards the downward direction. So, bending moment has to be calculated here. So, bending moment M at a distance of x from D, you see, from the fixed end.

So, you see we have, M is equal to minus because it is going to in the downward direction towards this particular direction. So, we have minus P of P into x because of the point load and because of the load intensity w UDL we have half of w into x square. So, this is the total algebraic sum of these, you know like, the bending moment because, you see, both are the responsible load is there for any kind of bending action on that particular beam.

Now, and its, you know like, derivative with respect to P is obviously, you know like, with the Castiglione's theorem dM over dP is equal to minus x. So, you seehere, with those equations, now we can simply, you know like, substitute for this particular value of M and then, you see, dM by dP will simply, as I told you, give you the minus x.

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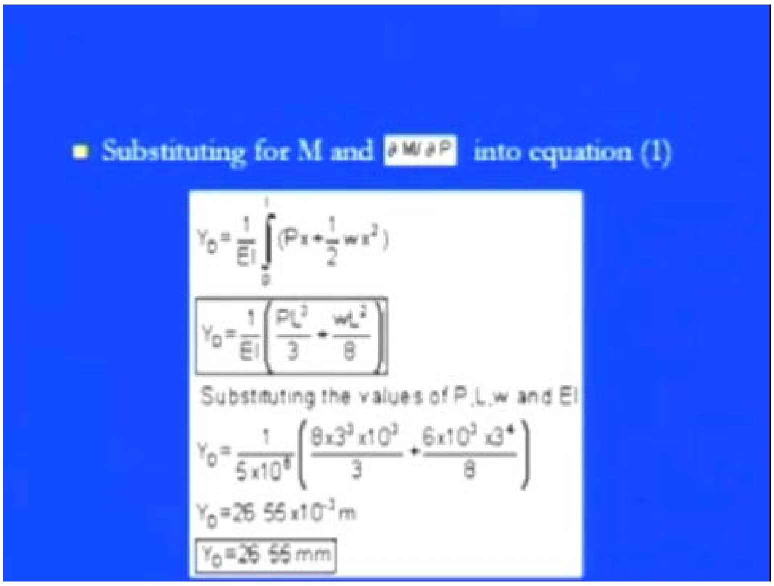

So, we have Y zero which is the deflection part is there is equal to 1 by EI integral zero to l and, you see, that the total moment was Px plus half of w x square. So, you see, with the integration and keeping of the value of L here, we have the Y zero value is equal to 1 by EI P L cube by 3 plus w L square by 8. So, now you see, we have all those values; means we have the value of point load, we have the value of total beam length that is 3 meter, we have the value of load intensity w, and we have the value of flexural rigidity. So, by keeping all those values in this particular Y zero, we have, you see, 1 by 5 into 10 to the power 6 and then in the bracket it is 8 into 3 to the power square because, you see, 3 meter is given as the length. So, you see, here we have 3 square into 10 to the power cube because it was in the meter part. So, you see, we need to convert it into that part divided by 3 plus 6 into 10 to the power cube into 3 to the power 4 because you see, you know like, this is there divided by 8.

After considering those things here, now we have Y zero which is equal to 26.55 into 10 to the power minus 3 meter or we can say that it is Y zero which is the initial deflection due to that load application is 26.55 millimeter. So, in this question, the key phenomena is when you see, when a bean is there and it is being subjected by UDL as well as the point load, then you see, it is very hard to calculate the deflection with using a standard phenomena; because again, you see, we need to consider, you know like, the direct integration method and go like that. But here it is straight theorem is there, that when the load application is there, then use the Castiglione’s theorem that, you see, the delta I is nothing but equal to del U by del Pi. Because, you see, the two different kind of loads are there. So, just consider the loads with the kind of moment and then, you see, you can simply calculate.

So, the real good feature of this Castiglione’s theorems says that if you have the number of loads on a simple structure, then it is pretty easy to calculate, if you can simply figure out strain energy. So, strain energy divided by the number of load the differentiation will give you the deflection at those points; only the condition is that the deflection and the load application is having the same direction.

Like you see, in this case we apply the load by UDL or the point load they are simply acted toward the downward direction and, you see, the deflection was also there toward the downward direction. So, with that consideration, you see, we simply apply the Y zero formula here by 1 by EI into, you see, the Px is there to the point load and w x square by 2 was there due to the bending moment and with the inclusion of both the things, we will have a clear feeling about that since both are the responsible, you know like, the parameters are there for having bending moment. So, Y 0 can be easily evaluated. So, this is, you see, the Y 0 part was there, in a simple case of cantilever beam where UDL and the point load was acted. Now, we are taken the question two, you see, for a simple truss problem is there. So, when we have truss as show in this particular figure in front of you, now we need to find it out the vertical deflection at point C.

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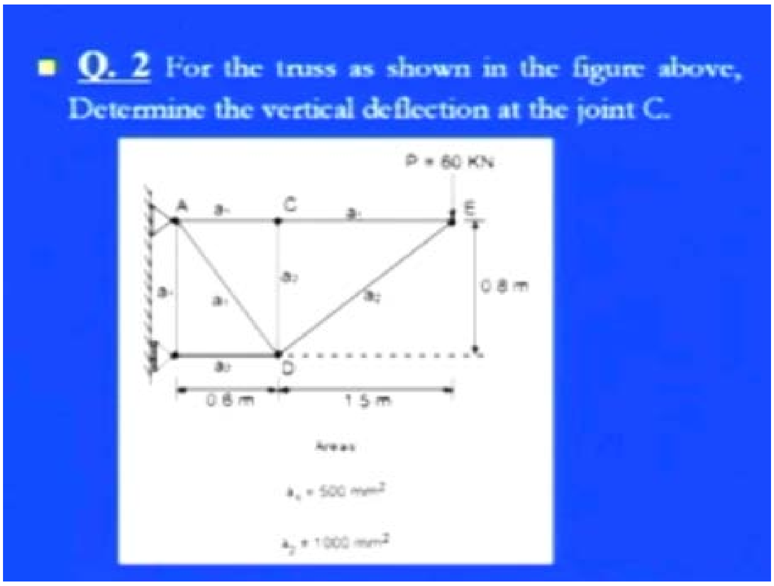

The point C is the intermediate point is there if, you see that a truss, then we have two rigid points, two rigid points are there, and this rigid points are simply shown by A and point B. So, you see here, these points are being rigidly shown up. And then, you see, there is a direct link is there with the distance A from a is there that is point C. And, you see, form point a 1 also we have the distance of point B.

And then, you see, you we have the linear, you know like, the distance is there in the truss problem, and you know like, from A to C that is 0.6 meter. And from point C to the free end, we have you see, the a 1 distance is there which is equal to 1.5 meter. And the vertical distance is where the two fixed rigid points are there is .8 meter.

Now, in this truss problem you see, you know like, we have the different coefficients a 1 and a 2 (s) are there and you see, you know like, we have given that a 1 a 2 are the area for different, different segments. So, we will find that many of the, you know like, the trusses have the similar kind of area somewhat you see, you know like, when the longer length is there we need somewhat more area, you know like, the means the cross sectional area of this trusses, so that it can be withstand some of the good load conditions.

So, you can see that when the base -when this particular base - is there we have a 2 area this base, this diagonal part is there this is also having the a 2 area, and somewhat you see, you know like, these things are having the a 1 area. And at point E which is the free point, we have a point load of having intensity of 60 kilo Newton at point E.

So, now in that, you see, we have the area a 1 which is, you see, like AB part, AC part, and AB part, and the CE part is nothing but equal to 500 millimeter square, while BD and DE part will be nothing but equal to this 1000 millimeter square area. So, after knowing the distances and the area, now our main intention is to find it out the vertical deflection at particular point C which is the intermediated point is there.

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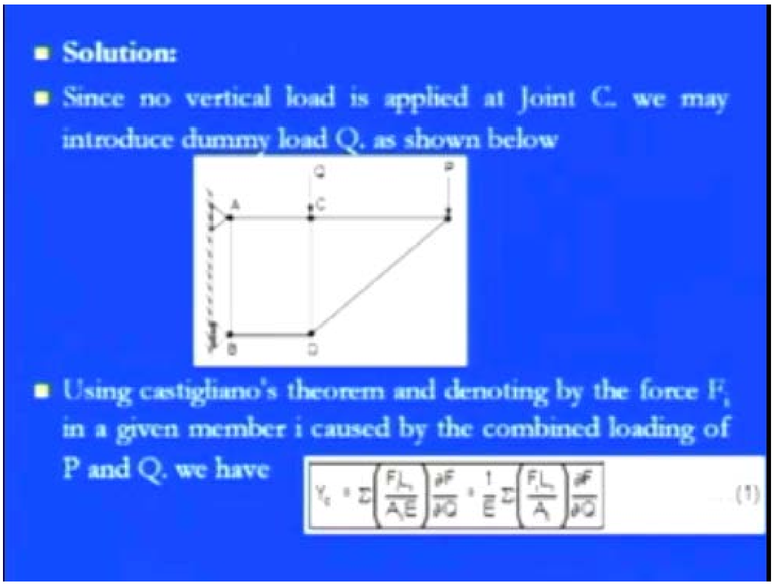

So, now, you see, in the particular solution part since there is no vertical load applied at point C, because you see, our main intention is to find it out the deflection, but there is no load - direct load - application is there. So, we have to introduce a dummy load there of load Q at point C. So, you can see in the particular figure, that you see, these are, you like, the A and B are the rigid points where we cannot apply any load, but our main intention is to find out the deflection part at this particular point C.

So, as per the Castiglione’s theorem they should be in any load application is there, then there is a direct link is there with the deflection part. So, there is an, you know like, the dummy load is being applied here at point C, and as you can see that at point E we have a vertical load is there already P which has, you know like, the magnitude 60 kilo Newton is there and these are all the other intermediate points are there.

So, now within this particular truss structure we need to use the Castiglione's theorems and we need to denote, you see, you know like, all those forces, like force F 1 is there in the given membrane, where you see, you know like, there are lots of different in… these locations are there through which we can say that the load application direct or indirectly is applied. So, we simply, you know like, apply the combined loading of P and Q, you see, here and we just want to see that actually what the other impacts are there of these loads at these other points.

So, with that now we have the deflection at point C, because that is our main concern, is equal to summation of Fi into this Fi Li divided by Ai into E, then you see, you know like, what exactly this domain change is there. So, del F by del Q because, you see, this is the dummy load, so what the real relation is there with the force to the Q? And then, we know that since it is a constant material term is there. So, we have 1 by E can be taken out as constant; then, you see, it is the integration or summation of Fi Li by Ai because, you see, at different, different locations it is there into dF by dQ.

So, this is our the first basic equation, by introducing the dummy load at point C, we could easily figure out that the deflection at point C. Because the Castiglione's says that if we have a different load location is there, then it is pretty easy to calculate, you know like, the deflection at those points if the direction of load and the deflection is in the same direction. So, you see, with that concept we simply write this particular equation.

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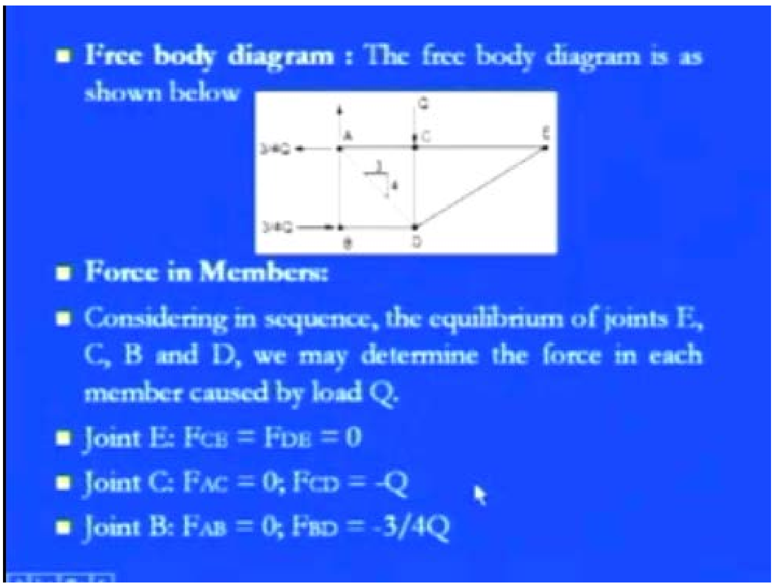

So, with that free body diagram now, if you see that, we what we have simply, you know like, we have load application is there at point E and this load is being under that point C. And this load is being shared at a point A and point D. So you see here, this is a perfect square is there, and that you see, you know like, if you are writing this, then we have, you see, the this base and the hypotenuse is like that and this length is there of 4, with 3 to 4 with that you see, we can simply segregate the force component towards the outward direction, as you see, when you apply the load, it will give direct impact to the fixed points. So, at the fixed points A and B we have, you see, the three-fourth of Q of this particular load - dummy load - and three-fourth of Q is being carried out by point D because of this location.

And then, you see, some of the load is there also on the top of direction at point A. So, with this particular free body diagram, now we have a real feeling that when you apply the load at point E and point C then, you see, this load is being simply, you know like, absorbed by some of the joint points.

And these joints points are, you see, we can see that the 75 percent of the load is being, you know like, coming at that particular rigid length. So, now, you see, with these forces in the membranes. Now if you are considering the sequence, the equilibrium of the point, you know like, at the joints E, C, B and D, we can simply you know like, means all those equilibrium points at either the point E or we can say point C or we can say point B and D. At these particular, you know like, positions where we are saying that the statically equilibrium positions are there with that we may simply determine the forces in each of the membrane by simply, you know like, applying the load Q. Because, you see, the load Q is a real, though it is a dummy load, but it has a real feeling about those things that you see, it will simply cause the kind of deformation or the deflection at these joint points.

So, if we are talking about the joint E, which is you see, you know like, these things are there, then we can say that the force which is coming from C to E is always zero which is equal to force coming out from D to E. Because, you see, it is the statically equilibrium point is there and if you are talking about point C which is, you see, you know like, the intermediate point is there, the force which is coming from A to C like this one, and you see, in the horizontal way, because you see, all those things are coming in the vertical part, you see here.

And then, you see, the force which is coming. So, it is equal to zero A to C, but the force which is coming from C to D; that means, you see, you know like, in the vertical downward direction. Since it is a dummy load is applied towards that part, so we have, so, you see, since it is coming from that part, so we have minus Q. And if you are talking about joint D, which is you see, you know like, a kind of rigid joint is there, there is no force can be transferred from A to B because, you see, they are rigidly linked to one of the joint.

So, we can say that F A to B is equal to zero, but if you are talking about the relative force terms between B and D; that means, you see, in the horizontal way we know that at this clear transformation is there of minus three-fourth of Q. Because you see, you know like, it is a transformation is there of the particular force. So now, you see, we have all those forces, you see, at the different, different locations, which are coming, you know like, causing from these things.

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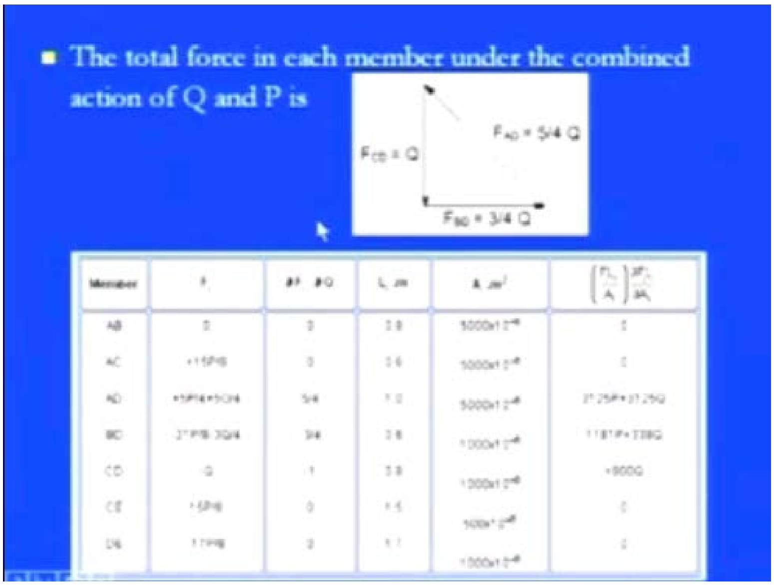

So, the total force in each of the member, under the combined load action Q and P, can be also, you see, shown here, that you see, F C to D is there which is Q, and F A to D is there which is the 5 by 4 because three-fourth is there, so hypotenuse is 4. So, with the taking of that particular triangular element, we have F A to D which is, you know like, towards the diagonal part is there 5 by 4 Q.

And towards this direction, you see, the horizontal direction which we discussed, you see, B to D transformation is there, that is three-fourth of Q. So, with that particular triangle now we can simply get the forces at individual membrane. So, now you see, and then also we can… once you have the forces, then we can simply, you know like, relate those forces with the dummy forces del F by del Q. And once you have these things, then you can simply figure out that what exactly the, you know like, the corresponding length and the area is there. So, what exactly the strain energy formation is there in these terms. So, if you are talking about the length AB, that means, you see, you know like, the two rigid links were there, you see, there is no force transformation is there in that term.

So, you see, here, obviously, this force divided by del Q is zero since it has the distance of .8 meter and we have, you see, the area which is 500, you see, 5000 into 10 to the power minus 6 meter square. So, you see, since there is no contribution is there because of the force applied is zero, there is no contribution in the strain energy term.

And if you are talking about A to C, means you see, where the dummy load is acting at C and A is rigid again, you see here since the force transmission is there from 15 by 8 P, but you see, here that since the this dummy load is acted on that point, so del F by del Q is zero. So, we can say that there is no contribution is there from this end. And since, you see, the distances already given as 0.6 meter so obviously, the area will come like that, but since, you know like, this del F by del Q component is zero. So obviously, it will have again the same similar kind of things are there, no contribution in the strain energy formation.

Now, come to the A to D form, you see, the diagonal form is there; obviously, you see, you know like, the straight transformation is there as you can see this particular part. So, what we have? We have 5 by 4 into P is there plus 5 by 4 into Q is there. So, the dF by dQ if you are simply taking dF by dQ, we have the 5 by 4 straight component is there and, you see, you can simply get that particular length is there by 1. And then, you see, the area can be easily figured out that 5000 into 10 to the power minus 6.

So, after doing all those component what we have? We have you see, you know like, the strain energy because of the P load and because of the Q load. So, we have, you see, 3125 P plus 3125 Q. So, you see, here this is the real, you know like, the intermediate link is there through which the force transmission is there and through which, you see, we can say that the strain energy contribution is there.

Now, if you are talking about the BD link; BD is the bottom link is there. So, we know that, you see, through which there is a force transmission is there. So, we can say that it is minus 21 by 8 P is there and minus three-fourth of Q is there because of this force transmission. So, after having these things if you differentiate out these terms, then we have minus 3 by 4 because it is a Q function is there. So, we can say that, you know like, there is a kind of contribution is there of this.

And then, you see, once we know that 0.6, you know like, this meter distance is there. So, we can also calculate the area that is 1000 into 10 to the power minus 6. And we can say that it is 1181 P plus 3308 Q is there in that particular strain energy contribution because of these terms.

Now, if you are talking about the CD which the straight part is there, you know like, at C we are applying the load and D part is there since, you know like, the force part is there as minus Q because straightway we are applying the load towards the vertical directions. So, this minus Q. So, we have this del F by del Q is minus 1; we know the distance of 0.8 we have the area. So, we can simply say that it is plus 800 Q. And then, you see, we are talking about CE which is, you know like, the straight part is there on this. So, in that we can say that the 15 by 8 P is the force application is there, there is no Q part is there. So, dF by dQ is zero. Once it is zero, so there is, you know like, and we know that the distance is 1.5 and the area is there, but since it there is no differential part is there; so obviously, the contribution from this particular link towards this strain energy is zero.

And simply, you see, if you are taking the diagonal DE which has the area of a 2 we know that we have the direct force in this transformation is there, that is the minus 17 by 8 P but you see, if you differentiate out those terms then it has to be equal to zero.

And then, you see, the length is, obviously, 1.7 and the area is there, but because of this particular term is absent, we have there is no contribution is there in the strain energy from this. So, if you look at these individual membrane contribution then we will find that only three contributions are there as far as this Fi Li divided by Ai into d of Fi by dQ contribution is there.

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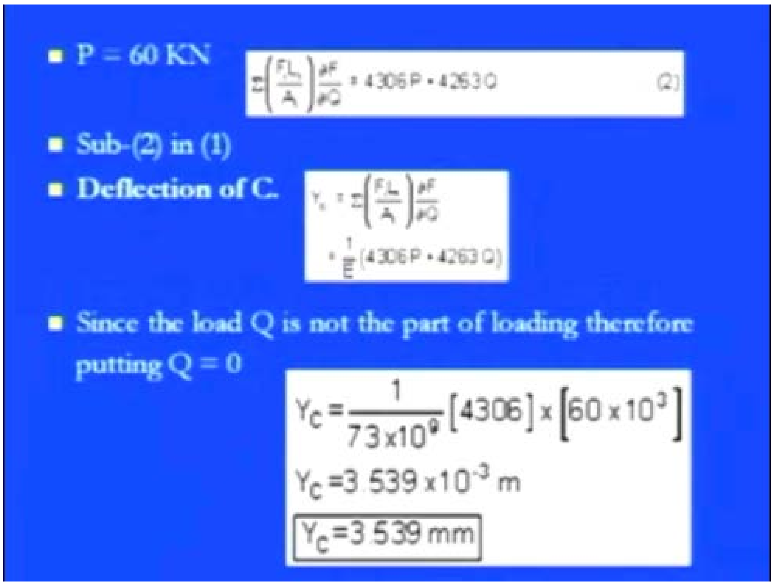

So, now with those particular combinations now our main intention is to find it out that what the deflection is at point C. So, we know that at point P, at point E the load application is there of the 60 kilo Newton. So, now, we need to replace that part. So, you see, if you sum up all three components from the table, what we have? We have the integration of Fi Li over Ai into dF by dQ is 4306 P plus 4263 Q. So, now, you see, here, we know the value P. So, we can simply put in those things in those numerical values. So, we have the deflection of point C YC is equal to summation of Fi Li divided by Ai into dF by dQ or we can say that simply, you know like, we know the value of that. So, 1 by E is into, you see, 4360 P plus 4283 Q. And, you know like, since the load Q is not a part of loading, because it was a dummy load is there. So, we need to keep the Q is equal to zero because the contribution is coming only from the load P.

So, within that constraint, you see here, we have the deflection at point C is equal to, you know like, the E value is already given as 73, you know like, into 10 to the power 9. So, you see, we need to replace that part. So, YC is equal to which is the deflection at point C due to the responsible load at point P of 60 kilo Newton is equal to 1 by 73 into 10 to the power 9. And then, you see, 4306 was there and P is 60 kilo Newton or 60 into 10 to the power cube. So, now, you see, after applying those things we have YC which is 3.539 into 10 to the power minus 3 meter or we can say that the deflection at point C is 3.539 millimeter.

So, in this calculation, you see, our main intention is that we want to calculate the deflection at the point where the load application is not there. So, what we need to do? We need to apply, you know like, a dummy load at that particular point. And we just assume that, under that particular load we have the deflection. So, under that particular dummy load what we need to do here? We need to first of all check it out that how the force transmission is there according to the distance and the load application.

Then we need to figure out that what the relation is there between the actual load and the dummy load, and then you see, you know like, in Castiglione's theorem it is saying that in all those terms it is dF by, you know like, means the total force with the dummy load is there, so what exactly the relation is there in between them. So, we simply make, you know like, individual terms that actually how the force transmission is there and which forces are being acting on which member.

So, corresponding to those relations we could easily figure out that these are, you know like, responsible parameters are there through which the force transmission is there and then after getting those things we can simply find it out - the deflection - by summing up those forces with the area and the forces with the relation of the dummy load. And then, you see, in this question also we did the same thing. So, you see here, you know like, these two numerical problems are clearly giving you that what actually what exactly the application of the Castiglione's theorems are.

Because, - if you check it out those things then - we will find that Castiglione's theorems are real, the real applications are there in which the number of float points are there. Just like, you see, in the previous case where, you see, the two different kinds of loadings are there - the UDL and the point load is there in the cantilever - and, you see, it was pretty simple to calculate those things, because you see, they have a direct relation with each other.

So, we can, you know like, simply make the relations between the point load and the UDL in that particular case and simply find it out the bending moment and the, you know like, due to point load and the UDL and we simply get the strain energy. And once you have the strain energy, once you have the load relation, you have the deflection at those points.

Similarly, you see, in this truss problem also, the truss problem it is simply that we have a number of various members and we need to calculate that what the force transmission is there when you apply the load at a particular point. So, in these cases also, you see, when the number of loads are there and due to the number of loads, the number of deflections have to be there. So, we need find it out the number of, you know like, what are the number of key locations and corresponding deflections. And then once you get those things, you have the deflection at the corresponding point.

So that is why, you know like, the Castiglione's theorem is an a real good application where the number of, you know like, the points are more and due to number of load points, we have the number of deflections, and then we can simply figure out these particular, you know like, the relations based on the Castiglione's theorems. So, this is the real application of the Castiglione's theorems for calculation of this deflection at the different points irrespective of the different load conditions.

Now, you see, here, you know like, that this is what you see, you know like, in the Castiglione's theorems which we discussed, in the various theorems, in the first theorem or the second theorem based on the load application as well as the bending moment application for calculation of the strain energy basis. Now, you see here, this is the last segment of this course curriculum, because you see, you know like, we started our journey right from the interaction of the solid object.

So, as per its name - strength of materials or the solid mechanics - our main theme was that when you see, we have solid objects and when they are interacting to each other, then always, you see, we will find that the kind of deformations are there. And, you see, the kind of, you know like, the force transformations are there or we can say that, you see, when we are talking about the static or dynamic force, then always it has a different picture to altogether, basis of whether they statically this equilibrium positions are there or the dynamic equilibrium positions are there. So, in all these cases, you see, with the various hypothesis and assumptions we started our journey with the stress part.

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So, the stress is simply defined as, you see, the force per unit area, but generally you see, you know like, we also we have the similar kind of parameter which is known as the pressure. But we need to clearly define that what exactly the difference between the pressure and the stress, because you see, if, you see, the definition, then you will find that either the stress or the pressure per unit area or Newton per meter square or the Pascal, but stresses are always… The real difference is that the stresses are always being induced in that object by an application of force.

So, that is what, you see, it has an great influence where the point of application of force is, while the pressure are simply applied part is there. So, stresses are always coming out from the objects, that is why, you see, we are generally refer the stresses as the intensity of restive forces. And that is what, you see, we are defining the stresses in the various types; like, you see, if you are simply apply the load along, you see, the axis, then we are saying that these are the normal stresses, because the normal forces are there towards that.

So, in the normal stress component we have the two, you know like, the types were there which we discussed: one was the tensile stress; one was the compressive stress. That means, you see, if you are elongating, you know like, any object, then you see, you know like, the kind of resistances which provided by material that was, you see, the tensile forces; so, tensile stresses are there. And simply when, you know like, in the axial form only, when we are simply compressing those part, then we have the compressive stresses. So, these are the axial form.

But if you have, you see, the normal form is there; that means, you see, you know like, if we are talking about the stresses not in a axial form, but in the plane. Then another form is coming that is known as the shear stresses. That means, you see, when the stresses are coming on the circumferential part of an object; that means, you see, when they are taking x and y - both axes, we can say that if it is taking an a plane, then the plane stresses are the shear stresses or we can say that the rotational part when we are talking about the shaft, then it is the torsional stresses.

Another form, which is coming out from the normal stress component is the bending stress. So, whenever, you see, we have… and that is what you see, in general application we will find that whenever a the beam is there and the load application is there, always there is a kind of deviation is there, the moments are there, and we are relating, you know like, this intensity of restive forces within that beam element is the bending stresses, within the bending form.

And then, you see, within that also when we discussed about that, we found that the another form of the stress is the thermal stresses; means, you see here, whenever the change of temperatures are there, from the room temperature, means either if we are going towards the higher direction more than 100 degree Celsius or even we are going towards the lower direction, then we will find that always the expansion or the contractions are there and the stresses are always being induced in the object due to the temperature variation and these are known as the thermal stresses.

So, these are 5- 6 types of the stresses were there and these stresses were introducing or inducing those object by the application of force.

Then, you see, the next term, which we discussed, which is closely associated with the stress was the strain. And we also figured out that stress is neither a scalar nor a vector quantity; it is a tensor quantity. So, that is what, you see, at least the 9 independent parameters are required to define the stress component. So, that is why you see, irrespective whether we are going in the normal direction or in the plane direction, always you see, you know like, it is shown by a 3 by 3 matrix and that is what you see, you know like, in the tensor quantity, we need the 9 independent parameters were there.

And that is what, you know like, sometimes when we are defining the stress, it is to be termed out as the tensor stresses are there.

And then, another term is the strain part is there, which is closely associated with the stress, because you see, when we apply the load, there is a kind of resistance is provided by the material, and due to that stresses are being formed, and along with that if any kind of deformation is coming, this deformation can be computed with the using of strain concept. So, strain is nothing but equal to as we discussed that actually the change of length or change of any dimension of the object divided by the original dimension. So, it is, you see here, that it is a dimensionless parameter was there. So, it is simply, you know like, measuring that how much deformation can be come up with that particular material under the action of those load.

And with these two important quantities in the strength of material we have, you see, the real phenomena about the material property. Like, you see here, when we have a simple tensile test, where you see, the load application is there within the stress and strain component, now if you draw the stress-strain component we have, you see, the variety of, you know like, the properties are there. Like you see, you know like, if you are drawing the stress-strain diagram for a simple ductile material like this mild steel or any high pid steel, high carbon steel or any kind of ductile material, they are exhibiting, you see, the elastic region and the plastic region altogether.

And, you see here, the clear difference between these two regions are simply, you know like, judging by the yield point. And then, you see, you know like, whatever the energy is being absorbed within the elastic region is always computed with the using of this modulus of resilience. And if you are talking about the whole elastic and plastic region; that means, you see, you are taking about the whole energy concept, means how much energy can be absorbed by a material under the stress-strain curve, then it is known as the modulus of toughness.

So, these are, you know like, the closely associated part is there and in that also we can simply figure out that the Hooke's laws are there and there are variety of parameters are there which are valid only for the elastic deformation. That means, you see, when we apply the load, the deformation is there, but when we release the load there is no deformation is there; means it simply regain their part. So, within that part, you see, we this law is known as - where the stress is propositional to strain or the elastic region under that particular law - the Hooke's is there.

And within that particular we have three main coefficients are there: one is the Young's modulus of elasticity; one was there - the bulk modulus of elasticity; and third one was there - the shear modulus of rigidity. And all three are dependent on that what kind of load application is there and what are the influencing stress component is there with the corresponding strain.

And apart from those things, then we simply discussed about that, you see, when you have the stress and strain, then you know like, all those types of stresses and all those kinds of applications are there, then we simply find it out whether, you see, the stress application is there in one directional or it is a biaxial stress this force action is there or in the triaxial part.

So, if it is in the biaxial form is there, that means,, you see, the stress is there in the x component, in the y component, and also along with that normal stress, we have a shear stress component is there, then the new term is coming as the principle stresses. That means, you see, we define the new planes on which some stress components were there and these new planes were known as the principle planes where, you see, there is no shear component was there.

So, you see, here, that means, what we have? We have the, you know like, by simply you know applying the load on this outer conditions, we can simply figure out the stress components. But if you want to find it out the stress at the oblique plane, that means,, you see, at the theta somewhere in between that, then also we could simply relate those parts, you see by, you know like, using the various theories.

So, that is what, you see, in the middle of section we discussed, that actually, if the oblique plane is like that, then what exactly the relations are there in the normal stress component, the sigma theta, tau theta, at where the theta, you know, the location is the oblique plane. So, that is what we discussed, and in that, you see, later part we discussed that we need to define certain, you know like, the planes where there is no shear stresses are there and these planes will be known as the principle stress component.

And then, you see, the principle stresses which we calculated as sigma 1 and sigma 2, generally if you note the if you know this notations, then you see, it is nothing but equal to sigma x plus sigma y by 2 plus minus square root of sigma x minus sigma y by 2 whole square plus a tau whole square xy 4 times of. So, if, you see that, principal stresses it has the, you know like, clear combination of these two normal stresses sigma x and sigma y, with this shear stress component. And once you have the principal stress component, the corresponding strains are there which are nothing but the principle strains.

We can also figure out straight away the principal stress components if you know the normal stress components at the oblique plane also, with the using of Mohr circle. So, this is the graphical techniques are there, through which we can calculate, if you know the this stresses at normal planes or we can say that the oblique plane, then we can simply find it out the sigma 1, sigma 2 or also we can find it out that what will the value of this maximum shear stress is there, sigma 1 minus sigma y by 2. Or also we can simply locate the location tan 2 theta where, you see, that these shear stresses are being what is the potential area is there ,where the maximum shear stresses can come. So, these were the basic formation was there, you see, about the strength of material.

Then what are the basic components can be introduced in strength of material to analyze the structure. So, you see, we discussed about the various types of stresses, strains corresponding, and then the oblique stress, oblique strains are there at the different locations of the theta. And then, you see, we have the principal stresses and principal strain correspondingly, and then you see, we can simply figure out by analytical method sigma 1 comma sigma 2 or by Mohr circle also which is the graphical technique.

So, that part we discussed, and after that, after discussing about the sigma x, sigma y, tau xy, now we simply discussed about the bending moment; means, you see, if we have the variety of the beams are there, means you see, you know like, the straight beam is there, the cantilever is there, the simply supported beam is there, and you see, in that also the different combinations are there; means in the pin joint we have, you see, the some movement is there. So, all those variety of combinations we discussed.

And in that, you see, again the variety is coming on the which type of loading is there. If simple point load is there at the extreme end or middle of the beam or anywhere or the

UDL is there - means the uniformly distributed load is there - then what the intensity of the UDL is there or the triangular element is there, then what kind of intensity is there, and how we can sum up that particular intensity, because it is in the triangular one, starting from zero one ending at the maximum, so how we can, you know like, just those element how we can take the uniform action of that part. And then, if we have this kind of triangular element, the triangular element of loading is there, then how we can figure out.

So, we discussed about the bending moment and the shear force diagram in this chapter, that you see, you know like, the BMD and SFD how we can calculate. Sometimes, you see, when the point load is there it is, you see, the this straight line terms are, when UDL is there the curved part is there. So, how, you know like, the bending moment diagram can be drawn; how we can calculate, you know like, the maximum bending moment at which location it will come. So, all those issues, which we discussed in that.

And corresponding, you see, we discussed about the stresses - bending stresses. That, you see, what will be the sigma y, sigma by y is equal to M by I is equal to, you know like, this tau by J. So you see here bending stresses, you see here, bending stresses which we calculated based on what the bending moment is. And what you see, you know like, the section modulus is there. So, what kind of structures are there and what is the material property is there.

So, we discussed about, you see, almost all types of, you know like, the cases of bending moment and the stresses. And within the bending moment we also discussed about the shear force diagram. So, shear is also closely associated with the bending because sometimes, you see, when we are talking about the bending it is, you see, kind of the combined action. So, you see here, we develop some of the shear, you know like, the theories, that what will be the, you know like, the shear stresses are there and how we can calculate the shear force diagram and all those things, along with torsional part, when we were discussing about the shaft.

So, you see here, in all of those either the bending moment and the shear, we discussed about the what the deformation are there and what the corresponding, you see, the stress components are coming within the bending stress or the shear stress.

And then, next component came with the deflection. That if we have, you see, you know like, the bending action or if we have the shear action alone, then what will be the deflection criteria are there. How we can get the deflection by direct integration method, by moment area method, or by Macaulay’s method. So, these three methods are very, very important, you see, and they have the individual, you know like, importance to study for calculating the deflection of any structure.

And you see, lastly when we when were discussing about the Macaulay method, which is quite fast, you see, simply you know like, we need to just that when the… You know like, the beam is there, when the load conditions are there on the beam are different and they have the different segments or the spans are there, then you see, what you need to do, you need to simply describe or make a generalized equation within those singularity function, and simply apply the boundary condition get the value of deflection. And you can also get easily that what is the potential area is there where the maximum deflection can come. So, that part we discussed.

And then, you see, we discussed about, you know like, the deflection when the combined loading conditions are there. That means you see, you know like, when we have, you know like, the couple with the UDL is there or when we have UDL plus point load is there, means you see, when the variety of, you know like, the load conditions are there, then how we can get the deflection under those conditions.

So, this was a real good, you know like, the applications are there because when we are talking about any beam in you see, you know like, either our structure, the house structure is there, or in any of you see, you know like, the kind of this real civil structure are there, always we have you see, you know like, this kind of load conditions are there, the combined load conditions are there, and we need to analyze those things accordingly. That which area is the, you know like, where the maximum deflections are coming, what the load conditions are there on those things, and the like that.

So, that is what, you see, this chapter was taken as in a, you know like, the separate chapter. And then, you see, we were discussing in the last - few of the last segments - of our course lecture, was a strut and the column. Because generally, you see, we have seen that, you know like, in our houses we have a strut, you see, the columns and the struts are there. So, how we can figure out and what will be the buckling load is there, what are the influencing parameters were there in that conditions. So, all, you know like, and what are the different loading conditions are there with those boundary part, you see, whether it is a rigidly fixed or pin joints or one end is free or one end is pin joint like all those things which we discussed, in the bending moment also we discussed the same thing in the column for a elastic stability. So, you see, here we discussed many things in those forms in the strut and column.

And lastly, you see, we discussed about the various form of energy which is pretty closely associated with the, you know like, the beam, the this kind of our applications. Like, you see, we have the strain energy; we have complementary strain energy; we have you see, the strain energy due to the bending action; the strain energy due to the various other actions like, you see, the load conditions are there; we have the modulus of resilience; we have the modulus of toughness; so, all these forms of the strain energies are very, very important.

And that is what you see, you know like, we discussed about these energy formation - that what is the domain is there; whether it is the shape, you know like, strain energy is coming due to the shape deviation or whether the strain energy is coming due to the volumetric changes are there.

And then, you see, in terms of the complementary strain energy or this elastic strain energy what is the domain is there. Means somewhat in elastic strain energy we had the common domain was, you see, the strain while you see, in the complementary strain energy we had the common domain was a stress. So, you see, here the energy form formation because, you see, whenever the deformation is there under the load application always store the energy, and since, you see, there is a stiffness, stiffness part is there in that, so obviously, whatever the energy stored is there, it is known as the strain energy formation because of the load application.

So, that is what, you see, right from our journey started from the stress and we finished up to the strain energy with the Castiglione's theorem that, you see, if you have the different load condition on structure, and you see, you know like, and all these load conditions are different at different points. So, we could easily figure out the deflection at the individual point with the using of strain energy concept. So, del U del Pi will give you this deflection point or, you see, del U by del M will give you the angular rotation theta. So, that was the last part is there.

So, I hope that you enjoyed with the whole journey which we started from the stress component. That what the stress is there, without having least stress in your mind, with you see, we discussed, you know like, the many of the numerical problems along with that, but you see, this subject is always gives you a quite comfortable position. Because, you see, whatever the design, which you are going to do in civil engineering, or in mechanical, this is the basic domain for any design.

Because, you see, until and unless if you do not know the basic material property under the load - given load conditions - then you cannot figure out that what is the factor of safety, how much you have to take. And then you see, you know like, how you can design, and what are the real load interactions are there, which are coming through the various components of the machine either or the structure part.

So, that is why, you see, the strength of material is the one of the basic subject of Engineering discipline. And, you know like, if you want be in comfortable position, just try to resolve those physical concept, means physics is very, very important which is quite associated with the some sort of small mathematics. So, I hope you enjoyed this particular subject, and I am wishing that actually, you know like, you will definitely do a good part later on.

Thank you, good day.