Strength of Materials
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Lecture -04

Solid Mechanics

Hi, this is Dr. S. P. Harsha from mechanical and industrial engineering department, IIT Roorkee. I am going to present today as lecture four of the topic of this, you know, like the strength of materials. In that you see, we are going to discuss about more of, you know, like the stress components, that, which we have discussed in the previous cases.

In the previous lectures we found, that if you want to describe the general state of stress in which you see, you know, like if, like, that there are total nine number of components were there including you know, like the three normal stress components and the six shear stress components.

So, if we have a regular structure like unit cube or parallel pipe, then it is pretty easy to analyze those things because you see, you know, like the three stresses are there, like sigma xx, sigma yy, sigma zz. And then, you see, we have six different components, like tau xy, tau yz, tau xz and the three remaining components. Mean to say, that you see, you know, if you want to describe the stress for an object or a component, then we need at least nine components. That is what you see, you know, like we, you know, like found, that we need, you know, like the tensile stress to describe the stress for a point. So, you see, you know, like this part, which we have discussed.

So, in this lecture the basic thing is, that if you do not have that kind of, you know, like the structure, means if we have, you know, like any radial structure in which even, you know, like we discussed, that the Cartesian coordinates were there or the cylindrical coordinates are there.

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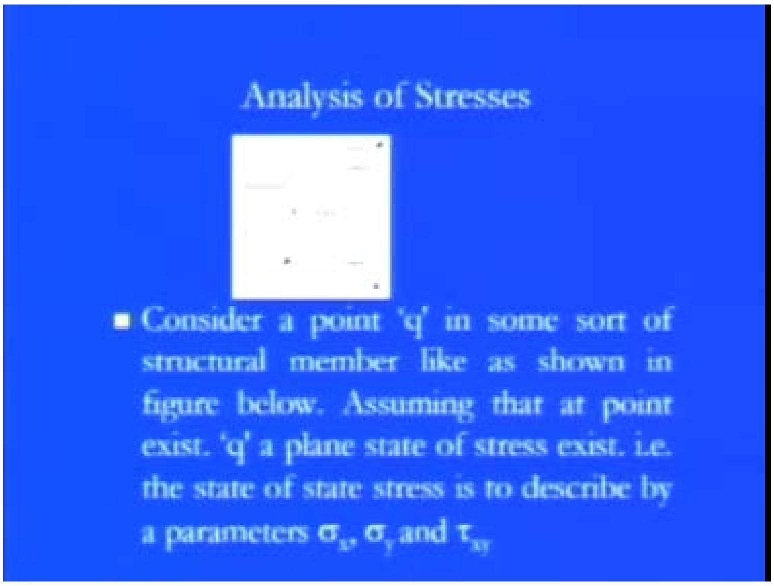

But if you are talking about a radial structure in which you see, if you are taking any point q or you see, you know, like the points are there in, that if you want to check the stress components, then how we can, or if we have an oblique plane means, that if you cut the sections not exactly perpendicular to x axis, perpendicular to y axis or perpendicular to z axis where you see, we define the stresses, if it is not there, then what is the stress. And how we can relate that stress to the main stress, like sigma x or tau yx or tau xy or all those components.

So, this kind of, you know, like the information, which we are going to dig, you know, like digging from this chapter. So, first of all we, we are starting from the analysis of stresses in which this diagram is shown.

In this diagram, you see, this we have, you see, you know, like the component, any machine component, which you see on our ((Refer Time: 02:46)), these forces are there. So, these arrows are showing, that where the force application is there.

And as we discussed, that the force is always defined by the point of application. So, you see here, these are the point of application where the forces are acting and this is the q. q is the origin, you see, you know like, or we can say, that this is the point where we just want to check it out, that what the stress components are there due to these, you know, like the forces. And you see, you know, like the stresses are nothing but the intensity of resistive forces. So, how this stress distribution is there all across this domain?

So, this is our domain, which is neither you know, like parallel or perpendicular to x, y or z axis, respectively. So, consider a point q in some sort of the structure, remember this, like you know, like as shown in this figure. Assuming, that at a point, you know, like we have q in which a plane, you know, like state of stress exist. That means, you see, now we are going to consider all those stresses, stress components, which are you know, like existing at this particular state. So, the state of stress is described by the parameters sigma x, sigma y and tau xy if you are considering the stresses is in x as well as the y component. So, this is the one, you know, like the figure, which we want to analyze those things.

So, if you go again back to the concepts, then you will found, that this, if we are talking about x-y plane, then what kind of stress distribution are there. So, these three stress components, sigma x, sigma y and tau xy are the dominating, dominating stresses and these stresses are always, you know, like distributing all across, you know, like the structure if you are considering the x-y plane.

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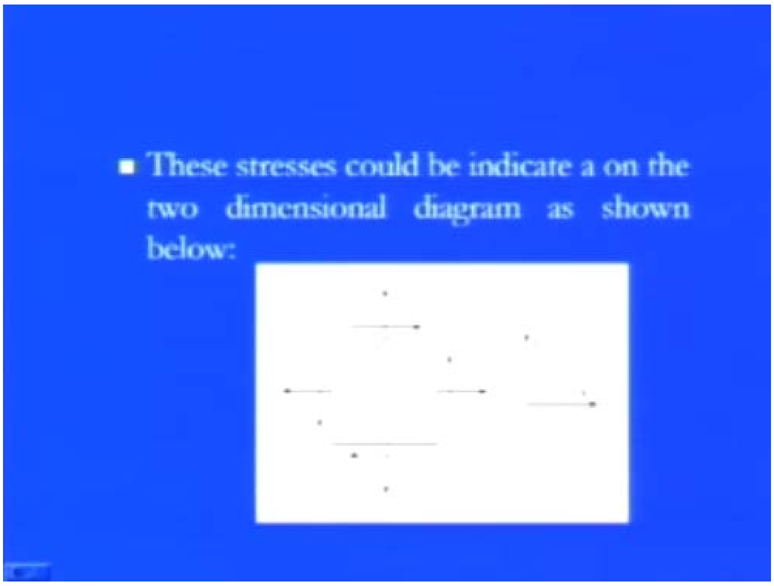

So, this diagram gives you a clear cut picture, that actually if we have this x-y plane, where you see this y and x is there. So, you know, like this diagram shows you. So, in this particular, you know, like picture we can easily, you know, like describes that.

We have sigma x all along this x axis. So, this is sigma x, which is of the nature of the tensile stresses and we have the sigma y, which is also of the tensile stresses, tensile stresses. And that is why, you see, we are considering, that this is a positive stress and remaining part, you see, like tau xy or tau yx we are considering here, that this is the tau xy, you see here, which tries to rotate this object in a clockwise direction.

So, we need, if you want to maintain the equilibrium of this object, then we need the complimentary stresses. So, they are coming from this domain. So, this is in this direction and these directions, these two, you know, like this direction shows, that the stresses are there, the shear stresses are there, tau yx. And they are always exactly equal and opposite to this stress component.

And that is why, you see, we can maintain the equilibrium within these objects. That is what you see, you know, like if you want to consider, that you see, the stresses are being formed because of the force application. And if under this stress or under the force application if this object is well maintained, you know, like the equilibrium position, it is only possible when all the summation of forces in x direction, y direction as well as the z directions is 0.

Similarly, if we consider, that if we have an origin point O, let us say here, and if you are taking moment about this point O, either by, this is the shear stress or by this shear stress or by this shear or by this shear stress, then we can say, that yeah, from all the four components of the shear stress, whatever the moments are there about this point O, they must be equal to 0. If we consider those things, then we can say, that yeah, this is well maintained, well equilibrium structure under the action of these forces, under these three stress components.

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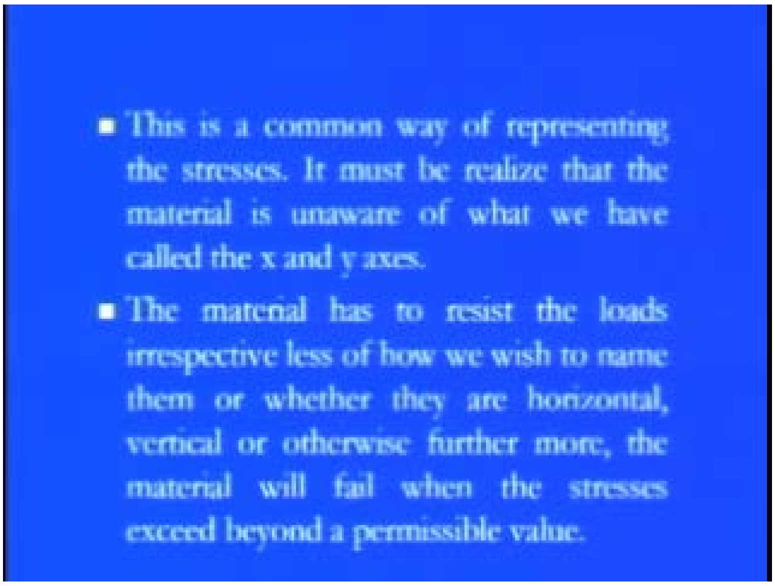

So, this is a common way to, you know, like common way of representing the stresses for x-y plane component because you see, if we are considering the three dimensional, then we need to consider, as I told you, that we need to consider all nine components, the triaxial form of these in which all three axis are mutually perpendicular. But here, only we are considering the two axis, x and y, where these three stresses are there because of the symmetricity of the object

We can say, that it is pretty easy to describe all the state of stress by, with the using of these stresses it must be realized, that the material is unaware of what we have, called the x and y axis.

So, whatever the material is there, irrespective of whether it is, you know, like this stainless steel, high carbon steel, high speed steel or any ductile material, it is irrespective of that, that what the stress are there. If you are saying, that it is a ductile material or it is a brittle material, like you see the cast iron or any hard material, we can say, that these are the stresses, which are always being formed if it is representing of this nature.

Means, you see, if you are apply those, you see, you know, like the stresses, like the normal stress component or we, we are saying that the shear stress component, then they must be there within those objects. They are well settled within this, those objects if these forces are their irrespective of what the material is. So, material is unaware of all those kind of these axis. The material has to resist the loads irrespective of less of how we wish to like give the name or we can say, whether they are horizontal, vertical or otherwise, you know, like we can say the material will even fail when the stresses exit beyond a permissible value.

So, material provide the resistance based on how much hardness is there, how much stiffness is there. So, it depends on, you see, that what is the limit of the loads are there or what is the limit of the stresses are there, the normal stress or the shear stress. So, this is the key feature; this is the inherent property of the material. But it is irrespective of whether it is a, it is going in x direction and y direction or z direction.

So, this is a real important phenomena about the stresses, that actually though the stresses, whether the normal stress or the shear stress in the any of the direction, they are the function of the material. But the axis are not, you see, you know, like absolutely depends on that what material is. So, you see here, if you are talking about any ductile material, then we have to be very, you know, like careful, that actually what is the stress limits are.

So, once you see, you define the material absolutely, you see the stress levels are coming, that ok, you can, we can go like, you see, if we have the normal mild steel, then we are always using, you see, 220 mega Pascal. That means, you see, this is the limiting value. Under that, you see, if whether we can go for the elastic deformation and if we are going beyond that, then we can say, we are going for the plastic deformation and that you see, you know, like for those limits we can define by the modulus of elasticity and other modulus.

So, there are, you see, lot many coefficients are there based on, which material you are using. But these stresses are absolutely the function of these material. But the material is, whatever the, this is a normal stress and shear stresses along these axis, they are not the function of the material itself.

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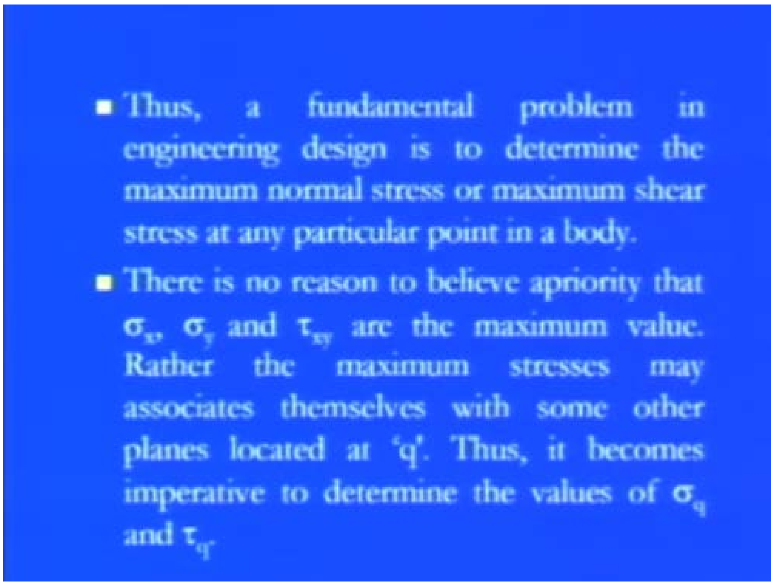

Thus, you see, the fundamental problem in any of the engineering design, because we are using all those kind of material and the different kind of loadings are there in that. So, this fundamental problem in the engineering design is to determine the maximum normal stress and the maximum shear stress at, at any particular point in a body. That means, you see, we, we do not know, that actually you see, if you are applying that load and somewhere, you see, the shear stresses are maximum, normal stresses are not maximum, whether, and whether this material will sustain or it will fail.

So, we have to be very careful or we need to design that component, whether this can sustain under these, these values of the stresses or under these types of stresses or not. So, the first aim in any of the component design is to know, that whether, what is the maximum stress and at what point these stresses are exerting or this, you know, like executing within those object.

And there is no reason to believe appropriately with that sigma x or sigma y or tau xy or the maximum value, you see. It is not, that actually they are somewhat, you see, they have some value because if you are talking about sigma x or sigma y, they are simply, you see, the normal stress components. So, irrespective of whether we are pulling or compression, they are simply the tensile or compressive forces divided by the effective area.

But if you are talking about the tau xy, even it is, you see, you know like this is nothing but the, as per the plane like tau xy is there. So, if you are considering this tau, this x and y plane, so these forces are parallel to the axis, but they are not having the maximum value. So, you see we need to be, you know, like chosen the maximum value out of which. So, rather the maximum stresses may associate themselves with other planes located at q. Thus it becomes imperative to determine the values of sigma q as well as tau q.

So, you see, once we know, that ok, now here in this particular object or whatever the component, we have a point where the maximum stresses can occur due to the variety of loading.

Then, our main focus is to calculate, that actually which stress is maximum first and what is the value or what is limiting value based on that, you see, you know, like we can you know, like design the factor of safety. And we can, you know, like we just decide the factor of safety and corresponding designs are there of the component.

So, the conclusion of whole discussion says, that actually, though stress is one point to calculate within the structure, but the distribution stresses are very, very important. And we must know, that what is the location of the maximum and the minimum stresses are there, the normal stresses as well as the shear stresses, so that we can appropriately design the engineering component, which are under the influence of various forces.

So, here, you see, now if you are talking about the shear stress, because you just want to know, that actually where the shear stress are there. And if we have a normal plane, then whether this plane can sustain under the maximum and minimum shear stresses or not.

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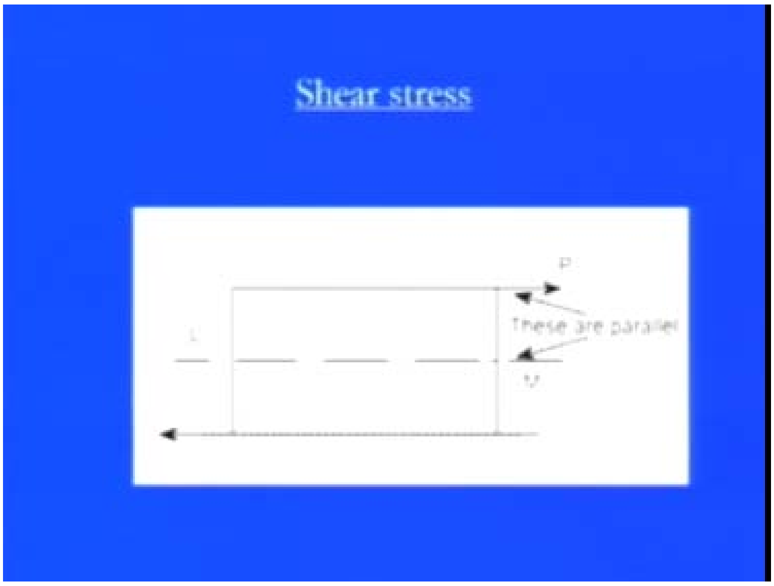

So, here you see you know like this Plane says that actually the force p if you are talking about these. So, these are you see the parallel you know like the planes are there and this force p is exerting in this direction and we have a datum over which you see there is a force which is going towards the this direction

So, if you see the nature of force then we found that these forces are the parallel forces along this plane

So, we have there is a plane or we can say the central axis and LM of this object and whatever the forces are applying here they are simply parallel to these things

And that is why we can say that whatever the stresses which are inducing due to the application of these forces they are the shear stresses and they, they are very much you know like we can say well established things are there within these regions

So, we must know that actually at which region within this object the Shear stresses are maximum so that we can easily design those components by taking the different value of the factor of safety.

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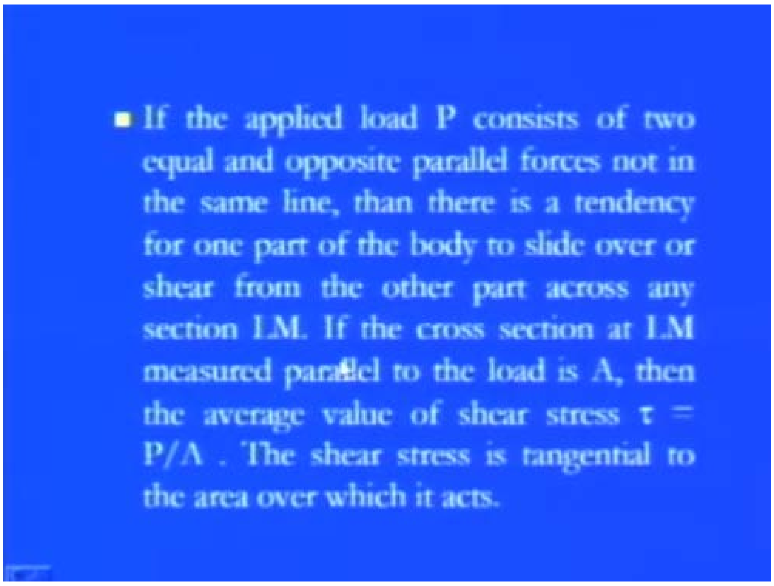

So, if the applied load P consists of the two equal and opposite parallel forces, as we discussed in the previous case, not in the same line because if the same line is there, then definitely, it is the axial force, we can say. And these are, due to these forces we have the normal stresses. But here as we found, that though they are the parallel and opposite forces, but they are not exactly on the same axis. So, we have the shear stress.

Then, there is a, you know, like tendency of, for one part of the body or another part to slide over the another and that is why, you see, the shearing is there or shear, shear from the other part across any section LM.

So, obviously, you see, this is the great tendency of the shear stresses, that actually they are always making the shear plane and amongst the central line or we can say, the neutral axis. And neutral axis is always, you see, the axis where this centroid is existing or we can say, where all layers are to be well settled or we have well equilibrium within this object.

So, as we discussed about the shear stresses, they are always set up in the object, in the parallel planes. So, because of these, you know, like the loads we found, that the shear stresses are there and they have a tendency to make the shear across this section and that section we are telling in this particular figure is LM section.

If the cross-section at this section LM, you know, like major parallel to the load, which is A, then we can say, that average value of shear stress is the load application, which is you see, the inducing shear stresses divided by A or we can say, P is nothing but the shear force or shear load. The shear stress, you know, like, is tangential to the area or which it acts. So, this is the great, you know, like the meaning of the shear stresses because as we have seen, that actually as far the normal stress component is concerned, they are, you see, always at parallel to the this surface, the area matter concerned.

But here, you see, whatever the area, which we are concerning or which these stresses are being set up or this loads are there, these are always, you see, the tangential to the whatever the these stresses are coming. They are always tangential to the area of concern.

So, which we need to very careful, that actually, once we, we are, just if we want to find out, that where is the maximum stresses, what is the minimum stress are there, then we, we have to very careful, that actually what is the area under which these tangential, you know, like forces are the stress, are coming within those object.

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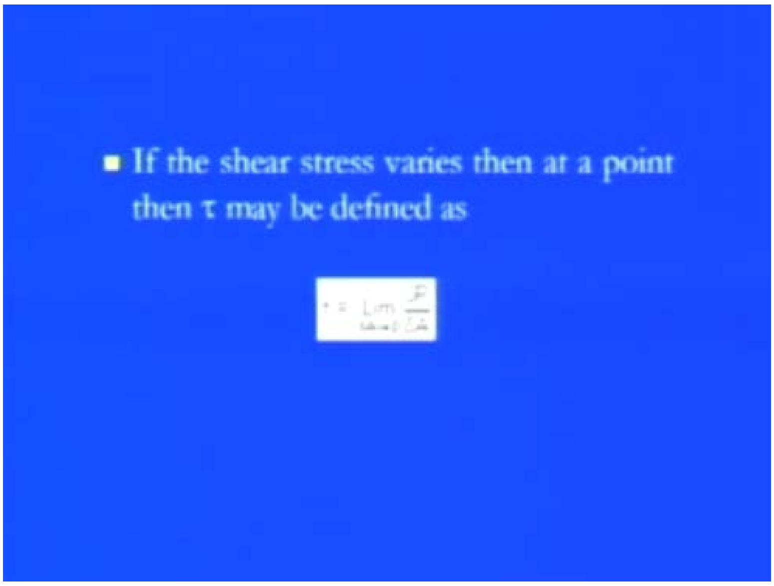

So, if the shear stress varies from point to point, as you see, you know, like we discussed in the previous lectures, that actually if we have irregular geometry where the forces are acting, but the forces are not uniformly distributed, because the different, different forces are there.

Like, if you have structure at some point, you see, some 10, 20, 30 Newton are there, at some point it is 50 Newton is there and it is of different nature. Some, some forces are of tensile nature and some forces are having the compressive nature. If this kind of, you know, like the structure is there and if you want to find it out the shear stresses of, you know, like within this structure, then as, as I told you, actually we need to discretize the domain into the different segments of the forces.

And then, once we know, that now these are the, you know, like for, if you have some 7, 10 segments what we need to do here? We need to calculate the shear stresses for individual segments, like segment one to segment ten and then, corresponding we need to integrate those things.

So, if, we have well defined regions, then there is no problem, you see, because the uniform, you know, like the structure is there and then the uniformly distributed forces are there. But if we do not have that kind of structure, then what we need to do? We need to define, that actually where is this area, of the area, the area is less or area is more or where is the stress concentration is there. If you see more stress concentration, then we need to take care of the, you know, like by taking more factor of safety and we need to design accordingly

So, if you are defining the shear stresses for those kind of regions where regularities are there, how we can define? We just define limit for individual regions and we are defining the delta P by delta F for that particular regions and then sum up to get the final value of the shear stress.

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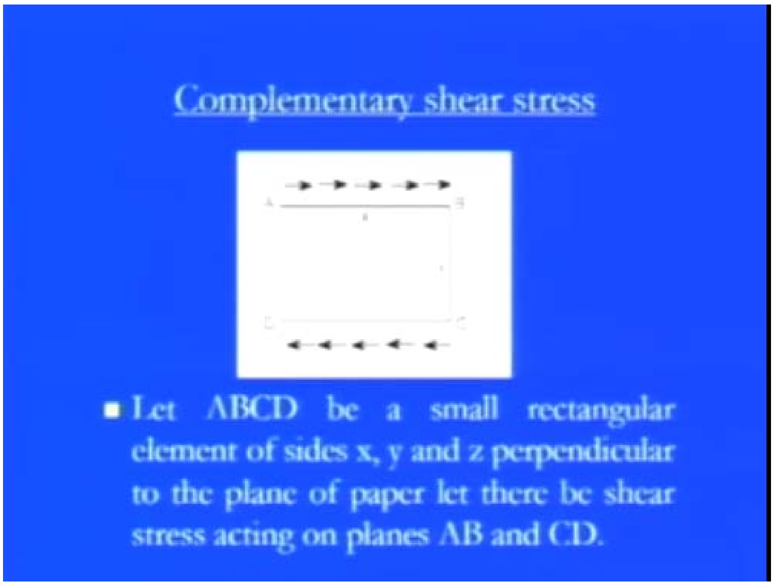

So, this is one meaning. But you see, you know, like if you are discussing about the general shear stresses, then we found, that always there is a tendency of the shear stress to move the object in one of the direction.

But if we want to maintain, as we discussed in the previous section, only that, actually if you want to maintain the equilibrium of an object, then we always need an equal and opposite stresses and those stresses are known as the complementary stresses.

So, if you see this figure, then we have the figure, you see, on you know, like the plane AB or CD, there is a stress component on this and this stress components if we are denoting by sigma or this tau, then these things are there. And these, you know, like the dash is showing, that the shear plane is, this is my shear plane on which this stresses are occurring.

So, if I am saying, that if I have ABCD, which is a rectangular element, which has the, you know, like the side of X, Y, Z, all these three mutually perpendicular sides to the plane of, you know, like the paper, then we can say, that the shear is, you know, like shear stresses are existing at this AB as well as the CD plane.

So, now you see, if we, if we exactly, you know, like figure out this particular problem, then we found, that this shear stresses always tries to tend this object towards the clockwise direction. You see, here you know, like this shear stress always influencing this segment of element just go towards this direction while this one is also, just tries to, you know, like tending this elements towards this direction, this CD portion.

So, that means, you see, you know, like if we combine both the effect, then we, you know, like we observed, that there is a tendency of this element under the influence of this shear stress, which is moving towards the clockwise directions. So, but if we want to maintain the equilibrium side, then we need the equal and opposite at these two phases, like at AD and CB, which is exactly equal and opposite to the applied stresses, applied shear stress sigma.

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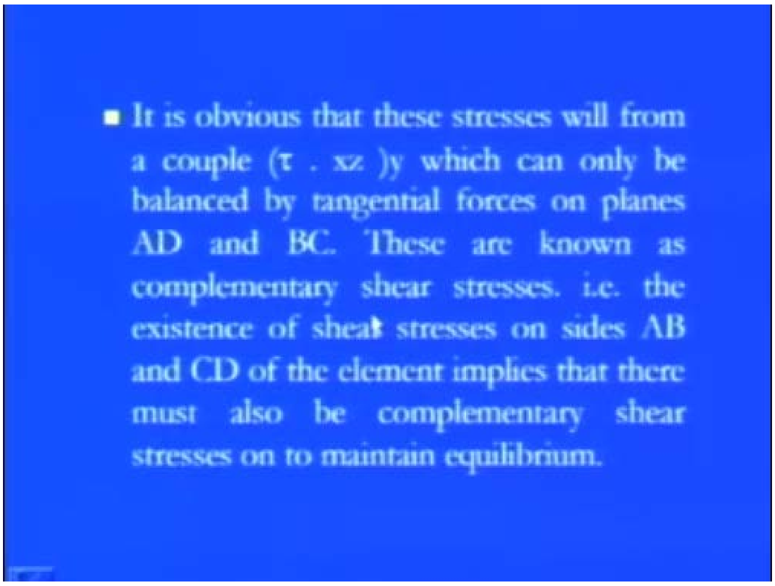

So, here you see, it is obvious, that these stresses will form a couple, as I told you that and the couple magnitude is this tau into xz, that is, the force into y. So, that is the couple. So, tau into xz will be the force exerting at exact plane into the y distance, which will give you the couple, always try to tend towards the clockwise direction, which can only be balanced by tangential forces on the plane, as I told you in the AD and BC, these two parallel force, these two parallel sides, and these are known as the complementary stresses.

Or we can say, the existence of shear stresses on the sides of AB and CD of the element implies, that the shear must be also, if shear stresses are there, there must be counter balance shear stresses. And those counter balance shear stresses are known as the complementary shear stresses, on the, on the other side of the element to maintain the equilibrium. So, this is, you see, the one form of shear stresses.

So, if you are, if I am saying, that the sigma is the shear stress, then sigma dash is the complementary shear stresses and if the sigma dash be the complimentary shear stresses inducing on the plane AD and BC, then we can easily maintain the equilibrium by taking moment, by you see, if we have on the one side of this sigma into xz.

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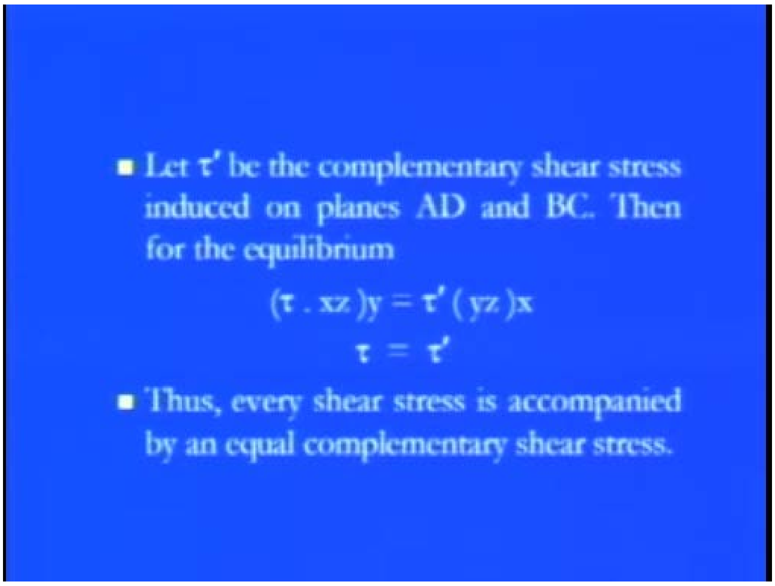

So, if xz is the plane and if we multiply, so sigma into xz is the force. If I multiply by y, then we have the total moment, which is exactly equal and opposite to the couple, which is coming due to the complimentary shear stress. So, it is sigma dash, which is complementary shear stress into y z. So, that is know as the total force in the yz plane into distance into x.

So, now you see, if you compare both the things, then we will find, that sigma equals to sigma dash. Thus, every shear stress is accompanied by equal complementary shear stresses. So, you see, we, we, we need a balanced part opposite which is equal and opposite to the shear stresses if we want to maintain the equilibrium of any component. So, you see, shear stresses and complementary shear stresses are you see, the two forms of a coin, which always there to maintain the equilibrium of an object.

Then, you see, you know, like till now whatever we have discussed, we were discussing about you know, like the regular geometry at the end, you see, we have the force application. Because of the force application we have the stresses, shear stress, normal stress component. Even in the shear stresses different, different shear stress are there. The different outer surface of those things by taking the oblique planes, by taking the normal planes and all.

But if we want to calculate, that you see inside those things, that means, if the, if you want to check, that what exactly is happening within, you know, like at different points in the object by under the application of forces, then you see, we need to check it out the stresses on the oblique plane. That means, you see, we need to cut the plane at an, a theta angle, which is not essential, that it has to pass through from the centre of the origin or we can say, the centre of mass of this object. It can pass from the initial part or the lower part or any part of that.

And if we cut that plane and if we want to check it out, that what the stresses are there, the normal stress component, shear stress component, that all the kind of information, which we are going to discuss under the heading of stresses on the oblique plane.

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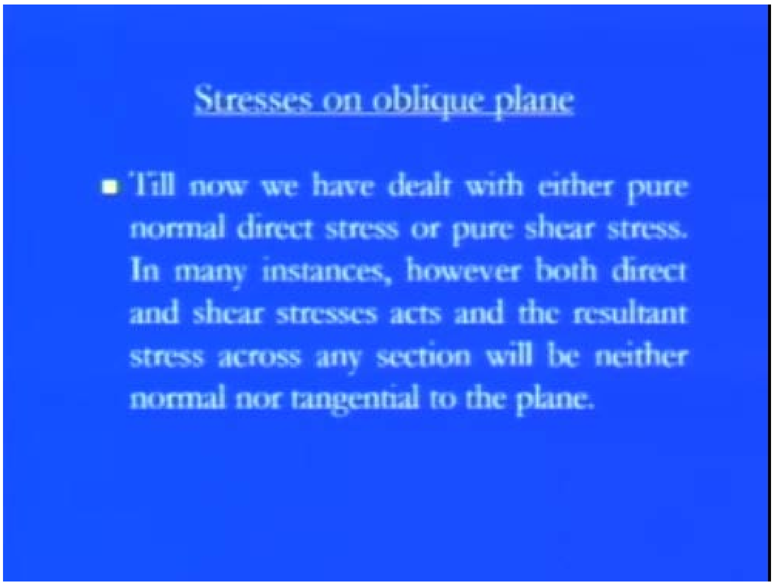

So, till now, you see, we have, you know, like deal with either the pure, pure normal, direct stress or pure shear stress in many of the instance. However, both direct and shear stresses acts and the resultant stresses across any section will be neither normal, nor tangential to any plane.

That means, this is the realistic situation, that actually if you want to check it out, that where is the maximum shear stress is there, where is the maximum normal stress is there, where is the maximum, the minimum, say normal stress or minimum shear stress, then whatever the analysis, which we have done in the previous cases, that is not sufficient or that is not, you see, the perfect to get the, all you know, like answers.

So, what we need to do here? We need, if we want to do the realistic analysis, then always we need to check it out it actually, what is the, what the stress levels are there under the oblique plane?

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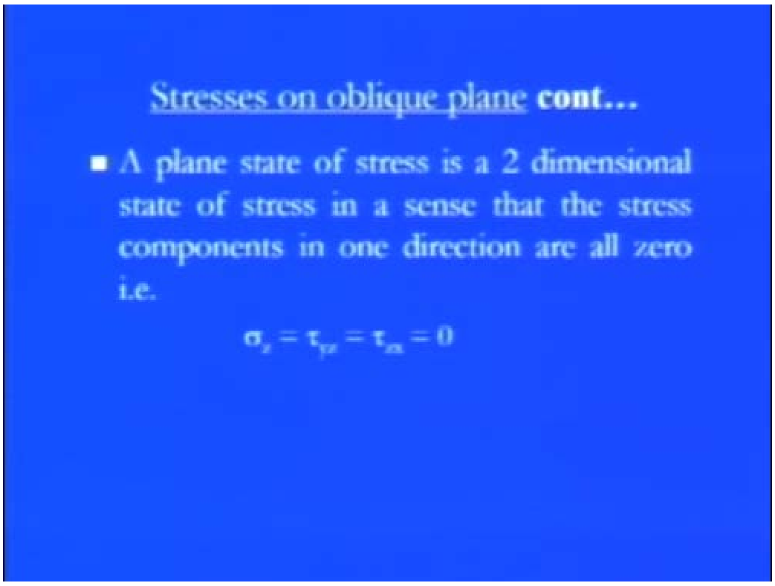

So, you see, here a plane of stress, a plane state of stress in any two-dimensional, you know, like the state of stress in a sense, that the stress component in one direction, you know, like all the stress components are there. They are all equal, equals to 2.

That means, you see, you know, like till now you see, if we discussed, if you just focused on that we discussed, that we have, that stress tensors in which the nine components are there. But and, but all, all, all nine components or we can say three by three matrix is valid if we have all, you know, like the triaxial state, the state of stress is there or we have all three mutually perpendicular axis.

But if we are taking about the two-dimensional state of stress, that means, you see, the one direction is gone. So, if you are saying, that if I am considering only x and y, this x direction and the y direction, that means, there is no stress. Means, I am assuming, that there is no stress component is there in this vertical direction or I should say, the z direction.

So, what has happened, you see, this kind of, you know, like the plane stress or I should say, that actually the two-dimensional plane stress is always calculating for these two directions and that is why, it is known as the plane stress. That means, for xy plane, yz plane or xz plane.

That means, you see, one direction is, just we are assuming, that there is a, this other direction is not existing. So, here you see, if it, if we consider this case, then we found, that actually, that this is you know, like the plane stress is there, which is only there in the xy, xy direction.

So, you see the sigma z or tau yz or tau zx means, either the normal stress component, z direction or the serious stress component in the z direction, which is coming due to the force in the z direction, like you see here tau yz means, the y is the domain and z is force or tau zx means, the z is the domain and this x is the force. That means, you see these stresses are not exerting, not existing in the plane stress.

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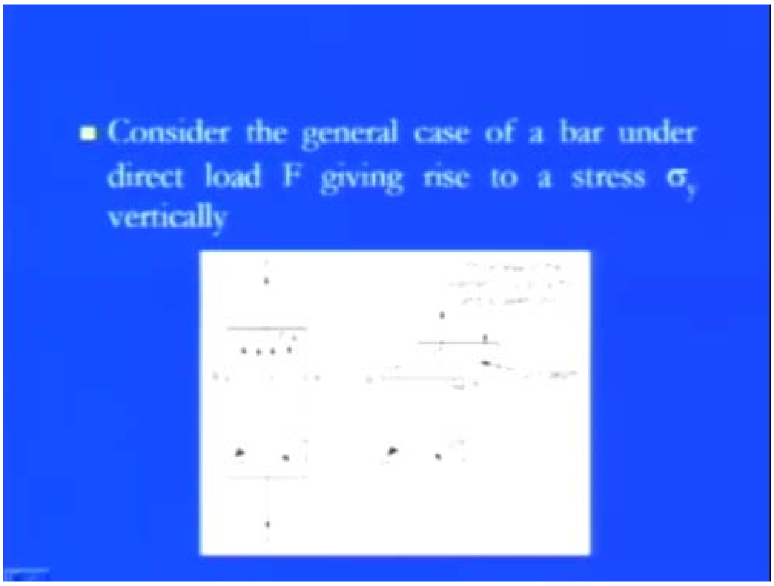

So, this is the plane stress, which is always there in the ((Refer Time: 25:15)) plane. So, now, you see, in under the two-dimensional state of stress or plane stress, consider a general case in which we have a bar, which is under the influence of the force in that y direction only.

So, if you see this figure, then you see here we have a two-dimensional object. This is a rectangular bar and the force is the tensile pulling is there in the y direction only. So, now if you see this thing, then we found, that actually these, due to these you know, like the force component, the y direction we have a tensile pulling, the tensile forces are there. Thus, and due to this tensile forces we have the tensile stress in the y direction only.

So, we have the sigma y. So, sigma y is nothing but equals to, if you, if, if you consider this BA is section, then we found that the sigma y is nothing but is equal to applied force divided by the area.

But if we want to check it out, the oblique stress is, of the stresses at the oblique plane, then we need to cut the plane at this BC. So, this, you see, now BA is a normal plane, which we have discussed in the previous cases. BC is an oblique plane. If you cut this plane by this section at an, at an angle theta, this.

So, now, we want to calculate, that actually what the stresses are there of the oblique plane. So, we can say this stresses at the oblique plane are sigma theta, that is, the normal stress component tau theta, that is the shear stress component and they are, you see, exerting or on these oblique plane by these. We can simply denote it by this sigma theta and tau theta.

So, you see, now if we cut this portion. So, now we have this oblique plane and we can simply denote, we can show by, you know, like if you have a unit cube, simply cut that unit cube and now we have, you know, like these all three directions. So, what we have? This is my ABC, you see, this is AB where this part was there or which we have the sigma y and we have this BC, over is the plane is cutting. So, this is my oblique plane and this is, you see, the straight line where you see, you know, like we are considering the sigma y or in the y direction.

So, thickness of this element in the z direction is always thin because we are considering, that there is no element is, you know, like observing under that and it is taken as the unity. So, because you see, you know, like this is the unit cube, so we can, we need, we have to consider, not we need to, we have to consider, that there is, there is a unit thickness is there all across these, all across this particular this structure. So, we have unit, unity means unit depth is there of these kind of things.

So, if you consider these, you know, like the plane, that if you are, you know, like if you are focusing on the main plane, then we have the sigma y, but if you are considering the oblique plane, then we have the sigma theta tau theta and if we dissolve these stresses, then we have the theta over this particular part.

So, now, you see we just want to set up the relationship between the sigma theta and tau theta, that means, the stresses, stress components at the oblique plane and the stress components at the outer surfaces. That means, we see the sigma y.

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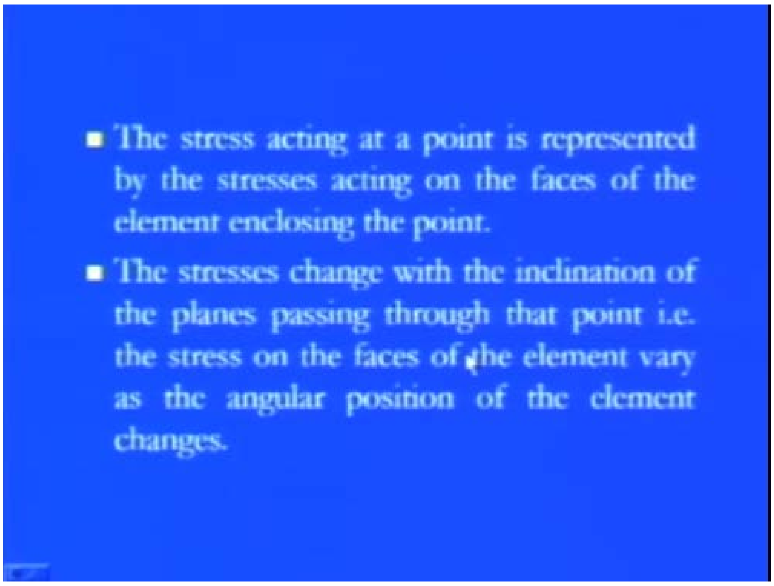

So, you see, the stress acting at a point, is represented by the stress acting on the faces of the element enclosing to the point. That means, see whatever the, you know, like the faces are there, we, we have to concern about all those stress components, which are there, which are existing all around these plane. So, the stresses changes, the stresses change with the inclination of the planes passing thought that point, that is, the stress on a face of the element vary as the angular position of the element changes.

So, obviously you see, you know, like because as we change the angular position, definitely there is a change in the magnitude in the, there, of the stresses. Because if you relate those things, then we found, that actually there are two resolution is there of the forces, in the x direction and y direction and it is always being focused by the cos or the sine theta. So, how, what exact relation is there and based on those relations we can easily found, that actually if you chase the angle there is, you know, like the difference is there in the magnitude of those stresses.

In the, in this figure, you see, you know, like we have seen, that the object is the under the influence of the tensile forces in the y direction. That means, you see, the tensile load is there under, you know, like the influence of this stresses. That means, we have the tensile stresses in the y direction.

And we just want to see, that if we cut the plane under the influence of this force, then what the exact relation is there of the, you know, like the sigma theta or tau theta, which is, which are nothing but the stress components at the oblique plane with only sigma y. That means, we are only considering there is no stress component is there, only we have this shear component.

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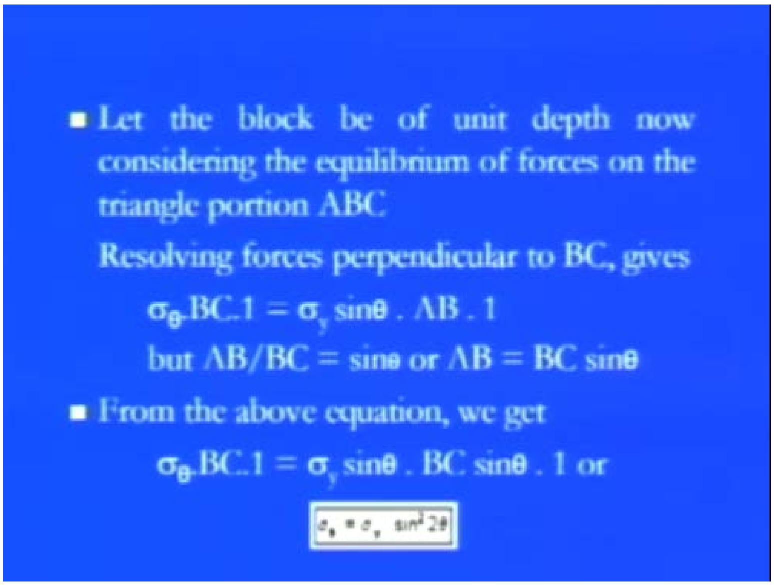

So, now if, if you are simply, you know, like carefully watching about those things, then we found, that there is a unit depth is there under the triangular portion of ABC. So, if we resolve the forces perpendicular to BC in the previous case, then we found, that we have, you know, like just in the perpendicular part sigma theta into BC, that is, the plane into unit depth because we are considering the depth is, unit is exactly equal to the sigma y into sine theta. Because now you see, only the normal force is there.

So, normal stress component is there in the y direction. So, sigma theta BC into 1 exactly matching by sigma y into sine theta of AB because AB is the plane where sigma y is sine theta is acting towards that into y. So, if you resolve those things, then we found, that actually AB by BC is nothing but equal to sine theta or we can replace this AB, which is you see, the plane of the y perpendicular to the y axis is nothing but equals to BC sine theta.

So, from resolving these equations we found, that we have sigma theta BC into 1 is equals to, you know, like if you put those things, that we have sigma y sine theta into BC sine theta into 1 or we have the stress at the normal stress component at the oblique plane is equals to sigma theta is equal to sigma y sine square sine 2 theta. Because if you resolve those things, then obviously, you see, the sine square theta is there. So, you can simply, you know, like resolve that part and it is equal to that or you see, you know, like BC BC will cancel out.

Meaning is pretty simple, that actually we can easily relate the stress component at the oblique plane, you know, like by whatever the force influencing is there of the object. So, again you see, this is the normal stress component.

Then, what is the shear stress component? We can again easily calculate by that tau theta into BC in the different domains or tau theta into BC, which is the cutting plane into 1 is equals to sigma y cos theta into AB sine theta into 1.

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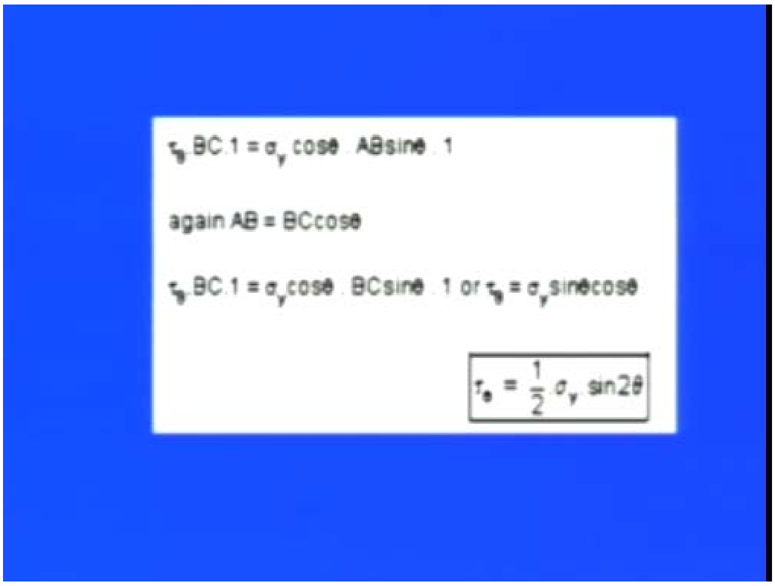

So, you see, you know, like as we have discussed in the previous case, that is, AB by BC is cos theta. So, we can simply replace AB equals to this BC cos theta or tau, this tau theta BC into 1 will give you sigma Y cos theta into BC sine theta into 1. Or we can say, that it is nothing but tau theta is equals to half of sigma y, sine, sine of 2 theta.

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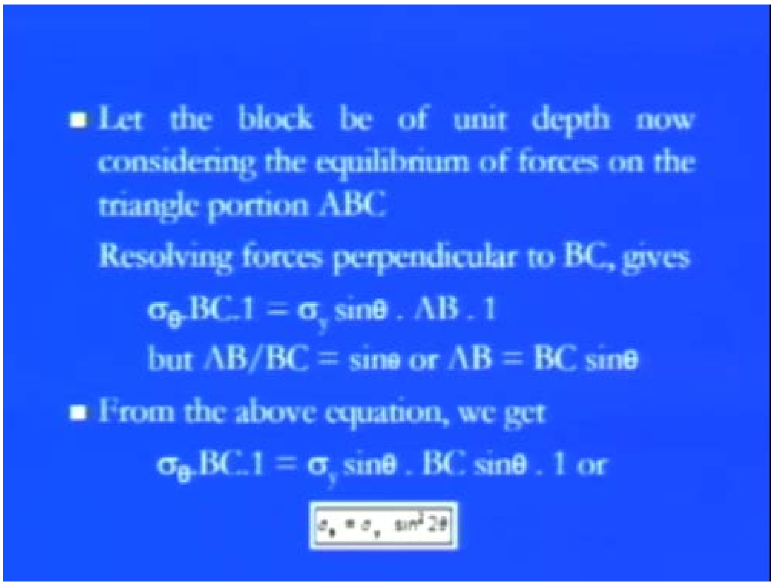

So, now, you see, this either the sigma theta, in the previous case, you see, which is this tau y sin of 2 theta the sine square theta. Or we can say, this tau theta, which is half of sigma y sine of 2 theta. They are only coming because of the influence of the pure normal stress. There is no shear component is there, but because of the pure, this normal forces, we have both the component at the oblique plane. Means, we have the normal stress component, we have the shear stress component and what the values are there we can easily get.

So, this is the beautiful, you know, like the meaning of these things are there, that we can get all those values though the shear stress. We are not, there is no shear forces are there, there is no parallel forces are exerting on the different layers of structure, but if only the normal stress is there. That means, if only pure, you know, like tensile or the compressive forces are there at any of the axis, we have both the component within the structure, like the normal stress component and the shear stress component. Then values are these.

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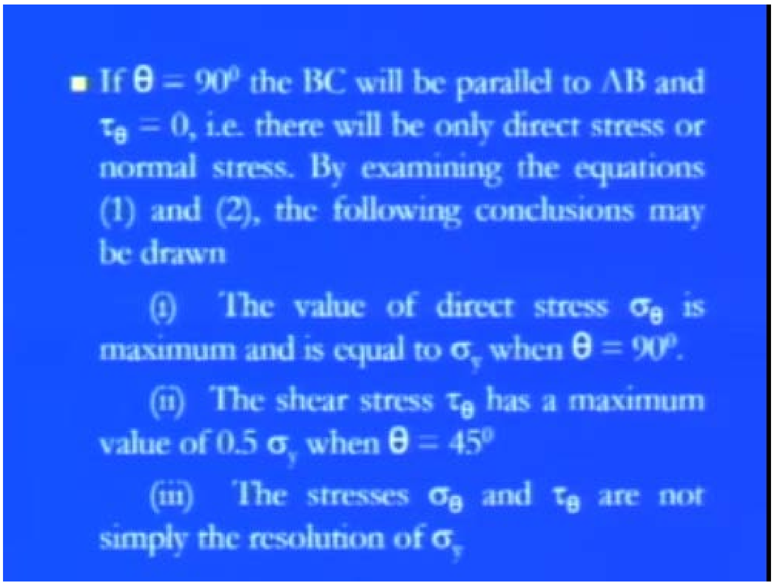

So, you see here, again we can resolve this cases by putting the plane at different angles, like if theta is at 90 degree, then whatever the BC, which is the cutting plane was there will exactly parallel to the AB plane or which the sigma y was exerting. So, this we can say, that this tau theta, whatever you see, if you put the theta 90, then this is tau theta, which is exactly equals to 0. That means, there is, there will be no direct stress or the normal stress is there.

Or by examining the equations 1 or 2, whatever you see, for the sigma theta and tau theta, we can simply conclude, that the value of direct stress sigma theta is maximum and it is equal to the sigma y when the theta is exactly at the 90. Means, you see, if you cut the plane, the theta exactly parallel to the BC, exactly parallel to the AB, then we have the exact value, what are the values are exerting the top of the surface, if, if you want to compute this normal stresses at the different layers of the structure.

And the second case is there, if the shear stress tau theta has a maximum value, which is equal to half of the sigma y at 45. That means, you see, if you cut the plane at exactly the 45 degree, then whatever the value of the shear stresses are there, these are half of the, half of the applied this normal stress.

And the third part, if the shear stress sigma theta and or tau theta and not simply, you know, like resolution of sigma y. That means, you see, you know, like if only these other two stresses, stress components are not exerting, then we cannot relate the sigma theta or tau theta with the sigma y.

That means, you see, you know, like we, we have to be very careful, that actually there are maximum or minimum values of normal stress and shear stresses are exerting even if the shear stresses are not been exerted on the outside of the surface. So, now you see here, we have a material, which is subjected under the pure shear stress. That means, in the previous case we are discussed about when the normal stress component is there and you know, like the material is subjected by that normal forces.

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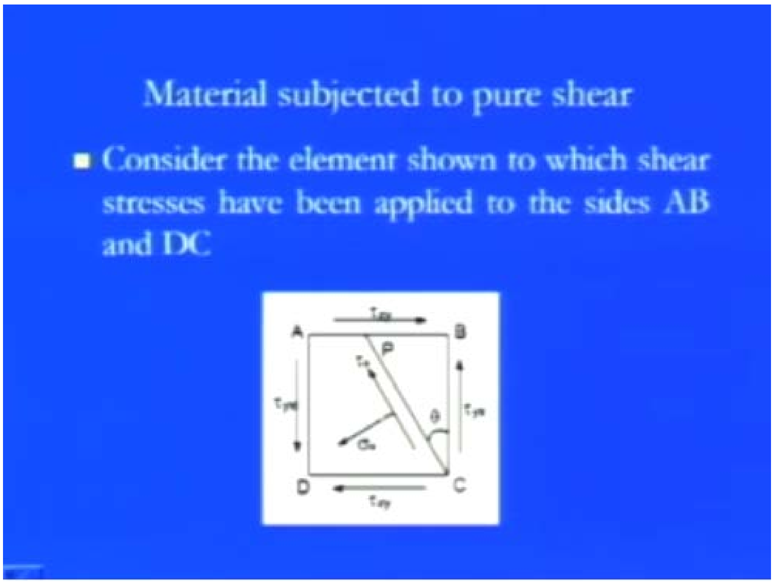

But here, you see, we are considering one element as you can see here in this figure, that this element is under the influence of the shear forces and the shear forces are being applied at these two, you know, like the sides, which is AB side, this tau XY is there and DC side, which is tau XY is there.

So, here, because of this tau XY, now the, see the stress distribution is there in this particular, you know, like portion. And we have another, you know, like as I told you, that actually the complimentary shear stresses are there, the tau XY, the tau YX, which is exactly equal and opposite to the these shear stresses, which are being applied.

That means, you see, here there is no normal stress component in this structure, only the purely shear stresses are there, complimentary shear stresses are there just to balance those things. And because of that, now we just want to check it out, that actually, that what the stress components means, the normal as well as shear stress components are there at the oblique plane.

So, you see, here we simply cut the plane at you see, the PC, this is the cutting plane, the oblique plane, I should say, and at the PC, you see. Now, we just want to check it out, that what the stress components are. So, we have the sigma theta and the tau theta at these PC plane, which is you see, the sigma theta in this direction and tau theta in this direction.

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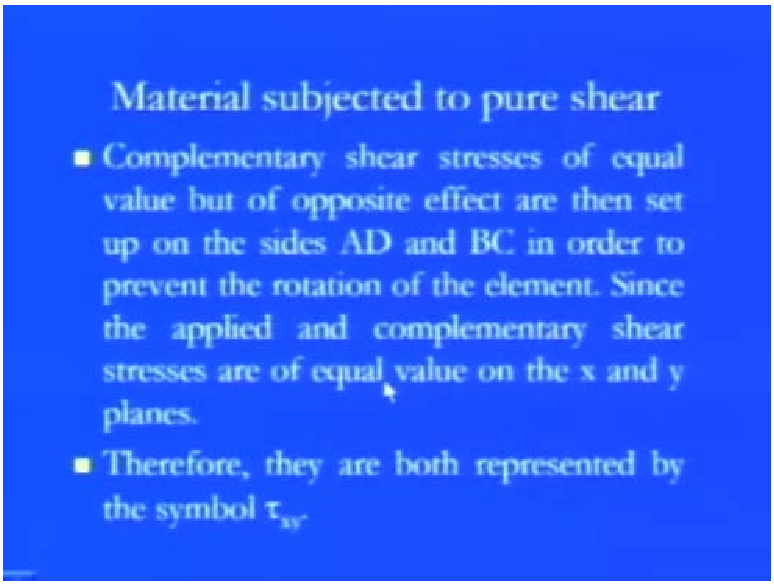

So, now you see, the complementary shear stresses, which I was talking about, is exactly equal and opposite, you know, like a fact is there against, that the applied shear stresses is on the two different sides of AD and BC in order to, you know, like the, prevent the rotation of the element.

Or we can see, we, just if we want to maintain the equilibrium of the element we have to, you know, like put the complementary shear stresses since the applied and shear stress, stresses are equal and opposite. Then, you see, you know, like of, of the value of xy planes. Then, you see, we can say, this object is under the equilibrium part.

Therefore, you see, you know, like both are being represented by the tau xy because the tau xy and tau yx, they are equal and opposite. So, it has the same physical meaning if you want to apply those things.

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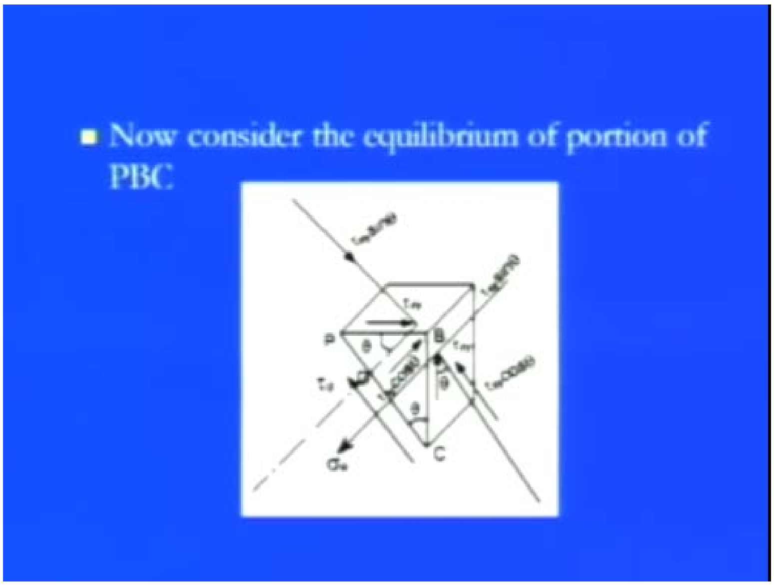

So, now come to, you know, like the equilibrium position of the PBC where PB is, you know, like the applied part is there, and PC is that part where the cutting or we can say the oblique part is there.

So, if you look at closely, then you find, that you see, we have the sigma theta, which is exactly at this point, you see, you know, which is coming out from this PBC plane and we have the tau theta, which is exactly, you see, the perpendicular to this thing. That means, the shear stress is there of this, the normal stress is there and if you want to check it out, the outer surface, that actually, what exactly shear stresses are being set up.

So, you can find, that you see, you know, like this, this component, that the tau xy cos theta and the tau xy sine theta are the resolved, you know, like the forces are there if you resolved at the theta angle. Because we have the tau xy, which is this one, you see, you know, like the tau xy is there, which is well set up on this plane or on this plane, you see, you can see this, this is tau xy, this is tau xy.

So, now, if you resolve this, if you resolve in this way, we have the tau xy perpendicular this y axis. So, if you resolve at the theta, you find, that this is the tau xy cos theta, this is tau xy sine theta in this direction. So, we have both components, tau xy cos theta, tau xy sin theta just to balance, just to counter balance the normal stress as well as the shear stress due to the oblique plane.

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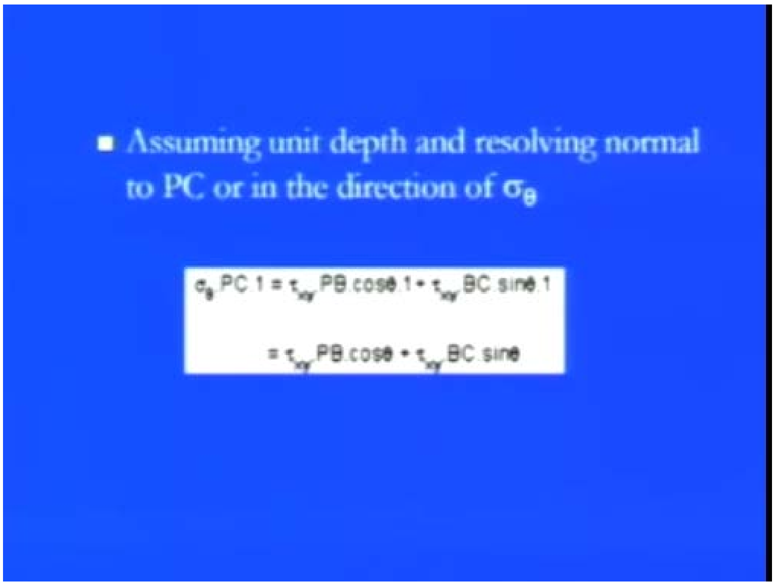

So, now, you see if you resolve those things, then you find, that assuming the unit depth and again, you see, we are assuming, that whatever the structure, which we have, it has the unit depth and if you want to resolve the, you know, like those forces, then we need to see, that actually what exactly the forces are there just perpendicular to this oblique plane PC.

So, you see, here we have the direction sigma theta, which is exactly normal to the plane PC. So, we consider the PC where the sigma, this is PC, you see, this is PC and sigma theta is exactly normal to this plane. So, if we resolve this plane, then you find, that sigma theta into PC into 1 because of the unit depth exactly equals to the tau xy into PB cos theta, which is exactly, you see, you know, like equating that part plus tau xy into BC sine theta. It is composition of these.

Or if you divide it, these things, then at the end, you see, what we have, you see, you know, like if you just remove those things we have particular sigma theta PC equal to tau xy PB cos theta plus tau xy BC sin theta. Means, that here we have only the application of tau xy. Means, only pure shear is there, but because of that, you see, we have the normal stress component at the oblique plane and we have the shear stress component at the oblique plane.

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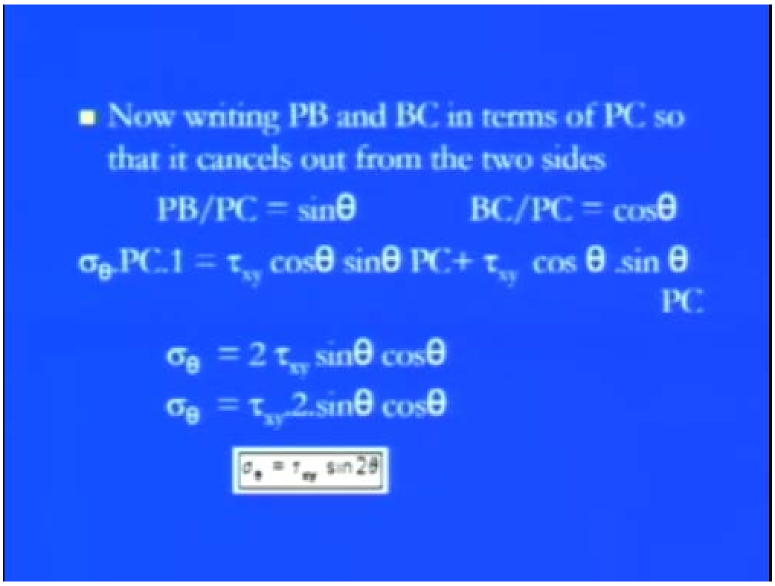

So, now, you see, you know, like if we dissolving these things, as I told you, that PB by PC equals to sine theta or BC by PC equals to cos theta. So, if you put those PB and PC value with the previous equation, then you found, that sigma theta PC into 1, at the 1, at the left hand side is equals to tau xy cos theta into sine theta into PC, which is equals to PB and plus tau xy cos theta into, in place of BC you can put sine theta PC.

So, now you see, PC PC will cancel out because it is exerting on both the side. So, we have, what, this, this sigma theta into 1 is equal to tau xy sine theta cos theta plus tau xy cos theta sine theta. So, if you sum up, then you found, that we have sigma theta equals to tau xy 2 times sine theta cos theta.

So, at the end we have the sigma theta, which is occurs, you know, like which is occurring in these element due to the purely shear stress tou xy is equals to sigma theta equals to tau xy sine of 2 theta. So, this is, you see, the normal stress component, you know, like because of the tau xy.

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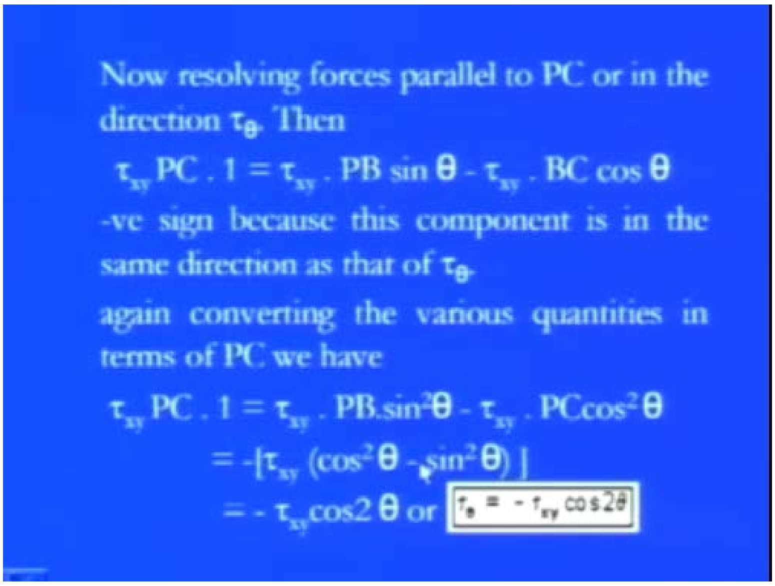

And again, you see, if you resolve the forces in this perpendicular plane, you know, like, then you find out the normal stress component is there. But if you found, that if shear stress component is there, like the PC plane is there, if the parallel forces are there and due to the parallel forces we have the shear stress component.

So, if you want to resolve the shear stresses, which is, you know, like at the oblique plane. And if you want to compare both the things with the oblique plane to the main plane, then you found, that actually we have this tau xy PC into 1 is equals to tau xy PB sine theta or we can, you see, minus tau xy BC cos theta.

The negative sign is coming because you see, you know, like the stress because you know, like we want to apply the complementary shear stresses, so that actually, whatever the rotation is there we can prevent that rotation, you know, because of the complimentary shear stress. So, always the negative sign is coming in resolving the shear stresses.

So, again converting those various quantities into, you know, like terms of PC, we found, that we have the tau theta, means the shear stress at the theta is equals to minus tau xy cos of 2 theta. So, the meaning is, that we have normal stress component, we have shear stress component, which is coming due to the influence of the shear stress tau xy and which has the different values of, like sigma theta is there.

Then, we have tau xy sine 2 theta, if the shear stress is there, sigma, sigma tau theta, which is equals to minus of tau xy cos of 2 theta.

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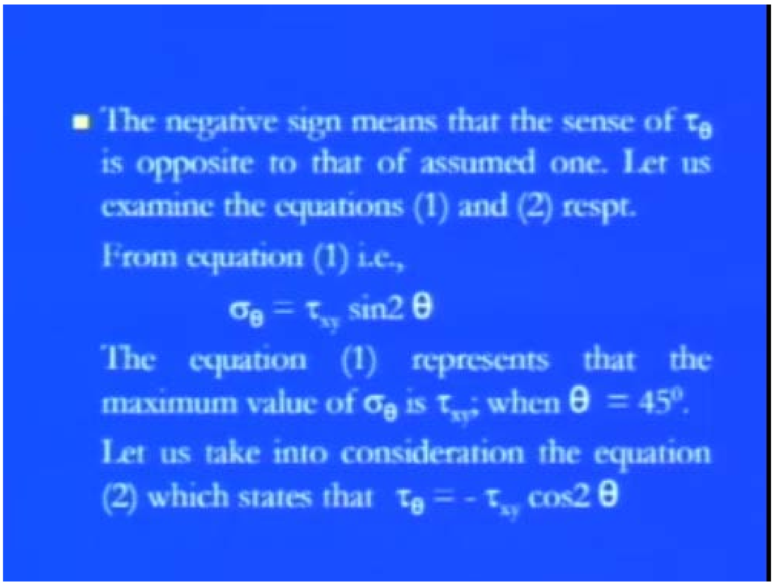

The negative sign means, that you know, like as I told you, sense, sense of the tau theta opposite to the assumed one. So, obviously, you see, you know, like always it comes, that actually if you are not concerning about the complementary shear stresses or the negative shear stresses, then you cannot say, that whatever the element, which have chosen, it is under the equilibrium position because of the applied stress.

So, always we need to assume that the applied stresses are there along the parallel, you know, axis. But we have, you see, the other side of these, you know, of the complementary shear stress, which can balance these stresses.

So, we can say from the other equations we have sigma theta, which is tau xy sine of 2 theta. The equation 1, this one represents, that the maximum value of sigma theta is tau xy. Obviously, this when the theta is at 45 degree. Means, you see, if you cut plane at 45 degree you have, you see, you will find, that the maximum normal stress, which is exactly equal to the shear stress, applied shear stress when you cut the plane at the 45 degree.

And if you, you know, like just introducing by considering equation of this, by taking, you see, tau theta is equals to minus tau x cos of 2 theta. If you put that particular thing, then you found, that actually this tau theta is exactly equal to 0.

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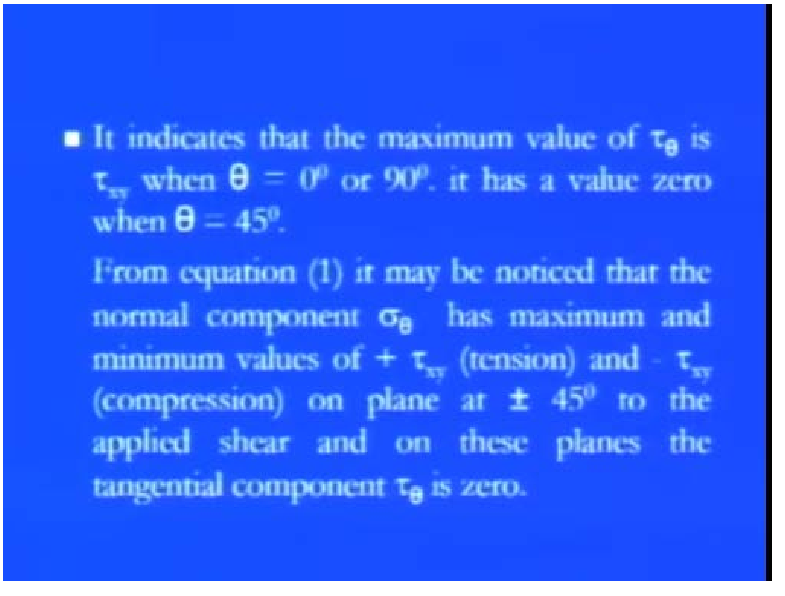

It indicates the maximum value of tau theta is tau xy when theta is exactly 0 or 90 degree. That means, you see, if whatever the PC plane, which we have chosen, if it is at just perpendicular, 90 degree, or if it is, that means, if it is exactly at the PB or if it is perpendicular to AB, that means, just you know, like these parallel part is there, then we have the maximum value of shear stress, that is equal to the applied shear stress, tau xy.

But if it is, you see, if you are cutting those planes at 45 degree, then only normal stress component is exerting, that is, at sigma theta there is no shear stress portion is there at the 45 degree of angle. From the equations all you see, either sigma theta in the one equation or tau theta, equation 2, you can easily calculate the maximum or minimum values of the normal as well as the shear stress component.

So, from equation 1 it may be noted, that the normal components sigma theta has maximum or minimum value of plus tau xy if the tension is there, you see, the tensile forces are there. Now, the minus tau xy if the compressive force are there on a plane where, you see, plus minus 45 degree of angle of rotation is there.

That means, you see, if you cut the plane at 45 degree of angle, whatever the planes are there, of these two planes if I say, that at this plane the normal stress are always maximum, the sigma theta, which is equal to the applied Shear stresses, applied. And these planes are tangential, you know, like if, if, if, if you want to check it out, you know, like shear stresses at the tangential plane.

That means, you see 0, or and then you will find, that the shear stress component has the maximum value and if you want to check it out, you know, like at this plane where these sigma theta is maximum, is the 45 degree, the tangential plane where the shear stress component is 0.

So, you see, you can, you know, like vary all those things. You can check minimum value as well as the maximum values of sigma theta or tau theta corresponding to the different planes.

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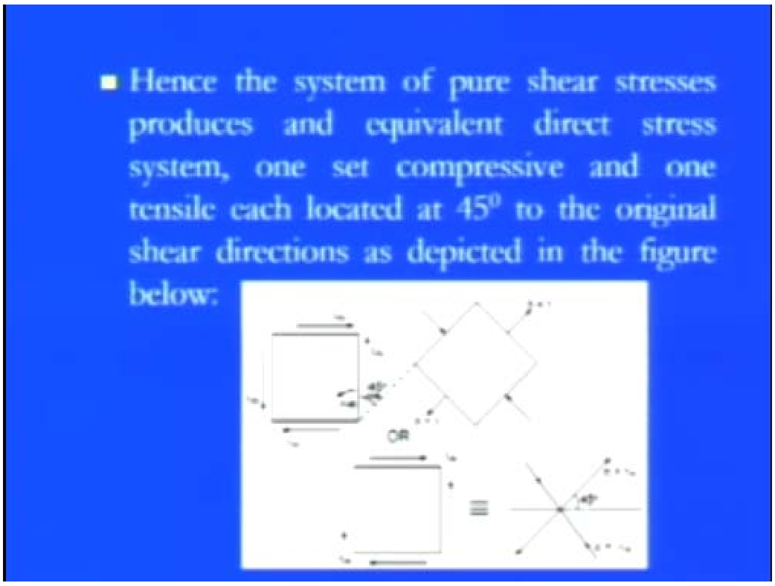

Hence, the system of pure stress, which we discussed recently, you see here, produce and equivalent, you know, like direct stress is termed system one, set of the compressive and the one set of the tensile each located at the 45 degrees of the original, you know, like this directions as depicted in the figure. So, if you see this figure, you see, we have, you know, like this component, which is influencing by only shear stresses, you see, these shear stresses are there.

Now, if I rotate at 45 degree. So, you see, now this is the rotated plane, which is exactly at the 45 degree. So, what we have? We have, all you see, at the 45 degree of angle, only these stresses are absolutely converted into the normal stresses. That means, you see here, stress are being applied, the shear stresses, counter shear stresses are there. But at the end, you see, if you rotate this element, simply by rotating this element, all the shear stresses are being exactly converted to the normal stresses.

Or we can say, we come down to these things, you see, we have this element, which is under the influence of the shear stresses. And if you compute those things, when you find, that you see at these planes where the 45 degree planes are there, this one sigma is, you know, like sigma theta, I should say, is tau xy or sigma, which is sigma theta also is minus tau xy.

So, we have both the, you know, like this stresses are there. But, but the shear stress, pure shear stress will give you only the normal stress component at the 45 degree of angle.

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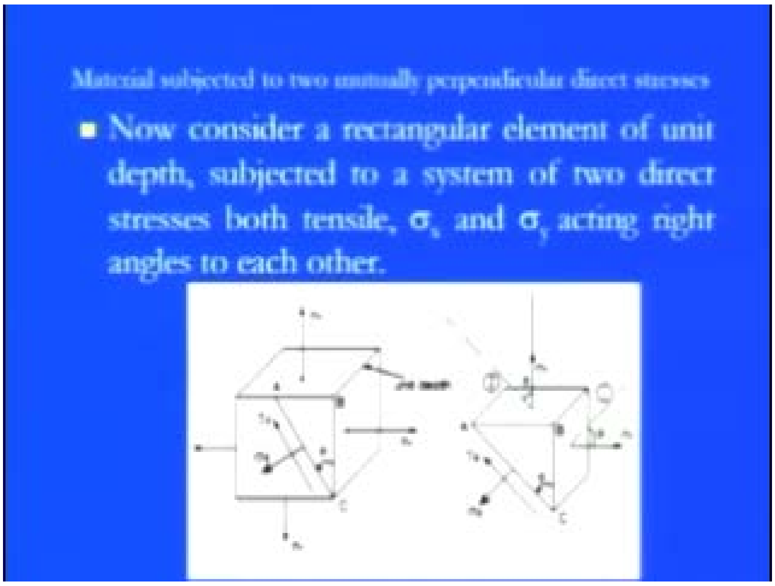

Now, you see here, this third case. The previous two cases, which we discussed, when the material is under the influence of pure stress and when the material is under the influence of pure shear stress.

Now, when material is subjected by two mutually perpendicular direct stresses, means, you see, we have direct stresses, direct stresses, but both are mutually perpendicular, like the x direction or the y direction, and they are acting in the tangential direction.

So, now you see, consider the rectangular element, which, which is being shown here in the diagram of unit depth, again you see, we are considering the unit depth subjected to a system of two direct stresses, both are tensile, as I told you, on the right angles of each other, means, both are mutually perpendicular.

So, if you see, that then you will find at this sigma x, sigma x is on this two, you know, like sides of the rectangular element sigma y sigma y. Again, you see, we have, you know, like these two, that this tensile stresses are in the y direction. So, now, you see, if you want to check it out because of the influence of these two mutually perpendicular stresses x and y, what is the, you know, like stresses at the oblique plane.

Again, cut the plane here by this AC and we just want to check it out, that actually what, what is the value of the sigma theta and tau theta is there at that. So, cut the plane and we have, now you see, the two different, you know, like reasons. At one reason, now we just want to check it out at AC, the sigma theta and tau theta is there. But other two plane, this is one we have sigma, you know sigma x.

So, how this, how you can resolve this things like sigma x cos theta, sigma xy sigma x sine theta or here, sigma y cos theta, which will come here, means, sigma y sine theta and sigma x cos theta in the other direction. So, now you see, we just want to resolve this stress components as per the value of these sigma theta and tau theta with the sigma x and sigma y.

So, only shear stress is there. There is no, only pure, this normal stress is there, there is no shear stress component.

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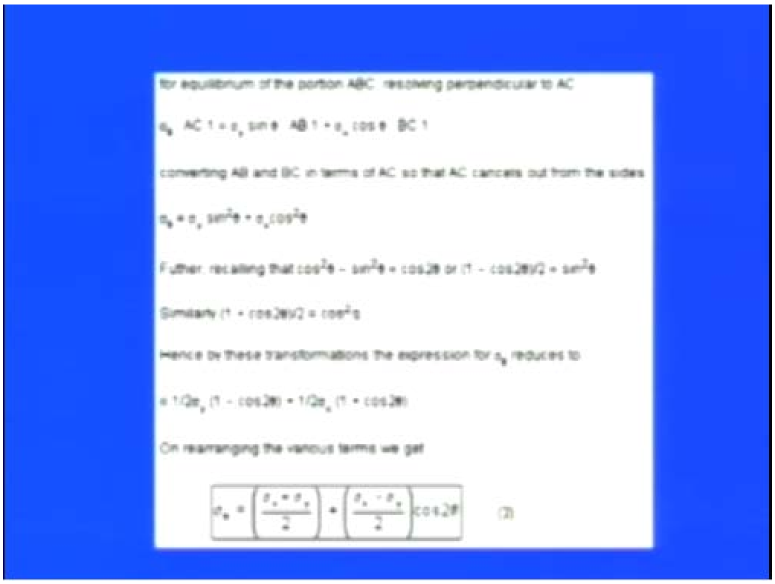

So, now, you see, within, within the equilibrium portion of the triangular part ABC, if you, if you want to resolve these forces, then we have first sigma theta is into AC. So, AC is, you see, the inclined part. So, sigma theta into AC into unit depth, that is, one is equals to sigma y sine theta into AB where this AB is acting into 1 plus sigma x, which is, you see, cos theta into BC into 1.

Now, you see, converting this A, this AB and BC in terms of AC, so that AC cancel out from this side. So, we can simply, you know, like resolve this forces. We have sigma theta is equal to sigma y sine square theta because AB by AC sine theta, BC by AC is cos theta. So, plus sigma x cos square theta or further we can say, that the cos square theta minus sine square theta is cos 2 theta, or we can say it is 1 minus cos of 2 theta by 2 is nothing but equals to sine square theta.

So, if you, you know, like do this kind manipulation, then we found, that at the end, that we have sigma theta is equal to 1 by 2, you know, sigma y cos of 2 theta plus 1 by 2 sigma x cos of 2. Or by rearranging those things we have the sigma theta because of the two mutually perpendicular stresses, sigma x and sigma y. We have the final stress at the oblique plane in the normal direction. Sigma theta is equal to sigma x plus sigma y by 2, which is not influenced by any theta. See, this, this is a constant quantity. So, here, so this is one constant quantity and one is influenced by the theta. So, once sigma x plus sigma y by 2 plus sigma x minus sigma y by 2 into cos of 2 theta.

So, we can, you know, like resolve these things, that actually what is the value of the maximum or minimum sigma theta is there if we have different angle, like if, means, if we are cutting the plane at theta equals to 0, 90, 45 or what, like that. So, you can get that values.

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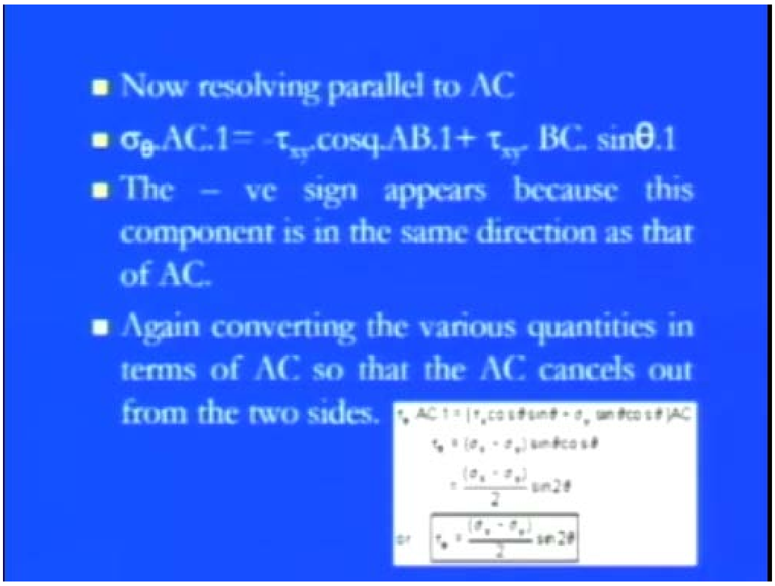

So, now, you see, resolving these parallel to AC, then we have sigma theta AC into 1 is equal to minus tau of that because now, you see, you know, like we just want to check it out the other component, which is parallel to AC. That means, the shear component is there. So, minus tau xy cos of q into AB into 1, which is, you know, like due to that tau xy in vertical direction AB, or we can say, tau xy BC into sine of theta into 1. The negative sign appears always, you see, because of the complementary shear stresses, so to balance those component.

So, again you see, converting those all variety of the components we found, that actually what exactly the value of tau theta is. So, if we resolve those things by equating the segment into the x as well as the y direction we have tau theta into AC into 1 is equals to this tau x. In this particular figure we can found out the tau x into cos of theta into sign theta minus sigma y sin theta cos theta multiplied by AC.

So, we can find it out, that tau, the tau theta is nothing but equal to sigma x minus sigma y into sine theta cos theta or you can convert into two theta parts. So, we have sigma x minus sigma y sine of 2 theta. So, we have the final value of tau theta, which is equals to sigma x minus sigma y into sine of 2 theta.

So, now you see, by you know, by applying the two different forces at mutually perpendicular axis, we can also get the sigma theta or tau theta at the oblique plane by amazing those values. By sigma x plus sigma y by 2 plus sigma x minus sigma y by 2 into cos theta, which will give you the sigma theta. Sigma x minus sigma y by 2 into sine 2 theta will give you the tau theta value.

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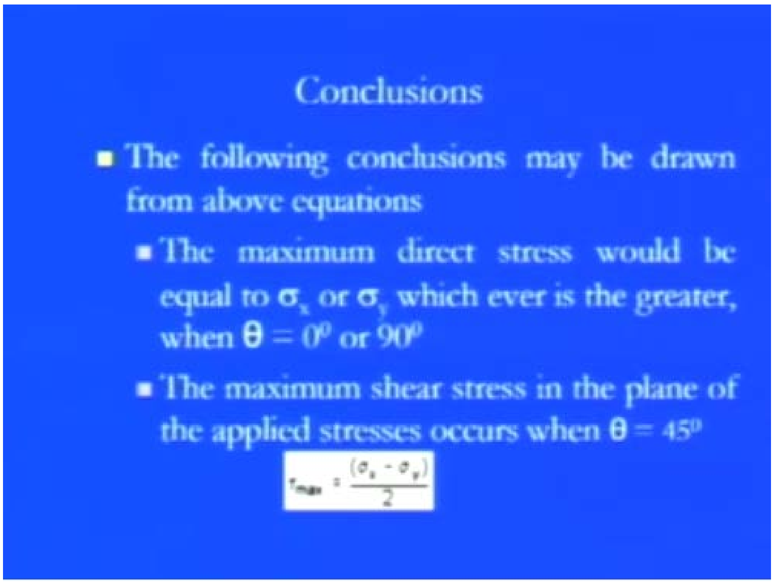

The conclusion of this whole discussion says, that the following, you know, like this conclusion, which we can draw, drawn that the maximum direct stress can come, you know, like because of, by applying theta equals to 0 or 90 degree, which is equals to exactly the sigma x plus sigma y by 2. That means, you see, if the mutually perpendicular stresses are there, the maximum direct stress will be of sigma x plus sigma y by 2, which is equal to, which is, which is coming when the theta is equals to 0 or 90 degree.

Maximum Shear stress is coming on the plane wave. The plane is applied, you see, at the 45 degree, means, you see, here now the things are changing. Two mutually perpendicular axis are there, stress are there, under the influence of these stresses we have the maximum value of shear stress at an angle of 45 degree, which is equals to sigma x minus sigma y by 2.

So, we have, you know, like the maximum value of direct stress as well as maximum value of shear stress at oblique plane if we apply these things. So, now, in this all discussion we discussed, that actually, you see, you know, like if the pure shear stress, pure normal stress is there, pure shear stress is there or two mutually perpendicular pure normal stress is there, then how we can get the stresses at the oblique plane irrespective of whether it is a sigma theta or tau theta.

So, in this lecture, you see, you know, like you found, that actually, we, we have to be very careful, that what is the value of the maximum and minimum stresses are there, normal as well as the shear stress.

So, in the next lecture we are going to discuss, that actually what, what is happened if the two mutually perpendicular axis are there and it has been influenced by, not only the normal stress, but the combination of the shear stress. Means, you see, now we have the combined forces perpendicular as well as the parallel forces, then how we can resolve the normal stress and the shear stress component to the both, you see, at the oblique plane as well as the reference plane.

Thank you.