Strength of Materials

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Lecture – 39

Hi, this is Dr. S. P. Harsha from Mechanical and Industrial Department, IIT Roorkee. I am going to deliver my lecture 39, on the course of the Strength of Materials, and this course is developed under the National Programme on Technology Enhanced Learning (NPTEL).

In this lecture basically we are dealing with the strain energy due to the bending and associated with, you see, the numerical problems and also we are trying to discuss some of, you know like, the Castigliano's theorem that, you see, if we have the strain energy, then what the exactly meaning of the complementary strain energy is there, you know like, and how the associated terms are there, through which we can calculate both of the term strain as well as the complementary strain energy.

But prior to the lecture discussion, I would like to briefly discuss about what we have discussed in the previous class. In the previous class, we mainly discussed about that if we have, you see, the structure and in this particular strut or in, you know like, any of the kind of thing is there like the column, and if any eccentric loading is there, then what the exact impact is there of that.

So, you know like, the in the previous lecture, we started with the eccentric loading in the strut and we found that when the eccentric loading is there, then there is an additional curvature part is there in the strut, and due to that there is an additional bending moment is there; so, we need to add that part. And we found that even at the beginning part, that means, you see, when it is started with the piping joint we have you see, you know like, due to this eccentricity, we have, you know like, the deflection, and this deflection is being, you know like, incorporated in that.

So, whatever we discussed in the Euler theory of the strut or column under the elastic stability, we found that the Euler theorem is not perfect in terms of the theoretical way. So, that is why you see, you know like, we incorporated the eccentricity in those terms and we found that after incorporation of the eccentricity, or we can say in other terms, after incorporation of the curvature or the bending moment due to that, we found that we are more realistic way, and if you want to design the strut with consideration of the eccentric load or due to the eccentricity, and due to that, you see, we have the bending moment we are having a realistic, you know like, the results through which we can say that if we are applying those things with those particular conditions, means the P by E E or we can say the M by Z in the whole, you know like, the moment, we always have the safer design and whatever the buckling load is coming it will come in the realistic way.

So, that is what, you see, we discussed and we found that there is a direct interaction is there of the applied load and, you see, this Euler crippling load is there when we are considering the eccentricity in those terms. Then, you see, the later part, you know like, of that previous chapter we discussed about the three main forms of energy under the axial loading.

The first one was there, that is the strain energy was there and the under the strain energy you see, you know like, what exactly the impact of the strain energy is there when, you see, the direct application of the load is there. And under the direct application of load, you see, the kind of deformation is coming. So, the first one, for the first form of energy was that.

The second form of energy under, you know like, up to the elastic region we defined as the modulus of resilience. So, this is the standard form is there, you know like, when the when we are drawing, you know like, the stress strain curve and we are only drawing up to, you know like, up to the elastic region, or we can say that in other terms that, if we apply the load and the load application is there up to only the yield point or the elastic region, whatever the energy is being absorbed or whatever the area under that particular linear stress-strain curve will give you the modulus of resilience, and accordingly, you see, we can simply calculate the sigma YP square by 2 E will give you the elastic energy within that.

The third form of energy, which we discussed under the tensile test, was up to the rupture part and that term is known as the modulus of toughness. So, you see, here when we are discussing about the stress-strain term, this relations, we found that there are, you see, the two main phenomena are there: One is the elastic and one is the plastic region; or we say rather that one is the linear relations and one is the non-linear relations. And if you are going up to the rupture point, both regions have to be considered.

So, in the modulus of toughness, you see, we considered both the region and what all the energy is coming is being absorbed up to the rupture is simply given by the modulus of toughness is there for that. Or rather we can say, that the area under that particular curve - right from beginning to the end of this curve - will give you the modulus of toughness for this kind of material. And we found that in all three form of energy, the key point is the material property - that what kind of material which you have taken, because, you see, corresponding the stiffer part is coming, the corresponding deformation will come under those conditions. And also we found that the cross sectional area and the applied load both have the equal importance for calculating the strain energy or modulus of resilience or modulus of toughness.

And then lastly, you see, we discussed about the numerical problem in the previous class that, you see, if we have the different load conditions altogether, then how we can calculate the strain energy for that, and how we can calculate the modulus of resilience based on, you see, this yield point limit.

So, you see, if you know the, you know like, the strain energy you can simply, you know like, get the yield point or if you know the yield point you can get the strain energy up to the elastic region or we can say that the modulus of resilience for that. And also if we can also solve the numerical problem based on, you see, up to the rupture part, you see, because in that we need to limit only up to the elastic and plastic regions. And then, you see, the corresponding, you know like, the energy will come, because if you can calculate the area under that particular curve, it can simply gives you the modulus of this toughness, or we can say that whatever the strain energy there up to the failure, that will clearly, you know like, can be easily calculated with the using of these terms.

Again, in those terms also, the Young's modulus and the strain part up to the fracture because, you see, E into epsilon R square by 2 will give you the modulus of toughness. So, in, you see, in that term also the Young's modulus of elasticity as well as the strain at the rupture point both are having the equal importance in that part. So, this is what, you see, we discussed in the previous class.

So, in this class see again we are going to discuss about the main strain energy in the bending action. So, that that is what, you see, in the previous terms only the normal action was there, but now here the combined loading is there; that means, you see, here the direct as well as the shear stresses are being you know present in that. And now we would like to see the strain energy phenomena in those actions. So, now, you see, here if the bending is coming always the beam is there in our mind. So, we have a beam, a simply supported beam is there, and then, you see, the loading conditions are there on the particular beam.

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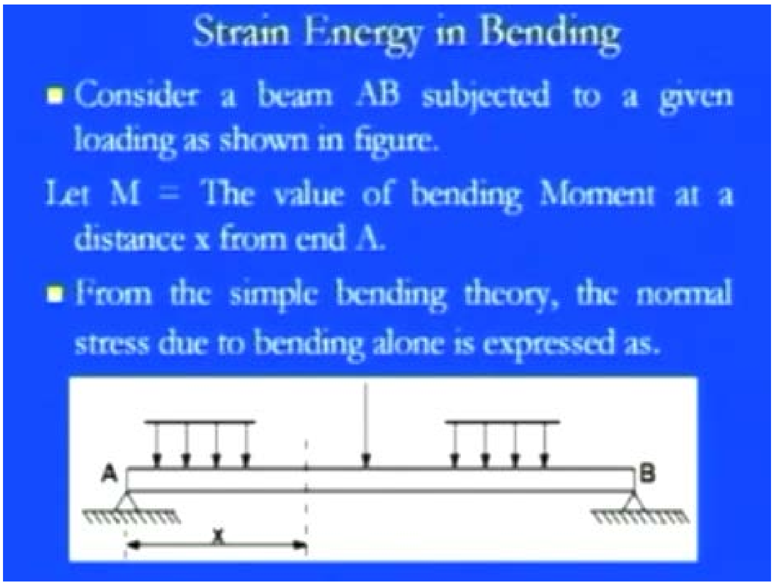

So, consider a beam, AB, which is subjected to a given loading, you see, here. So, in that we have the M - the value of the bending moment at a distance x from A. So, you see, the A and B are the two simply supported free points are there, where the pin joints are being acted. And, you know like, there is x distance is there from this point A, and there is a point load is acting at exactly center point of that, and from the simple bending theory the normal stress due to the bending alone is expressed in terms of that. So, we would like to express those things, but prior to that, you see, we would have a feeling about that, that you see, here under that what we have? We have the two different regions altogether in this and then under the bending action, you see, we have both the shearing action is there, this normal action is there, the normal direct stresses are there, as well as the shear stresses are there along with that.

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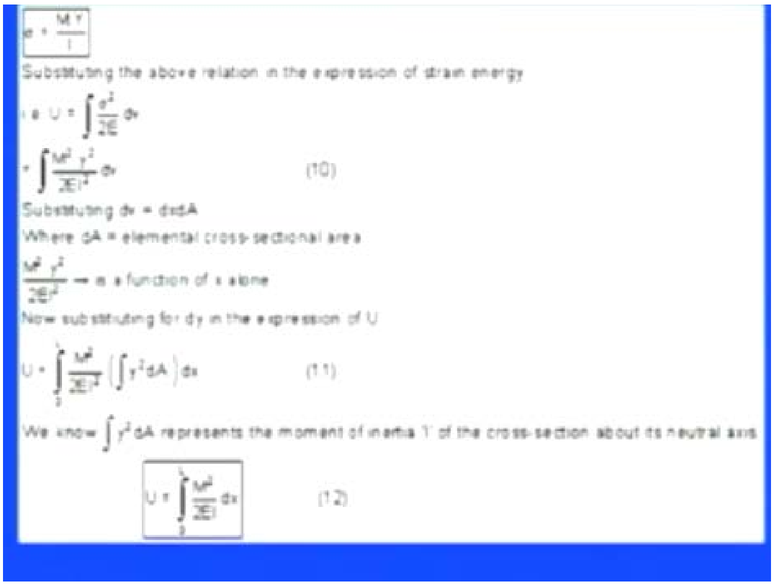

So, with that, you see, now just go back to the bending theory we have sigma by y is equals to M by I. So, through which we can calculate the bending stress sigma, which is equals to M into y divided by I. Now, substitute the above relations in this particular expression for gaining the strain energy. So, we have the strain energy U which is, you see, for entire beam the integration of sigma square by 2 E into dv.

So, now, you see, we know that the sigma is nothing but equals to M into y divided by I. So, we need to replace that part. So, we have the integration for entire domain of the volume, integration is equals to integration into M square y square divided by 2 E into I square into dv. Or we can say that simply we need to substitute that volume in that particular domain. So, what we have? We have dx because the variation is there in the dx only because sigma x is there in the xl form into dA. So, dA is the elemental cross area is there for that particular beam.

So, now what we have? We have the M square y square divided by 2 EI square which is simply a function of x only. So, you see, these are all the constant terms are there. So, you know like, with that particular thing now, we are simply manipulating those terms in strain the energy part.

So, we have strain energy U which is equals to integration of zero to L and square divided by 2 EI square, because it is a function of x alone; that means, the distance alone. So, we are keeping with this zero to L integration. And so, what we have? We have the two main integration part: one is with the distance terms which is varying with the distance; so, we have zero to L M square divided by 2 EI square.

And the another form is integration of zero to A, whatever you can say, the area part y square dA into dx. So, now, you see, here this middle part which is y square dA within the area domain, represents the area moment of inertia or you can simply replace by I also with this. And the cross sectional area about this particular neutral axis because, you see, in which, you see, our main focus is on the dimension property of that and what will be the center of, you know like, the mass is there through which we can calculate the cross sectional area. So, based on that, now we can simply calculate the strain energy of, you know like, this particular cross sectional.

So, what we have? We have U is equals to integral zero to L because, you see, the M square y square divided by 2 EI square was only the function of x. So, again zero to L M square by 2 EI into dx is there, because y square into E dA integration will give you the I and II will be canceled out in that term. So, now we can simply calculate the strain energy under the bending action, which is absolutely focused on what the bending moment is there and what is the flexural rigidity is there of that particular material.

So, these are the two key parameters are there through which we can simply say that how much energy can be absorbed or how much energy can be, you know like, put in that particular material based on what the bending moment is applied part is there. And what is the flexural rigidity? That means, what is the material properties is there of that. So, U is equals to integration of zero to L M square divided by 2 EI into dx. So, this is, you see, the form of the strain energy for the bending action. Now, we would like to solve some of the numerical problem based on that.

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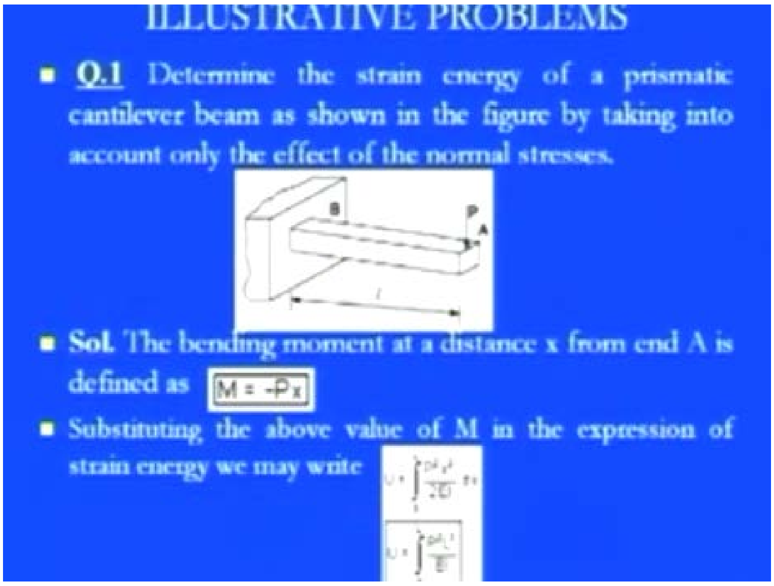

So, you see, here first the numerical problem is now we to determine the strain energy of prismatic cantilever beam. So, you see, here we have a cantilever beam is there as shown in this particular figure by taking into account only the effect of normal stresses. So, again, you see, here somewhat we can say that certain assumptions are there that only our main focus is on the normal stress component; so, we are ignoring the shearing part here. So, whatever the bending action is happened under the action of, you see, this eccentric loading or whatever, you see, the normal stress are being formed and we need to calculate on the bending part.

So, look at this particular picture, we have a cantilever beam and, you know like, it is being rigidly fixed or at this particular wall - at point B; and at point A we have a point load which is being acted on that particular part, and we have the total length of this particular prismatic bar, the cantilever beam is L.

Now, you see, the bending moment at a distance x from this particular end A is simply defined as M is equals to minus P into x because, you see, we simply taken as certain distance from that particular point. So, P into x will give you the bending moment and now we need to substitute the above value of M in the particular expression of the strain energy which we discussed in the previous section.

So, what we have? We have, you see, the strain energy U which is equals to zero to L P square into x square divided by 2 EI into dx. So, we know that what exactly, you know like, because in the previous case we found that the total responsible parameters for calculating the strain energy is the flexural rigidity EI. So it is already there and the second part is M.

So, we know that the bending moment is absolutely in this particular example is P into x. So, M square is being replaced by this P square x square. So, U is equals to integral zero to L P square divided by 2 EI into dx. Now, you see, within that particular frame of mind we have, you see, the total strain energy if you this integrate that part we have U is equals to P cube P square L cube divided by EI.

So, you see, here because the x square dx will be x cube by 3. So, you see, here, you know like, we can simply replace that part and we have, you see, the total terms are like this. So, we have, you know like, the strain energy and that again you will find that the strain energy in these kind of bending action also there was responsible parameters, because we are only considering the normal stress component, so the responsible parameters are the load application and the flexible rigidity of the material. So, this is one point. Now, we would like to see the next question. Next question is again, it is pretty simple, because the first one was there the cantilever beam, now our main focus on the simply supported beam.

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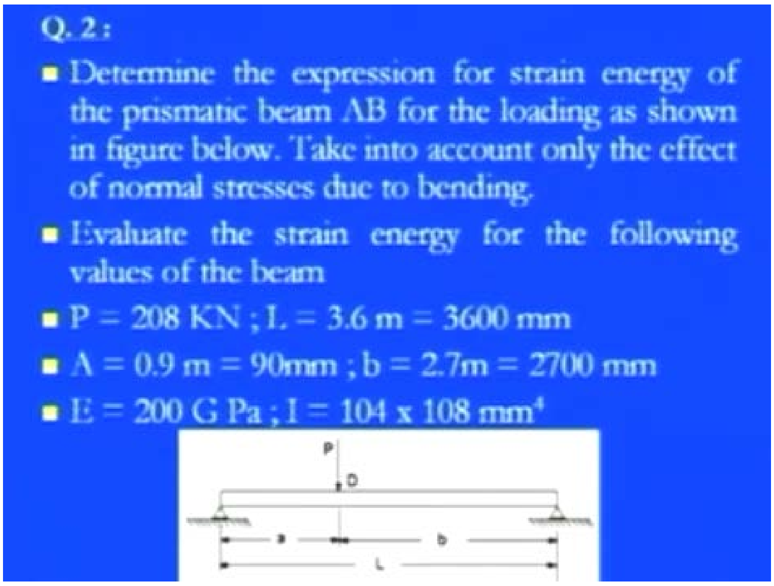

So, determine the expression for the strain energy of a prismatic bar AB for the loading conditions as shown in this particular figure and taking into account only the effect of normal stresses due to bending. So, again, you see, our main intension is to focus on this bending action only; we are simply ignoring the shearing action here. So, whatever the normal stress component will come we need to consider those components only.

So, evaluate the strain energy for the following values here. So, here the values are given as the load P which is acted at point, you know like, from point A of the left end is 208 kilo Newton. And the length - total length - is given as 3.6 meter; that means, that 3600 millimeter.

The total A cross sectional area of the beam is given as this 90 millimeter square and then we see, we have, you see, in that we have, the b which is equals to, you know like, 2.7, that means, you see, the total is 2.700 millimeter. So, we have both the distances here. And then, you know like, we can simply, because we have the total L, we have the b, so we can calculate the A and we can simply figure out that what is the point of location is there of the load.

Then, you see, we know the materials. So, we can simply get the part E; E is nothing but equals to 200 giga Pascal, and then, you see, once you know the cross sectional area, you can simply calculate the mass moment of inertia. So, we have I which is equals to 108, you know like, into 10 to the power 8 millimeter to power 4. So, that is what you see, if through which we can simply calculate that part.

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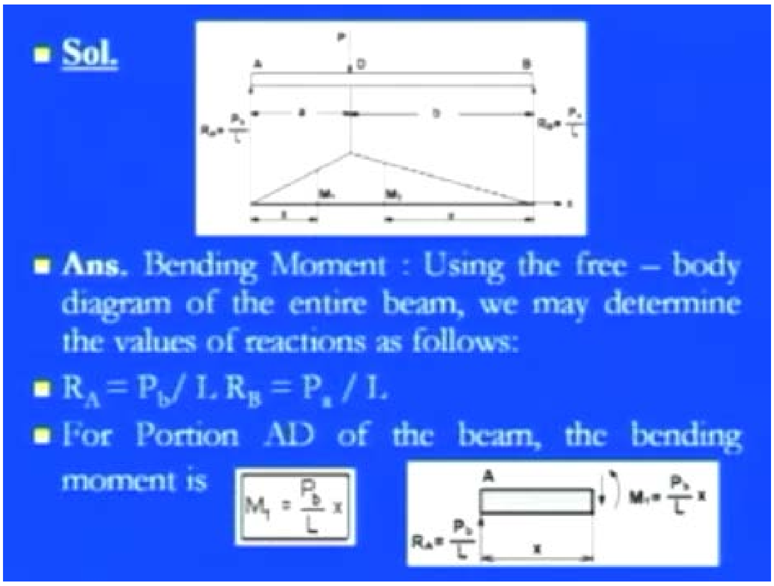

Now, come to this particular real feasible figure of that. What we have? We have you see, you know like, this simply supported beams are there. So, our both of the simply supports we have the reaction forces RA and RB at a point A and point B and, you see, the point load is acted towards the downward direction. So obviously, you see, we can simply get the moment - this bending moment diagram - for that, for which, you see, first we need to calculate the reaction forces. So, for first the reaction forces at point A is nothing but equals to Pb by L because, you see, the distance is B is given to us. So, you see, here the load into distance divided by L and then you see, you know like, we can simply calculate the reaction at point B also which is nothing but equals to Pa divided by L.

So, with those consideration, once you get it, you know like, those reaction forces at point A and point B, now we can simply figure out that what will be the moments are there under the action of load P at point D. So, with that you see, you know like, we are considering the distance x at this particular point. So, we have M 1 and M 2 - these two bending moments are there - and we need to find it out these M 1 M 2 with a different configuration.

So, bending moment using free body diagram, you see, now of the entire beam we need to determine, you know like, the values of M 1 M 2 with the using of following relations.

So, first as I told, you see, RA and RB is there which can be pretty easily calculated by Pb by L and for RA and Pa by L for RB. Now, the portion AD, you see, which is, you see, the left end portion of the particular beam and it is under the bending action. So, you see here the bending moment Ma is nothing but equals to the reaction, you know like, this RA into the distance x, because it is acted at distance x. So, we have RA into x or we can say that Pb by L into x which is nothing but the M 1.

So, now you look at this particular free body diagram of this. Now what is there? This RA is acted on the top one direction and, you see here the due to this point B it is acting in this direction. So, you know like, this bending action is there and due to that we just want to balance that part. So, we have the bending action, you see, M 1 which is just, you know like, going in the counter clockwise direction and which has the value of Pb by L into x; that means, you see, whatever the RA, RA is acted towards that or just opposing that part. So, RA into x is there. So, this is one form. So, you see, through which we can now calculate the moment 1 which is nothing but equals to Pb x by L. But the real phenomena is that again that when, you know like, the bending action is there, our main focus is that now you have the M 1, so now, you can calculate the U 1 for this one.

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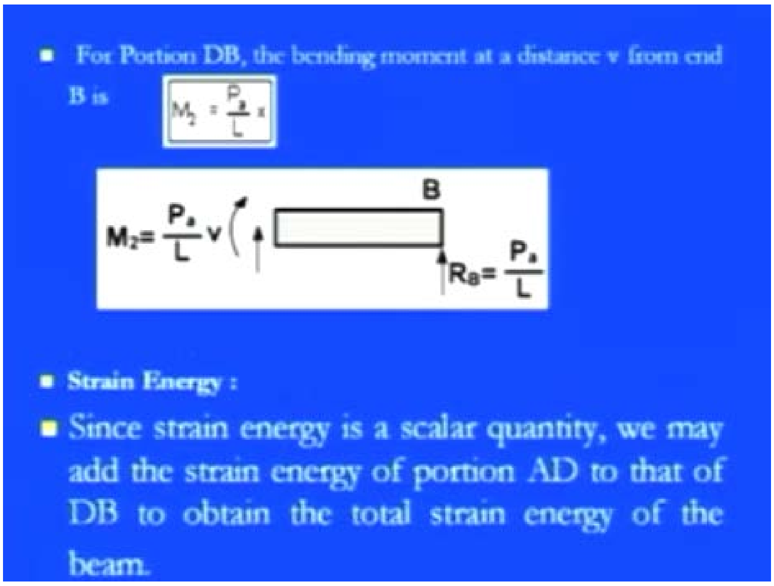

So, correspondingly, you see, now if you go to towards the, you know like, the other - another - portion that is, you see, the B to D. So, DB portion is nothing but, you know like, is having the influence of the load P which is towards the downward direction and the reaction forces at the extreme end of B. So, you see here, with that particular… the bending moment at a distance of that B from, you know like, the free end of B is nothing but equals to M 2 is there. So, what we have? We have the next bending moment M 2 which is, you see, coming as the RA into x. So, what we have? We have Pa divided by L into x.

Or coming to the point, you see, now what we have? We have now M 1 which is nothing but equals to RA into x; M 2 which is nothing but equals to RB into x. And with those inclusion of that thing we have a real feeling about how the moment being transferred from this AD part to BD part.

So, with those, you see, now you can simply look at the free body diagram of this, then you will find that what we have, we have this particular point, you see, the B at only the reaction forces are there on the top up direction. And at the free end, we have, you know like, this particular, the loads are there which is just going on the top ward direction. And corresponding, you see, the bending moment M 2 is there which is trying to oppose the reaction forces RB which is nothing but equals to Pa by L. And which is the main, you know like, the responsible parameter is there which is inducing the moment in that particular beam. So, you see, here this M 2 is nothing but equals to, you know like, this RB into you know whatever the distance is there. So, we have, you know like, the RB into x is the M 2. And we have the v which is the, you know like, the shearing part is there because we are ignoring that part. So, only we just want to see that how these, you know like, the bending stress is being distributed amongst, you know like, this particular beam under the loading actions. But our main focus is to calculate the strain energy for that.

So, you see here the strain energy is a scalar quantity. So, we may simply add the strain energy of the portion - of these two portion - M 1 and M 2 is there in which, so AD to that of DB to obtain the total energy of the beam. So, U is nothing but equals to U of AD plus U of DB.

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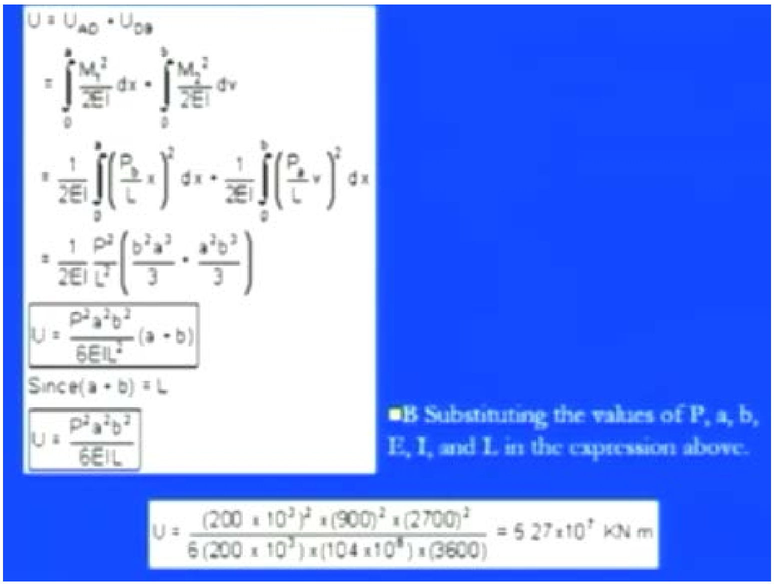

So, now, you see, here simply, you know like, what we are going to do here simply, you know like, we have simply segregated this particular beam into two different regions right from zero to a for which, you see, only the point load is acted on the left end portion. And, you see, the another region is there for zero to b in which, you see, the DB portion is there. So, and zero to x section is being influenced by a moment M 1, and zero to b is being influenced by moment M 2.

So, now, you see, here with the using of that moment equation for the strain energy what we have? We have, you see, UAD is being replaced by zero to a M 1 square divided by 2 EI dx and UDB is being replaced by this integration zero to b M 2 square divided by 2 EI dv. With those things, you see, now what we need to do here? We need to again replace that part. So, here what we have simply taken out this 1 by 2 EI, taken out; we have zero to a Pb by L, because again, you see, we need to replace that part of moment, which is M 1 is nothing but Pb by L into x whole square into dx plus 1 by EI is taken out. So, we have zero to b this Pa by L into, you know like, the x whole square into dx.

So, you see, after doing this particular simple integration part here what we have? Finally we have 1 by 2 EI P square by L square b square into, you know like, in the bracket itself, you see, after keeping those values for limiting condition zero to a and zero to b. We have b square a cube by 3 plus a square b cube by 3 or, you see, by simply adding that part, you see, I am taking out a square b square we have, you know like, the strain energy within that component U is P square a square divide a square b square divided by 6 times EI L square into a plus b.

Or we can say that, you see, a plus b is nothing but it is a total length of the beam L. So, we are replacing that part and we have, you see, the final strain energy for, you know like, under the bending action where the simple point load is there at a distance a from the left end is U is equals to P square a square b square divided by 6 EI into L. So, you see, here, you know like, simply we have all those values, we have value of P, we have value of a, we have value of b, E I, and L.

So, after, you know like, keeping those values in this expression what we have? We have the value of strain energy U which is nothing but equals to 200 into the load; 200 is there. So, 200 into 10 to the power cube whole square; and then, you see, we have the a value 900 square; we have the b value 2700 square and divided by 6 EI, you see, all those things which we have, we are simply keeping those values and ultimately we have the total strain energy is 5.27 into 10 to the power 7 kilo Newton meter.

So, you see here, it is pretty straight, you know like, the expressions are there for calculating the strain energy; only we need to focus that when, you see, the bending action is there, so which area is influenced by which moment, and corresponding, you see, the moment which is, you know like, coming out who is the responsible parameter for the moment.

So, you see, in that case, you see, in the just now which we discussed, the M 1 is coming due to the reaction force RA, and M 2 is coming due to the reaction force RB; then, you see, just after incorporating all those forces, we have a simple formula of U, and we can simply replace those values and get the final value of strain energy. So, you see, here this is the part where, you know like, simple discussion was there about the strain energy due to the bending action. Now, we have one more energy term that is known as the complementary strain energy. So, you see, here in one of the diagram I have shown you that when we are drawing, you see, the stress-strain curve we have the two main, you know like, the regions are there. One region is the strain energy; one region is the complementary strain energy.

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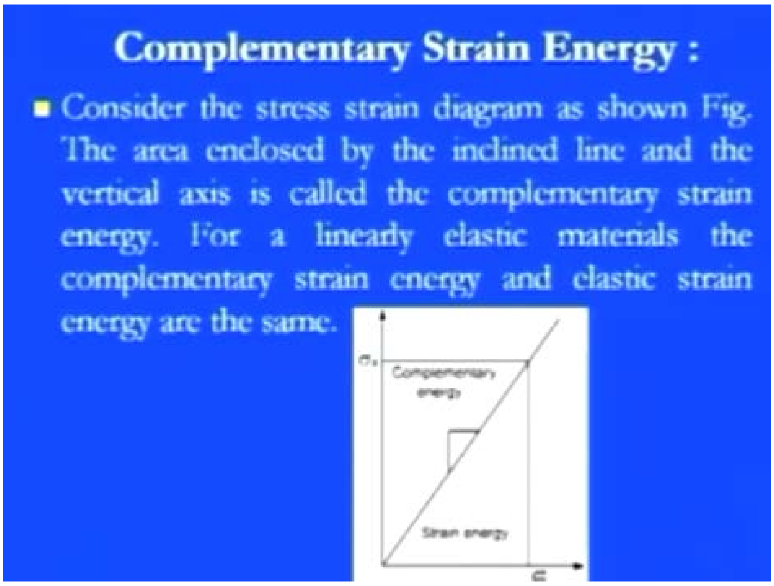

So, consider the stress-strain diagram as shown in this particular way also; the area enclosed by, you know like, the inclined line and the vertical axis is called as the complementary strain energy. And for linearly elastic material, the complementary strain energy and the elastic strain energy are same.

So, you see, here if you are, you know like, applying the load and if you are going up to the elastic region only, then we will find that the real physical sense of the elastic strain energy and the complementary strain energy is the same. But if you are going in the non-linear region,; that means, you see, where if you are going in the plastic region where the permanent set of deformation is there, then things are real different.

So, here, you know like, if you see, in this particular diagram, what we have? We have the elastic region and this is up to the, you know like, the yield point is there. We can simply see that even this particular region or this region, they have a similar kind of nature. So obviously, you see, you know like, whatever the energy which is being stored in terms of, you know like, this complementary or the strain energy, they have the same value or they have the same physical sense in that way.

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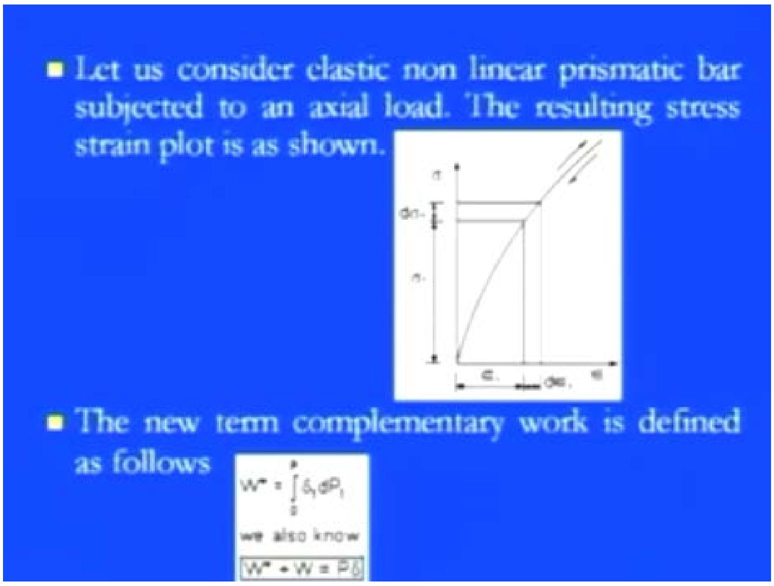

But now, you see, if we consider the elastic non-linear prismatic bar and it is subjected to an axial load, so you see, here, there is some sort of differences are there. So, the difference is like that we have, you see, the sigma 1 and we have the epsilon 1. And in these terms, you see, we have, you know, that we have a slopey curves are there.

So, within the slopey curve what we have? We have you see, you know like, the d sigma 1. Now if you are taking a simple element of this particular slopey means the non-linear term. Then with that consideration, if you have you small segment d sigma 1 and d epsilon 1 in the sigma epsilon curve, so you see here, after focusing on that part, we could simply for, you know like, check it out, that you see, under these particular region where, you see, the strain energy is there, and out of these particular regions where the complementary strain energy is there, what you see, you know like, the significance is there.

So, the new term which is coming as the complementary work is absolutely to be defined as, you see, the zero to P because the load application is there of the P which we are limiting that part delta 1, as you see, what the deformation is there into dP 1. Or we can say that the total work which is coming in that particular deformation is W star plus W is equals to P into delta.

So, you see here, that the total work which is coming as the complementary plus the real work and which is equals to or we can say which is responsible for the load application and corresponding deflection is.

So, here the things are, you know like, when we are discussing about the non-linear regions, we could easily figure out, you see, from this particular diagram that, in the non-linear region we have, you see, you know like, even if you are talking about this particular range, then this complementary strain energies is somewhat different than this particular regions.

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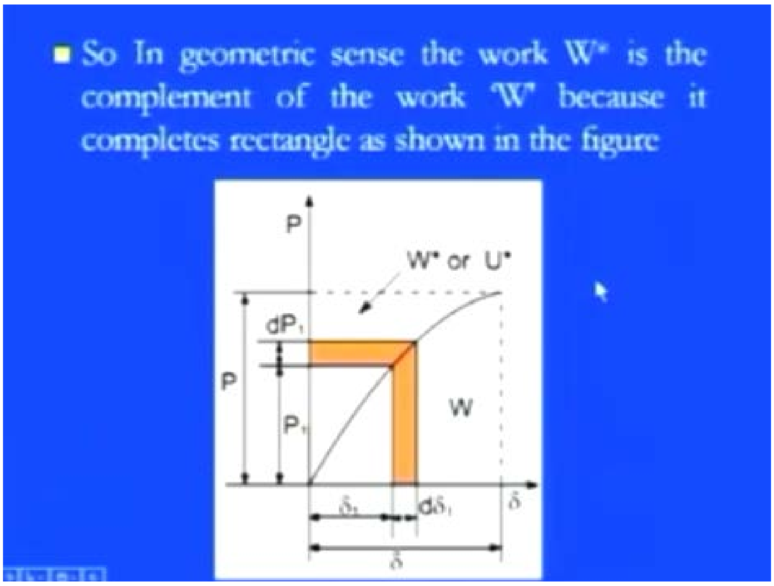

So, now, you see, in the particular geometric sense, of this particular work, W star which we were discussing about the complementary part and, you know like, the W because of the normal work done. Now, if you simply look at that particular figure of the non-linear nature of this load verses deflection curve, then we can simply figure out that starting from the zero, now we have, you see, the P 1 curve, and within the P 1 curve, we have the delta 1 region.

So, whatever the deformation is coming or it is to be computed it has, you see, the delta 1 due to the load application of P 1. Now, you see here, if you go up to the real P and the delta, we can simply see that it is simply covering the total this, you know like, the area of the non-linear region also and in that now we have the two different, you know like, the work done is there.

This work done is simply for the strain energy is responsible part, and this W star is responsible for the U star, and this U star is the complementary strain energy. And then in that, you see, if you are considering a small segment dP 1 which is responsible for, you see, the delta, this deformation d sigma 1, so now we would like to, you know like, simply setup the basic relationship between those.

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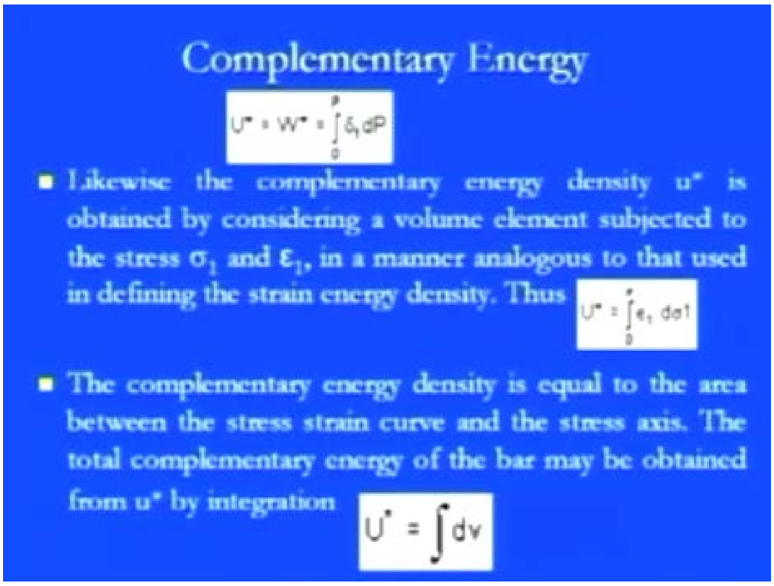

So, come to that point U star and I told you that the W star, which is the complementary work is there, is responsible for complement strain energy is equals to zero to P because, you see, that a limiting load is P, so zero to P delta 1 into dP.

Likewise the complementary, you see, energy density U star is obtained by considering the volume element; because you see, you know like, this is the domain is the volume for any kind of strain energy, and you see, either the complementary strain energy is also having the same domain. So, by considering the volume elements subjected to the stress sigma 1 and epsilon 1 in a manner analogous to the use the same, you know like, the definition for strain energy we have U star is equals to integration of zero to a sigma epsilon 1 into d sigma 1. So, complementary strain energy is equal to the area between the stress-strain curve and the stress axis.

So here, you see, this is the key feature. So, the complementary strain energy is always coming under the area of a stress-strain curve irrespective of the linear and the non-linear, but the stress axis is there. So, the domain here is somewhat different than the normal strain energy, because in the normal strain energy, it is again, you see, the stress strain curve is there, but the domain or we can say the main, you know like, the reference axis is the strain, the strain axis, but here the main domain is the stress axis.

And, you know like, and that is why, you see here, the total complementary energy of a bar which can be simply calculated, you know like, on the basis of U star is equals to zero to sigma epsilon 1 into d sigma 1. So here, you see, we are considering a real small difference of this sigma 1. So, how the d sigma 1 is, you know like, changing with the corresponding, you know like, the strain is.

So, here our main focus for the complementary strain energy is absolutely focused on what exactly the variation is there in the elemental part of the stress, and then, you see, what the corresponding changes are there in the strain part. So, you see, here after considering of all sigmas, small, small elements where you see, there is a variation is there in the stress part, we can simply get, you see, the U bar by the integration U star is nothing but equals to integration of all those volumetric part.

So, you see here, for all complementary strain energy we found that the domains are somewhat different than the complementary strain energy and this normal strain energy or we can say elastic strain energy. And that is why, you see, we were discussing about in the initial phase that, if we are talking about the linear elastic region, then the complementary strain energy and the elastic energy - this elastic strain energy - must be same. Because in that, you see, whatever the domain is there, they will simply have a linear relationship irrespective whether it is a strain domain or stress domain, and since they have the proportional relationship, so there is no difference as such.

But when we are talking about the non-linear relationship then, you see, both of the domain; means if you are talking about, you know like, the strain or if you are talking about the stress part, you see, here they will provide the different domains altogether. And whatever the corresponding strain energies are there, based on the deformation, they have to be real difference in that sense.

And that is why, you see, in the non-linear relations, we altogether we have a different relations or, you know like, this complementary strain energy and the elastic strain energy. Now, we are going to discuss about the Castigliano's theorem. And the Castigliano's theorem is basically focused on the complementary strain energy and the elastic strain energy in which you see, you know like, if any slight change is there within the elastic region then, you know like, which energy is dominating.

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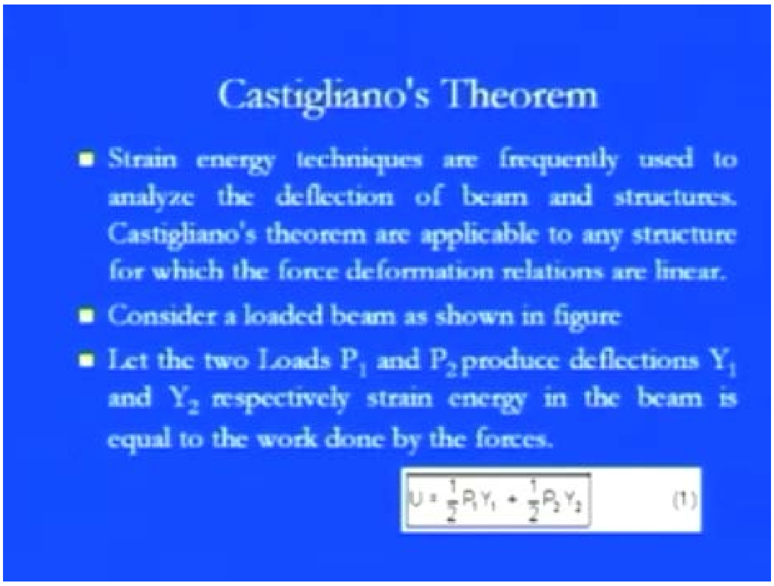

So, coming to the strain energy technique frequently used in, you know like, analyzing the deflection of beam and the structure as we discussed. So, Castigliano's theorem are applicable to any structure for which the force deformation relations are linear. So, that is what, you see, I told you that if within the elastic region, if any slight change is there we can simply go with this particular theorem.

So, now, consider a loaded beam, which I am going to show you in the next slide. There are, you see, I mean two loads are there P 1 and P 2 which are producing, you know like, the deflection - two different deflections - Y 1 and Y 2 respectively. So, you see, the strain energy within the beam is equal to the work done by the forces since, you see, we have the two different forces and both the forces are equally responsible for corresponding deflections. So, we have the strain energy U is equals to half of P 1 Y 1 plus half of P 2 Y 2.

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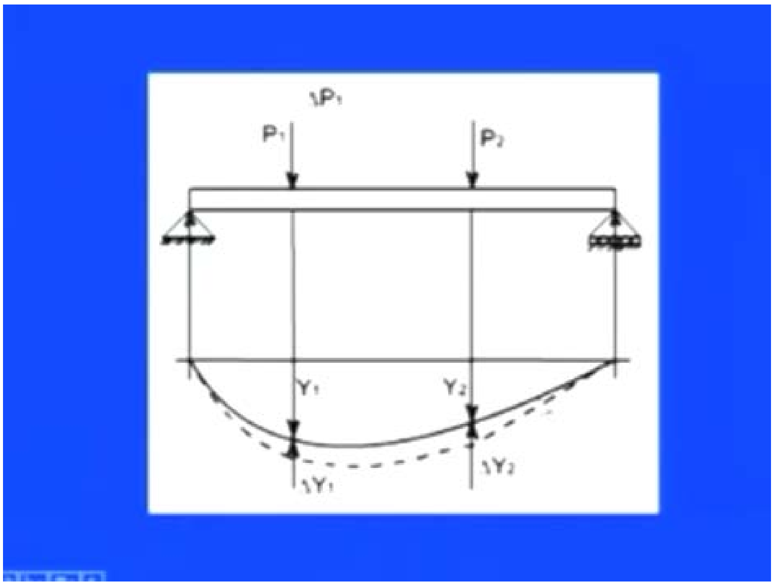

So, now coming to the figure, we can simply see that it is a simply supported beam, but you see here, we have, you know like, simply we are giving a kind of flexibility, so that through which it can simply move in the actual direction. So, when we apply the load P 1 and P 2, you see, at the corresponding locations, we could figure out that we have, you see, this particular straight part is there. And we have, you see, the deflections are there, Y 1 corresponding to P 1 and Y 2 corresponding to P 2. But if any slight change is there in the load P 1, we know that very slight change will corresponding coming in terms of, you see, the delta Y 1 and delta Y 2 because, you see, if addition or subtraction any kind of, you see, the variation is there in the P 1 always deviate this particular position here.

So, if it is simply deviating this particular position, there is in the additional phenomena is phenomena is coming which is, you see, coming as delta Y 1 and delta Y 2. So, you see, what the exact impacts are there of these things, we are going to consider here.

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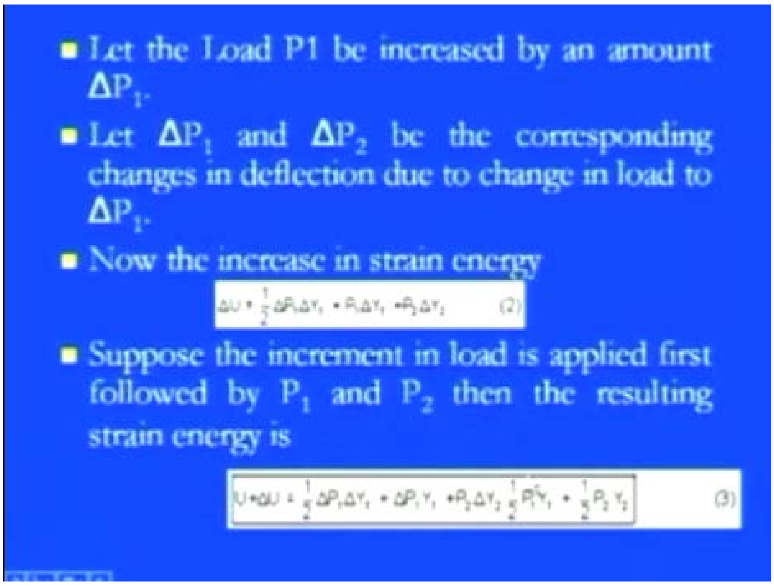

So, let us say the load P 1 is being increased by an amount of delta P 1. So, let delta P 1 and delta P 2 with the corresponding changes in the deflection we know like due to the change of load delta P 1. So, you see, here now we have the different, you know like, the deflection terms are there because of, you know like, the additional load of P 1, and which is will be considered a considerable as delta P 1.

So, you see, the increase in the strain energy due to the additional part is nothing but equals to this delta U which is you see, you know like, the incremental part is equals to half of delta P 1 delta Y 1. Because of you see, you know like, the additional P 1 is coming from that particular part, and then, you see, you know like, due to this additional part what we have? We have simply, you know like, the additional deflections are there, delta Y 1 and delta Y 2. So, you see, the total part is P 1 delta Y 1 plus P 2 delta Y 2. So, now, you see, we have the new term of the strain energy that is the incremental form is equation 2.

Suppose the increment in load is applied to the first followed by P 1 and then P 2, then the resulting strain energy is somewhat different. That means, you see here, now we are increasing the load just at P 1 first, and then, you see, the simultaneous increase is there in the load capacity in the P 2 also.

So obviously, you see, the total, you know like, the strain energy is coming out as U plus delta U is equals to half of delta P 1 delta Y 1. Delta Y 1 is coming due to the delta P 1.

So, that is what, you see, it is like that plus delta P 1 and this delta P 1 and the Y 1 is there plus P 2 into delta Y 2. And then you see, you know like, half of P 1 Y 1 is there plus half of P 2 and Y 2. So, you see here, what we have? We have several components altogether. The one, you see, because of the increase in load. So, delta P 1 is there and due to this increase in load we have the increase in deflection. So, you see, the two different phenomena are there with the delta Y 1 and delta Y 2.

And then, you see, the corresponding you see, you know like, the P 1 and P 2 are equally responsible for Y 1 and Y 2 deflection. So, these component have to be there in that particular inclusion. So, that is why, you see, if you are computing the total strain energy then, you see, we have almost all kinds of components in that particular expression.

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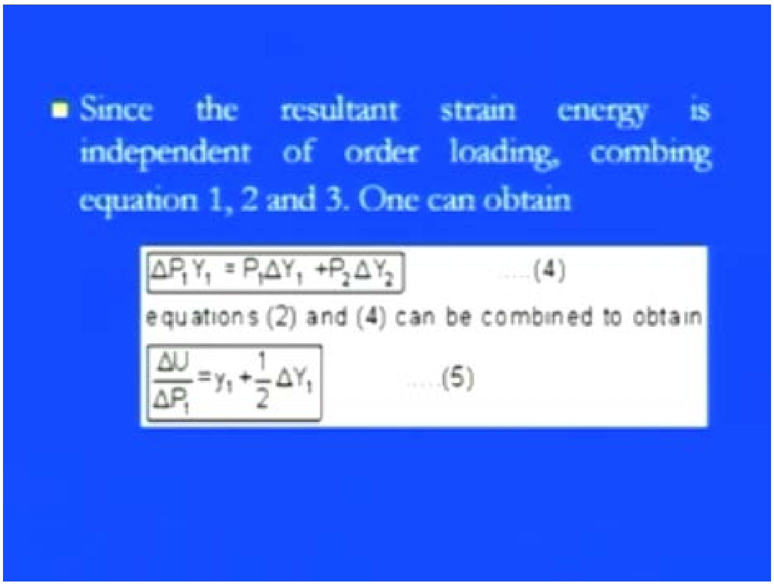

Since the resultant strain energy is the independent of the order of loading, you see, whether it is, you know like, first it is coming to the P 1 and then it is going to the P 2 or first it is coming to the P 2 or going to the P 1 it does not matter you see. So, the equation, you see, whatever the equation which I have shown you here it can be simply, you know like, equated both the part. And then, you see, we can simply obtain delta P 1, delta P 1 into Y 1 is absolutely equal to P 1 delta Y 1 plus P 2 delta Y 2. That means, you see, here even with the increase of load in the P 1 it is simply responsible for having the displacement at Y 1 location and Y 2 location and that is what the equation 4 is showing.

So, you see, here we can again say that delta P 1 is coming means that is an additional off load is there and the P 1 part responsible for P 1 into delta Y 1 plus P 2 into delta Y 2.

And equation 2 and equation 4 can be combined and we have, you see, delta U divided by delta P 1; that means, you see here, the change in the strain energy with the change in the domain of the load is equals to Y 1 plus half of delta Y 1. That means, you see here, whatever the changes are there in the load formation, the corresponding changes will come in the strain part with the deflection original and with the slight change in the deflection. So, that is what, you see, the equation 5 is telling.

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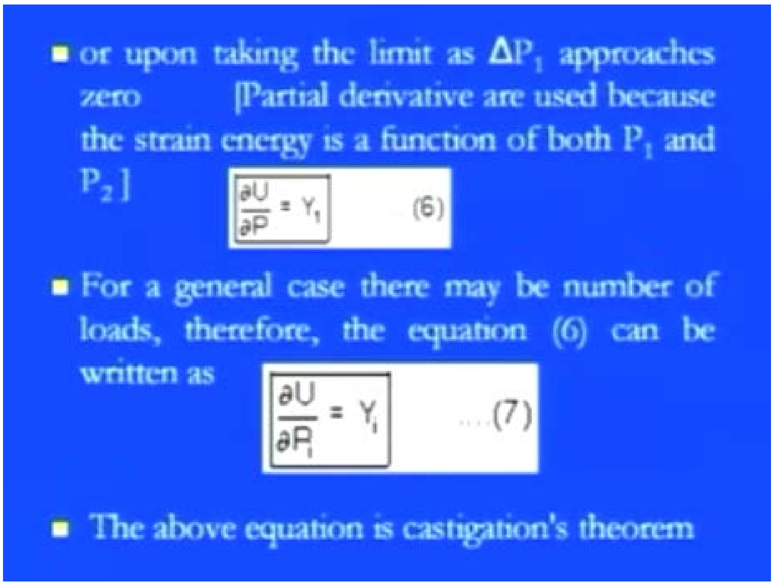

Or upon taking the limit of delta P 1 approaches to zero; that means, you see, there is a very slight change is there in the load P 1. So, partially derivative I used because the strain energy is a function of both P 1 and P 2. So, when we are saying that delta P 1 is tends to zero. So, you see here, obviously, delta Y 1 will be in a zero in the equation 5. So, we can say that the delta U divided by del P in the partial derivative term, as I told because it is a function of strain energy part is there. So, delta U by delta P is equals to Y 1.

So, for general case there may be, you see, the number of loads are there and we can simply write this equation as delta U by delta Pi because, you see, here we are saying that there are tons of loads are there. And due to that we have, you know like, the various segments of strain energy is there. So, the strain energy within that particular domain delta P 1. So, delta U by delta P 1 is equals to Yi. So, this is, you see, the equation is known as the Castigliano's theorem.

So, Castigliano says that we have n number of deflections are there and these are equal to the domain of the load with respect to this strain energy. So, dU by dPi will be exactly equal to Yi; means that the strain energy within the domain of all the loads, the summation of those things will give you this deflection for individual point.

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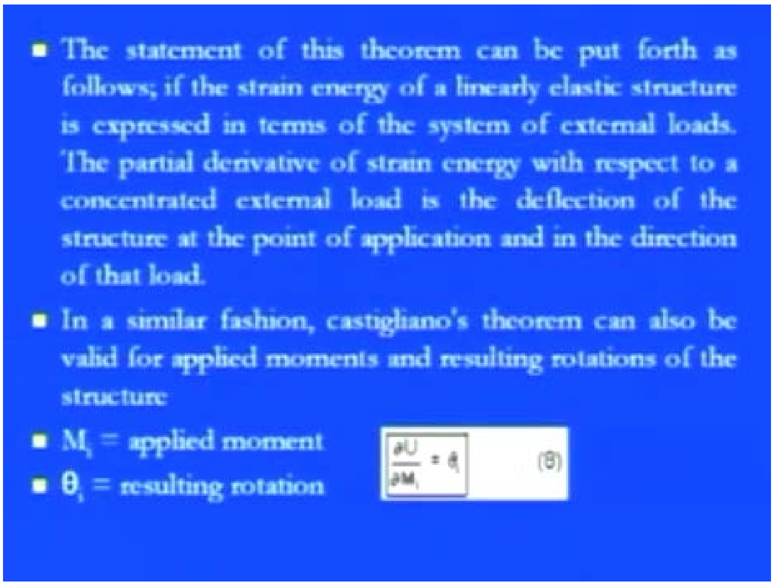

So, this is, you see the, you know like, the Castigliano's theorems are there and if you simply separate out these terms, then we will have the statement. The statement of this theorem can be put, you know like, as the follows like: if the strain energy for a linearly elastic structure, because again, you see, this is the first condition is there that the Castigliano's theorem is only applicable to the linear elastic wave where the stress is proportional to strain. So, for a linearly elastic structure it is the strain energy is expressed in terms of the system of external loads.

Means whatever the external loads are there, you see, P 1, P 2, P 3, P 4 the partial derivative of strain energy del U with respect to concentrated external loads del Pi is the deflection of the structure at a point of application and in the direction of that load only.

So, you see here, a partial derivative of the strain, this strain energy with respect to concentrated load will give you the direct deflection within that particular direction at a particular application. So, this is what Castigliano says that even you have number of loads you can simply, you know like, differentiate those things with respect to the strain energy, and they will give you the deflection of those points towards the application of the load.

And in a similar fashion, Castigliano's theorem can also be valid for the, you know like, the applied moments and the resulting relations of the structure. You see because he has given, you know like, 4-5 theorems are there, but generally, you see, we restrict our self within those theorems. So, you see, here within those moment also we can say that the partial derivative of the strain energy with respect to the concentrated moment, not only the external load, with respect to the moment - concentrated moment - for all those you see, external moment applied, will always give you the resultant rotation of the structure at a point of application and in the direction of the moment only.

So, this is, you see, the another theorem; again I am repeating the important, you know like, the statement that the partial derivative of the strain energy with respect to the, you know like, applied moment it always gives you the resulting rotation of the structure at those particular points of application where, you see, the moments are applied in the direction of those moment only. So, this is you see, you know like, with those moment we can simply get the resultant rotations and, you see, if we are taking the strain energy with respect to the applied load we have direct deflections.

So, you see, we can simply figure out that what exactly the deflection is there and what exactly the resultant rotation is there, based on what the applied load and the applied moment is. Now, you see, we are going for the first theorem again.

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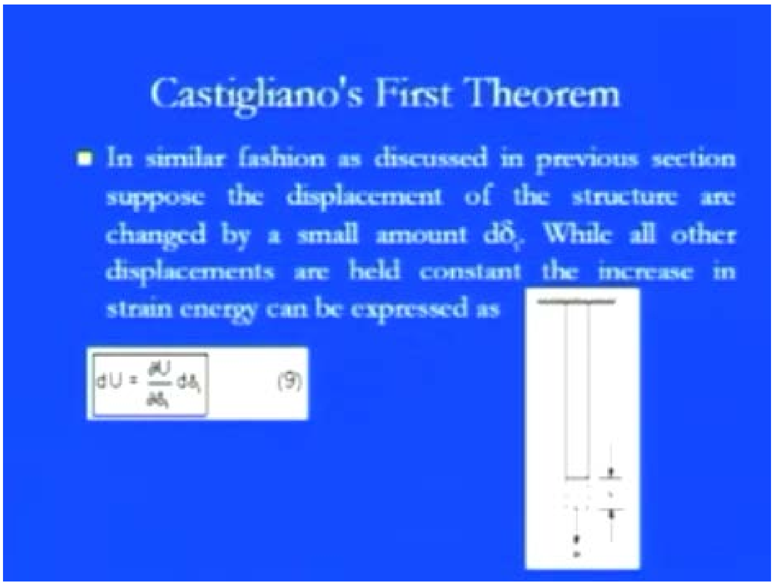

In a similar fashion as discussed in the previous section, the displacement of the structure are simply changed by small an amount of this delta d delta. While another, you see, the displacements are simply, you know like, held constant and there is an increase in strain energy is there which can be simply expressed by dU is equals to del U by del delta into d delta.

So, you see here what we are doing here? We are simply taking the small amount where, you see, the this d delta is there and now we are simply taking that what exactly the increment is there. So, if you look at this particular figure then you will find that what we have? We have a simple membrane here, with particular, you know like, the basic, and when you apply the load here, you know like, this vertical load we have, you know like, the del d this delta deformation is there, and within this del delta what we have? We have there is a direct increment is there in the strain energy and this strain energy is simply computed on the basis of d delta.

So, with that, you see, now if you want to compute the strain energy, then this increase in a strain energy is nothing but equals to dU into this epsilon, this d delta and then you see, you know like, since this is the domain of this dU by d delta, and now, we are going for all the small amount. So, it is d delta.

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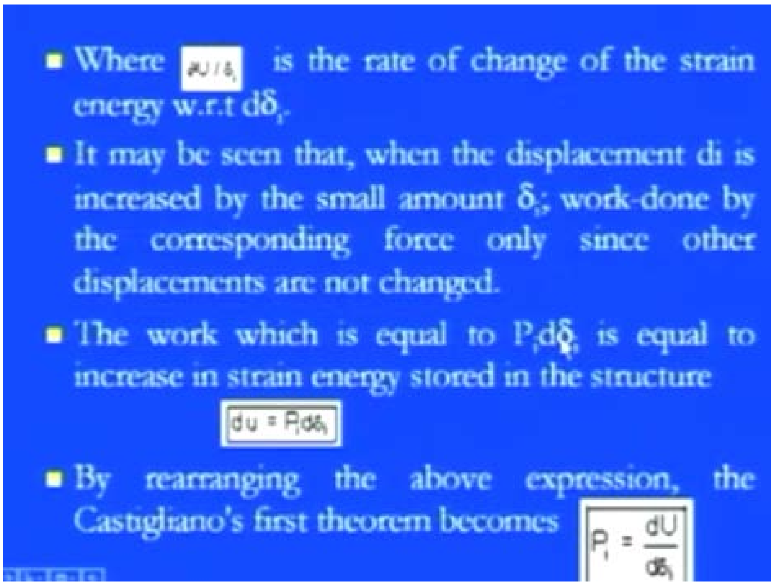

So within that particular theorem now, we can say that this d delta by dU by d delta is the rate of change of the strain energy with respect to that small deformation. And it may be seen that when the displacement - this displacement di - is increased by the small amount of this delta work done by the corresponding force only since, you know like, within that particular displacement, and other displacements, other displacements are not being changed. So, only we are simply focused on the small amount of d del I only. We are not, you know like, considering the other displacements for that particular figure.

So, the work which is equals to Pi into d del I is always equal to the increase in the strain, you know like, which is being stored in the structure. So, we can say that the dU is nothing but equals to Pi which is the responsible part into d del I.

And by increase in the, you know like, simply, you know like, rearranging those things, we can say that now using the Castigliano's theorem that the external point load can be easily calculated if we know that the increment in strain energy within the domain of the change in their displacement. Or we can say that whatever the small amount of displacement change is there or a increment is there in that domain whatever the strain energy is coming it will simply you know locate this force application.

So, you see here, in this chapter we basically discussed that you see, you know like, if we have the two different domains are there; that means, you see, first of all if we have the energy form, the strain energy due to the bending, then you see, how we can simply figure out you see, you know like, that under the action of, you know like, the different, different forces, what are the different segments are there, and how we can figure out, you see, the strain energy component within that bending action.

Then you see, you know like, we were discussing about, you see, the different energy domains using, you see, the complementary strain energy and the elastic strain energy. And in that you see, you know like, we found that this Castigliano's theorems are really important because, you see, if we are talking about the externally applied load and there is a change in… the corresponding changes are there in the strain energy, then you see, the change in strain energy within the change in externally applied load will give you the displacement at that particular points, and these displacements are there within the load application point only.

And similarly you see, if we are talking about the another theorem of Castigliano's part then we will found that if, you see, the change in this strain energy is there, within the change in the domain of bending moment, then we could easily figure out that how the angular rotations are being there within these del U by del Mi which will give you the theta.

So, you see here, with the arrangement of these two theorems we will find out that when, you see, when we are talking about the small change in the strain energy within the domain of small displacement, straight away we will get the load at these particular points where the small, small displacements are there.

So, if we are talking about the first point, then dU by d delta 1 will give you P 1; if you are talking about the nth term then we have Pn is equals to del U by d del delta n is there. So, through that, you see, we can easily find it out, you know like, that what the load applications are there at these particular points, which are equally responsible for the strain energy at this point and the displacement correspondingly.

So, that is what, you see, we discussed in these things and Castigliano's theorems are really very important where, you see, if we want to find it out, you know like, the deflection or the moment at that particular point with the using of strain energy concept, because, you see, you know in that, you see, the complementary form and elastic forms are there and as we discussed that, actually within the elastic region since they are equal, but if we are talking about, you know like, this non-linear phenomena where the plastic regions are there, then somewhat it is considerable in different terms as we discussed. So, that is what, you see, we discussed in this particular lecture.

And now, you see, in the next lecture, our main, you know like, focus is there that now within those Castigliano's theorem how we can really figure out those problem; that means, you see, how we can calculate the load conditions or, we can say, how we can calculate the deformations within, with the using of these strain energy concepts.

So, some of the numerical problems are there within that, and then, you see, we would like to conclude, you see, because the next lecture is the last lecture for whole of the course, so probably we are going to discuss about what we have discussed. Because you see, you know like, in the whole structure we have discussed many of the key points and, you see, as this is… the subject itself names as the strength of materials, so what is the physical justification of this course within this particular limit or we can say that actually what exactly the importance has a mechanical engineer or is or as a technocrat what you feel about this particular course? That whether it is really useful to study and in what sense it is useful, because, you see, for many of the technocrats, they should know about, you see, the stress, what the meaning of the stress and strains are, and then, you see, if somebody wants to design some of the components, then which stress or strain components are really important to figure out those things.

And then, you see, generally, you know like, for many of the disciplines, the beam theory or the struts or the columns or there are many other things they are really important for designing part. And, you see, this strength of materials subject will provide the basic input for all the kind of designs So, we are going to discuss in the next lecture all these terms.

Thank you.