Strength of Materials
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Lecture - 38

Hi, this is Dr. S. P. Harsha from mechanical and industrial engineering department, IIT Roorkee. I am going to deliver my lecture 38 on the course of strength of materials, and this is course is developed under the national program on technological enhanced learning.

Prior to start of the lecture we would like to discuss briefly about what we have discussed in the previous lecture. We discussed about you know like the strut and the column that how we can differentiate about the strut and column and what the importance is there of the buckling load. So, you see we derived many things you know like in the different cases like you see if the column or the strut is there with the pin joint or you see with the pin. And the fixed rigid joint is there or completely rigid joint is there or you see if there is a free end is there, then you know like how we can get those crippling load and all. And then you see you know like we found that there is Euler theory is there which is absolutely computational theory, and you know like using that particular theory we can simply get the crippling load that is known as the Euler crippling load.

And also we found that if we have different modes of this buckling then how we can get exactly those numerical values, and how we can replace those things in terms of you know like that wavy feature in this particular strut and column. So, that we decide and also that we discussed, and also we discussed about that what are the key factors through which we can simply characterize you know like the properties of the strut. So, that part you see we discussed in you know like the previous lecture, and also along with that we discussed that if we are going in a realistic way; that means you see if we have an experimental part then what exactly the difference is there in between the experimental and the computational theory of this Euler.

Then we found that actually if we are doing in experimental way, then there is a failure due to buckling well before than what we have calculated in the Euler theory. That means you see whatever the assumptions, which we made in the Euler theory that was not correct. So, we need to improve you know like theory. So, that is what you see in the previous lecture. We suggested few of the modification in the Euler theory like you see you know like that we are considering the axial force exactly along the axis of this particular strut. But in realistic way you see always there is some sort of eccentricity is there and, due to that eccentricity what we have? We have an additional moment is there in the strut and you see when we are increasing the load.

So, with the gradually procession we will find always this moment is playing in key role in the failure of the system due to the buckling. So, you see here in that particular modification we adopted that additional curvature part is there the R 0 which is you see you know like 1 by R 0 is nothing but equals to d 2 y 0 divided by dx. So, this additional you know like y 0 is being added before you see any load application; that means you see we are assuming that actually there is a kind of deflection is already there you know like in terms of y 0, and then you see it can be easily added. And then in the whole theory again you see we derived using these numerical values and we found that it is pretty close to the experimental part.

Along with that you see we found that there is some means you know like with this eccentricity some sort of nonlinearity is there which we are assuming in linear way. Like you see you know like the first thing is that we have the material nonlinearity or geometric nonlinearity; means you know like we are assuming that the strut is absolutely straight during the load application, but in realistic manner it is not like that. So, we assumed you see all that kind of phenomena using straight line parabolic path or whatever you see you know like, and then we found that after application or after inclusion of all these theories or all these you know like the modification, we found that, yeah, we are pretty close to these things.

So, with those inclusions you see we discussed some of the issues in the previous lecture that you see you know like if both joints are pin joint, then you see you know like how we can develop with the modified way. So, along you know like with all these discussions, now in this lecture we will start from that that if we have you know like the strut with the pin joints or whatever then how we can define those things. So, these are you know like few of the cases which we discussed and also the we would like to discuss one more case and then we will go with some energy methods that you see you know like if we have a kind of any solid mechanics, then what exactly the energy methods are there in the those things and what are the associated terms are there.

And what is the physical and their relative terms are there and the physical significant is there in order to check it out those property and in order to check it out the failure theories of the object or we can say the solid mechanics part. So, you see here in this lecture as I told you we are continuing with that strut part with eccentric loading.

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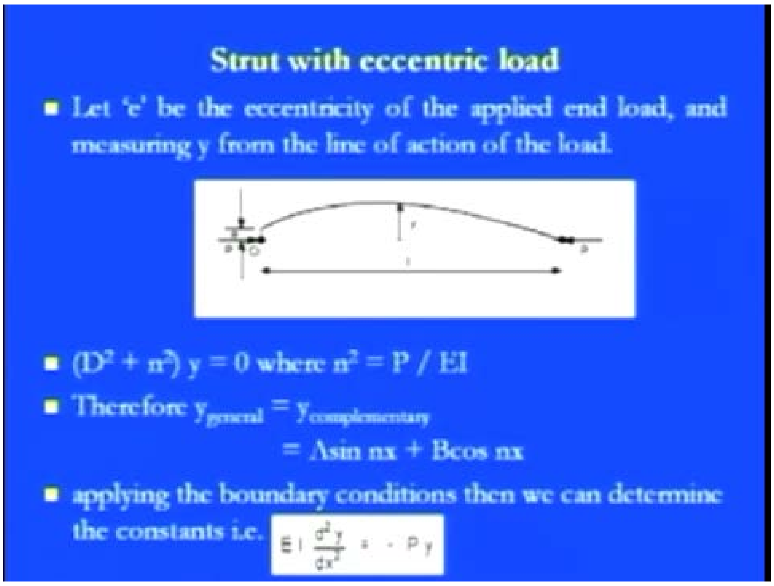

Because you see you know like as I told you you know like in the Euler theory we are assuming that there is no eccentricity is there, but in realistic way we will find that there is always a kind of eccentricity is there. Because when we apply either the compressive or any kind of load it is not exactly in the same you know like the stream length or we can say the centroid of the particular strut. That means you see you know like we need to introduce some sort of eccentricity to get the real feeling of the strut in experimental way, because as I told you in few of my lectures that whatever we are discussing about a real nature or the experimental part, all real systems are highly nonlinear or highly complex in the real nature.

So, if you want to predict the failure theory or behavior of those kind of systems we need to be very close to the real nature; that means you see we need to introduce some sort of the nonlinearity or we can say this kind of eccentricity. So, you see here if you are saying that you can look on this particular screen the figure, it is simply a strut is there, the pin joints are there. And we have you see you know like a simply introducing here the eccentricity e which is you know like at the particular applied load part.

So, here you see you know like this load is being applied here, and we have an eccentricity e part here at this particular point o. And just we are measuring y; obviously, in this whatever the deflections are there in longitudinal direction, and then you see this strut is flowing in a this transverse direction is there. So, along with this longitudinal and transverse direction our main intension is to find it out that what exactly the impact of this e is on that particular behavior of our strut. So, again you see the same procedure is there which we discussed in the previous lecture, what we have? We have the strut is there and then you see the total length of the strut which we are assuming here is l and then the same elastic properties are there.

So, modulus of elasticity is e, and this area moment of inertia is I is there. So, we can simply calculate the flexural rigidity for this E I, and then you see you know like again the same the second degree of freedom is there. So, that is what you see we have you know like the second order differential equation is there. So, again if you want to write in the derivational way, then we have you see you know like D square plus n square into y is equals to 0. So, what we have? We have D square is nothing but equals to d 2 square divided by dx square and n square as usual you see we have P over E I.

So, we have second order equation. So obviously, we have the generalized solution. So, in that you see we have the complimentary function is there. So, if you want to calculate the complimentary function same procedure is there A x sin x plus B cos x is there. So, again you see since it is simply flowing with the n. So, we have A sin n x plus B cos n x is there as in the generalized solution. So, you know like to get the coefficient value A and B we need to apply the boundary condition.

So, after applying the boundary conditions in the pin joint where the load application is there only the key feature is that we have eccentricity at point o, while at the other end of this particular pin joint there is no eccentricity is there; we are assuming that it is the exactly at the centroid of this strut. So, with those conditions what we have? We have you see you know like EI in to d 2 y by dx square is equals to minus P times y the simple theory you see which we discussed.

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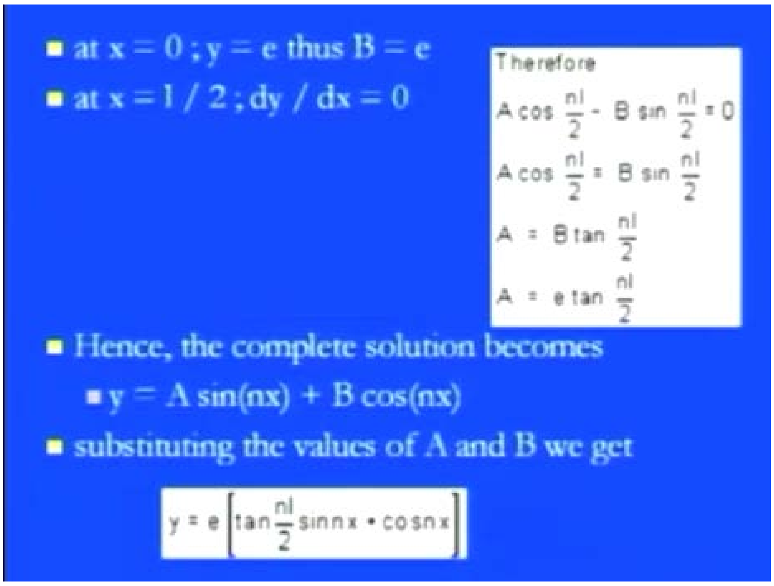

Now with the boundary conditions at the x equals to 0 y equals to e; that means you see already we have even this eccentricity might be in terms of microns or even in terms of millimeter, but it is present at x is equals to zero. So obviously, you see we have now this time B has some value. So, B is equals to, e and if you are keeping you know x equals to l by 2 there is no slope is there because the maximum deflections are there. So, dy by dx is equals to 0.

So, keeping those values at x is equals to 0 and x equals to l by 2 we have A cos n l by 2 minus B sin n l by 2 is equals to 0, or we can simply calculate the A which is equals to B times into n l by 2. And you see here we already calculated the value of B which is eccentricity e is there. So, we have the coefficient A value is e times tan of n l by 2 So, you see here what we have? We have the complete solution in terms of y is equals to A sin n x plus B cos n x. So, if we are keeping those values A and B, what we have? We have the complete generalized solution of the equation in which you see the pin joints are there, but the eccentricity is there. So, eccentricity is also playing in key role in the generalized solution as you can see the equation y is equals to e times.

So, you see here this e is present exactly in the basic term of the solution. So, you see how great influence of this e is on the generalized solution, and you see generally in the Euler theory we simply ignore this term. So, that is why you see you know like whatever the buckling load is coming and the failure is there that is a great difference, because we were ignoring this e in that case. But with the consideration of e, now we are very much close to the real nature of the failure. So, that we whatever the prediction there about the failure theory due to the buckling it is pretty accurate. So, y equals to e times tan into n l by 2 sin n x plus cos n x.

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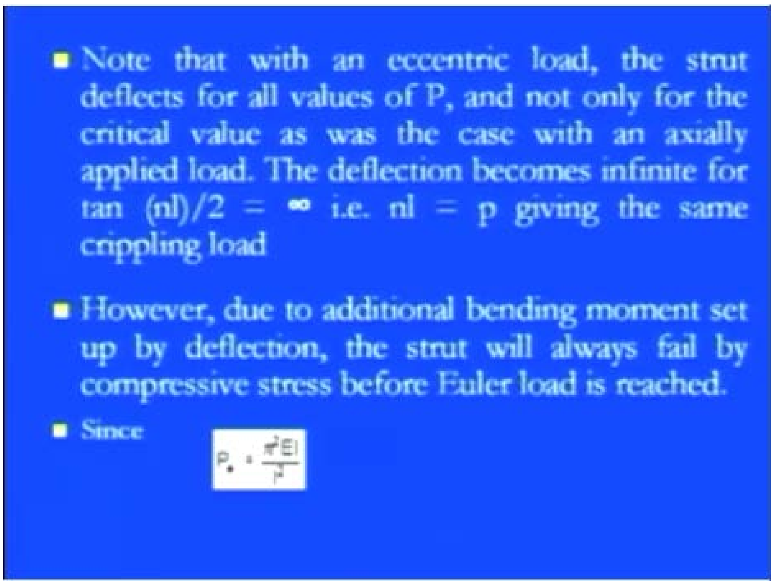

Now you see here we have you know like the complete equation or complete solution is there of that. Note that with an eccentric load the strut deflects for all values of P not only for the critical value as you know like the case with the axially load applied. That means you see the deflection becomes infinite for tan n l by 2 which is equals to infinite or we can say that n l is equals to P giving the same this crippling load. So, you see here means it is valid for almost you see tan times tan into n l by 2 to the infinite part. So, if you are talking about a general case then we will find that you know like this eccentric load, under the eccentric load the strut will deflect not only for you know like the critical load values but also for variety of the values.

However, due to the additional bending moment setup for you know like the deflection, because you see there is always due to the eccentricity we have an additional bending moment is there, and we need to consider in order to calculate the real value of this buckling load. So, the additional moment is being add up, and the strut will always fail by compression stresses before the Euler load is reached because of the additional part. So, you see here what we have? We have the crippling load of Euler P is equals to phi square E I by l square, and you see if you are calculating the P you know like for buckling load failure if P e then you see the value is quite maximum. But you see we need to add up something as we discussed just now see that the additional bending moment has to be added in this case.

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So, now come to the main point the generalized solution y was equals to E into tan into n l by 2 sin n x plus cos n x or we can say that now we have the real values of this. So, we have you know like the maximum at x equals to l by 2 the deflection is there, because it is exactly at the centre position. So, we can say that e into tan n l by 2 sin n l by 2 plus cos n l by 2, or we can say that if we simply manipulate those things when we have sin square n l by 2 plus cos square n l by 2 divided by cos of n l by 2. Simply you know like we need to change this tan by sin over cos, and that is why you see sin square plus cos square that is 1.

So, we have the maximum deflection which is exactly at the midpoint of that particular strut X equals to l by 2; we have the value 1 by cos of you see you know like n l by 2, or we can say that e times of sec of n l by 2. So, the maximum bending moment in this case would be exactly equals to you know like a maximum which is you see due to the axial load P in to y maximum, or we can say that now since we know the y maximum which is exactly at the centre point of the strut. So, we can simply replace in this particular value and we have the maximum bending moment in that case is P into e into sec of n l by 2.

So, now the maximum stress it can be easily obtained by combining those things with the direct strain. So, what we have because you see we know that there is an additional moment is there of this nature. So, we have you see sigma which is P by A due to you know like the stresses due in to the axial loading plus stresses due to the bending moment. So, the total combined stresses which is you know like in the nature of the maximal stresses are P by A plus M by Z. So, stress due to bending moment is pretty common that is sigma by y is equals to M by I or we can say M is nothing but equals to sigma into this I by y, or we can say that I by y you know we can simply replace by the section modulus of the elasticity.

So, we can simply replace sigma maximum is M by Z. So, by keeping you see you know like this M by Z we have the combined strut the stress is there, but we have a real feeling that whenever you know like strut is there under the action of these axial forces it can be you know like in the buckling load can be easily calculated with the using of two main impact. So, if we are talking about the stress then we have you see the two mainly component in that; one is due to the bending moment M by Z you see and one is due to the applied load is.

And that is why you see we have a clear feeling that you see the second term is coming because of the bending action is present in that, which we were ignoring in the basic form of the Euler theory. But in this case you see now we have you see you know like the bending action is there and the axial action is there in that case.

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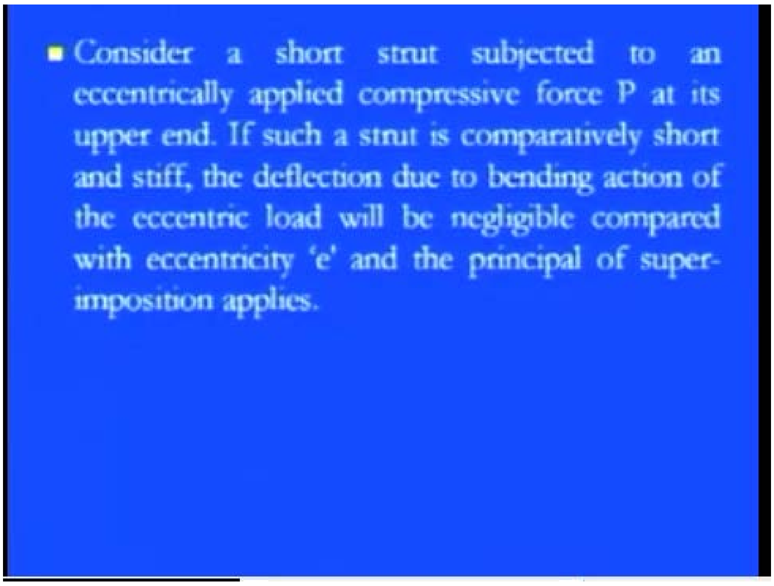

So, consider a short strut subjected to an eccentric applied compressive force P at its upper end as shown in the first diagram. If such a strut is you know like comparatively short and stiff, the deflection due to the bending action of eccentric load will be negligible as compared to the eccentric e, and you see the principle of super-imposition applies. That means you see here if the strut is pretty short and it is very stiff, then the bending moment due to this particular eccentric load having less impact. So, in that case you see you know like somewhat this Euler theory which we derived is comparatively, okay. But you see when we are talking about somewhat the column part; that means you see some what you know like even the short distance is there; the short length is there, but it is not much stiff.

Then obviously, you see there is kind of you know like the curvature part is there because of less stiff part. So, then we need to consider you know like the M by Z as an additional component, because the bending moment is always applicable because of the eccentricity. So, you see here you know like these theories are always somewhat having real feeling about what exactly is going on with the kind of strut or the column. But if the strut is assume to have a plane of symmetry; that means you see it is simply in there x y plane is there like that. And the load P lies in this particular plane like you see we are considering at a distance E, then you know like we need to consider this, the impact of this eccentricity and that is what you see exactly it is following the centroidal part of the axis o.

And then in such a case you see the loading may be replaced by its statically this equivalent centrally applied compressive load which we can simply show in the next diagram, because you see you know like we know that since this load is exactly not at the centered part but somewhat you see it is displaced by point e. So, it has there are two main impacts are there. So, you can see in this diagram.

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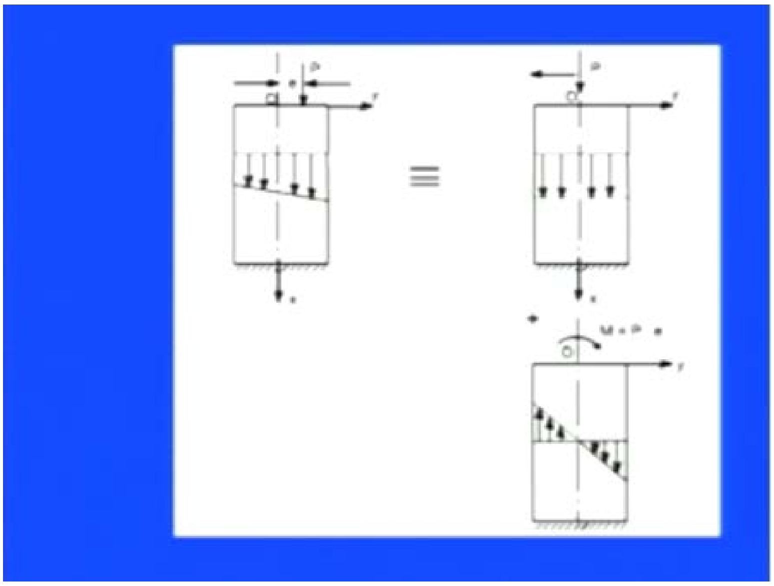

This you see you know like we have the structure simply you know like this is there, and it is simply flowing along the x direction; simply this is the rigid part is there and this is the centroid you see. We can see this particular central line or which the centroid is there. So, instead of keeping the load at centroid let us say it is it is being displaced by e as I told you. So, it can be replaced by two main things; one you see the load is there. So, you shift the load at the centre point. So, like you see we need to shift this P load at this particular centre point.

So, this is the centre point here; along with that we have the moment which is coming due to this eccentricity, and this moment M is nothing but equals to P into e and this has a nature in the clockwise direction. That means you see this moment is always tending to move this particular element in a clockwise direction. So, you know like we have the two main components are there if eccentricity load is there; one is the axial load that is P which we are considering, and assuming that actually it is now on the centroid part. And second is there due to eccentricity we have this bending moment which has the magnitude P into E.

And now this bending moment will tend to move this object in a clockwise direction and then you can see that you see the nature, what the tendency is there. So, if you look at this particular only axial load then only it is just going or we can say it is simply flowing with the flow of this axial load, but while if you check it out the bending moment. Now we can see that at this circumferences we have the maximum value you know like because the tendency is just to move the circumference of this particular way. So, if you combine those things then the eccentric loading will give you the real feeling of the load is like that with the straight and having tendency at the maximum of the circumference. So, this is you see the real feeling of the eccentric loading is.

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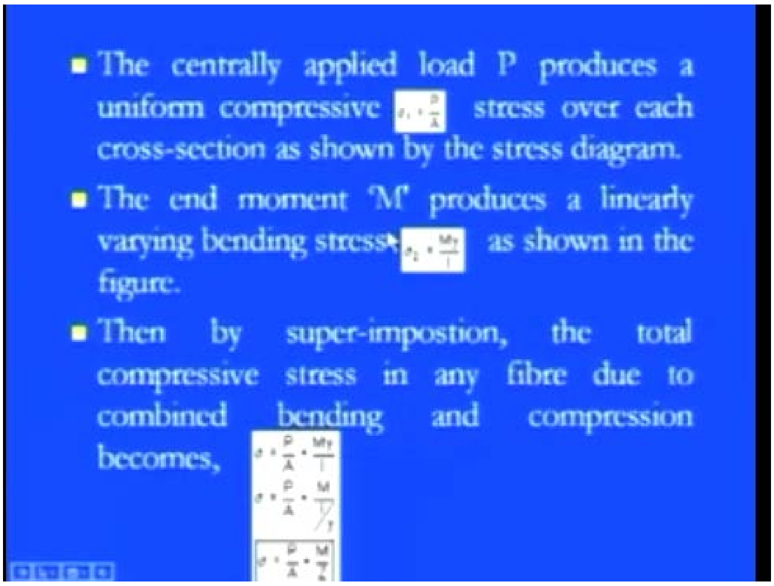

And that is why you see you know like the centrally loaded applied load P produces the uniform compression as I have shown you the sigma 1 which is P by A stress over which each cross section has shown you know like having the same stress diagram is there with the uniform way. While you see the end moment this bending moment is there M produces the linear varying bending stresses sigma 2 which is M y by I or we can say M by Z as shown in the other figure you see. And if you combine those things then we have you know like the superimposition figure is there, the total compressive stresses in any of the fiber due to the combined loading bending as well as the compression; we have you see sigma is equals to P by A plus M y by I, or we can say that sigma is equals to P by A plus M over one this I by y, or we can say sigma, which is exactly you know like the real value is there which is P by A plus M by Z.

So, this is you see you know like the real procedure to calculate when you know like the eccentric loading is there in the strut and it is simply applied along the axis, but keeping some eccentric part and due to that you see we have an additional component of the bending. And that is what you see you know like in the particular strut part always when we are going for experimentation; it is always fill before the Euler crippling load because of this object was missing in the Euler part. So, you see here in that particular theory this is all you know like we discussed about the strut and everything you see now. But we found that actually there are some of the key parameters which were you know like absolutely missing and which has to be focused.

So, that is what you see now we pick up some of the part here and that is what it is starting with the energy methods. In that you see we have variety of categories, and we would like to see that what are the important you know like the energy parts are there or the terminology is there which is very much closely associated with the kind of material property. And how they have an important influence for designing you know like these materials when they are subjected by a force application.

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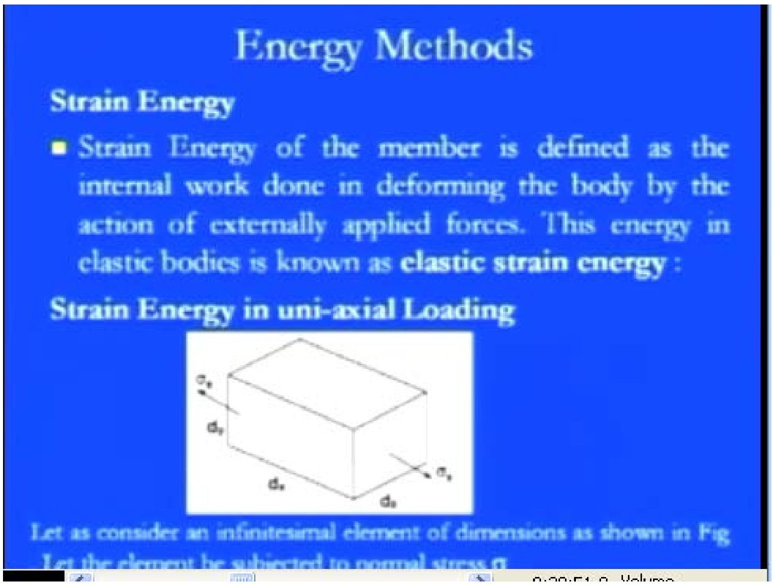

So, starting with the strain energy we have you know like the strain energy of a member is defined as the internal work done in the deforming body by action of externally applied forces. So, when you see here you know like the load application is there or point of the application of the force is there, we need to see that actually what the kind of deformation is and whenever the deformation is there; that means you see when there is an external excitation is there the energy is being transferring so where is this energy going.

So, the energy is absolutely absorbing by these objects, and this energy in the elastic body is known as the elastic strain energy. So, if you look at this particular uniaxial loading part we have you know like a kind of cube. So, you can see that there is you know kind of you see the sigma x is there; that means the axial forces are there in the x direction. So, you know like the stress component is there along this particular x direction in a tensile way. So, sigma x and sigma x is going in this way, sigma y sigma z and you know like all these small small segments are there. And if you are you know like going for an infinite decimal element of a dimensions of dx, dy, dz, then our main focus is just look at the sigma x.

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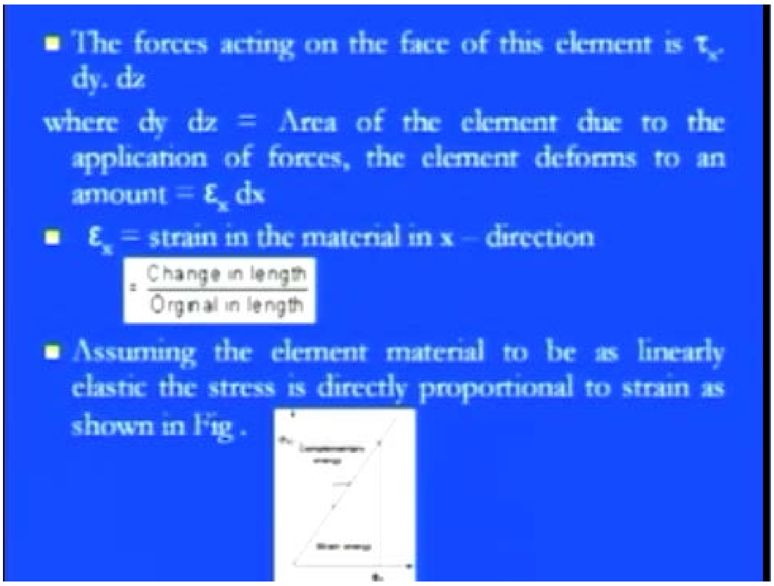

So, you see here this is now you know like the forces which are acting on the face of this element is tau x dy you know like because the stresses are just in that particular this x axis form. So, we have you see the different kind of forces tau x into dy into dz, because you see the domain of the force in the x only. So, the area is dy dz, and you see the shear stress is there which is the plane stress. So obviously, the shear force which is there on the x phase is this tau x into dy into dz, where dy dz is the area of the element due to the application of forces and element deforms to an amount is this you see whenever the deformation is there under the action of any force is epsilon x into dx.

So, this is you see you know like the deformation is there in that epsilon is nothing but the strain in the material in the x direction. So obviously, it is nothing but equals to the ratio of change in length divided by the original length. So, now what we have? We have elastic deformation, and in this elastic deformation now you see here when the load application is there we have certain kind of deformation. So, that means you see when an applied load is there in x direction the kind of deformation is there. So, we can straightway define the strain energy in this way. That means you see you know like the strain energy is nothing but equals to half of sigma x into epsilon x.

So, sigma x is coming as the natural you know like the normal strain component the stress component due to the action of force, and due to the action of force the deformation is there which is to be computed in terms of the strain. So, epsilon x is there and both are equally responsible for the kind of strain energy in an object. Now assuming the element material to be linearly elastic; so you see here all these theories are pretty close to that what exactly you are assuming. If you are assuming that the load is only applied up to the elastic deformation; that means you see whatever the relations are there in between stress strain or you see what are the other terms are there like Young’s modulus of elasticity, shear modulus of rigidity or whatever you see, they are absolutely valid.

And in that case also we have linear elastic you know like the stresses are there which has you see you know like they are directly proportional terms are there with the stress strain. And you can see here with that particular figure that we have you see the stress strain diagram and in that there are two main types of energies are. One the downward part of the straight line; that is the strain energy and another one is there in the recovery form is there; means on the top of side in which you see you know like we have the base of sigma and the domain; the domain is sigma end, the base is tau. So, in that case you see what we have; we have the complimentary strain energy is there in that case. So, you see here both. Both are there, but the part of you see you know like the lower part is strain energy and the upper part is the complimentary energy. We can also say that the complimentary strain energy in that case.

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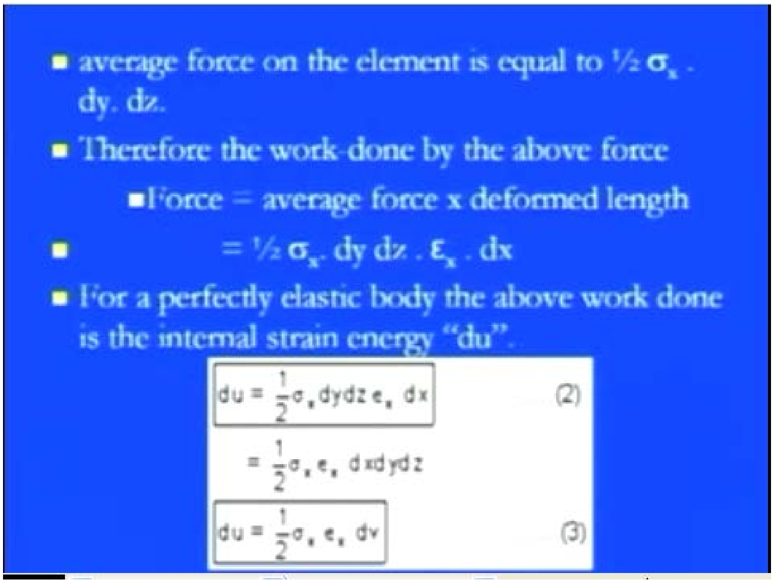

So, average force on an element is, obviously, you see since if we are taking about the average because we are assuming that again with the very important aspect is we are assuming that under the action of the force, stress distribution is uniform along the material. So, hence we can say that the average force on the element is half of sigma x into dy into dz, Therefore, you see the work done on above force is you know like this; obviously, it is always coming due to the average force into the deformed length. So, half into sigma x dy dz into epsilon x into dx. So, now you see you know like for a perfectly elastic body the above work done is you know like the internal strain energy.

So, du can be easily calculated because whatever you know like the work done is there, this work is being done by the application of force, and the equal amount of energy is being stored within the element. And that too you see we are always considering that it is elastic body. So, we can simply replace this work done by this internal strain energy du. So, du is equals to average force into the deformed length or we can say that half of sigma x into dy into dz. So, this is you see the force and we have the deformed length that is epsilon x into dx. So, n combining those terms we have half of sigma x epsilon x into the whole unit volume of this element. So, we have dx, dy and dz.

And if you are talking about the du then we have half of sigma x epsilon x into dv, or we can say that the strain energy per unit volume is nothing but equals to sigma x epsilon x by 2. That is what I told you; that if you are talking about an elastic body then the stress sigma x and epsilon x strain is equally responsible for the kind of strain energy in storage. So, you see here and since you see the strain energy is having the domain of volume. So, that is what you see the du is always calculated as half of sigma x and epsilon x into the dv that what exactly the volume is there, and how it is being deformed under the action of applied load.

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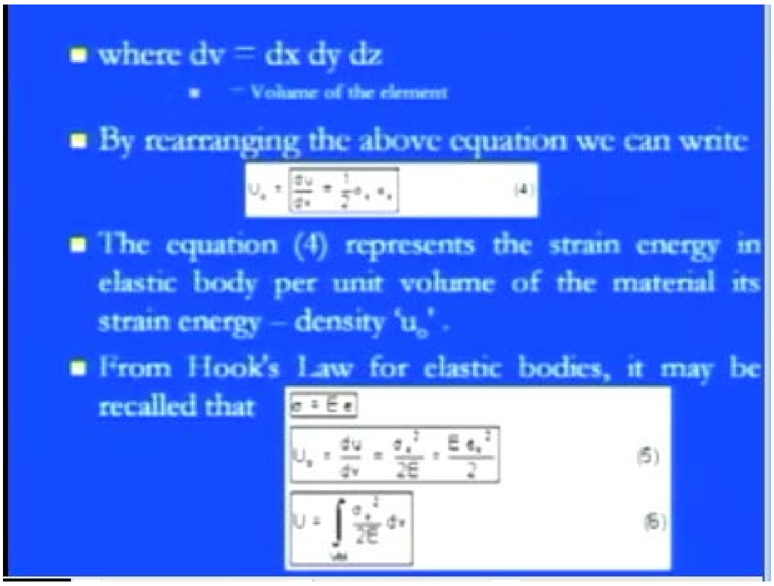

Where you see dv is nothing but equals to dx dy and dz which is you see the volume of the element. So, by rearranging those terms, what we have? We have you know like the u 0 or we can say this du by dv is nothing but equals to half of sigma x and epsilon x. So, this equation represents the strain energy in the elastic body per unit volume of the material. And you know like its strain energy density u 0, but that means you see you know like when we are you know like saying that it is per unit volume is there. So, we can say that it is strain energy density that is u 0 is there. And since we are considering only the elastic body so obviously, whatever the deformation is there, this is absolutely under the Hooke's law.

So, for Hooke's law the elastic body it can be simply sigma equals to E into epsilon where E is the young's modulus of elasticity's is there. So, with that consideration now and since you see you know like there is a linear relationship is there in between the stress and strain, and you see you know like it is directly proportional is there. So, we can simply replace this equation in that particular form. So, what we have? We have u 0 which is du by dv. It can simply replaced by sigma x square divided by 2 E or we can say that since the sigma is nothing but equals to E into epsilon as we can see in this previous derivation. So, we can simply replace E into epsilon square by 2.

So, you see here now since it is you know like the elemental part is there. So, if you consider whole body. So, you need to simply you know like integrate that part whole small small segments are there if you integrate in that particular same domain. So, we have you see the internal energy is equals to the strain energy within that internal part is equals to the integration of whole volume into sigma x square divided by 2 E in to dv. So, within the volume domain you can get the total energy by summing up or by integrating those terms.

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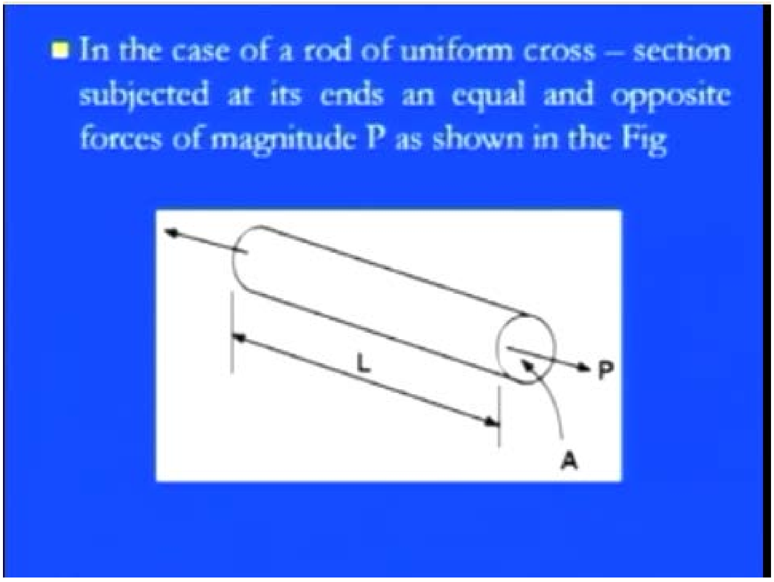

So, in the case of a rod of uniform cross section subjected to its end and equal and opposite forces is being there. So, now you see here after you know like going through from this particular internal energy, we have clear feeling that whatever the load application is there like you see in that particular diagram in front of you, we have equal and opposite load is there. So, we know the P; once you know the P and you know the cross sectional area that is you see the A is there, what you have? You have a straight you know like this stress components are there you know the material properties. So, you have the value of E.

So, sigma x square by 2 E and then you see you know what the domain is there of this particular element; that means if it is cylinder, if it is you see the cross sectional area is square or any rectangular; simply take the volume of that and calculate what exactly the strain energy is there of whole element.

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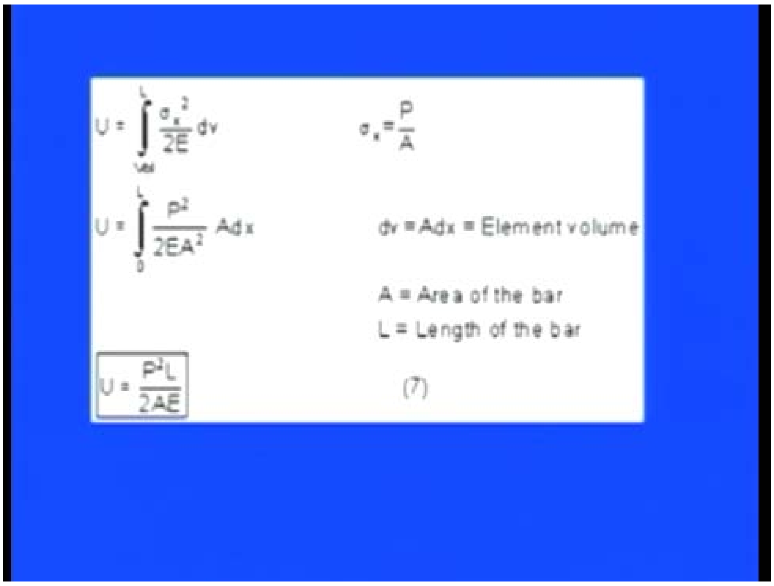

So, you see here for this particular diagram. Now we can simply calculate the strain energy as this U is equals to integration of this volume with this particular L sigma x square by 2 E into dv or we can say that, since if we absolutely replace from that particular area part, because you see in this particular cylindrical part it is pretty easy to calculate the area. And since area in to dx will be the volume. So, you see here the U which is 0 to L this sigma x square. So, sigma x is now replacing with the force because this axial stress is coming due to the application of force. So, this sigma x is now P by A. So, if you are replacing this sigma x by P is equals to P by A, we have P square by 2 E into A square into now dv; dv since we have a cylindrical element.

So obviously, it can be easily replaced by area into dx, because it is simply you know like varying the length in the x only distance. So, we have dv is equal to A in to dx which is the elemental volume. So, we have you see P square divided by 2 E into A square intoA times of dx. So, with the manipulation, what we have? We have the strain energy for that simple you know like the cylindrical element in which the axial forces are there; we have P square L over two times of A into E. So, if you look at this particular point then you will find that two or three you know like the parameters are really you know like for the importance to calculate the strain energy.

One is that material property; what exactly material which you are using, because you see whatever the load application is there if material is more stiff or material is less stiff, accordingly the deformation is there and then you see we can say that, yeah, it can be deformed like that. So, it may be having you see the more and more the strain energy is there in the elastic form. Second property is the area that what exactly the dimensional part is there; means strictly what exactly you know like the cross sectional area is there, correspondingly, you see the energy storages are there in the fibers of the layers.

And third which is more important part is directly related to the strain energy is the load. Maximum load is there, maximum you see the deformation is there, and there is a great chance of having the strain energy storage is. So, that is what you see. If you are checking out the U then U is you know like equals to P square. So, it is in the terms of the P square is there. So, these are you see some of you know like the main parameters are there through which we can say that we can simply calculate this strain energy if we have any kind of element you see simply you know like we need to calculate the area. We need to check it out what the material is there corresponding Young's modulus of elasticity's is there.

And just you know like we know the length, and just the load according to the varying load we have the different elastic energy, or we can say the strain energy in the elastic form. Okay, so in the first part of that we discussed about the strain energy in which we found that you know like these are the key parameters to calculate the strain energy of any of the element irrespective of whether it is a size or material or whatever. Now you see here the second part is the modulus of resilience.

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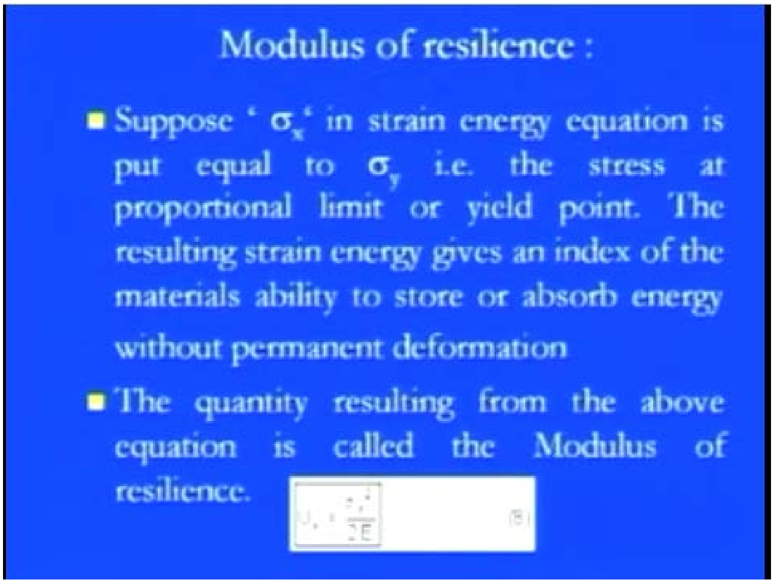

Now suppose let us say if we have the sigma x in the strain energy equation is put equal to sigma y; that means you know like stress at proportional limit or yield limit is there. So, that yield point is coming now, and this stresses approaches up to that point which you know like clearly finds the limit of elastic and the plastic region. The resulting strain energy gives an index of material ability to store or absorb energy without permanent deformation; that means you see now we are approaching up to the maximum value of you know like the strain part up to the elastic limit. If you go beyond that there is a certain chance is there of the kind of elastic deformation.

So, up to that point whatever the stored energy is there or what are the absorbed energy is there, that is you know like is considerable. So, the quantity which is you know like coming up to you know like this yield point resulting from the above equation is called the modulus of resilience. So, you see here this limits this absorb energy or we can say the stored energy limits the elastic region and the plastic region So, we can say that actually if the stored energy is up to the particular point we have an elastic region. So, with that you see now the different form of energy, which is known as the modulus of resilience always shows that the stress is there up to the elastic limit.

And this you see can easily calculated d U y is equals to sigma y square divided by 2 E; that means you see the stress up to the yield point sigma y divided by 2 E. And then you see this energy absolutely depends on how much force application is there up to the yield limit and what is the elastic property is. So, now you see if you want to draw these things we can simply limits that particular part.

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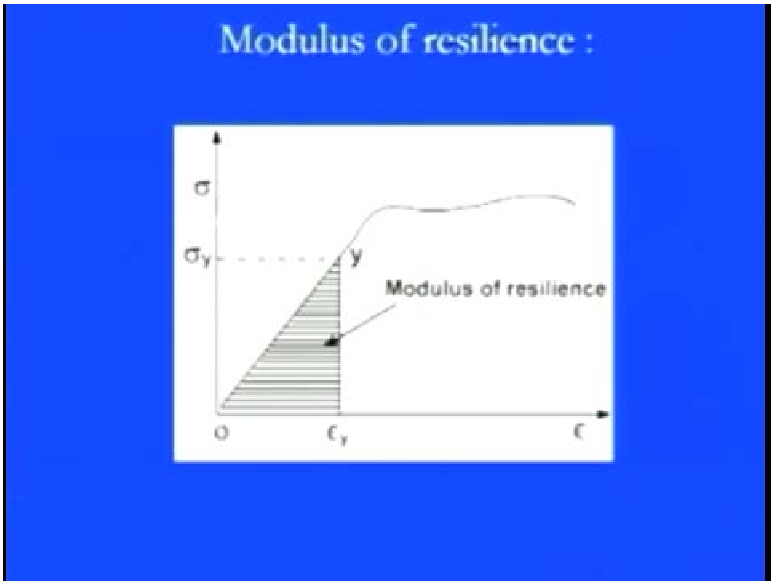

That we have a simple tensile test you know like diagram is there the stress strain diagram, and you know like one of the key features is there in that; you always draw the stress and strain in this way. Can you tell me why you see the stress is always being drawn on the y axis and strain is there on the x axis? The basic reason is the strain is always a measurable part and the controlled part. Let us say you see if you want to check it out you know like we apply the load. And we just want to check it out that how much stress you know like induction is there if we simply go up to the deformation of 0.1 millimeter or 0.2 millimeter or you know like 0.1 micrometer; we can simply check it out. So, it is a measurable part, and it is a controllable part.

So, that is what you see whatever just like you see the time displacement is there or time velocity is there or the time acceleration is there; always time is coming on the x axis. So, which domain you want to analyze those things? So, here also our domain is how much deformation is there, what is our parameter is there of the deformation is? So, that is what you see here on the x axis or on the x domain, always the controlled and the measurable part is coming while the dependent part is always coming on the y axis. So, similarly you see you know like come to this point again; we have this modulus of resilience is there.

So, in that case a simple tensile test part is there and you see now we just you know like it is a very straight diagram is there that you know like when you start the loading it is the elastic deformation. And then you see the yield point is coming upper and lower yield point is there and then the non-linear relation starts forming. So, in the non-linear relations now the sigma which is now directly proportional to the epsilon; that means the stress is not directly proportional to the strain. Then you see all those you see you know like the maximum tensile strength is coming and the failure is there up to the end.

But we are talking about you see when it limits the yield the elastic deformation region. So, this is the yield point is there. So, up to this if we apply the load and you see this sigma approaches to sigma y here. You see whatever the area under the curve; that means you see what are the energy is there being absorbed by the material under this particular load application, this shaded area is showing the modulus of resilience. So, you see this modulus of resilience limits the elastic region and then you see we can say it limits the linear strain region is there with the linear stress part. So, this is you know like second term is there. Now the third terms of the energy are the modulus of toughness.

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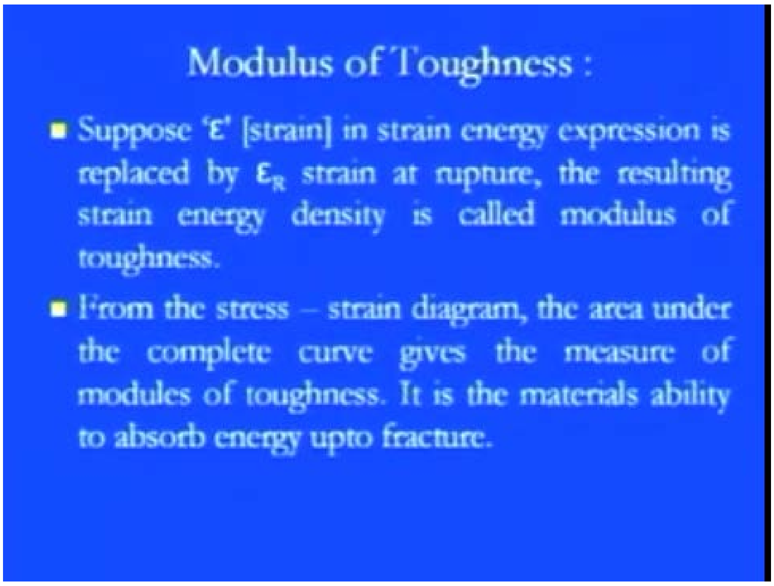

Now suppose you see the strain, the epsilon in the strain energy expression is replaced by epsilon R; that means you see the strain at rupture or the failure. That means now we are talking about the nonlinear relationship between the stress and strain, and this strain is now you see replaced by the strain at the fracture or the rupture. The resulting strain energy density is called the fracture modulus of toughness. So, now this is the region is slightly shifted; earlier you see you know like the modulus of resilience, we simply keep the region within the yield point only from the starting point.

Now, we are going up to the rupture point; that means you see it includes both elastic as well as the plastic deformation So, from the stress strain diagram which I shown you in the previous one, the area under the complete curve gives you the measure of modulus of toughness that how tough material is, and it is the material ability to absorb the energy up to the fracture. So, this is the key difference between you see you know like this from this modulus of resilience and the modulus of toughness. So, you see here from all these terms you know like, what we have concluded? We simply concluded that even if you are talking about the strain energy, we need to start from the basic element, integrate those things and we can get that part.

And then another you see we can simply segregate the total curve of the stress strain diagram simply by two term. When we are taking about you know like from this starting point to the yield point means the elastic region, the modulus of resilience is there; whatever the energy is being absorbed up to the load in the elastic region. But if you are going up to the field whatever the energy is being absorbed by the material, this energy is being known as the modulus of toughness.

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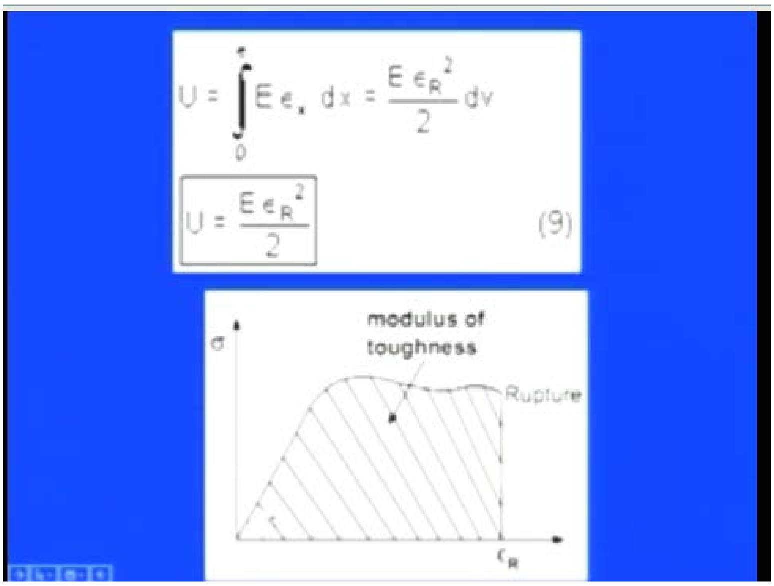

And then you see in this diagram you can simply see that this is the strain diagram. This is the elastic region you see the directly proportional region is straight line region is there; we can say the linear region. And this is the nonlinear region, and up to this you see we have the rupture. So, you see whatever the energy which is being absorbed under that, this will give you the modulus of you know like the toughness and if you want to calculate up to that and we have you see U which is you know like integration of 0 to E. So, this E into this epsilon x into dx or we can say that since now this epsilon x is being replaced by epsilon R.

So, E into epsilon R square divided by 2 into dv the whole volume now because up to the failure it is there. So, we have U which is you see the modulus of toughness is equal to E into epsilon R square divided by 2. So, that is what you see here. Again in the modulus of you know like the toughness, the material property is very very important the E the Young's Modulus of elasticity is there. And the second is there that how much deformation is there at the rupture epsilon R. So, the strain part is there at the epsilon R is there at the rupture point. So, you see here these are the two key parameters for calculating the modulus of toughness you know like. And then you see you will find that these two main parameters which are really having a great influence of these things.

So, you see this is the another reasons of the modulus of toughness and the first one was there the modulus of resilience. So, you see these three parts which we discussed right from you know like strain energy, the modulus of resilience and the modulus of this toughness we simply categorize these energies as that how much you know like the energies are being associated according to the application of load, and what exactly we want? We want only the elastic deformation; we want to plastic deformation. And we just to want to see a kind of deformation, and how much load is applied is there and what the total you see you know like the stored energy is there within the particular volume domain that will give the strain energy. So, these three basic forms of the energy you know like were there, and we discussed all these parts. Come to now this some of the numerical problems.

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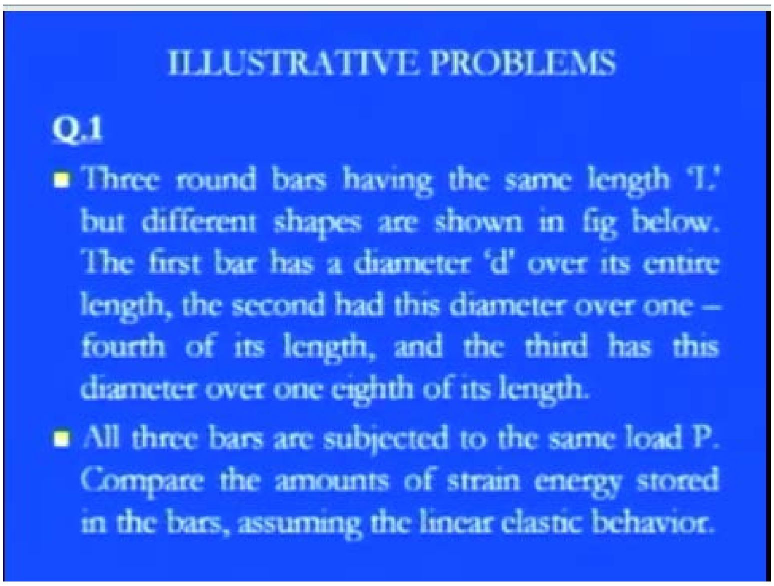

So, the first numerical problem we have taken up and you see this problem is we have you see the three rounded bars having the same length. So, you see all three bars are having the same length L, but the only key change is they have the different shapes. I am going to show you in the next slide the first bar has a diameter d or its entire length. So, you see it has you know like the uniform cross sectional diameter is there. And then you see simply the diameter d is there. The second head you know like its diameter over one-fourth of its length, and the third one is the diameter over one-eighth of its length.

So, that is what you see now they have given the same relation with the diameter and the length, and you will find that they have the different kinds of shape with the length and diameter ratio. All three bars are subjected to the same load p; that means you see the external excitation is the same. Compare the amount of strain energy stored in the bars assuming that we have simply applied the load within the linear elastic region only. So, that is what you see in that they are behaving as you know like linearly elastic one.

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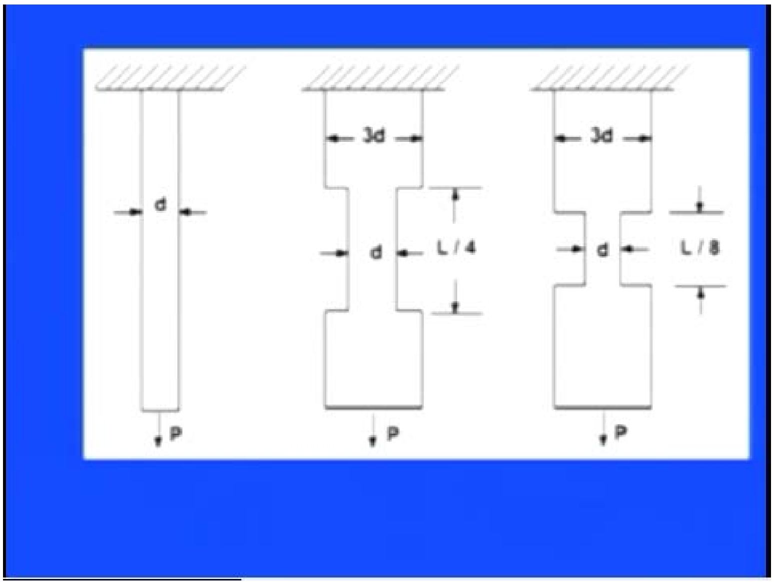

So, the same diagram you see we have all three bars; one is the straight cross uniformly cross section is there load application P, and this is the diameter d is there. And then you see again this is the second one which has the same length; there is no change in the length and the applied load is P, but there is you see you know like the dip is there in between that. So, the first diameter is 3d, last diameter is 3d, but in in between you see the one-fourth of length is having diameter d only. Then you see we have one-eighth of length is just diameter d, but other parts are you know like thrice of that. So, 3d and 3d is there in the remaining two regions. So, you see if you compare we will find that this is the entire length L is having the diameter d, one-fourth of this length is diameter d, and one-eighth of this length which is having the diameter d. Then you see here now what we need to do?

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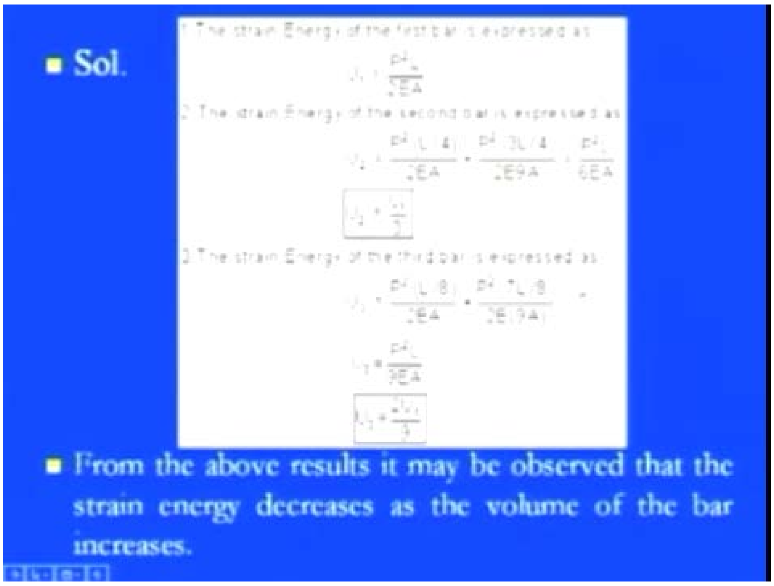

We need to simply go with the same because we want to calculate you know like the energy. So, the total strain energy for the first bar is pretty simple, because you see we have straight relation. So, U 1 is P square L by 2 E A. As I told you see you know like this strain energy absolutely depends on what the load application P square is there and E into A. So, this is very straight you see simply by keeping the values you can have the strain energy of that bar. But the second bar is very important because you see it has a different altogether you see the regions.

So, the strain energy in the second bar can be simply expressed by the two different ways. One is with the diameter in which the d diameter is there. So, that length is L by 4, and another one is in which the diameter is you know like we have the 3d. So, you see the corresponding area will be changed, and the total length of that is 3 L by 4, because you see 1 L by 4 is going in to the d diameter 3 L by 4 will be there with the 3d diameter. So, U 2 is first for L by 4 this length in which the diameter is d; we have this P square L by 42 in two E into A.

So, A is say let us you see you know like it is a simple uniformly cross section bar. So, pi by 4 d square is there. But now you see here in the other term we have you know like this 3d diameter is there. So obviously, the area will be coming up with the nine times of the previous one, and the total length is 3 L by 4. So, we have P square 3 L by 4 in to you know like divided by 2 E nine times of A. So, that is what you see you know like if you compute those things then we have the total P square L by 6 times of E A. So, B is that you see if you know like simply increase the diameter, the strain energy storage is one-third of the previous one.

So, you see it has a clear meaning that even if you increase the diameter of the bar under the same load application, the deformation is quite less and then the stored energy is somewhat you see you know like one-third of the first one. Now come to the third case in which you see you know like only the one-eighth of area is having diameter d and the other diameter is being increased by 3 d. So, that means you see one-eighth is there and the seven by eighth this you know like the diameter or the length is now having 3 d diameter. So, come to the total strain energy for that particular region, what we have? We have U 3 which is P square of P square divided by 2 E A in to L by 8.

So, this is the one-eighth area where the diameter d is there. And now you see P square seven times of L by A divided by 2 E. Now since the diameter is 3 d. So, we have the nine times of A. Compute all that part, what we have? We have U 3 which is equals to P square L by 9 times of E A. So, U 3 is nothing but equals to 2 by 9 U 1. So, you see here means if you again you know like reduce this particular length with this particular you know like diametric changing, we will find that the number of you know like with the increasing of the diameter, the total stored energy within that element under the same load application is somewhat reduced by 2 by 9.

So, you see here what the great influence is. Initially you see it is being you know like just reduces by 33 percent, then now it is being reduced by 22 percent. So, you see here when you know like we are increasing or decreasing the diameter, what the great influence is there of the stored energy or the strain energy under the same load; application this is of a great importance that how stiff that part is. So, from the above result it may be absorbed that the strain energy decreases as the volume of bar increases.

And then you see you know like if we want to like the significance of this strain energy accordingly we will take the cross sectional part of these elements. So, this is you see the difference between a uniformly cross section bar and the non-uniformly cross section of bar.

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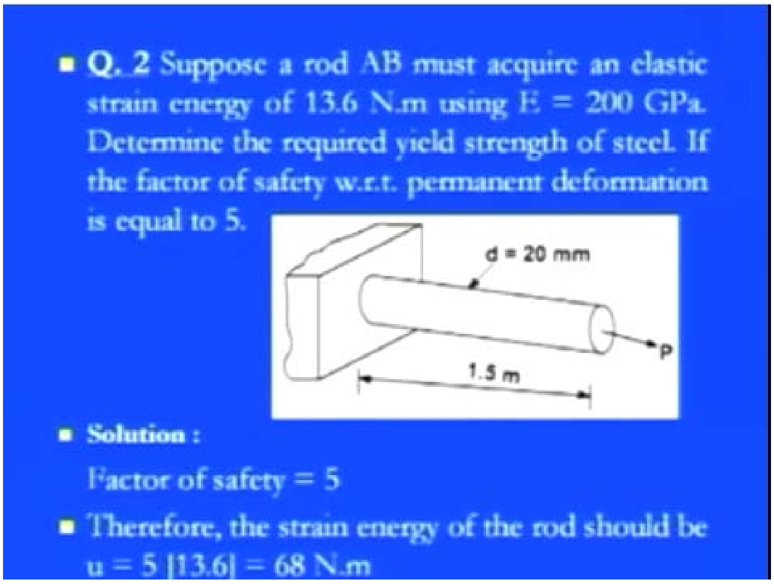

Now we are going with the second numerical problem. Suppose we have a rod AB you see this rod AB is there, and it must acquire you see elastic strain energy of you know like 13.6 Newton meter, with the using of Young's modulus of elasticity 200 Giga Pascal. Now we need to determine the yield strength of the steel if you see the fracture just you know like we are going up to the rupture or fracture up to that and in that the factor of safety which we are considering under the permanent deformation, the fracture which I am talking about is equal to 5.

So, we need take the factor of safety 5 to avoid the rupture or the fracture of this failure part. So, now you see we are going with that. Now since the factor of safety is given already 5, and then you see now we would like to calculate to see you know like that what exactly the yield strength is for which the strain energy is given as 13.6. So, you see here now the strain energy of the rod; it should be now this U which is equal to five times of 13.6. That means you see to avoid the failure with the using of this much factor of safety, we should have the strain energy 68 Newton meter, because you see this factor of safety factor is always given. So, that is what you see we can absorb the energy or we can store the energy up to 68 Newton meter to avoid the failure.

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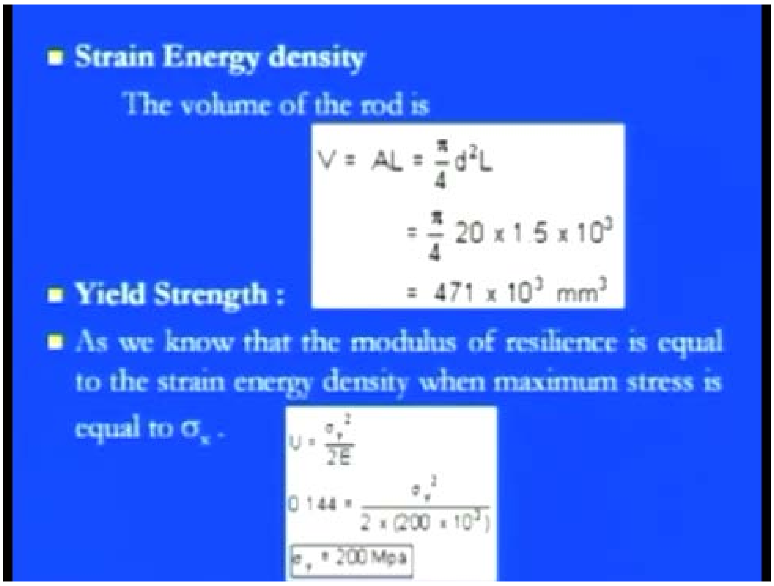

So, strain energy density you see you know like for with that particular basis now we have the volume of the rod is V which is A into L or we can say that pi by 4 d square L. So, now you see we know that the diameter is given already, and the length is given. So, by keeping those things we have pi by 4 in to 20 into 1.5 into 10 to the power cube, because you see you know like we have it is in the meter. So, now after converting those things we have 471 into 1000 millimeter cube. Our main aim is to calculate you see yield strength. So, as we know that the modulus of resilience is equal, because you now we are keeping up to only the elastic regions. So, for that whatever the energy is coming, it is coming in terms of the modulus of resilience.

So, it is equal to the strain energy density when the maximum stress is reaching up to sigma y P. So, U is equals to sigma y square divided by 2 e. We can say that since U, which we have already calculated in that particular term is 0.144 is equals to sigma y square divided by 2 into 200. The E is given as 200 Giga Pascal. So, 200 into 10 to the power cube. So, after calculating that part we have sigma y which is equals to 200 mega Pascal. So, we have now the yield strength which is being calculated on the basis of modulus of resilience, and it is coming as 200 mega Pascal.

So, now you see we would like to conclude this chapter you know like we started with the strut which is having you see the pin joint and consideration of the eccentricity. So, you know like we go to know that eccentricity is a key phenomenon which we were ignoring at the time of Euler theory. So, that is what you see after consideration of that part we found that actually we are very pretty close to the real experimental values. And they are exactly you know like somewhat matching, because here also we are considering the additional bending moment is there and due to that certain kinds of curvature part is.

So, we started with that part, and then we found that some of the energy associations are there when the material is being subjected by in a kind of loading. So, we started with the strain energy that what exactly the impact of the strain energy is there when the load application is. And the second part was coming as the modulus of resilience when you see the load application is there, and we are only keeping up to the elastic region. So, this modulus of resilience will give you the energy absorption by the material and the load application is there up to elastic region. And whatever the energy is coming that will clearly define the limit of the elastic region.

And then you see the last part was there the modulus of toughness in which we are simply going up to the failure part. And whatever the total energy which is being absorbed till fracture or the rupture is always known as you know like the modulus of toughness, and it simply gives you that how much energy or material can be withstand up to you know like prior to failure. So, these three you know like the energy terms which we discussed in that particular way, but in that you see all these things are there under the simple action of loading.

And then you see we discussed lastly about the two numerical problems that you see and how we can calculate those part especially which is based on the strain energy and based on the modulus of resilience. Now in the next lecturer we would like first you see discuss about the strain energy you know like in the bending part; means you see we when you see the bending actions are there then what exactly the strain energies are there and what you know like how we can calculate those part. And then will discuss about the numerical problems associated with the strain energy.

Thank you.