Strength of Materials
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Lecture - 37

Hi, this is Dr. S. P. Harsha from mechanical and industrial department, IIT Roorkee. I am going to deliver my lecture 37 on the course of the strength of materials, and this course is developed under the national program on technological enhanced learning.

Prior to start you see this is the next lecture of the 37 you see I would like to briefly discuss about the previous one, because you see in this also we are going to use the same theories, which we discussed. Maybe in the previous lecture we discussed about the column and the strut that how we can difference it out you see the columns and the strut, and we could figure out that you see; there are two main criteria's through which we can you know like make a differentiation between those column and the strut.

One was the dimensional parameter, the length you see if it is short and thick, then it is to be at a column treated as, and if it is longer cylinder member and you see thin part is there then probably we can use this strut is there. Another which is very important phenomena is this buckling part. Means you see when you apply the load, and if you know like the buckling is there then we can say that it is a kind of strut part is there. So, that means, you see here this buckling is you know like is a key phenomena to differentiate out the column as well as the strut in between.

And then you see you know like we simply discussed about that that what are you know like the root cause for the buckling in a kind of strut part. Then we found out that there are two types of nonlinearities are there through which we can say that, yeah, the buckling can be happening. One is the material nonlinearity; that means you see you know like when a material is not uniformly distributed it is you see a kind of isotropic part is there, and you see if one end of this strut is you know like acted perfectly in another end you know like which is having a sort of more deflection. Then probably you see you know like the kind of buckling can be happening.

The second was geometric nonlinearity is there; that means you see you know like if this strut is not uniform, there are you see the step part is there or some irregularities are there in the straightness of you see the strut, then probably there is a good chance of having buckling. And the third one was there you see when the axial load is you know like due to the axial load you see, if they are not exactly on the same line; they are not exactly axial. Due to that there is any kind of an eccentricity is there, and there is a great chance of the buckling.

So, you see here these were the key phenomena's through which we can say that the buckling can be happening, and due to the buckling we can say that this is the kind of the strut. And then you see you know like we simply you know like impose certain conditions and then we derived the Euler theory of you know like the Euler theory is there for calculating the crippling load where the buckling can be happening. So, you know like analyze the three different cases. The first case was when we have a pin joint; that means you see you know like both ends are pin joint and the load is being applied there, and we are you know like assuming that it is in the x domain and whatever the deflections are coming it is in the y domain.

So, you see here you know like we simply set up the relation, and then we found that the crippling load where the buckling can happen which is known as the P e which is the Euler particular load is equals to pi square E I over L square. So, in that we see the sensitivity of the two main parameters as we discussed also that one was the E I that is the flexural rigidity and one was the length of this column or the strut. So, this was the case where you see the pin joint was there, and we found out you see if you go for the higher modes of the buckling. Then it is you see if you are going for the second mode then we have the two waveforms are there in the particular strut, and this buckling load is four times of this pi square E I over L square.

And if you are going for the third mode of buckling, then we have you see the crippling load is equals to the nine times of pi square E I over L square. So, all these you know like these modes can be easily calculated depends on what numbers of you see you know like these crippling loads are you know like being considered here. That means you see how many number of you know like the degree of freedoms are there for that kind of thing and corresponding you see you know like we have the value of this two, one, four, nine, sixteen, like that according to that pi, 2 pi, 3 pi, 4 pi like that; means, in which mode which you are you know like being to analyze the buckling load.

Next you see the case which we considered that when we have a rigidly you know like we have the strut, one end is rigidly fixed up and another end was free. So, you see we simply apply the free you know like at the free load the load is there; obviously, we have the maximum deflection at this particular load and which we you know like computed that actually if we are going up to a mean level. Then you know like the Euler crippling load for this kind of situation where one end is free, one end is rigidly fixed up, we have this P e is equals to pi square E I over four times of L square.

And then you see the third case which we discussed that both ends are rigidly fixed up; means you see you know like the rigid walls are there, in between we simply fix up the straight the strut, and then you see we simply apply the load. Then we found that the crippling load for this particular condition P e is equals to four times of pi square E I over L square. That means you see here if you compare the pin joint and if you compare the rigidly fixed up joint means you see the rigid ends and pin joint ends. Then you see the rigid ends it is always inducing the crippling load four times more than the load is there in the pin joints. So, now you see here we are going to consider in this particular chapter again we are carrying out the same kind of analysis, but the boundary conditions are different altogether.

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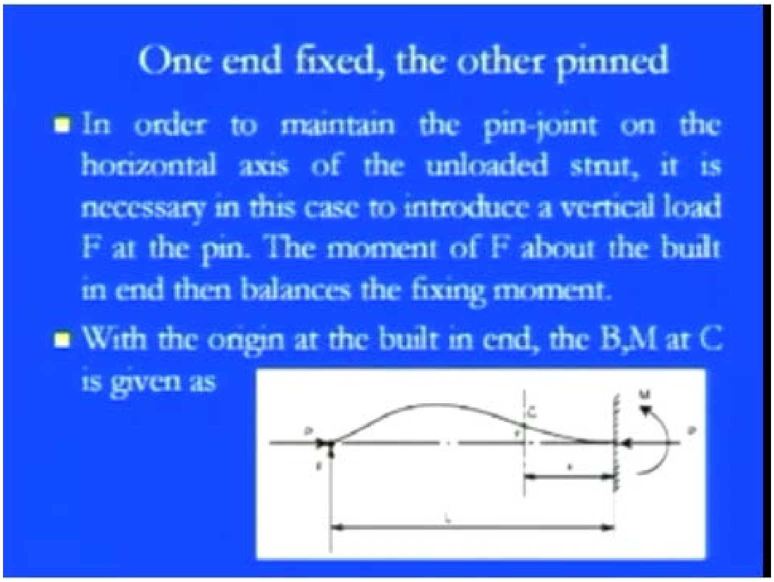

So, the first case you see where the one end is fixed and another end is pin joint; means you see here the combination says that at one end where the fixed end is there, there is a chance of the moment, and you see when the pin joint is there, there is no moment application is there due to the pin joint of that. So, in order to maintain the pin joint on horizontal axis you know like of unloaded strut it is necessary because you see here; obviously, you know like we have the pin joint. So, there is a kind of you know like the unbalance can be happening. So, we need to keep this unloaded strut is there; it is necessary in the case to introduce the vertical load F in the pin.

So, that you see you know like we know that at one end is fixed and one end is you know like this pin joint is there. So, definitely you see we have different kind of cases altogether here. So, the moment of F about you know like built in this particular case then balance and you see it is simply you know like going towards this fixing moment is there towards the fixed end. So, now you see we are always you know like taking the origin at you see the fixed end is there and by taking that particular part we know that the P load is applied at both the ends. So, see here these end these ends and the moment is applied at this particular part, since, we are taking the origin at this particular point.

So, the x distance is there where the C point is there, and we have the deflection in y direction as usual which is y and the total length, which we have taken here the L; only the main case is that here not only the axial load is there. But we have you see the vertical load is there just to you know like put the necessary conditions for unloaded strut, because you see one end is free. So, that is what here we are keeping these conditions. So, we can see the strut with these boundary conditions where the load applications are you know like the axis as well as the vertical part, we simply want to find out the equilibrium moment here. And we know that you see the moment can be there due to the external moment you see the M and there is also due to the force this F.

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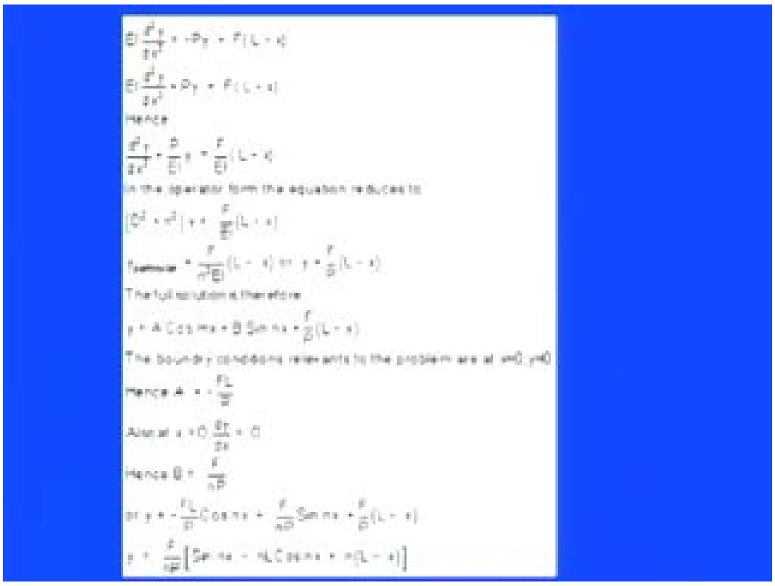

So, now you see the same standard this bending equation if you go, then we have E I into d 2 y by d x square is equals to minus of P into y because of the axial load is there, and we are considering at point C you know like the y deflection. So, minus P of y plus you see here since this up to point C that means the distance x; this is the main influencing parameter, and then you see there is also as you saw that in previous figure that the F was the vertical load was there. So, we have another moment due to this particular vertical load, but the effective distance is L minus x.

So, we can simply write the equation E into d 2 y by dx square is equals to minus of P into y plus F times of L minus x. So, if we rearrange that part then we have E into d 2 y by dx square plus P times of y is equals to F of L minus x, or we can say that you know like by simply dividing this E I we have d 2 y by dx square is equals to plus P over E I into y is equals to F over E I into L minus x. So, now you see here we want to convert you know the same procedure which we need to follow here that by converting this you know like the equation in the operational form, we have you see the equation D square plus n square into y is equals to F over E I into L minus x where you see the n square is nothing but equals to pretty common stuff P over E I.

So, now you see here what we have? We have simply you know like the particular solution y particular is nothing but equals to F over n square E I into L minus x or we can say that this y is nothing but equals to you see you know like since it is already being there this P. So, we have F over P into L minus x which is equals to y of particular integral. So, now you see we need to convert to get the full complete solution as in form of generalized way. So, y generalized or we can say the y complete solution is nothing but equals to A cos of n x plus B sin of n x plus this F over P into L minus x.

So, now you see we have the full solution, but again you see there are you know like constants are there A and B. So, with the application of the boundary conditions of you see this where the one end is pin joint, one end is fixed joint. Again you see we need to calculate the value of A and B by keeping those conditions. So, here you see the first boundary conditions you know like which is relevant to the first case where x equals to zero; that means the fixed end is there, we know that there is no deflection because of the rigid end.

So, by keeping that at x equals to zero y is equals to zero, we have the first coefficient A is coming out as minus F times of L divided by P. So, these two you know like vertical and horizontal forces are there with the corresponding distance of L. Now again you see again the same at fixed end we know that since there is no deflection, there is no slope also because of the free fixed end. So, by keeping X equals to zero dy by dx is equals to zero, we know that the value of P is coming out as F over this n times of P. So, you see here we have the value of A, you have value of B, and you have this y particular end this P I.

So, you see by keeping all those values here we have the complete solution y is equals to minus F L over P cos of n x plus F over n P sin of n x plus you see this y P I which is F or P L minus x. So, you see here in these the F over n P is common. So, you see here you can simply say that this F over n P, the complete solution y is F over n P sin of n x minus you see n into L, because the n will be going on the top up side; so n into L cos of n x plus n times of L minus x.

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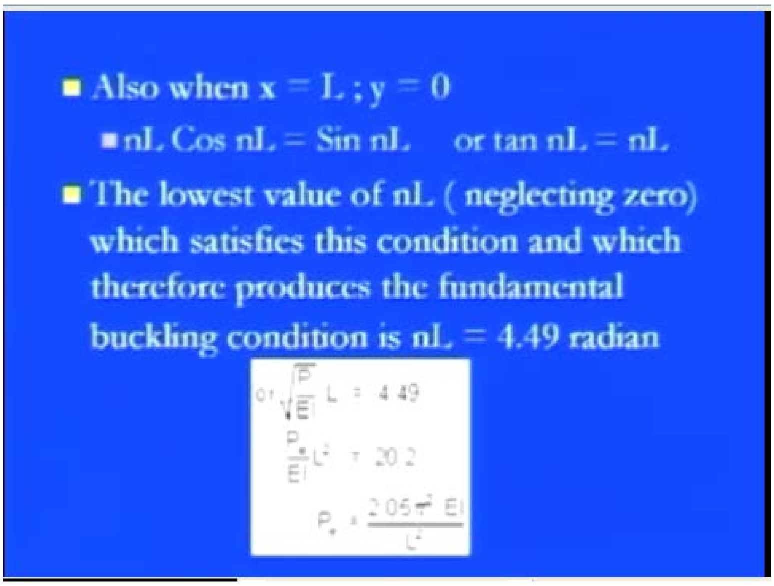

So, see here we have these particular conditions. So, now just by taking another condition where the pin joint is there we know that there is no deflection at the pin joint. So, at x equals to L y equals to zero. So, we have n L cos of n L you see the cos of n L because the x is now coming out as L. So, n into L or in to cos of n L is equals to sine of n L, or we can say tan of n L is equals to n L. And we can say that you see by keeping those you know like the values which are you know like equating, the lowest value of n L just you see we are not going with the 0.

But the lowest value of n L which satisfy this condition and which, therefore, produces the fundamental buckling conditions under the situation where one end is fixed and one end is pin joint is n L equals to 4.49 radian; just you see by taking inverse part and all those things. So, now you see here by replacing this part what we have and we know that n is nothing but equals to square root of P over E I into L we know you know like that actual the length of the total cylinder part you see the strut So, we have the square root of P over E I in to L is equals to 4.49 you know like, and then you see you know like by squaring our turn and to calculate the P e value which is the fundamental Euler crippling load for this kind of situation where one end is fixed and one end is pin joint.

And you see here to make it you see the unloaded situation at the strut part you see where the pin joints are there, the vertical force is being added and due to that the fixed end you see the moment is there with inclusion of all those conditions the crippling load is coming out as 2.05 pi square E I over L square. So, now you see here you have a clear comparison in the two main situations. One situation was there where the both ends were you see the pin joint So, in that case you see the crippling load was there pi square E I over L square. That one situation was there where we have both end as rigidly fixed joint. Then the crippling load was there actually as the four times of E square pi square E I over L square.

But when we see there is an combination; that means you see you know like when the slender members are connected in such a way that at one point we have to do this pin joint and at one end you see the extreme rigid end is there that means the wall is there. So, if this kind of situation is there then you see the buckling can be happening at the crippling load that is 2.05 pi square E I over L square So, this is you see you know like the situation was there So, in all four cases. Now we found that there is you know like only the variation is there of the one part; otherwise, you see it absolutely depend on the pi square E I over L square.

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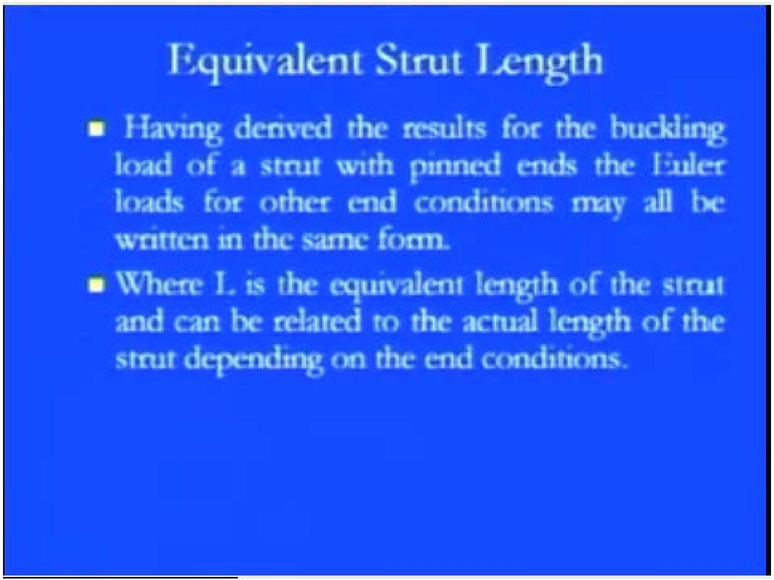

Now we are interested in calculating the equivalent strut length; that means, you see having you know like derived the results from the buckling load of a strut with pin ends the Euler load for other end condition may all or you see it can be you know like written in the same form, where you know like the L is the equivalent length is there of the strut and can be related to the actual length of a strut depend on the end conditions. That means you see you know like if you are saying that this pin ends are there and you see the other end is of any situation may be pin joint, may be fixed joint, may be anything you see the free also. So, what will be the equivalent strut length is there through which we can calculate the crippling load is. So, you see it absolutely depend on what the end conditions are there of the other side.

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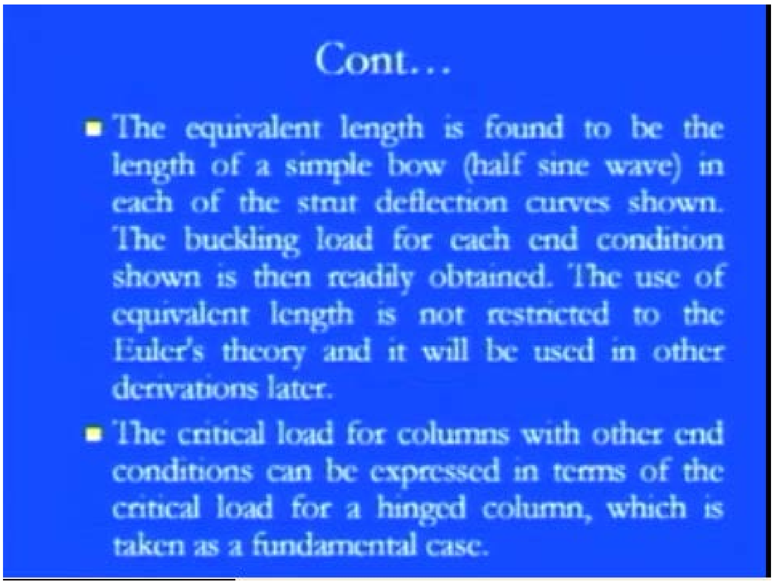

The equivalent length is found to be you know like the length of the simple bow half of the sine wave you see in the first condition you know like where we know that the strut deflection you know like is the one curve is there. That means you see we are saying that the first mode of the deflection is there in the strut where we know that only the one sine wave curve is there. The buckling load for each end condition is shown; therefore, readily obtained because you see depends on you see if you are saying that both the pin ends are there and if you are calculating those things. Then we know that in one case it is simply pi square E I over L square is there; in the second case where you see you know the full sine wave curve is there the two main you know like the heights and the peaks are there.

So obviously, it is four times and if it is we are going in the third term then it is you see it is nine times of pi square E I over L square. So, all these you know like the solutions under the buckling load these conditions are readily available. The use of equivalent length is not restricted to the Euler theory, and it will be used in any other you know like the deviations part so which we are going to discuss later part. The critical load for columns with the other end conditions can be expressed in terms of the critical load; for the hinged column you see when the pin joints are there which is taken you know like as in a fundamental case.

As you see we started from that that if you see both the end is hinged or pin joints are there, then what exactly the column is there; as you see we have seen in the strut case. And then you see corresponding changes are there depends on what the boundary conditions which we are taken like you see if we have a pin joint or if we have a you know like the fixed end is there.

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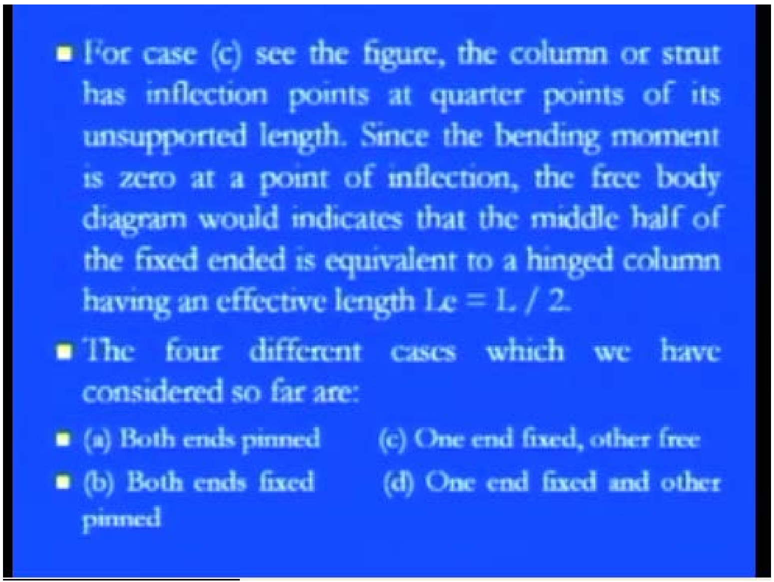

And for c case where you see you know like as we discussed in that where the column or the strut has you know like inflection point is there at the quarter point of its unsupported length. That means you see you know like in the c case where we discussed that one end is rigid and one end was freely you know like the free was there. And you see in that there is an inflection points are there because of you know like the free end load conditions are there. Then we need to you know like check it out that exactly you know like how the deflections are coming out within the unsupported part.

Since, the bending moment is zero at a point of inflection at this particular way the free body diagram would indicate that the middle point of means actually the half of that particular fix ended is equivalent to the hinged column having the effective length of L e is L by 2. Because you see let us say if you are comparing with the main thing that is the both ends are pin joints, we have pi square E I over L square. And if you are comparing with the one end is free and one end is rigid you see then we have pi square E I over four times of L square.

So, if you are comparing the equivalent length then we will find that effective length will be of L by 2, because you see the four is coming in the denominator way and since the L is coming as you know like the total in terms of you know like the square term. So, it is obliviously you see L by 2 is there. So, the four you know like the different cases which we have discussed here so far in that as I told you many times that when first case which was the beginning part was there, when the both ends are pinned the second was there the one end fixed and other end is free; when the both ends are you know like rigidly the pin part was there. So, fixed part is there. So, this b condition and the d condition was when the one end was fixed and another end was pinned in the previous case. So, all you know like the possible cases which have been already discussed and that you see we can simply figure out the equivalent length by comparing one of them.

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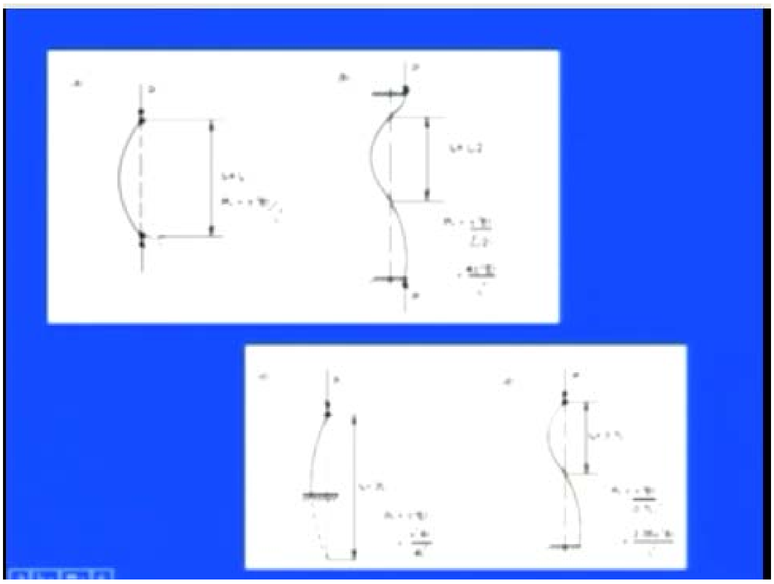

So, like you see here the first case which we discussed the pin joint was there and you see the axial load was coming out. So, LE simply because the P e is nothing but equals to pi square E I over L square. So, we can see that L E is nothing but equals to L. The second case you see where we discussed about the both ends are rigidly fixed up. So obviously, you know like effective length as we discussed in the previous slide that L E is nothing but equals to L by 2. So obviously, the P e is nothing but equals to pi square E I over this L by 2 whole square, or we can say that the four times of pi square E I over L square is there.

And the c case in which you see the one end you know like which we discussed about the free end, one end is rigidly fixed up, then you see in that case we have L E is equals to two times of this original length. So, we can say that the P E which is nothing but equals to pi square E I over four times of L square. And the final case you see here in which one end is pin joint and one end is rigidly fixed up, we need to calculate you know like the effective length you see you can see that this is the effective length in this case which is nothing but equals to 0.7 times L.

So, we have you see the PE if you are comparing those things pi square E I over 0.7 L square; we have 2.05 pi square E I over L square. So, the equivalent length is always important to check it out the buckling part that you see how the buckling can be happening, and you see what kind of sign waves are coming under the kind of load of actions are there. So, this was you see you know like we simply compared all those four situations based on the load application and what the effective length is there to check it out that part based on the Euler theory.

Now we would like to compare the Euler theory with the experimental results. So, we find out that there are because you see you know like these are the computational theories are there; so obviously, always it incorporates certain assumptions in that So, we can say this assumptions as I told you in the very first part in the strut and column that you see if we are going towards the natural system, always it is having more and more nonlinearity is there. But if you are comparing on the mathematical way we always incorporate certain types of assumptions So, this assumptions you can say that these are some sort of the limitations of all those theories So, you see here this Euler theory is also having some sort of limitations in that.

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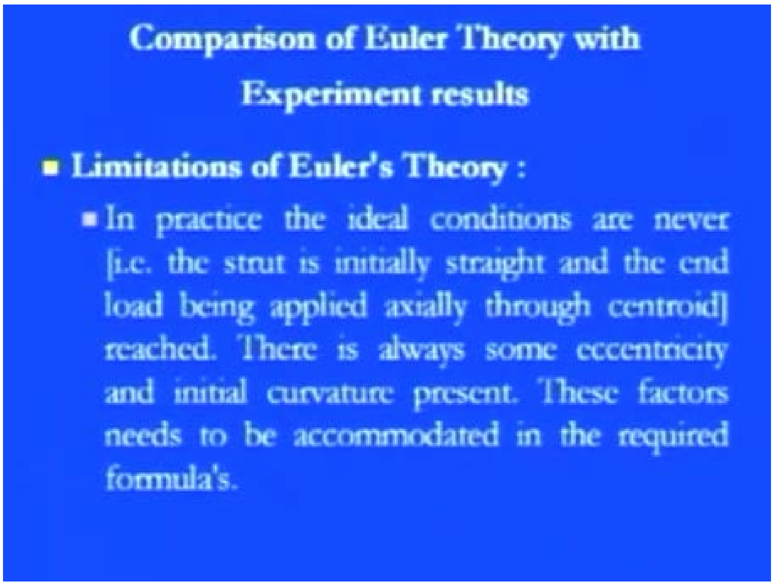

So, in practice the ideal conditions are never; that means you see here the strut is initially straight, and the end load being applied axially throughout the centroid is reached; that means it is not possible that you see you know like in mathematically by keeping those parametric way that. Now this is initially it is the straight part is there, and the load conditions are applied axially throughout the centroid part means you see when you apply the load, it has to be applied on the centroid part, and then it will you know like the point of application is towards that part only. Theoretically it is quiet you see you know like logical, but experimentally you see it is not that feasible as such.

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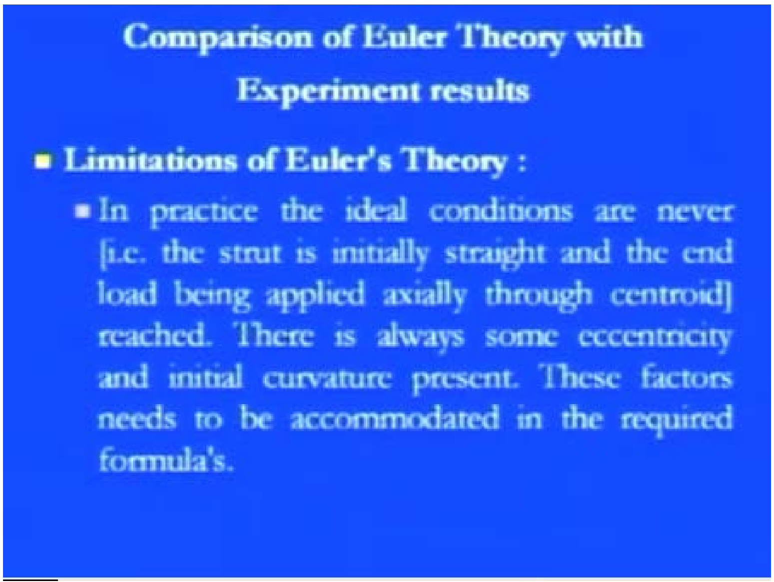

So, therefore, it is always somewhat eccentricity is there and the initial curvature is always coming due to the axial load So, that is what you see I told you that if you go in the realistic way then always there is sort of the eccentricity is there, and due to the eccentricity some sort of curvature is there and the buckling are always coming in terms of that. And these factor needs to be accommodated in the required formula which was not there you see. So, you see in all four cases which we discussed we found that the things were quiet idealist way.

We assume that the load application is there at the pin joint or the fixed joints, and it has to pass through from the centroid and there is no you know like the initial curvature is there. That means there is you know like the kind of eccentricity is there under the load applications. But you see this part is absolutely missing in the Euler theory because it is simply based on the sudden assumptions.

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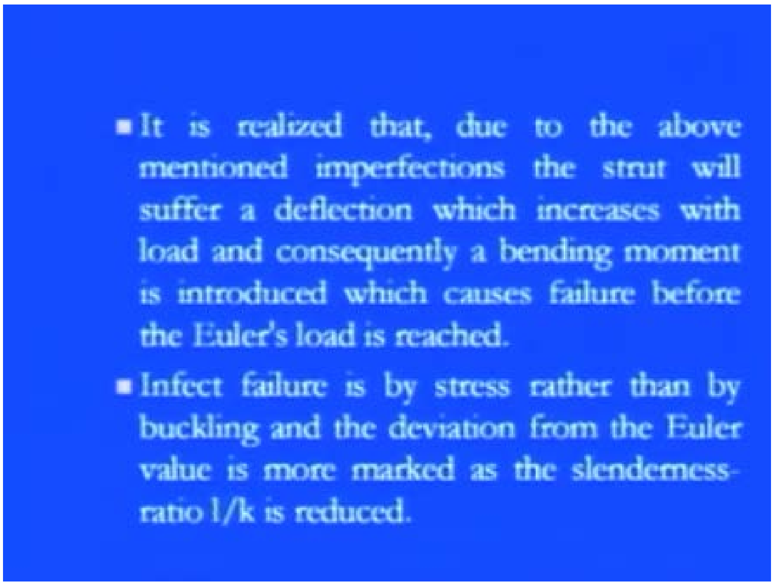

It is realized that due to above mentioned imperfection; this is a kind of imperfection only because of the eccentricity. The strut will suffer a deflection which increases with the load, and consequently, the bending moment are introduced which causes failure before Euler load is reached. So, you see here we are keeping you know like the crippling load at particular way, and we are saying that when the load is reaching up to this P E value the buckling can be happening, but in a realistic way it always happened before this Euler load.

Why? Because you see because of the eccentricity we know that there is a additional curvature is there, and you see when this load is gradually increasing it adds you see the bending moment and you see when there is an additional moment is there which we are not considering in our formula. But in practical it is being added when the real application is there, it additionally causes you see the kind of moment and then you see there is a definite failure is there before Euler load. So, that is you see you know like the realistic condition is.

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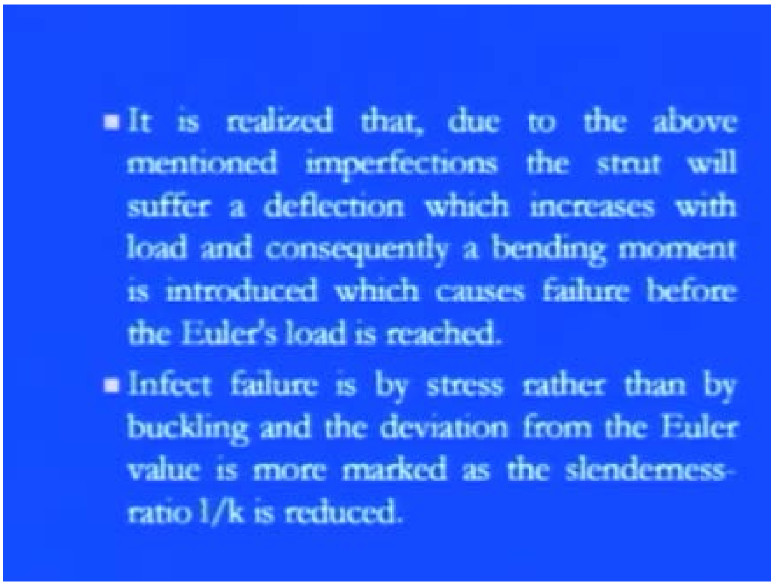

And in fact failure by stress rather than you see by buckling and the deviation from the Euler value is more mugged as this slenderness ratio you know like is simply slender ratio means 1 by k is reduced. Because slenderness ratio is somewhat you see you know like we are taking L and divided by you see the radius of gyration 1 by k this L by k is there. So, we have to be very, very careful that actually you know like this failure is what the basic cause of this particular failure is. So, somewhat we will find that you see the stress, rather than the buckling is you know like having more deviation and due to that you see you know like the Euler value is you know like more marked as compared to that particular part and you see the slenderness ratio is always reduced in this kind of cases. So, if you visualize these conditions then you will find that these are the realistic situation which we are ignoring in our you know like Euler theory or in general theories and we have to be considered in almost all these kind of cases.

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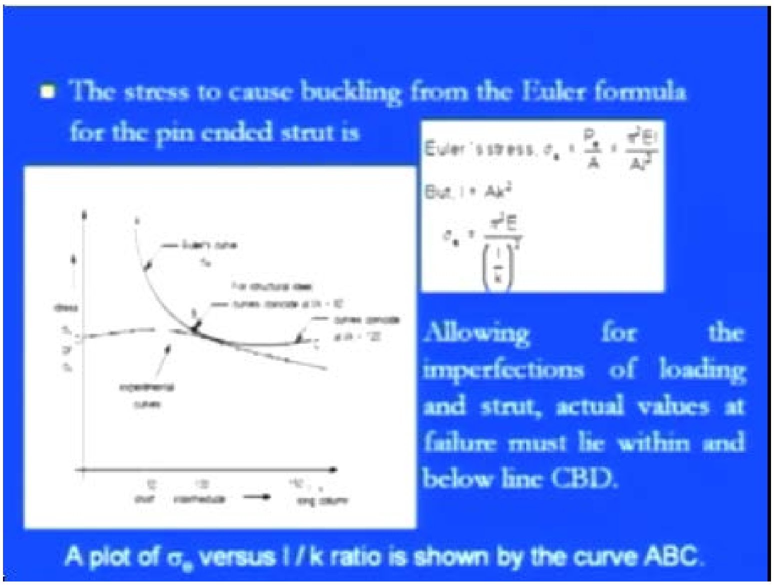

So, the stress to cause buckling from the Euler formula for the pin strut is somewhat you know like this sigma P which is you see P E over A or we can say that the pie square E I over this L square or we can say the A k square somewhat you can say. So, if we are replacing this I which A k square then we have the sigma E which is nothing but equals to pi square E divided by this I by k whole square. So, now what we have? If you look at these points, so we discussed that not only this E I and L is important, but also you see this radius of gyration the I by k the slenderness ratio is again very very important.

And then if you are saying that apart from the buckling if the stress is also one of the key feature and you see due to that if the L by k is reducing, and if it is reducing L by k it is in the square term and it is in the denominator you can see that the sigma E is increasing drastically. So, along for the imperfection of the loading and the strut, the actual value of failure must lie within you see you know like the condition which you know like the figure is showing here where we have the Euler curve is there and we have the experimental curves are there. And you can see that you know like a kind of particular material for the structural steel you see we have this I by k is this 80.

So, there is an interaction of this particular Euler curve and the experimental part at point B where you see you know like for in a specific case of this structural steel where this I by k is value of A t; we could simple figure out that in this particular plot where the sigma E verses I by k ratio is there. You know like this experimental part is pretty realistic way and below this C B line; that means you see this D point here and the C point here and this B point. Below this particular line we can say that these are the real actual values at the failure, and they have to lie within this particular range.

But if you are talking about you know like this computational part which is you see not realistic part but it is absolutely based on the computational one with all those you see sort of assumptions we can see that we have the high range of this you know like the sigma E, which is somewhat you know like we can say that if you approach up to that, but because of the realistic nature always there is a good chance of failure within this particular experimental way. So, this is the real term we can say the experimental terms are there; within those we can say that, yeah, there is a good chance of failure within that. So, here you see as we move further means you see the L by k ratio starting from 50 to 150, you see we have the different regions altogether for checking out the failure theories in that.

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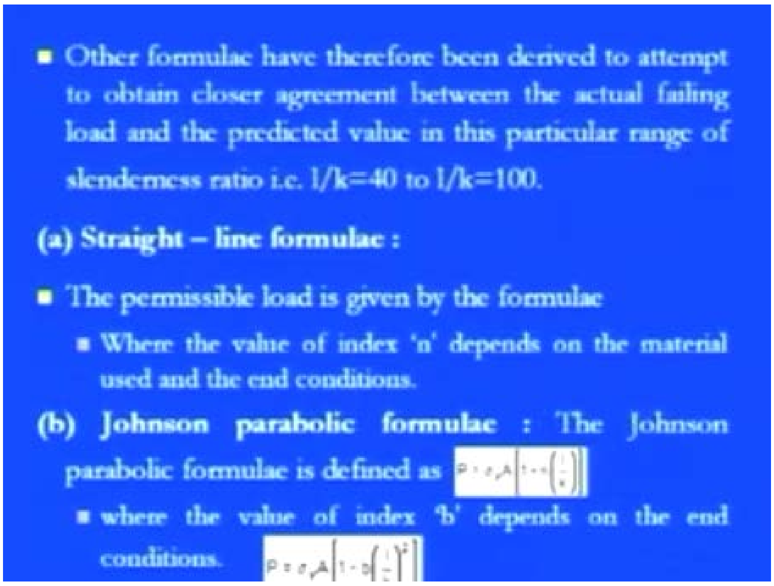

So, the other formula, which you know like being derived to attempt the closer agreement between the actual failing load and the predicted values in this particular range of the slenderness ratio. That means you see this I by k of 42 I by k of 100 is the real region where we can say that; yeah, there is some sort of you know like the closeness is there with this particular predicted values of failure with the experimental as well as the analytical one.

But again you see you know like with all those sort of the non-linearity or the realistic nature, we need to incorporate the kind of non-linearity in those terms for calculating the crippling load in the Euler theory, or we can say that we need to incorporate some sort of you see this eccentricity. So that we know we would easily figure out that, okay, if this in terms of in the microns or in terms of even the millimeter, these eccentricity is there. Then how it will introduce the moment, and how it will you know like add up in the final part of the crippling load so as to calculate the realistic way of the buckling part So, in that you see you know like for that we have the first part is the straight line formula you know like in the other formulae.

So, first when we are going with the straight line formula, the permissible load is given by the formula where you know like the value of the index n depends on the material used and the end conditions, because you see will find that n is nothing but equals to P square over E I. So, we know that in the E I always there is a material properties are there and also what the end conditions are there; that means how you see you know like whether it a pin joint or whether it is a fixed joint or whether it is a free end like that. So, this is the first simple thing is that that we need to calculate the permissible load according to that particular formula in which it absolutely depend on the n. So, that you see because in almost all the equations in which you see we are simple writing in form of the operational D square plus n square is there.

So, n is an important part in that, so that you see the straight line may come as far as the permissible load is concerned. Second part is there that is the Johnson parabolic formula in this curve where you see the Euler part is there. So, in that Johnson parabolic form since as its name is there, it has a side kind of parabolic nature. So, we can simple define this P as you see the sigma y a into 1 minus you see n into I by k, where you see you know like we have an index beam because you see in that you know like we have some sort of the index is there which is absolutely depends on the end conditions and through which see by introducing this n which is being replaced by this index.

We have some sort of you know like the value for this Johnson parabolic equation as P is equals to sigma y into A 1 minus B; now it is being replaced, because n is nothing but equals to B times of I by k. So, when we replace this index you know like it has you see that final formula sigma y in to A 1 minus P times of I by k whole square. So, it has you see a kind of parabolic in the nature.

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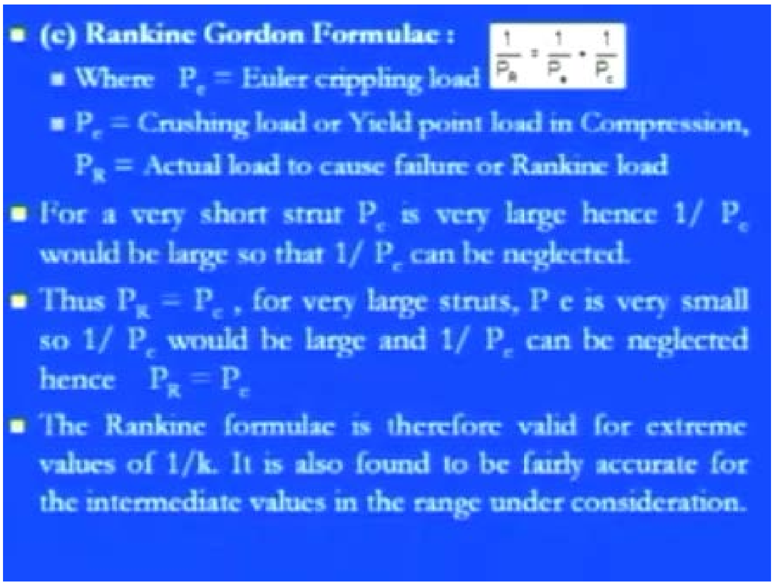

The third formula which you see you know like in that particular approximation is the Rankine Gordon formula. So, you see here Rankine Gordon formula says that you see this one of the P Rankine is nothing but equals to we have 1 by P e plus 1 by P c. So, P e is nothing but you see as usual we have taken that as a Euler crippling load, but in that you see we need to add the crushing load or we can say the yield point in the compression, and P r is the actual cause of failure. So, you see here what this Rankine says that you need to add certain additional part in the realistic way, because you see it will always be there; the crushing load will always be there under the compression, or we can say that there is an additional yield point is there due to the compression part.

So, by incorporating that now you know like we can go up to a closer part of the realistic part. So, that is what you see 1 by P r which is the real or we can say the actual load is there which cause the failure or we can say the Rankine load is nothing but equals to 1 by P e plus 1 by P c. So, for a very short to you know like this strut where P e is very large; hence, 1 by P c would be large, so that 1 by P c can be you know like easily neglected in that term. Because we know that 1 by P e is always an important feature is there in general cases; thus, P r is equals to P c for very large strut.

So, again you see now we have to be very, very careful that if you are talking about a large strut then realistic always be equals to P c, because you see you know like whatever you know like the crushing load is there, it is the main responsible for the large strut which incorporated you see the buckling part. So, P e is very small. So, 1 by P e would be large, and we can say that you know like this Pc can be easily neglected. So, we have P r is equals to P c for the large strut, but as far as very small struts are concerned we have you know like we need to you know like focus on this particular P c part and you know like he 1 by P e would be you know like an important consideration in these cases.

So, you know like either the last strut or the short strut how these you know like the crushing load or the Euler load is responsible for these kinds of things. So, they have to be calculated in the actual load which is causing the failure in the strut. So, the Rankine formula is, therefore, valid for extreme values of this I by k, and it is also found to be fairly accurate for the intermediate values in the range under the consideration, because you see it is always related to you know like that for this small or for large which load is real responsible for causing the buckling parts. So, that is what you see it is somewhat more accurate than other formula.

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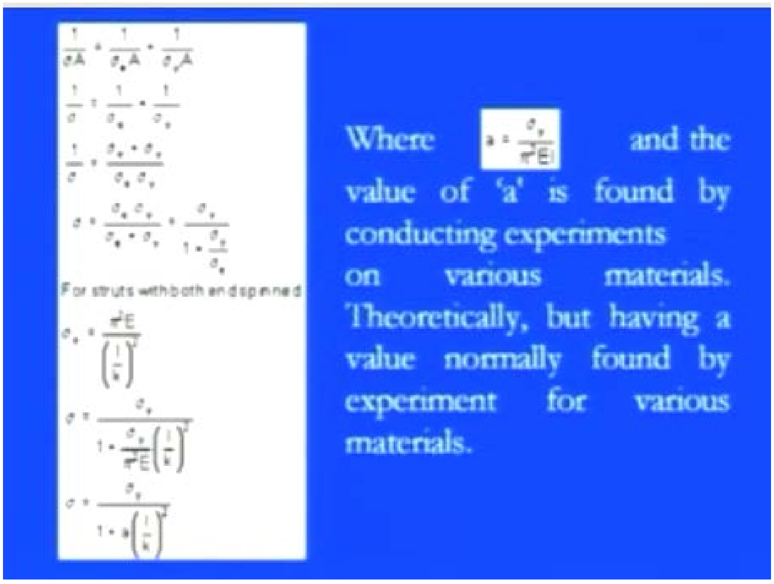

So, with that you see now if you are going for you know like 1 by sigma into A which is equals to 1 by sigma E into A plus 1 by sigma y into A. So, now you see you know like we are fairly in that particular portion, so that we can say that 1 by sigma is nothing but equals to 1 by sigma E plus sigma y, or we can say that we can simply calculate the responsible sigma in those cases in the Rankine formula. So, we have sigma which is nothing but equals to this buckling stress is there is nothing but equals to sigma y divided by 1 plus sigma y by sigma E.

So, you see you know like these are the crucial things are there in that cases. So, now for the strut with the both ends or you know like the pin part we know that it is nothing but equals to sigma E pi square E I in to pi square E I by L square was there. So, now by replacing those things what we have? You know like this I by k formula we have pi square E divided by I by k whole square, or now you see if you are keeping that part particular here the sigma, then we have sigma is nothing but equals to sigma y over 1 plus sigma y pi square E into I by k whole square, or we can say now if we are replacing this by a constant A, and this constant is nothing but equals to sigma y over pi square E I.

And you see now we have the final value of sigma which is a realistic way you know like as compared to the Euler one sigma is equals to sigma y which is you see sigma y is the yield point you know like the stress divided by 1 plus A times of I by k whole square. So, where A is equals to sigma y or pi square E I, and you see the value of A is found by conducting experiments on the various materials because you see E I depends on the material. So, what material which you are using and you know like what kind of shapes are there. So, it is absolutely sensitive to the material property which we are using theoretically, but you know like having a value normally found by experiments for variety of the materials.

So, you see here in those cases we simple calculated that you know like if we have means you know like the experimentations are there and the theoretical values are there, if they have the deviation then how we can you like match those values Then we simply concluded that, yeah, if you see like there is a comparison is there in between those terms. Then we can simply add some nonlinearity or we can simply add some correction factor to keep those values in a closer way. So, that is what you see all these either the Rankine or either you see this parabolic term are either you see the straight line terms; all these terms are being added just to verify those results.

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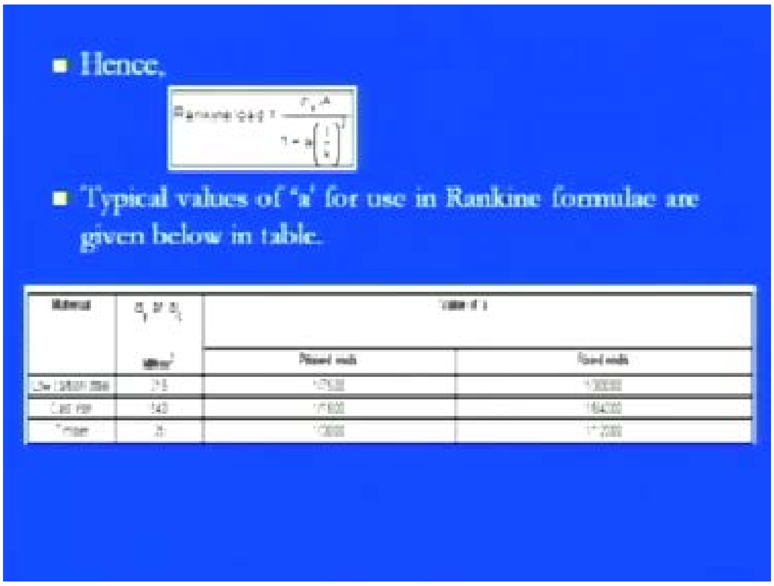

And hence we can say that we have the Rankine load; this is nothing but equals to you know like divided by area it is there. So, Rankine load is equals to sigma y into A divided by 1 plus A over i by k whole square. And typical values of a which is real constant is there in the Rankine formula are simply we can take as per you see what kind of load conditions are there and what is the material property. So, if you if you look at this particular column then you will find that we have the material column here. So, we are using three different materials; the low carbon steel is there, the cast iron is there, and we have the timber. So, you see you know like right from ductile to brittle material we have all those kind of things are there, and we can simply use you know like sigma y or sigma crushing parts is there in terms of mega Newton per meter square.

So, you see it has the value clear of sigma y which is 315 in case of low carbon steel, 540 in case of cast iron in the real you know like the brittle material, and in timber also it is a least value that is the 35. So, now you see here and again as I told you since it is a absolutely the value of A depends on what exactly you know like the load conditions are there. So, if you see you know like if we have the pin joints at the extreme ends of these. Then you see we have the value of a is 1 by 7500, and if it is for the low carbon steel and if this value of a for cast iron is 1 by 1600 and 1 by 300 is just for the timber part.

So, this is the condition where the pin joints are there, but if we have the fix ends means the resulted fix ends are there, then 1 by a value for you know like these materials are much much higher than these, because you see now we have the rigidly fixed ends are there. So, obliviously you see the kind of moments in those terms which we are assuming in the numerical values is real different than the actual one, because you see some sort of moments can happens; some depends on what kind of you see rigidness is there. So, for that you see you know like we need more you know like values in those terms.

So, we have you see you know like we will find that for low carbon steel we have 1 by you know like the 30000 is there when the fix ends are there for low carbon steel of value a; for cast iron the value of a for fixed end is 1 by 64000. And you see for timber we have 1 by 12000 is there of value of a for these things. So, depends on you see how much you know like the flexibility is there more. So, it depends on the flexibility of part like we can see that if more you know like rigidly fixed joints are there, then probably see it will not have more flexibility towards that.

So, you know like we can keep the value of a will be you know like lesser position while in the pin joints you see more and more flexibilities can be introduced in this particular kind of systems. So obviously, as computed the fix joints we have you know like the higher values of these things for different kind of materials, but you see you know like as far as the low carbon steel is concerned based on the microstructures and based on the kind of load conditions, always we have higher values of these values as compared to the timber part. And cast iron since you know like it is providing you know like it is brittle material. So, it is more harder material is there.

So, somewhat you know like it has not exactly you know like we can say the lesser values are there as compared to the ductile material of this low carbon steel of the value of a. So, you see here. So, this was the correction when you know like this when we know that there is you know like the n is not exactly matching with those particular values. So, the a will come, but now you see if you know like because of the eccentricity we know that there is an additional kind of moment is there. So, we need to add that part. So, you know like introducing the initial curvature in the strut then you now like we again go towards you know like realistic conditions that; yeah, now because in the realistic way we have the initial curvature is there or because of the eccentricity. So, by adding directly in the numerical values, can we really go towards these things? We just want to check that part.

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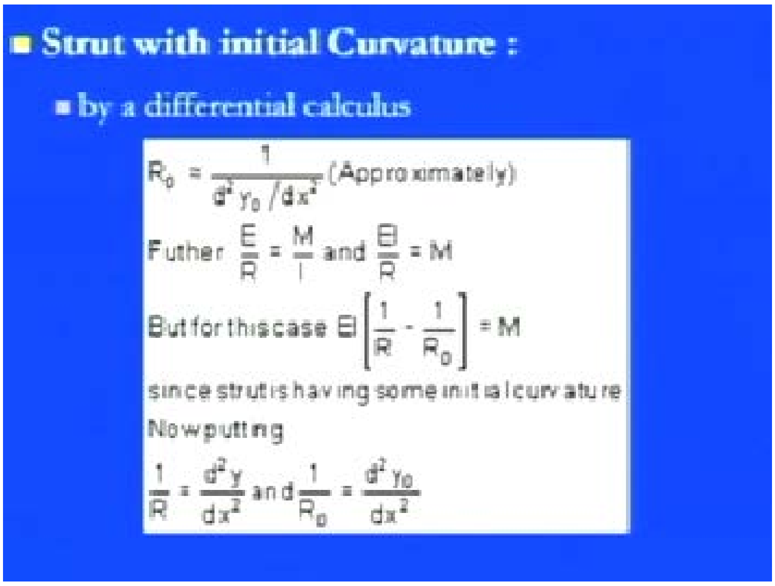

So, here you see in this case we have strut with the initial curvature. So, you see here we are simply approximating the curvature radius R 0 which is you know like equals to 1 by d 2 y 0 by dx square. Initially you see you know like we are considering d 2 y by dx square at the y 0 condition; means you see when there is no load conditions are there and we have initial curvature. So, by adding that you see we have this R 0 values, and now by taking the same conditions of the bending moment. M is nothing but equals to you know like M by I is equals to E by R in this particular curvature length in the bending of beam.

So, E I by R will give you exactly the moment which is beam there initially you know like. So, with that consideration here we have B over 1 by R minus 1 by R 0 is equals to M. Since, the strut is having some initial curvature as we have considered here, now we need to put 1 by R which is d 2 y by dx square or 1 by R 0 which is d 2 y 0 by dx square. So, we can see that this R which is the realistic condition you know like is coming under the load conditions and R 0 which simply introduce because of you see the initial conditions just to compensate that you see when the load is not exactly on the centerline, and due to the eccentricity the things are there.

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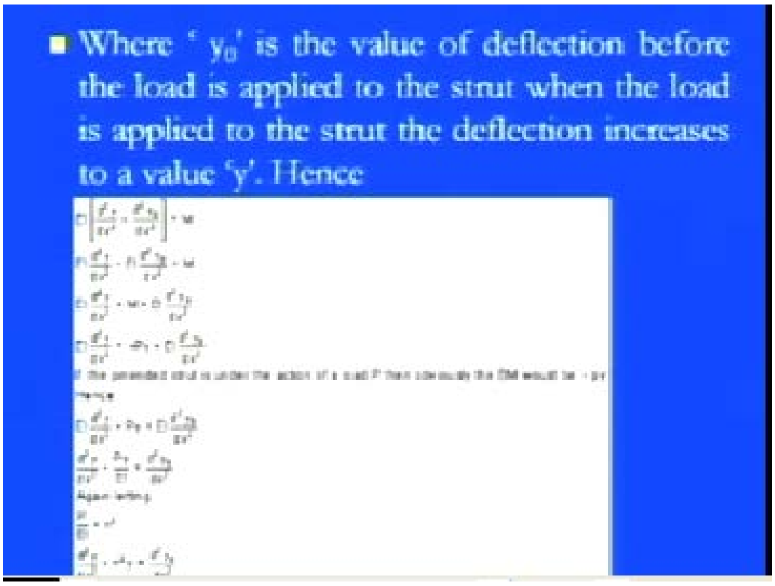

So, now you see here where the y 0 which dy by dy zero by dx square was there for R 0; y 0 is the value of deflection before the load is applied to the strut and when the load is applied to the strut we have you know like the deflection which increases to the y. So, with that consideration what we have? We have the resultant part is there. So, E I into dy by dx square minus d 2 y 0 by dx square is giving you the real value of M or if you multiply those things we have E I into d 2 y by dx square minus E I into d 2 y 0 by dx square is equals to M or if you compare those things, then we have probably E I into d 2 y by dx square which is exactly equals to M plus E I over d 2 y 0 by dx square.

So, now you see here if we are saying that you know like the pin joints are there, and due to the axial load condition we have the moment which is P minus P into y. So, now if we are comparing those things, then what we have; we have E I into d 2 y by dx square. Now we can simply replace this M by minus P I. So, it is equals to minus P I plus E I into d 2 y 0 over dx square. So, in that particular pin strut where you see the pin joints are there, the bending moment is always you see it is coming in that particular you know like this way. So, we need to consider minus P y in terms of the M.

So, with that particular consideration what we have? We have now d 2 y by dx square you know like plus P into y over E I is equals to d 2 y 0 over dx square. So, we know that the n square is you know always P over E I. So, by equating that what we have? We have the second order differential equation in terms of d 2 y or dx square plus n square y is equals to d square y 0 over dx square. So, this is you see the new second order differential equation which is having you see the initial slope plus the slope increases due to the load conditions.

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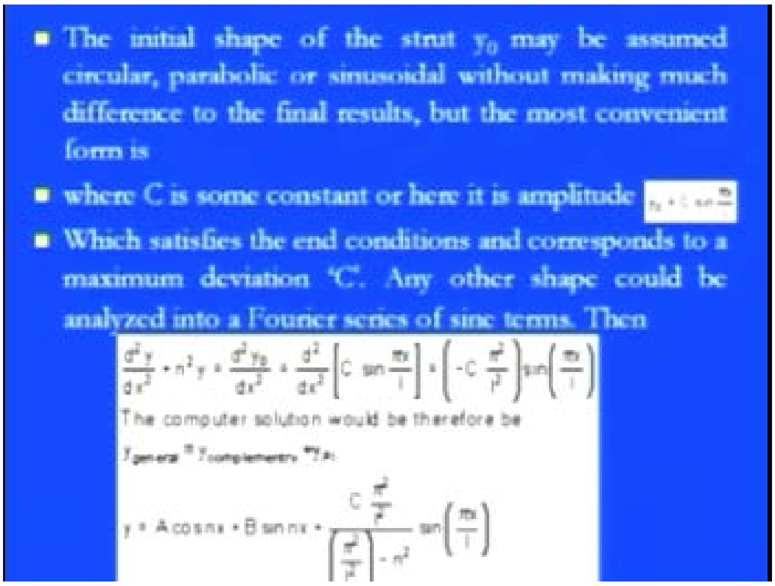

So, the initial shape of the strut y 0 may be assumed to be circular, parabolic or sinusoidal without making much difference to the final result, but most convenient form is you see here in terms of the C which we are considering y 0 is C times of sinusoidal this n into x over I, which satisfies the end conditions and the corresponding to a maximum deviation at point C. So, any other shape would be analyzed also by Fourier you know like the series is there. Then we can say that d 2 y over dx square plus n square y is equals to d 2 y 0 over dx square. So, by you know like in the most convenient form of the sinusoidal form because there is the periodic phenomena's are there. So, what we have? We have d 2 over dx square in terms of C into sin of pi x by I is equals to minus C pi square by I square sin of pi x by I.

So, you see the computational solution which can be easily got by you know like the two forms; one the general solution is y complementary function and y this particular integral. So, you see by keeping those things what we have? We have y equals to A cos nx plus B sin nx plus this you see which we have discussed C into pi square over I square divided by pi square over I square minus n square sine times of pi x over I.

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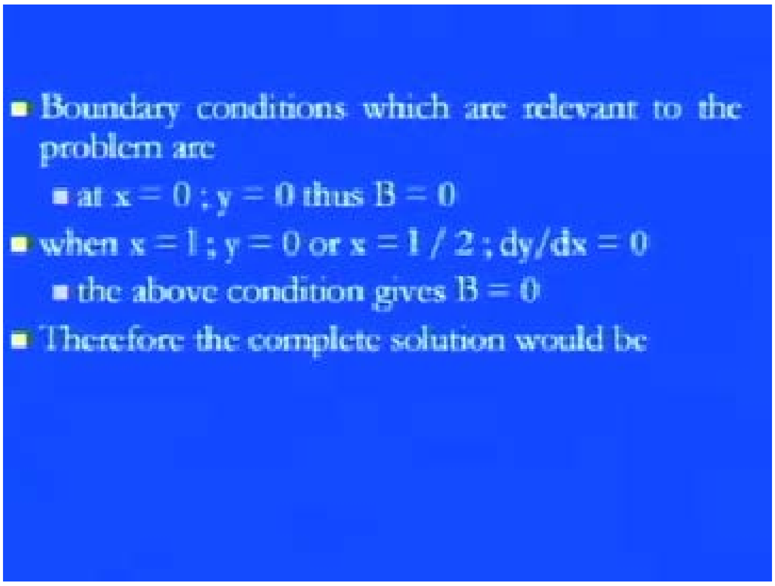

So, now you see here by keeping those boundary conditions we can simply get the real solution of the generalized part. So, we have boundary conditions in the relevant problem is at x equals to 0; obviously, there is no deflection is there. So, it gives you a clear-cut indication of B equals to zero where you see where x equals to l, the extreme condition we have y equals to 0. So, by keeping those conditions we have or we can say at x equals to this L by 2 also, there is no slope conditions are there. So, for all those conditions we have the B equals to 0.

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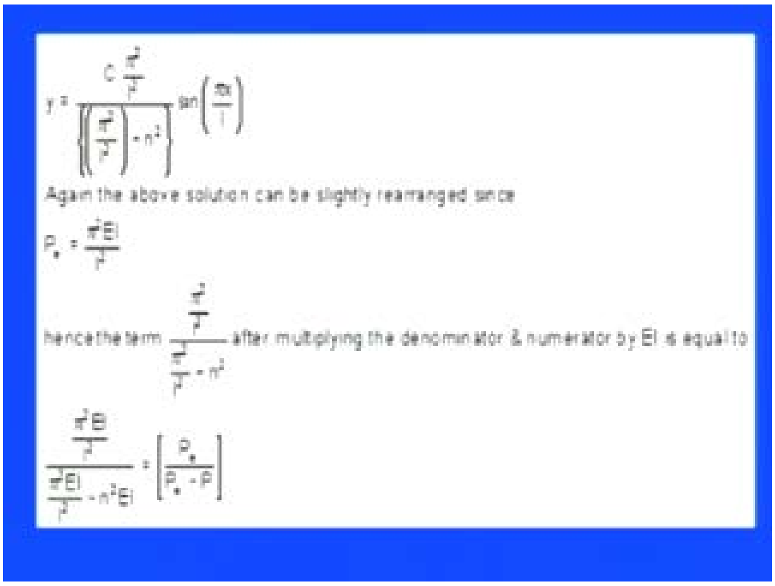

So, the complete solution will give you y equals to C times of pi square over I square divided by pi square over I square minus n square sine times of pi x over I. So, you know like the basic equation was the P e which was pi square E I by l square was there. So, now we just want to you know like compare both the equations in those terms So, here we have the new term in the first equation which was y equals to C into pi square y l square divided by that So, what we have in those terms? We have pi square over l square divided by pi square over l square minus n square.

So, in that case only just we are missing about the E I. So, we need to multiply the E I with denominator and this numerator. Then we have the new form of the equation which is pi square E I over l square divided by pi square E I over l square minus n square E I. So, you see which is equated by you know like both of the condition P e over divided P e minus P. So, what we have in this case? We will find that in this particular you know like the crippling load there is a difference part is there in the denominator part.

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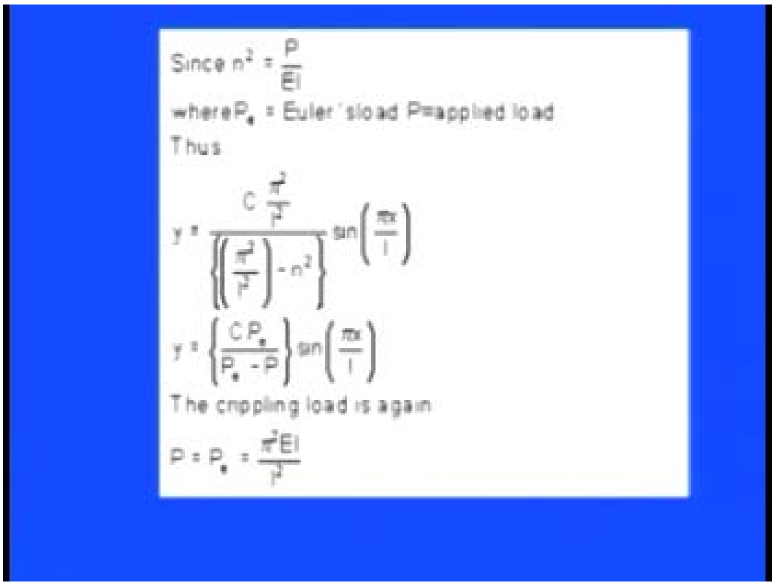

So, by comparing those n square which is nothing but equals to P over E I we have the Euler load P e which is you know like the crippling load was there, and the P was the applied load. If you compare all of them then we have y which is the new term y is equals to C times of pi square over l square divided by pi square over l square minus n square sine times of n x over this l. So, you see here if you compare both the things then what we have? We have C times of P e divided by P e minus P sin of pi x over l, or we can say that the crippling load which was there in that case was pi square over E I into l square; P you see here is the applied load is there that how you know like the axial load which is to be there which is causing the deflection in this cases.

And you see here we have n square which is nothing but equals to P over E I. So, incorporating all those values we have the new term, which is coming due to the initial deflection consideration that, okay, now we have the initial deflection, the radius of curvature R 0 is there at d 2 y 0 over dx square. With the all the consideration we have the new term, and that is why you see always in the realistic term we found that the failure is there before this particular part.

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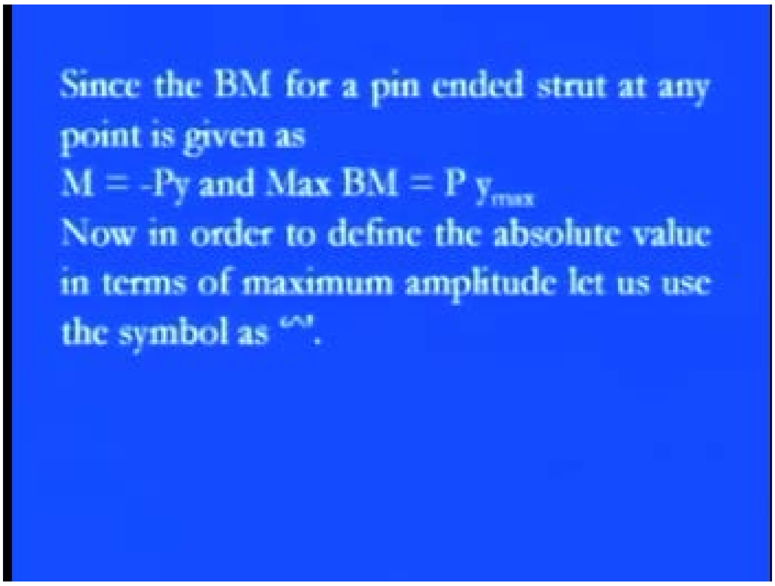

So, here in this case the bending moment for a pin ended strut at any point can be easily given as minus P times of y, and the maximum bending moment is always at somewhere where the maximum deflections are there So, P times of y maximum. So, in order to define the absolute value in terms of the maximum amplitude, now we are using the kappa word, the cap is there.

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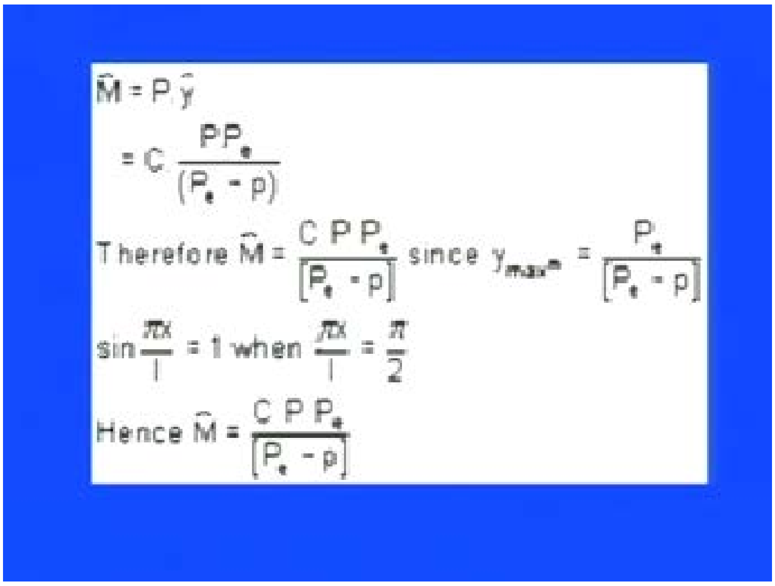

So, in terms of the cap if we are incorporating that part here, so what we have? We have the realistic way M cap is equals to P times of y cap, and we know that it is nothing but equals to C times of P times of P e divided by P e minus p, because you see we got that particular term in that case in the initial curvature conditions. So, we have M cap is equals to C P P e divided by P e minus p where y maximum is nothing but equals to P e over P e minus p. And you see here sin of pi x by l is equals to l equals to 1 in the maximum loading condition case. So, we have pi x over is l is equals to pi by 2. So, we have the real you know like the bending moment case in which the initial curvature is being considered is C P P e divided by P e minus p.

So, in all these cases we found that you see here that if we are comparing the experimental results with this particular Euler theory, then there is a great difference is there in between all. So, again you see in the next lecture we would like to continue these cases, so that you see what other parameters are there which can be incorporated to calculate the realistic values of you know like the failure. So, that you see when we are predicting about the failures of the strut or the column, then always we are reaching to the real values of that rather than you see the numerical value or the computational value which is for ahead from the real or we can say the experimental value of the buckling load.

So, you see here you know like in this lecture we mainly discussed about those, and we found that there were two main you know like the phenomena's were there. First you see there is always an eccentricity is there in these structures and due to that the additional bending moment is always applied, and you see you need to you know like consider those additional moment is there which we were ignoring in the Euler theory. So, here you know like the additional R 0 was added, and we found that this M value was C P into P e divided by P e minus p.

So, you can see that how you know like what the great influence is there of the difference P e minus p, because you see here whatever the values are coming they have the direct influence on the bending moment. And the second case which we discussed you know like which was not considered in the Euler theory was you see you know like what is the material property is there. So, based on that you see you know like these you know like the slope conditions are always deviating from that and that is why you see the 1 by a; means you see in the different formulas we are considering you know like in the Rankine formula or in you know like this Gaussian formula. So, whatever the things are coming accordingly we need to consider with those things.

So, in the next lecture you see we are going to discuss about more you know like the realistic nature of these Euler or beam theory, this Euler and strut theories are there for calculating the real values of these buckling load. And then you see whatever the safer designs are there accordingly we need to consider the factor of safety for that.

Thank you.