Strength of Materials

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Lecture – 36

Hi, this is Dr. S. P. Harsha from Mechanical and Industrial Engineering Department, IIT Roorkee. I am going to deliver my lecture 36, and of the course of Strength of Materials, and this course is developed under the National Program on Technological Enhanced Learning.

Now, you see here prior to start this lecture, we have almost done about the beams; beam theory you see what the bending stresses are there, what torsional stresses are there in the beams, and then you see there are lots of you know like the theories are there through which we can calculate not only the deformation, but also the deflection part and then, you see you know like in the last part of you know like we discussed about the theories of failures, and we found that when a beam is subjected by both of time. That means, you see when the combined loading is there, then there are you see lot many ways are there to describe those stresses, and that is you see there are various theories of failures are there as we have seen you know like lecture 34 that when the combined loads are there.

That means you see when a beam is subjected by bending stresses as well as the tensile stresses, then we could figure out the equivalent stresses like in terms of the bending or in terms of these shear or rather we can say in terms of the torsional part. And then, you know like in the previous lecture, you see we discussed about when there are five different ways to describe the failures of theories, and you see all those theories you know like they invented by some of the mathematician or the physician.

So, that is what you see you know like we know these theory by their own name like you see the Rankine or Haigh or there are you see you know like the Tresca’s theory. So, there are lot many, you know like things are there in that. So, start from the maximum principle stresses that you see you know like when we equate that part which is equivalent to the maximum principle stresses in a particular plane under the variety of you see the load conditions, then you see it is going to be failed. And then, with that you see we ended with up you see with the total strain, the maximum shear strain energy per unit volume in which we also found that you see how the relative terms are there of this normal stress component with you see because you see it is absolutely based on the maximum shear stress part, maximum shear strain energy.

So, how much energy is being gained under the load, different load conditions, so that part we discuss and you see along with that you see we discussed about the maximum principle strain part, the maximum shear stress part and then, you see you know like maximum strain energy per unit volume. And then last one was there, the maximum shear strain energy per unit volume in which you see when the shearing is there. Due to the shear action, we have the strain energy and you see under the action you see what the relative reference frame is moving with respect to the sigma 1 minus sigma 2 or sigma 2 minus sigma 3 or sigma 3 minus sigma 1, and how we can compute and equate with sigma y P. That means the yield point in that.

So, these were you see few of the five theories of failures were there which we discussed briefly about you know like to calculate about the ductile material because you see this is very important to rectify because all these theories were there under the elastic failures. So, you see we need to clearly define the reasons of elastic as well as plastic, so that we could easily figure out that now within these elastic failures, this could happen if it reaches up to sigma y P. Due to maybe you know like the principle shear stress, may be due to principle stress, principle strain, maximum shear stress strain, energy maximum shear strain energy per unit volume. So, all these things you see you know like these five theories always just try to approach up to sigma y P, and then we are equating to get the real feeling about the failure theory under those prominent parameters. So, this you see you know like we discussed in the previous lecture.

So, now, this is almost done. We are done with the kind of beam part you see you know like now we are going to focus on the column. So, instead of beam, now if we have a column, then what will happen? So, you see here this lecture is mainly focused on as I told you the elastic stability of a column, but you see again we need to define the boundary like which can clearly identify that what will be the column or the strut is.

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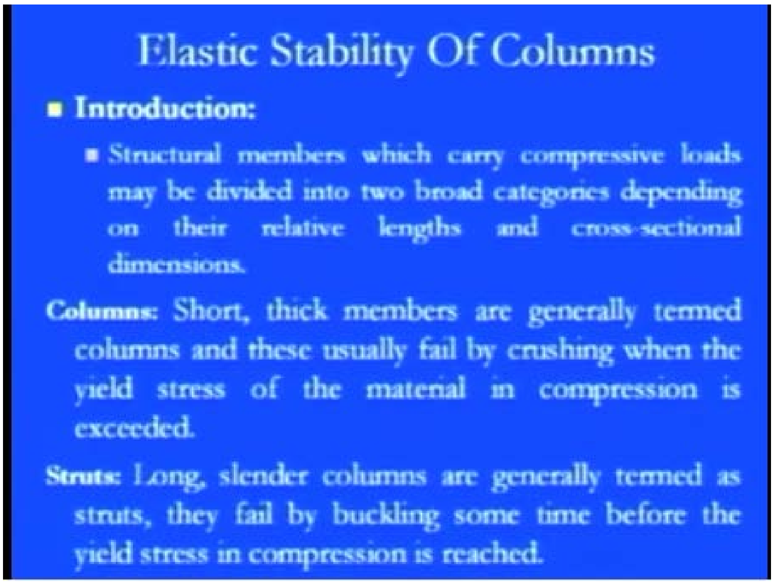

So, you know like in this particular case, we have the structure members which carry compressive loads may be divided into two broad categories depending on their relative lengths and the cross-sectional dimension. So, that is what you see here these are the two influencing parameters through which we can simply you know bifurcate that which one is the column and which one is the strut, and these two important parameters are what the relative lengths are there and what is the cross sectional dimensions. That means, you see what the dimensional parameters are there through which we can say that yeah, we can keep this thing in a column, we can keep this thing in a strut part.

So, the first thing is the column, the short thick members you see. So, if we have a short member, you see again we say very relative term. Sometimes you can say short is long, but you see because now days we are always talking about the nano or micro levels, so obviously you see this is somewhat you see the millimeter or centimeter is you know like tall or long terms, but in relative frame you see we can say the short and thick members where the applications are there are generally term as the column, and these you know like usually fail by crushing when you know like the yield stress of material in compression is exceeded.

So, you see here when you are simply compressing and you see due to the compressive part if there is a crushing, it means the layers are you know like exacted crushing to each other. They are simply overlapping to each other, and if the yield stress is approaching up to that point, we have a failure part in these columns. So, you see here you know like we are defining in terms of their dimensional part and the second part is the strut. Struts are nothing but you see you know like the longer, the slender columns we can say are generally termed as the strut. This means you see they have the long, not thick. They are the thin part you see and you know like we can say that whatever you know like these theories are coming, they have to be spread out in along like you know like the transitional way.

So, they fail by buckling because you see they have the longer part. So, obviously the buckling is the important parameter to check out the failure. So, they fail by buckling sometime before yield stress in compression is reached. So, this is pretty sensitive part that you see when we are defining you know like when we are keeping a thin line in between the column and the strut, then we have to be very careful that if we are taking a length. And you see if they are you know like failing due to the buckling prior to reach the yield point sigma y P, obviously, it is in the strut part. So, this is you see the criteria through which we can say that, yeah this is not a column. This is the strut because they have, you see they are failing under the buckling load irrespective what are the kind of things are there.

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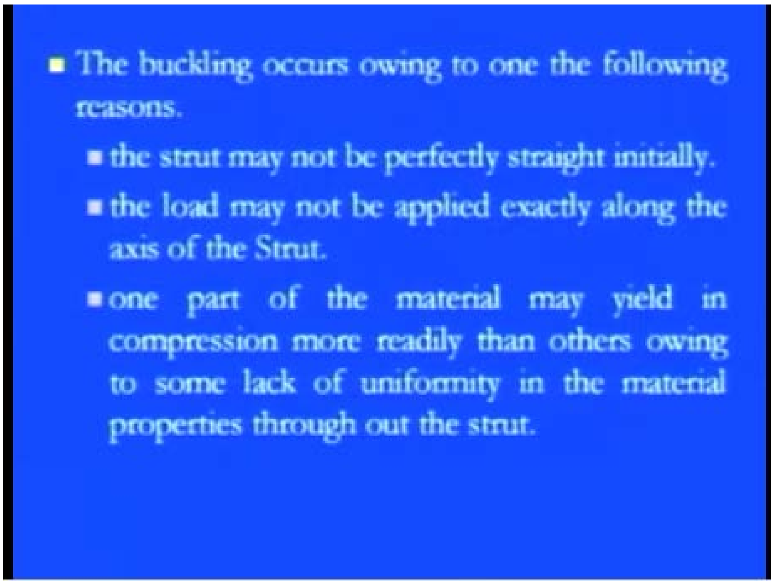

So, what are the kinds of other parameters? So, now you see the buckling occurs owing you know like to one of the following reasons because you see you know like obviously when strut is failing by buckling, so we have to be you know like as I told you, it is very sensitive thing. So, we have to be very careful that what the basic reasons behind it are. So, one of the thing is that strut may not be perfect straight initially. So, you see if the strut is not perfectly straight, it means if you see you know like the stepped part is there or if any irregularity is there in the shape of the strut, obviously you know like this could be you know like figure out in the strut concentration part, or we can say that there may be a chance of failure due to the buckling part.

Second, the load may not be applied exactly along the axis of the strut. As I told you see in the trans was direction, if the load is not exactly applied through the axis like the compression area talking like that. If they are not exactly on that particular axial part because you see as I told you that this stresses like the compressive or the tensile. They are the normal stress components and they have to be you know like along a particular line because these are the axial stresses. So, if they are not exactly along the axis of strut, obviously you see you know like it will you know like just ending towards the eccentricity and due to the eccentricity like we have different kind of loadings which we have discussed already in the earlier chapters.

So, this is the second reason. So, the first reason is that if they are not perfectly you know like the straight one or we can say the rigid one, and the second thing is if they are not, the load application is not along one axis, and the third one is that if one part of the material may yield in you know like the compression more readily, then the other you see owing to some lack of uniformity in the material property throughout the strut. So, this is known as the material non-linearity.

That means you see if the material is not isotropic, if material is not behaving perfectly and uniformly all along you see the length of the strut, then obviously you see when we are applying the load at some part, we have more distortion, we have more deformation as compared to the other part. And obviously, you see due to that we have a great chance of failure because of this material non-linearity because there is no uniformly transformation of the material or we can say that whatever the micro structures are there of this particular beam or sorry this strut, it is not behaving in a perfect you know like these isotropic part or we have you see the different due to the different boundary conditions of the material non-linearity. We have you know like great chance of failing part.

So, these are the three you know like reasons in which we found that the more of it is you know like on the geometry. So, we can say it is a geometric non-linearity or more of these, it is on the material. So, it is a material non-linearity. So, material non-linearity and third part, it is you see the force or the load action is not along the same line. So, these are you know like if we are talking about a general sense, we will find that these are three of the key you know like the parameters through which we can say that there is a great chance of having the buckling in the strut part.

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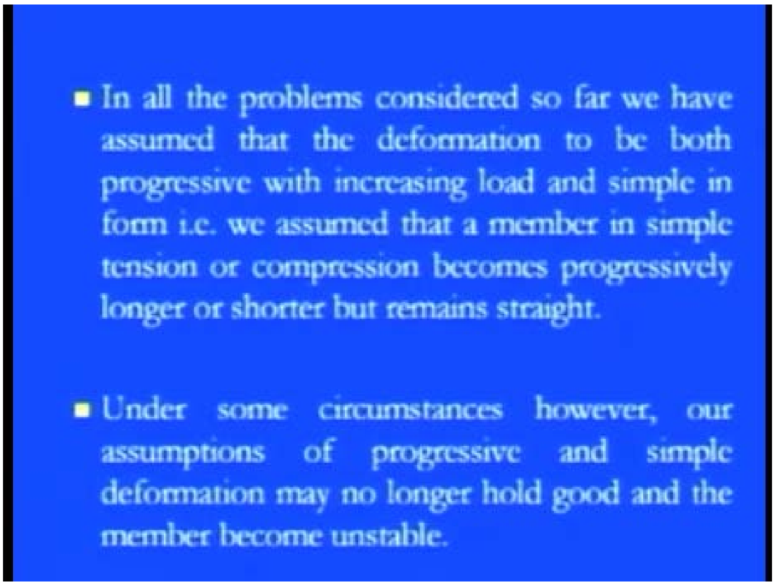

In all of the problems considered so far we have assumed that the deformation on both progressive you see with increasing of load. That means, you see then you know like increase the load we are assuming that this stress or the strains which are coming due to the application of load, they are uniformly distributed. That means you see they are progressive. They are increasing you see linearly with the linear increase of the load you know like simple inform. That means, you see if we assume that a member in simple tension or compression means, the uniaxial part becomes progressively longer or shorter, but remains straight.

So, there are two things you see. Here we are not considering any kind of non-linearity, but you see when we found that of a column you see or either strut, strut is having buckling. There is geometric non-linearity, they are material non-linearity and there are you see the force, which is not exactly on the same axis and due to that we have an eccentricity or due to eccentricity, we have buckling in that particular structure.

So, all these three terms are perfectly realistic way, but you see always we assume to make the simplified you know like applications that now you see these are the simplified assumptions through which we can generally you know like deriving the equations. So, in this particular segment says that if we assume that a member is in simple tension or in a simple compression, the progression means you see the stress in even you know like the elongation or the compression. Whatever the progression is there in the dimensional part, or we can say in terms of the strains you know like they have to be you know like go linearly, and in under the load application, this object has to remain in the straight part which is you know like it is somewhat we can say the under lowest take in some of the higher load joules, under some circumstances. However, our assumption of progressive and a simple deformation, again the progression and the simple deformation may no longer hold good and the member becomes unstable.

So, this is the perfect theory of instability because you see you know like as I told you in a realistic way always nature produces maximum non-linearity in terms of many you see like you know like as simple example is that. The water is flowing from higher to lower or is it lower to higher nothing is that because if you want to do this kind of thing, we need to go against the nature law. That means you see we need to apply an external agency to do this nature. So, here also you see if you replicate that particular concept, here also will always find that if we apply likely see we are holding a beam, and we are you know like extending that part or compression is there. Always you see sort of layers are being distributed from their own position, and due to that the object is always tending to move in terms in sometimes microns. Also, they are not be there in the straight part you see at the straight beam is there before apply application of load.

So, you see here whatever the theories which we applied in term of the deformation or the deflection, it will not you see you know like remain in the same way, some sort of you know like the distortion is there due to this particular unstable part, and that is why you know like we need to go with the stability. And that is why you see sometimes the buckling is there and buckling is an important phenomena to check it out when a longer membrane is there under the application of load.

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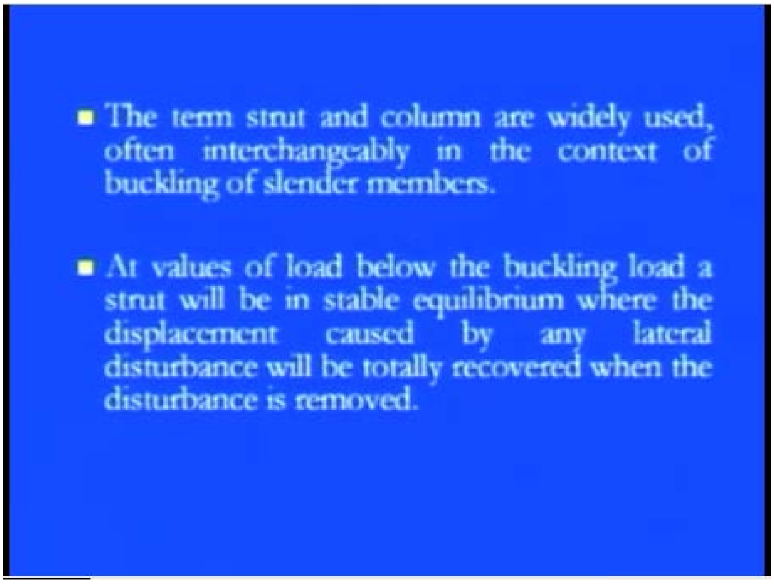

The term strut and the column are widely used because generally you see we are termed as you know like this is the strut are the column, particular often interchangeably in the context of buckling of slender members. So, you see here as I told you in the very first slide that only buckling is the key phenomena through which we can simply bifurcate that you know like this slender member is coming under you know like the column, and this slender member is coming under the part of the this strut. So, at values of load below the buckling load, a strut will be stable equilibrium where the displacement you see caused by any lateral disturbance will totally recovered when the disturbance is removed.

So, that is what you see here we applied the load and generally, that is what you know like we can say that form this statement that we apply the load and the elastic deformation is there, and you see whatever, even the deflections are there if we release the load body, its original phase without any permanent set of deformation whatever the theories which we developed. That means, the Hooke’s law and what you see the elastic coefficients are there like Young’s modulus of elasticity, or the shear modulus of rigidity or whatever you see. The bulk modulus all you know like these theories are well applicable because in that we can say that they hold the good relations between the stress and straight the strain. They also you see there is no permanent deviations in the beam or the column or any kind of membrane.

So, we can say that for that there is no buckling within that elastic deformation, or we can say this is statically equilibrium position prior to apply the load you see you know like and after the application and load up to the elastic limit. So, that is what you see you know like we are defining of the regions. Then, you know like if the load application is there below the buckling part, the strut will be the equilibrium position and then, you see whatever the displacements are there or you know like whatever the deviations are there like due to this particular load application, we can say that it can be regained once we realize the load, and that is what you see the theory. What are the theories are coming within that particular reason. They are holding you know like assumptions, and we can say that this is absolutely up to that region and the buckling.

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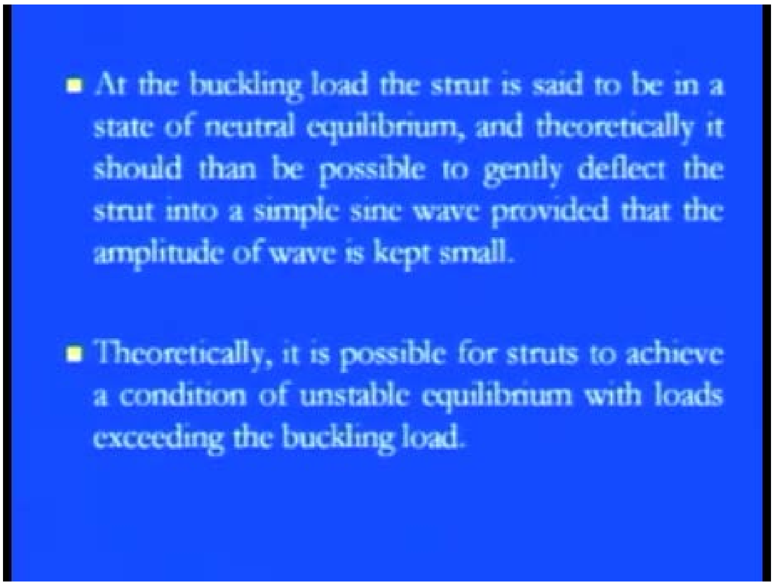

Now, you see here at the buckling load, the strut is said to be in you know like state of natural equilibrium because now we are in the buckling load, and buckling is always coming as I told you due to the non-linearity part. As I told you that non-linearity is always there or I can say that non-linearity is well tuned with the natural phenomena. So, we can say that at the buckling load means when you see the strut is there under the action of buckling, the strut is always to be in the state of natural equilibrium because it is now well tuned to the nature and theoretically, it should then be positive to gently deflect the strut in to a simple sine wave provided that the amplitude of wave is kept small.

So, now you see here once we know that the buckling is there and due to buckling, whatever you know like the layers are there of this particular strut is now under the deflection or the deformation, and they are forming you see as per the kind of load. So, generally we are saying that now this strut is whole divided into the sinusoidal form because this is well periodic phenomena is there, but you see we always keep in mind that you see what are the displacements are there of this particular layers from their own positions. They have to be very you know like small amplitude. There only you see if it is more than obviously you see you know like we are in the danger zone, and there is a good chance of failure.

So, we are just trying to keep this amplitude else more as we can, so that you see whatever the phenomena’s are coming at the buckling load, it can be easily evaluated. Theoretically, it is possible for strut to achieve a condition of unstable equilibrium with load exceeding the buckling load. That means, you see you know like if in the theoretical way, again you see in the theoretical is somewhat you know always associated with some certain assumptions. Realistic way is somewhat different than those things because you see in realistic way we have some sort of real nature, non-linearity is there in those processes.

So, if we are talking about the theoretical, then you see it is always possible for a strut to achieve kind of condition in which the unstable equilibrium is there under when we exceed the load of buckling part because you see once we go beyond that, then you see we never know that what is the prediction is there of these, you see how you know like these layers are being behaved under these particular exceed load of the buckling. So, somewhat we can say this is an unstable equilibrium phenomenon if you know like applied the load beyond the buckling load.

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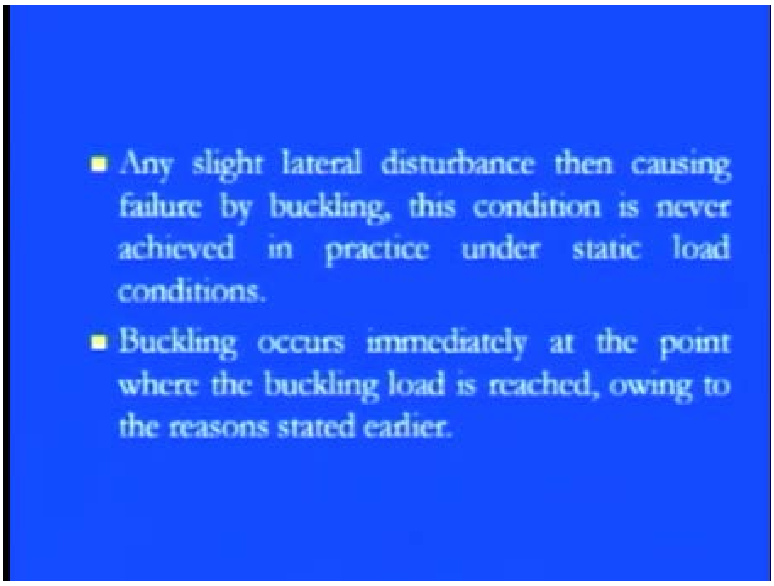

Any slight you know like this lateral disturbance then causing failure by buckling because you see we cannot predict the behavior of the layers under these heavy load which is beyond you see the buckling. So, there is great chance that actually if we do and that is what you see you know like we are sometimes saying that if it is deterministic phenomena, then what exactly the initial parameter is and due to that, it is very sensitive part because you see we are already in the danger zone and now, what we are doing. So, whatever the action is there, against or in favor of that thing, lead to failure.

So, any slight lateral disturbance even you see you know like, so how sensitivity you see simply causing a failure by buckling, and this condition is never achieved in practice under the static load condition. Because you see under static load condition, we know that when you apply the load in esthetical manner because all these assumptions are being carried out with this under static load. So, under the static load you see you know like we know that whatever the phenomena's are coming, they are not in the transient nature. They are not in the dynamic nature. So, we can you know like keep all these things and we can look at all those phenomena. As I told you that you know like straight we can simply figure out under the static load.

Now, this is the elastic region, this is the plastic reason, this is the ultimate tension strength or compressive strength, whatever you see, and then there is a fracture in those phenomena, but when we are talking about a buckling part and if you go beyond that, then you see again under the dynamic load condition where you see you know like all these deviations are there in the sinusoidal form and beyond that, there is great chance of failure, and you see in a realistic manner, you see always in the dynamic nature. It may happen, but if it is a static load condition, then you see we can simply you know like control this particular part. Buckling occurs immediately at the point, where you see buckling load is reached.

So, that is what you see always buckling is very closely associated with the buckling load, and buckling load always gives you that. Now, you see this is hidden danger part is there. If you reached up the buckling load you see of a particular strut or column, you are always in the danger zone and if you go beyond that, then there is certain chance of the failure under the dynamic load condition also. So, you see here buckling always occurs immediately at the point of buckling you know like owing to reasons stated earlier that actually whatever the reasons are there due to geometric non-linearity material, or you see when the forces are not exactly on the same axis, and due to that the eccentricity is there. So, all these are the key reasons through which we can say that the buckling make certainly will happen to the strut part.

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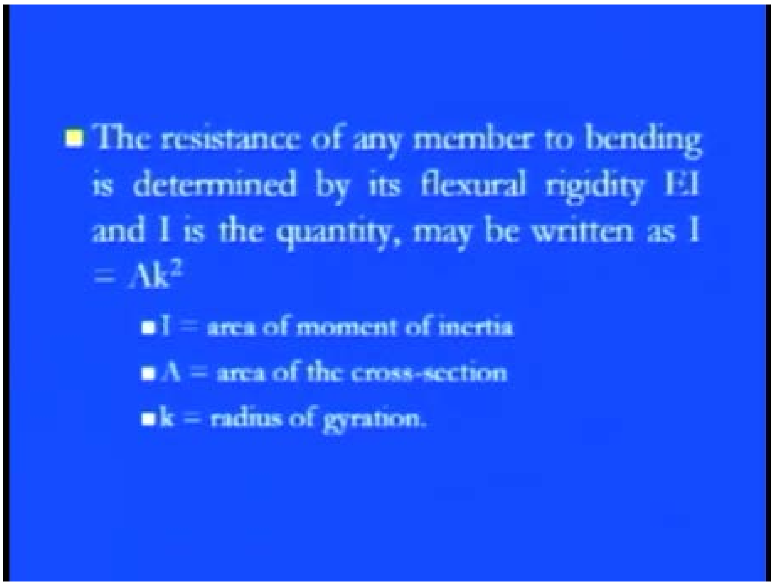

So, you see the resistance of any member to bending is determined by its flexural, this flexural rigidity. The flexural rigidity is always having two main key parameters. One is you see the E, the Young's modulus of elasticity. The Young's modulus of elasticity will give you a clear reason about the elastic deformation once you define let us say some values there. So, once you define the value of E, this will give you that now this material is having this much you know like the relation of the stress and straight in the linear region. If you apply you see the load, and you see under the load application we have the stress and strain, and this you see the relation of the stress and strain is going beyond certain part and certain part means now going beyond the value of E. That means, you see we are now going into the permanent set of deformation. I is nothing but you know like this is the area of cross, this is the area moment of inertia.

So, again you see in that we have to be very careful that actually what exactly you know like the shape of these objectives because it is based on the shape. We know that how the fibers or the layers of these fibers are being set up. So, if you apply you know like the load, let us say the compressive or the tensile or the shearing part, then we know that how these fibers will react because of the shape of that. So, shape is also an important aspect which we are computing you see in terms of I which is nothing but the area of moment of inertia.

So, this you see the whole term E into I is known as the flexural rigidity, and this is a limiting region I should say you see under the load application through which we can say that yeah. This is you see the resistance can be provided by a material. So, again you see this EI is a very sensitive the material which you are using and what is the shape. So, E is the Young's modulus of elasticity as I told you and I is the quantity which can be written as Ak square. This area of the moment of inertia is equal to the area of cross section whether you are using the circular cross section, or whether you are using the rectangular cross section or the triangular cross section. Whatever you see the kind of column or the strut is there, accordingly we can simply use this area, and k is the radius of gyration. That means, you see where you see you know like the mass moment of the center is there, and through which how you see the moment will be acted in this particular way.

So, we could easily figure out that these are some of the sensitive regions, where you see you know like the stability or instability can happen based on the value of k. So, this is you know like EI which will provide you the basic domain for having you know like the material property, or we can say that how it can be withstand of kind of load up to elastic or plastic regions.

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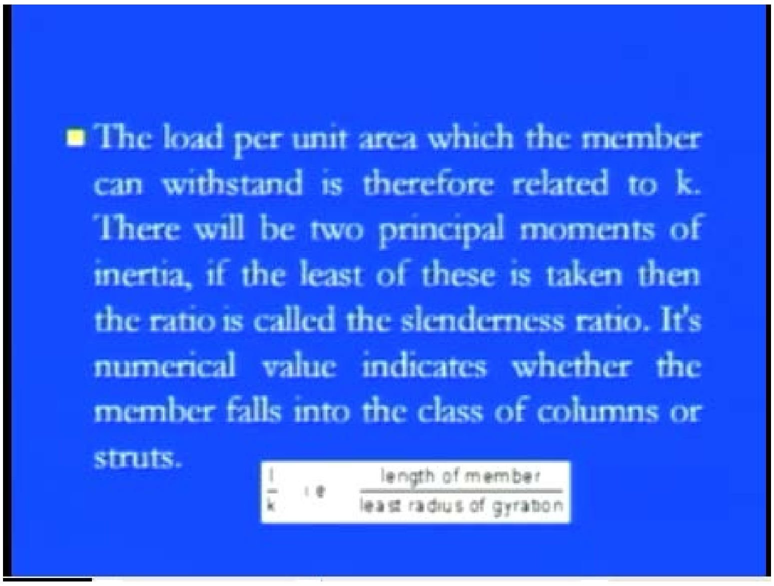

Then, you see the load per unit area which the member can withstand you see the load per unit area for which a member can withstand is therefore related to k which is the radius of gyration, and there will be you see two principle moments of inertia which we discuss if the least of these is taken, then the ratio is called the slenderness ratio. So, you see here one is the mass moment of inertia; one is the area moment of inertia. So, we have to be very careful that actually what kind of load application is there, and whether we are going in the mass. That means you see mass is the influencing property, and due to that the inertia forces are dominating in the nature and then, you see if we want to calculate the stresses due to that, then we have to be very careful about the mass distribution and the mass moment of inertia, but like in previously you see if we are talking about you see that what kind of shape is there.

So, the flexural rigidity we go through which we are defining those you see because it has an isotropic material property, and you see the area is under dominant parameter in those kind of things. Then, we have to be very careful to check out the area moment of inertia in those cases. So, you see here if we are taking at least you know like in one of them, then the ratio is called the slenderness ratio of these, and you see it is a numerical value which indicates whether the member falls into the class of column or the strut. So, you see this is the another criteria to check it out whether you see the member is going because you see as I told you, the dimensional parameter is very important.

So, you see we are computing the dimensional parameter through you know like the slenderness ratio. So, 1 by k that is you see the length of member divided by the least radius of the gyration. That means, what is the minimum radius is there of the gyration through which you see you know like if you are dividing those things, it is known as slenderness ratio. So, based on the value of slenderness ratio, we could easily figure out that whether the member is falling in the column or whether the member is falling in the strut part. So, you see here this was the prime theories, you see that what exactly the meaning of the column and the strut is, and again you see when we describe about the column and strut, we could find out that actually you know like the buckling is their key phenomena.

So, that is what you see what are the basic reasons of the buckling which we discussed and then, you see we found that again. Not only buckling is the criteria to differentiate out the column and the strut, but also you see slenderness ratio, which is based on this radius of the gyration is also one of the key factor through which we can say that this member is now falling in the column or this member is falling in the strut. Now, you see here we are going in the Euler theory.

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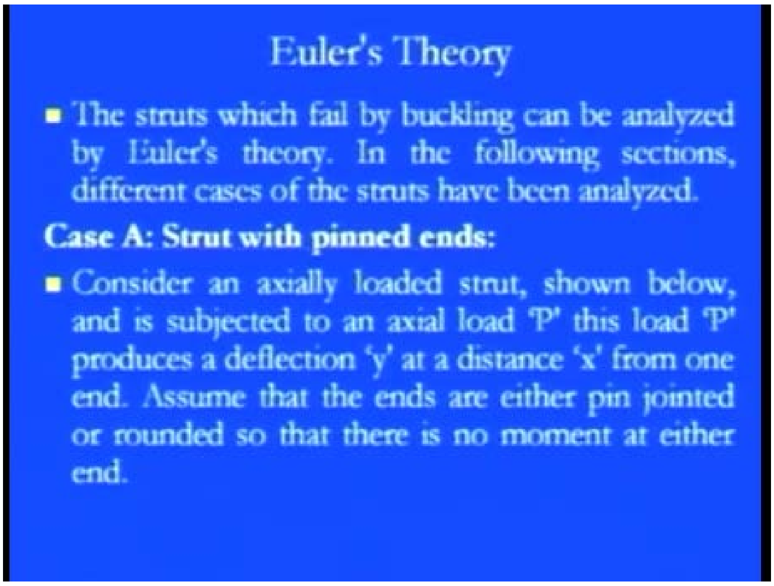

So, you see here the strut which fails by buckling can be analyzed by Euler's theory. So, Euler is given the theory through which we can say that yeah what is the basic reason, how we can analyze that region using the Euler theory. In the following sections, now the different cases of the structure you know like the strut have been analyzed. So, what we are doing here now as I told you that you know like what the boundary conditions are there, and through which now how the buckling will happen and how we can stop once we know that now this is we have the member and this is under these conditions.

So, we could easily limit the conditions through which we can say that now this is before the buckling load, and this is the buckling statement is there. So, in that category you see in the Euler theory, now we have taken the first case as the strut with the pinned end. That means, you see it is a kind of you know like the beam with simply supported part. So, here what we have? We have the strut with the pinned ends at the extreme ends of these struts. So, consider an axially loaded strut. Now, you see the loading is simply axial towards you know like the longitudinal part shown below in the next figure is subjected to an axial load P, and this load P produces and deflection. Obviously, when you apply the load along the particular action, this axis we have you see a kind of deflection which can be measured in terms of y as a distance and x from the end.

So, you see here we are taking you know like since you see this strut is always lying in the x axis, so always whatever the deformations or whatever you see the flow of this strut is there, it is always along the x section and x axis, and the deflection is there which can be measured in terms of y axis because you see both ends are pinned. So, when you apply the load in terms of you know like the compressive or extension, you see whatever the deflections are coming, they always come in the perpendicular direction. So, you see the deflection can be measured in the wide axis. Assume that the ends are either pin joint or you see rounded, so that it is you see either you can represent by any of the way. You can simply say that these are the pin joint. It says that actually it has only one degree of freedom which can go in x direction or we can say you can simply show by you know like the dot part or rounded part, so that you see there is no moment at either end.

So, that is generally you see we are using that when simply supported beam is there, the supports are there from the bottom part, but here you see we are saying that you know like these are the pin joints. So, this strut is just you know like hanged under the pin joint action, and the load application is there towards the axial direction.

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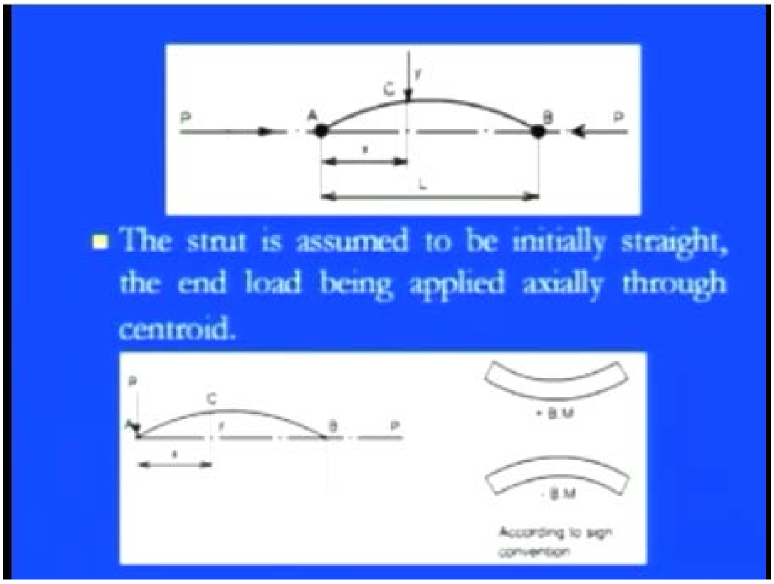

So, you see here in front of you have this particular diagram which I have described in the previous one. You see here this is the strut, and these two extreme joints are there. The pin joints are there which is you know like showing and the rounded one, A and B and you see you know like these axial forces are being acted, and due to that since it is in the compression nature, you see here we have a kind of deflection in this way you see the curvature shape and we are measuring at the x distance. We have you know like the deflection is y and the total length of the strut is L. The strut is assumed to be initially straight, that is the very common phenomena you generally see in the beam or in any of the slender member. We are assuming that initially there is no deformation. There is nothing within the structure of the beam.

So, it is well stabilized esthetically equilibrium part. So, it has an initial straight part and the ends loads being applied axially through the centroid of the beam, the strut. So, you see here if you look at this point, then what we have? We have the same. You see you know like at the point C, where we can say the maximum deflection is there of the y and again, you see if we are talking about this bending moment. Then, according to the sign convention if it is you see the concave nature and if it is a convex nature, then you see the corresponding plus and minus signs are there for these convex on concave shapes of the beam.

So, you see here you know like the sign conventions which we are using for the strut also. If it is going in this direction you see, then we have the positive. If it is going in the convex direction, then we have the negative part. So, you see either one of them you see if it is going, then we are always going for plus and minus, but in this case, you know like we have you know like this shape. So, obviously the next one which is the minus bending moment is there. So, we have to adopt that particular sign convention.

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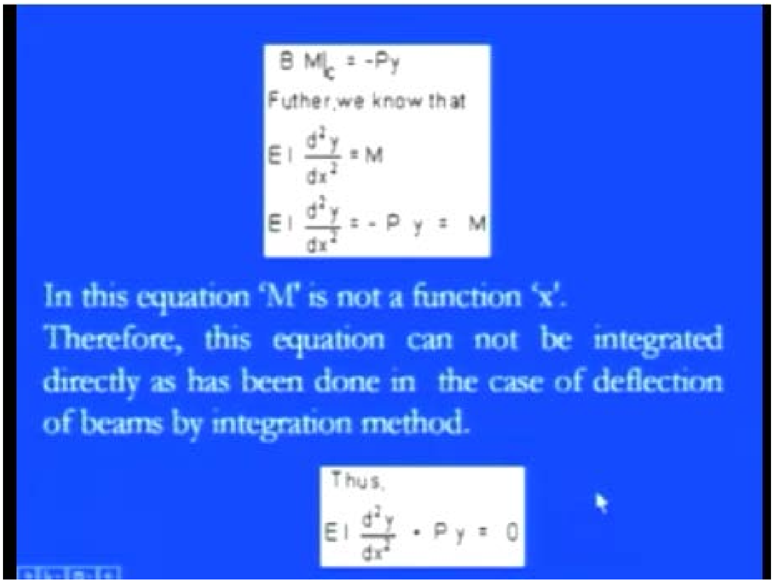

So, you see the bending moment at point C you know like in the previous figure as I have shown, you at the x distance where the point C is there, the deflection which we assume is the y. So, bending moment is obviously the load application P into y, and you see here you know like by simple bending theory, we know that we have the simple bending equation using direct integration method which we discussed is nothing but equals to M equals to EI into d 2 y by dx square. So, again the same theory which we applied here, EI d 2 y by dx square equals to M and M like if we are calculated at point C, this is minus P y.

So, again if we are replacing this M here, we have EI into d 2 y by dx square equals to minus P into y, and as I told you that minus sign will be there according to the sign convention. So, this equation you see M is not a function of x because you see when you applied the load, there is a great chance of deviation in the deflection in terms of only y. So, you see here whatever the change will come, it is a function of y only. Therefore, this equation cannot integrate directly because you see here whatever this domain which we have taken d 2 y by dx square, it is in the domain of the x, but since you see here this M is not the function of x. So, we cannot directly integrate those things.

So, you see here the direct integration method is not applicable in this case and then, you see the deflection of beam can be you know like using the different part. So, here as far as this equation is concerned, EI into d 2 y by d dx square which is there in the x domain plus P y equals to 0 because you see there is no direct integration in that way.

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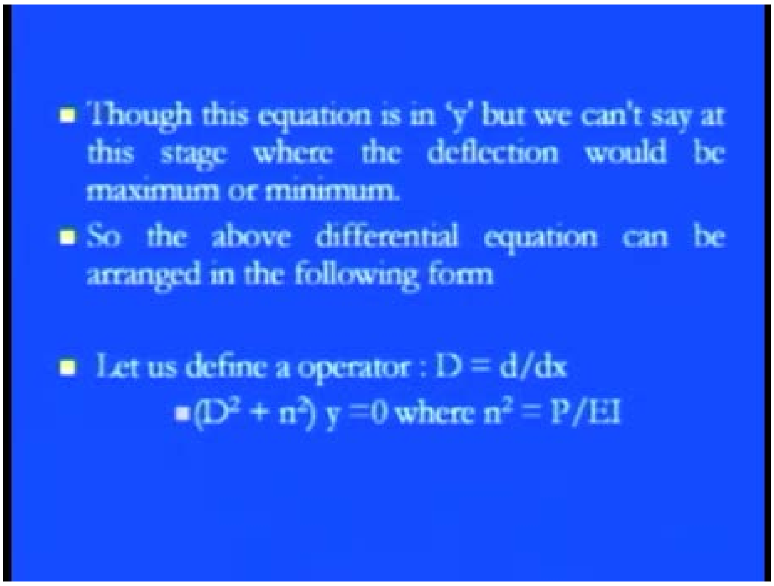

So, though this equation is in y, but we cannot say you see at this stage where the deflection would be maximum or minimum. Because you see here we never know that actually how the forces will act on that particular part, and what is the basic area through which we can say that yeah this is the potential area, where the maximum or minimum deflection is. So, the above deflection equation can be you know like arranged in the following terms. So, you see here now we are rearranging the equation, so that you see we could easily figure out that what are the potential areas where the maximum or minimum deflection is there. So, for that you see now what we are simply using in an operator is d by dx is an operator. So, if you put at that part, particular part, then we will find that this D square plus n square into y will be equal to 0, where n square is the new term which equals to P by EI. P is the applied load, and EI is the flexural rigidity. So, with using of this operator and with using of the term n square, now you see we have the new term.

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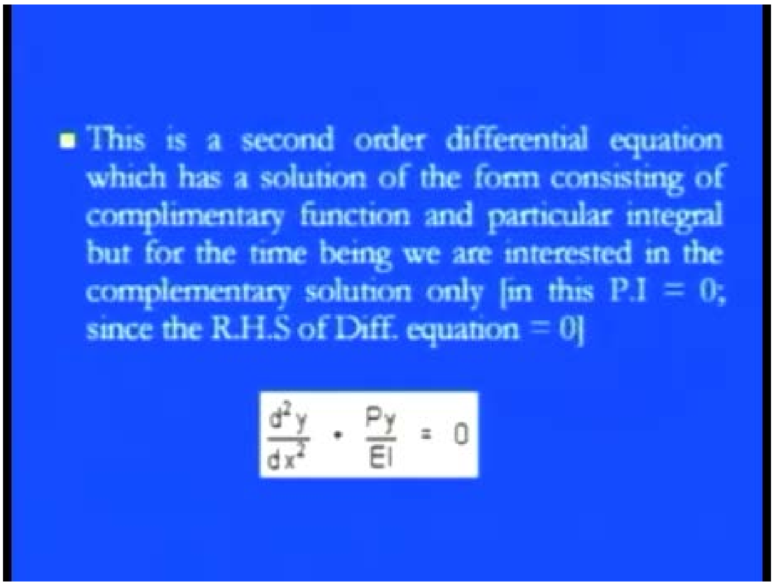

This is the second order differential equation D square plus n square into y which equals to 0 which has you see you know like because this D square is there. So, obviously, it is d 2 by dx square. So, we have a second order differential equation which has the solution of form of consisting complimentary functions and a particular integral because you see you know like whenever we are talking about a linear second order differential equation, we have a CF and PI which is very common you know like phenomena to calculate those things, but for time being you see what we are doing here. We are interested in the complimentary function because you know like in that case, we have PI equals to 0 in you know like the differential equation is equating to 0.

So, you see here come to the main equation, what we have? We have d 2 y by dx square which was you see d square plus P into y by E y because you see the n square was there. So, d 2 y by dx square plus P y divided by EI equals to 0. So, now this is you see the second order differential equation.

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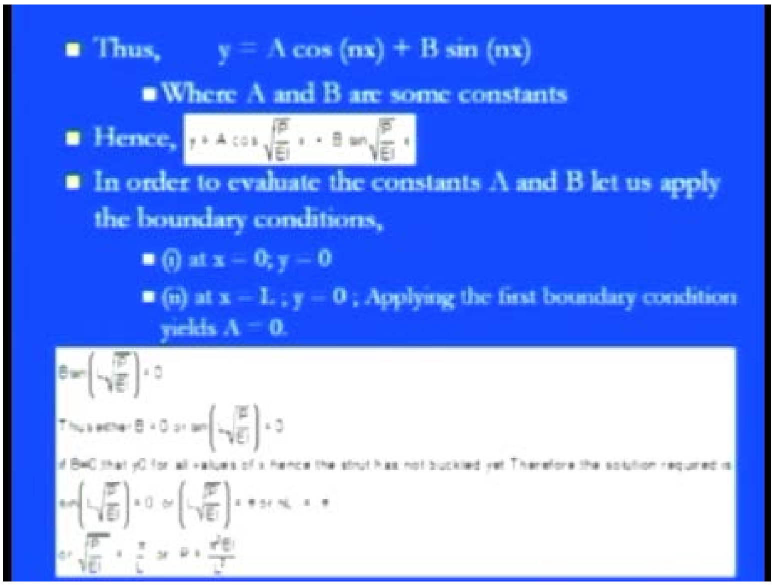

So, now you see our main intention is to find out the complimentary function. So, you see here simply you need to this y equals to whatever the displacement, the deflection is there in the y direction y equals to A cos nx plus B sin nx, where A and B are you see it is an sinusoidal form. It is in a periodic form. So, we can simply replace. This is very common. The standard solution is there for any kind of second order differential equation. So, this y which is the deflection input, deflection is there and we have A cos nx plus B sin nx.

So, you see here with the application of all those load conditions, what we have? We have y equals to A cos square root of P by EI into x. So, you see you know like n square as we discussed A was nothing but equals to P square by EI. So, if we can simply replace this n by square root of PEI, so we have y equals to A cos square root of P divided by EI into x plus B sin of square root of P over EI into x. In order to evaluate the coefficients A and B because they are you see coming according the equation. So, we need to find it out what exactly the meaning of this A and B is. So, we need to apply the boundary conditions of the strut. So, you see here if you look at the strut, we have the pin joints. So, at the pin joint you see you know like in simply supported beam, we know that there is no deflection when the pin joints are there.

So, you see at x equals to 0, we know that there is no deflection. So, y equals to 0 and at x equals to L means the extreme other corner you see there is no deflection. So, y equals to 0. Also, in that case applying the first boundary condition, where x equals to 0 and y equals to 0, we simply get A equals to 0 here where you know like x equals to 0. So, it is cos 1, so cos 0 is 1, but sin 0 is 0. So, obviously, A equals to 0. Now, you see here we have B sin like when x equals to L with another condition, where you see x equals to L and y equals to 0. So, you see here what we have since A has already gone? So, this A cos n L will go out and we have B sin square root of P by EI into L is equals to 0, and you see you know like we can say that either B is equal to 0, or another function which is sin into L into square root of PI is equal to 0.

So, either of that you see if we are applying this B equals to 0, then obviously you know like y 0 which is the initial means you see the initial condition values will come, and you see you know like the strut will not be buckled yet because you see it is the initial condition. Therefore, the solution required in that particular way the sin of you see L into square root of PI P over EI is equal to 0 because we always want B initially. We have already assumed that there is no change in the dimension is there, or the straight part is there. So, there is no you know like importance. So, there is no worth to keep the B equals to 0.

So, obviously the next term will be equal to 0. So, we have sin of the other whole term is 0. So, we have L into square root of P into EI equals to pi or we can say you know like n into L, so the pi. So, you see here if we are keeping those things, then what we have? We have square root of P into P over EI equals to pi by L, or we can say that the load where the buckling point is you know like there, it is equal to pi square EI over L square. So, this is the load condition through which we can say that again it depends on the pi. The constant term EI is the flexural rigidity and that is what you see in the previous slides. I told you that EI is very important phenomena, and the another important phenomena is the length. What is the length is there on of the strut part. So, we can see that this EI which is an important parameter as we discussed in the previous slide.

So, again you see here in the load particular to check it out, the buckling load, the EI will be again the flexural rigidity will be considered, and the L is an important phenomena because you see this is the dimensional parameter through which we can say that whether it is categorizing the strut or the column. So, the L square is there in the term denominator.

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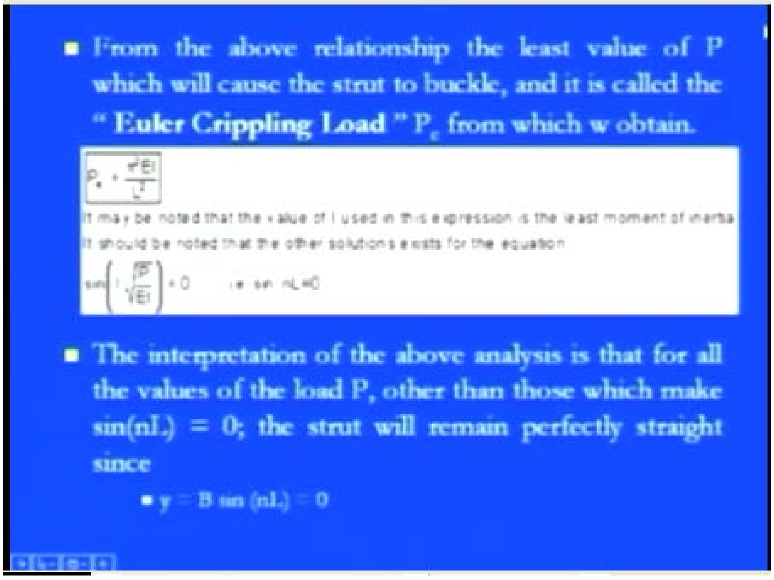

So, from the above relations you see which we have discussed, the least value of P which will cause the strut to buckle is you know like called the Euler crippling load. That means you see you know like this is the limiting part through which we can say that if we go beyond this, then the buckling is certainly there. If you keep you see the load application within that particular part, we have like there is no buckling in that. So, that is why you see since this is the limiting case for the strut, so it is known as the Euler crippling load from which you see we can say that the PE which is the Euler crippling load equals to pi square EI divided by L square, and it may be noted that you see the value of I which is you see like in the expression is the least moment of inertia should be you know like taken perfectly. So, that is what exactly the structure of this particular beam is there corresponding because this is the area moment of inertia.

So, we have to be very careful that what exactly the shape of that particular column is, and it should be noted that other solution exist in the equation is also you see if it equals to 0, sin nL is equals to 0. That means, you see when we are keeping you know like this square root of P by EI with M, so you see here we have sin A of nL equals to 0. So, we can interpret these above analysis in that sense that when the load within the value of the load if other than those you see you know like the values which make sin of nL equals to 0, the strut will remain perfectly straight. Since, you see you know like we know that this is one of the conditions, where we have sin of nL equals to 0 or we can say that Pe equals to pi square EI by L square.

Here you see there is a chance of the deviation. If you go you know like beyond that, there is a chance of the deviation of the shape of the strut, but other than this situation, we have you see in almost all kinds of load, where the sin of nL not equals to 0. We can say that the strut will perfectly remain in the straight position, and we can say that sin y of which is B of sin of nL which equals to 0. We can say that now in this condition you see we just try to avoid.

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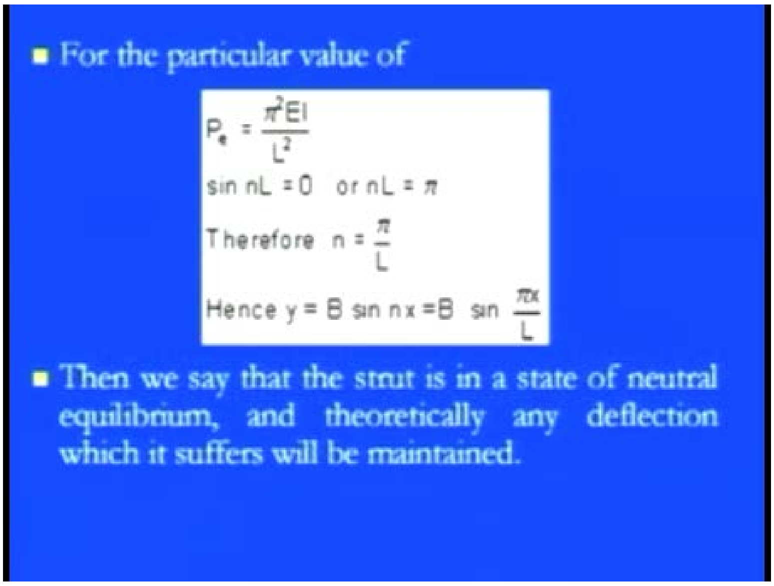

So, if for a particular value and in that we have Pe equals to pi square EI by L square or we can say that in another form sin of nL equals to 0 or nL equals to pi. So, we have n which is you see you know like the square root of P over EI equals to pi by L. So, we can say that another solution for you know like this for a particular value, y equals to B of sin of nx or we can say that B into sin of nx pi L. So, you see here what we have? We have another solution by you know like keeping that what exactly the meaning of the load calculation and to check it out, the limiting part of buckling in that. Then, we can say that the straight strut is in the state of natural equilibrium and theoretically, any deflection which you know like it suffers will be easily maintained without any change of its straightness. We can say that these are the limiting conditions to keep this phenomena in the naturally equilibrium.

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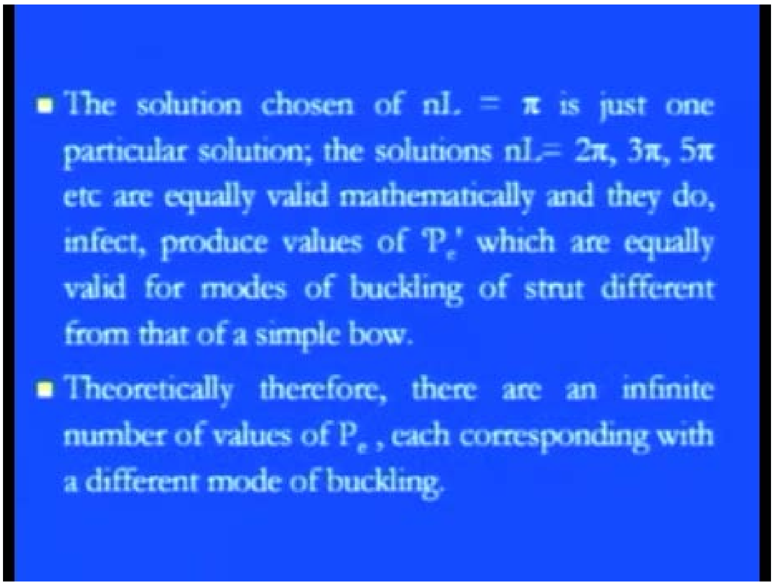

The solution chosen for this you see which we have sin of nL is equal to 0. So, we can say that this nL is equal to pi is just one particular solution. The other solution which may be you see where you see the sin of nL is equal to 0 is nL equals to 2 pi, 3 pi, 5 pi and each are you know equally valid mathematically, and they do effect like infect, produce the same values of P e, which are equally valid for modes of the buckling of the strut at different from that of a simple bow. That means, you see here as we go in a further analysis, then we will find that you see there are another modes, where there are you see the different kind of just like it is a simple phenomena of vibration, where we know that at first natural frequency, there is one mode. At another natural frequency, second or third or fourth, you see we have different vibrational modes.

So, similarly you see here when we know that nL equals to pi, where you see the buckling is there or simple mode is there, but as you are going in 2 pi, 3 pi, 5 pi, then we have a different you know like the model parts are there. That means, you see we have different modes of these struts is there. Theoretically, therefore, you see there are infinite numbers of values of P e, each corresponding with different mode of buckling. So, that is what you see here if we have an infinite means n number of you know like the value of Pe is there, there are n number of modes of buckling within that structure, and that is why you see you know like we can say that once we calculate the Pe, that means, it is if if we are calculating EI and L square, these are the significant terms for calculating the buckling in terms of the first mode, second mode, and third mode. Then, if we are going in a higher range of mode, certainly you see you know like the failure chances are great and we can say that there are certain parameters, which are very sensitive to go in a higher range of modes of the buckling.

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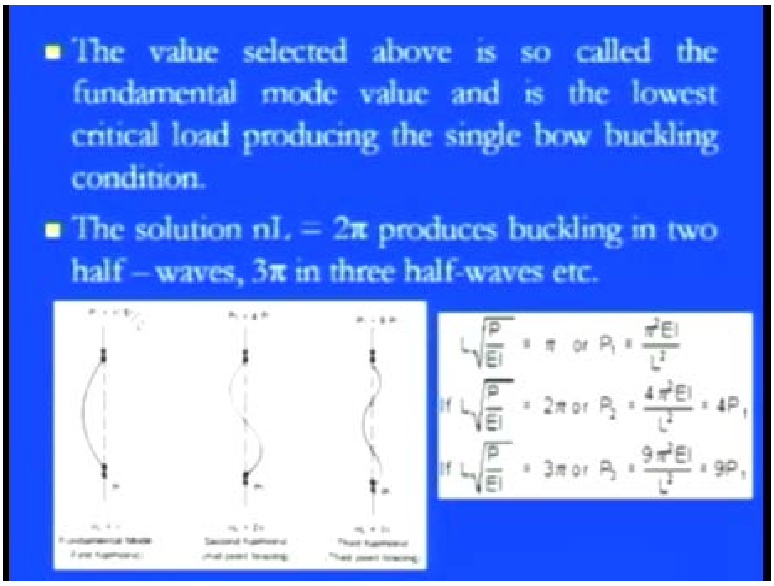

The value selected above is so called fundamental you know like the mode value, and it is lowest critical load producing the single bow buckling condition. Obviously, you see when we know that just like in the vibration, we have the fundamental frequency or you see you know like the first mode is there at the particular thing. So, same thing here when we know that at this particular first mode, where nL equals to 0 or we can say nL equals sin of nL equals to 0 or nL equals to pi.

This is the first mode. We can say the fundamental model value and this is also known as the lowest critical load Pe, which is pi square EI divided by L square is the fundamental crippling load is there at the lowest part, and at this particular part, we have a single bow buckling as you can see in this particular part. So, you see here at nL equals to pi, what we have is single you know like the bow is there and only one deformation is there which is P 1 is pi square EI divided by L square. Second, you see here now if we are keeping the solution as I told you that for higher buckling modes, then we have nL equals to 2 pi produces buckling in a two half-waves. That means, you see you have two different vibrational modes at these particular points.

So, similarly you see what we have, we have you know like the P, the P 2 which is nothing but the four times of P 1. That means, 4 pi square EI divided by L square always giving you two different waves. So, we have two halves of that. At one you see the perfect you know like the wave is there, and another one, it is second form. So, you see here both the nodes and antinodes are being formed and you see the orthogonality conditions are there in different modes of the buckling. Similarly, you see if we are keeping the 3 pi which equals to nL, so obviously P 3 is nothing but equals to 9 pi square EI divided by L square because now it is in terms of the square terms.

So, what we have now? We have you see the three different modes are there. So, you see the third harmonic terms are there. Harmonic means you see the multiple of these natural or we can say the mode of this buckling. So, you see here if we are talking about the second, then we have the second harmonic. If we are talking about the third-one, nL equals to 3 pi, then we have the third harmonics. This in critical load is there at the buckling condition. So, you see here we have three shapes in which all the nodes and antinodes are being formed at these conditions, and you see here, this crippling load is there which has the value of 9 into pi square EI divided by E pi square EI divided by L square.

So, now comparing those terms, we know that this nL which is equal to at pi in the first condition, we have n which we need to replace. So, L into P square root of P over I is pi. So, we have P 1 which is pi square EI by L square. So, in the second case, you see we can simply see that when this nL which equals to 2 pi, then if we are keeping this n which is you know like square root of P over EI and by keeping L, we have L into E square of P over EI which is equal to 2 pi. So, we have P 2 which is four times of P 1 or we can say 4 into pi square EI over L square. Similarly, for third mode where you see this nL equals to 3 pi, we have P 3 which is 9 times of pi square EI over L square or we can say that 9 times of P 1.

So, as you move further, that means, you see if we are going for higher modes of the buckling, we have always you see the n square terms. That means, you see if we are going for the two or three or four, you see here always it is going in terms of geometry, geometrical progression 1, 4, 9, 16 like that. So, you see here we can say that whatever the shapes are coming, that means, whatever the wavy features are being approached to the strut, it is very hazardous because you see all the layers are in the crippling part. So, we have to be very careful to design those buckling just to avoid going in the higher modes you know like for a failure analysis.

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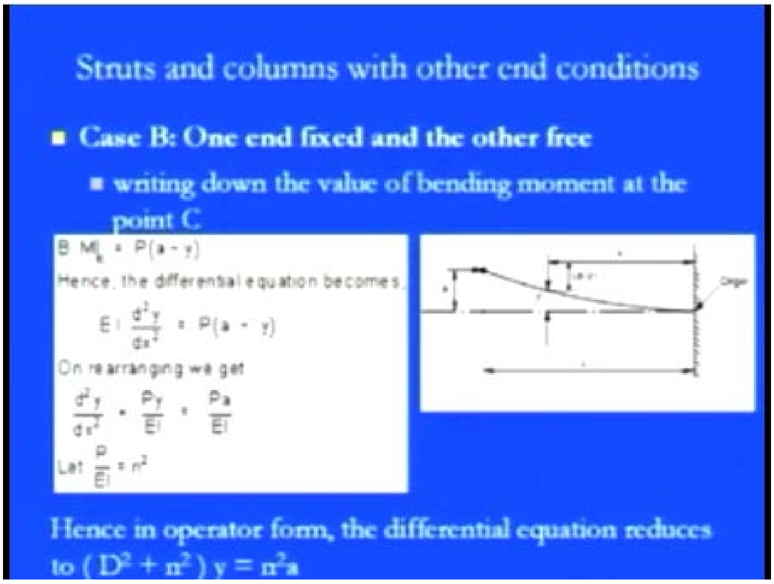

Coming to the second case, where you see you know like we have one end is fixed and the other end is free. That means, you see you know like just we have a rigid end is there and one end is absolutely free. So, you see here simply we need to write the equations at point C against the same case, which we have taken here. The deflection is there in the y direction with the load application P you see here at these particular points. So, here you know like this P, the applied load is there. This is the fixed part. So, we can say that the region rigid part is there. So, it is sort of you know like the column which we have discussed. When you apply the load, you know like it is a vertical deflection is there. So, we can say that the maximum deflection can be always coming at the free end.

So, we have let us say the deflection and the y deflection is there at the x, you know like x distance. So, we have you see the x distance from the free end. So, this is there. So, the maximum bending moment you see here at, or we can say the bending moment at point C with the difference of these two which is nothing but equals to P which is the load application into a into y. So, again you see after the application of those basic equations of the bending moment, we have E into d 2 y by dx square which equals to P into a minus y. So, just you see again we need to arrange that part. So, we have d 2 y by dx square dx square equals to P over EI plus you know like these P, this Pa divided by this

EI. So, if you rearrange this, then we have the final part is d 2 y by dx square plus E P times of y into y over EI equals to P, P into a over EI.

Now, you see here again we need to compose those things that n square which is pretty common, P over EI is there and by keeping the differential, this originator means if we have you see B which is equal to d 2 over dx square the operator, we have you see in the operator form D square plus n square whole as into y equals to n square a because now you see we have an additional deflection. The free end due to the load condition, and it is obviously we have n square into a.

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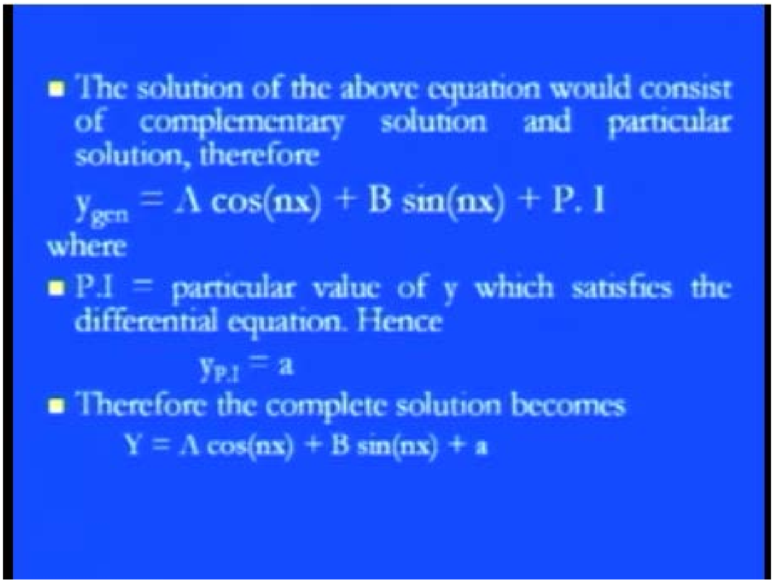

So, now you see with this equation, we would like to formulate the solution. So, the solution of above equation which is d square plus n square as a whole into y equals to n square a. This equation would be consists of the complimentary solution will be having the CF and the PI value, the complementary function and the particular solution. So, you see here for the particular value and the complimentary functions, again we need to generate that particular generalized solution. So, y generalize is equal to A cos nx plus this B sin nx plus PI, the particular integral. So, particular integral value of y which satisfies the differential equation is always you see you know like it is free end is there.

So, P y of particular integral is equal to a, which you see has the free end amplitude. So, therefore, you see we can simply write the complete or the generalized solution of the equation above y equals to A cos nx plus B sin nx plus this A because the PI has a value of a.

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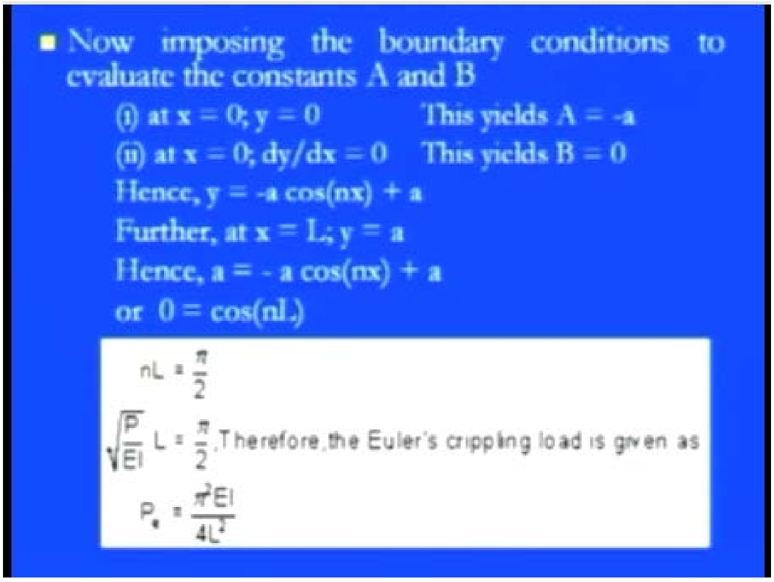

Now, you see we need to impose the conditions of the boundary because we have one end is rigidly fixed up and one end is free absolutely. So, obviously, you see now we have different conditions altogether here. So, first if we apply x equals to 0, where rigid end is there. We know that there is no deflection because of the rigid fixed end is there. So, you see here when we keep that y equals to 0 at x equals to 0, we have the value of a coefficient a, capital A equals to minus a.

Now, you see here if we are coming to the same condition, where x equals to 0 is the fixed end. So, we obviously have the slope 0. So, again we can say that at x equals to 0 dy by dx is equal to 0 which will give you another coefficient value B equals to 0. So, by keeping those conditions, we have the generalized complete solution of the equation y equals to minus a cos nx plus a, and you see here another condition which we can apply here at the extreme free end. So, you see here at x equals to L, we have the free end. So, deflection will be equal to a, which we have already computed that part. So, by keeping that value we have a, which is equal to minus a cos nx plus a. So, if we are keeping at x equals to L, we have this y values a.

So, you see here cos of nL is equal to 0, obviously see cos is always 0 at pi by 2. So, nL is equal to pi by 2 n is we know that this n square is P over EI. So, square root of P over

EI into L is equal to pi by 4 pi by 2, sorry therefore the Euler crippling load for this condition, where the one end is fixed and one end is free. The Pe is equal to pi square EI over 4 L square. So, you see here now by applying the different boundary conditions, the crippling load is absolutely different, but the main parameters are again the same. EI is E, EI and L are the responsible parameter for these kind of conditions.

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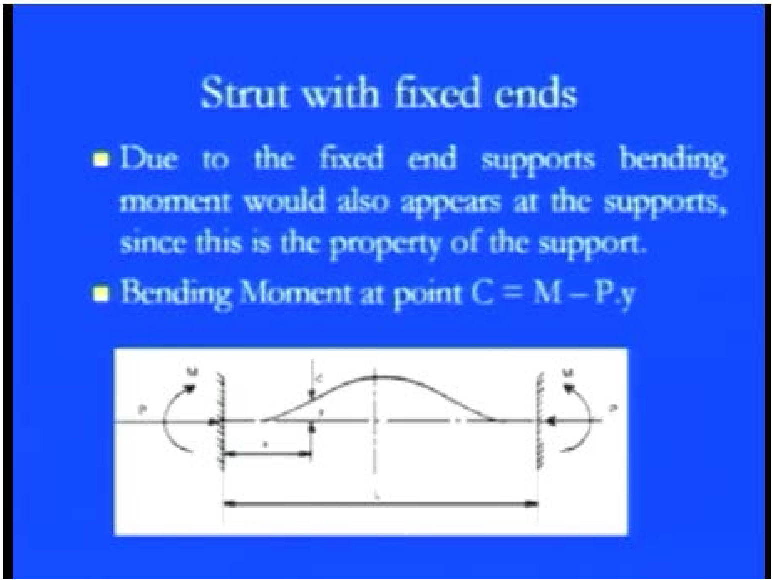

The third part is when the strut with the fixed ends is there. That means, you see here we have a strut instead of the one end, free both ends are fixed. So, due to the fixed end supports bending moment will also appear at the supports, since this is the property of the support. So, they will apply simply the moment because this is not the pin joint. They have a rigid end. So, obviously, you see the bending moments are there. So, we need to consider. So, you see here the load application is there at the two extreme ends, and because of the load application, these have the fixed rigidly supports.

So, you know like this will be you know like uniformly distributed, and we have you see the deflection like in these particular y direction, and if we are computing at point C from the x distance you know like from the one end. So, we have the y deflection is there, but it is also having the bending. This moment is there at this particular extreme end. So, we can say that the bending moment equilibrium at point C is nothing but equals to M at the extreme end minus P into y.

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So, you see here with that consideration, again you see the same equations which we need to apply E into d 2 y by dx square which is equal to M minus P y. So, this is the equivalent you know like the bending moment is there. Again what we have? We have the same equation d 2 y by dx square plus you know P over EI equals to M over EI. Now, we know that n square is nothing but equals to P over EI, this P over EI. So, we need to convert this in the operational of the operator form. So, we have D square plus n square into you know like y is equal to M over EI. We have you see like the two main things. One is the complimentary and one is the particular integral form. So, you see here y PI is nothing but equals to M over n square EI plus M over P.

So, you see here with that we can simply replace this thing in terms of the complete solution or we can say the generalize solution. So, by keeping y PI which is nothing but equals to M over n square EI or we can say M over P. Here we have y equals to B cos nx plus A sin nx plus you see here M over P, or you see with applying the boundary condition because both ends are rigidly fixed up. So, at x equals to 0, you see we have y equals to 0. So, we have B which is equal to minus M over P. So, you know like by keeping that condition, what we have? We have simply one coefficient which has a value of minus M over P and by keeping at x equals to 0 because there is no slope. Because of the rigid boundary conditions, we can say that another coefficient a is equal to 0, and then you see what we have is y which is equal to M over P 1 minus cos of minus nx.

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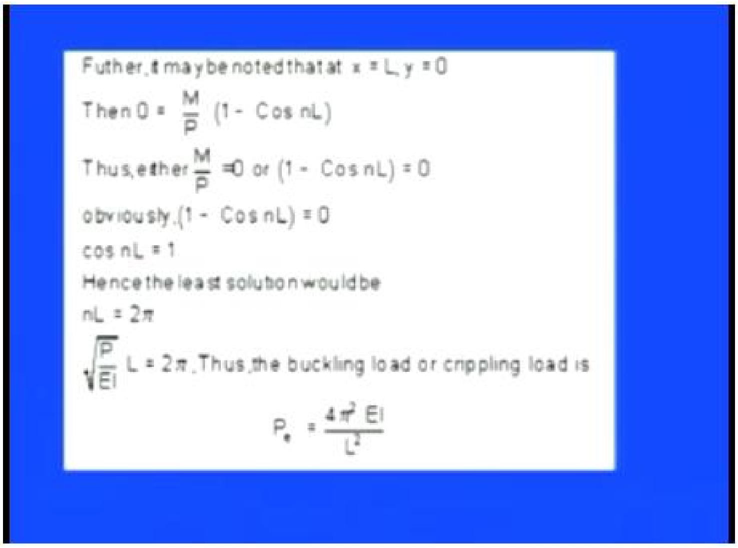

So, you see here it may be clearly noted that at x equals to L, that means, again another fix rigid end, there is no deflection as well as the slope. So, you see by keeping those conditions, we can simply figure out that we have M over P into 1 minus cos of nL is equals to 0. So, either of the term is 0. So, we can say either M over P is equal to 0 or 1 minus cos of nL is equal to 0. So, obviously, you see the moment and the load cannot be 0 because due to that, only the deflection is coming. So, we can simply ignore this condition. So, we have the final condition 1 minus cos of nL equals to 0 cos of nL is equal to 1. So, obviously, you see it is equal to 2 pi. So, we have square root of P over EI into L is equal to 2 pi.

So, buckling load or we can say this Euler critical load, when both ends are fixed are Pe equals to four times of pi square EI over L square, so it is you see here when we have you see both ends are fixed, the pin joints are there, then we have pi square EI over L square. When one end is free, then it was one end was rigidly fixed up, and the P load is there, then the crippling load was there pi square EI over four times of L square, but when both ends are rigidly fixed up, then we have Pe is 4 pi square EI over L square. So, now you see here we are going to discuss that when one end of these pin joint and one end is free, then what will happen. So, all you see you know like the different boundary conditions are to be you know like incorporated, and we just want to describe the crippling load, so that you see whenever the realistic applications are there, we can avoid this buckling condition in the way to safely design about the strut part.

So, in this you see the lecture we discussed about the main strut and column, and also you see that what the importance of the buckling load is there, and what are they influencing or we can say the dominating parameters are there in that. Also, you see what are the root calls for buckling and apart from that, we also discussed about the main boundary conditions in the realistic way that if we have both ends rigid or both ends are pin joint, and if one end is free and one end is rigid, then you see what the value of the crippling load is. So, in the next lecture, again we would like to check this crippling load for the different boundary conditions and then, you see you know like we will go for the numerical problems of that.

Thank you.