Strength of Materials
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Lecture - 35

Hi, this is Dr. S. P. Harsha from Mechanical and Industrial Department IIT, Roorkee. I am going to deliver my lecture 35 on the course of The Strength of Material, and this course you see you know like developed under the National Program on Technological Enhanced Learning.

In this lecture basically we are trying to focus on the theories of failures in which you see there are various theories based on what kind of strain energy is there, what kind of stress part is there, the principle stresses are there or the maximum shear stresses are there. So, based on various aspects of the stresses which are inducing in the object, we are trying to analyze the theories of failures and you see who ever is given this kind of theories, the name is there. So, these theories are basically named with the inventor of these theories, but prior to discuss these things, actually we would like to briefly discuss about the previous lecture.

In previous lecture, we basically discussed about the beam analysis and one of the method which we discussed about the bending moment or shear force was the Macaulay method. In that we know that if simply supported beam or cantilever beam is there, and if it is subjected by different kind of loadings, then how we can get you see and in these kind of loadings where different spans are there you know like the different segments are there, then how we can do the analysis. So, in that the basic theme was to give a generalized singular, generalized expression with the similarity function. So, you see you know like we derived those parts, and we have in the last lecture, we have given the kind of examples in which you know the couple is there, only pure couple is there in the simply supported beam or cantilever.

Then how we can get the singularity function or how we can express those things, and how can we get exactly that which part of the beam is subjected by the maximum bending moment or shear force, and then we analyze about the UDL and a point load and both are acting like in the different segments of a beam of simply supported beam. Then, how we can get you see the expression first of all and then, the maximum potential area about this bending moment as well as shear force.

So, these kinds of two examples we discussed briefly in the previous lecture and then, we discussed about the key part that actually if a beam is subjected under you know like the bending as well as the shear. That means, you see when the bending stresses and the shear stresses, both are occurring together, then what kind of combine part is there, and you know like if you remember quickly, then you will find that in the first few lectures, we discussed about you see when a beam is subjected by normal stress as well as the shear stresses. Then, how we can get the principle stress? So, similar kind of discussion we made in the previous lecture also, that when a beam is subjected by bending moment as well as the torsion is there or a couple is there or any kind of moment is there. That means, due to that we have a torsion stresses and then, how we can get a principle stresses you see on the principle planes, and how we can get the maximum shear stresses out of that.

So, you see here if you focused purely on the bending moment, or you see the bending stresses are there, then we found that the equivalent which means you see here when you know like the bending moment is there and the torsion is there, if we you know like designate by M and T, then we have the equivalent bending moment in that particular beam which we calculated you see by half of M plus square root of a M square plus T square. And similar you see we also discussed about the equivalent torsion stresses are nothing but equals to square root of M square plus T square.

So, these two discussions, these two conclusions which was made and then, you see finally, we discussed about when a shaft is there and you know like it is in between, it is in the connection of the parallel or we can say it is in the series part, then you know like how you can get the torsional stresses in both of the connections, and what is equivalent, the torsional stresses are there or we can say shear stresses are there. Then, you see we can see clearly visualize that which area is maximum, which area is influencing maximum under the action of these shear stresses or we can say the bending stresses. So, this kind of discussion was made in the previous lecture.

So, you see as I told you, you know like in this lecture we basically focused on the theories of elastic failure. That means you see if we have you know like the elastic deformation, then there are various theories based on what kind of stresses are being inducing, or we can say rather that what kind of point of application of the forces are there on these particular beam structure. Accordingly, we can simply get the kind of theories of failures.

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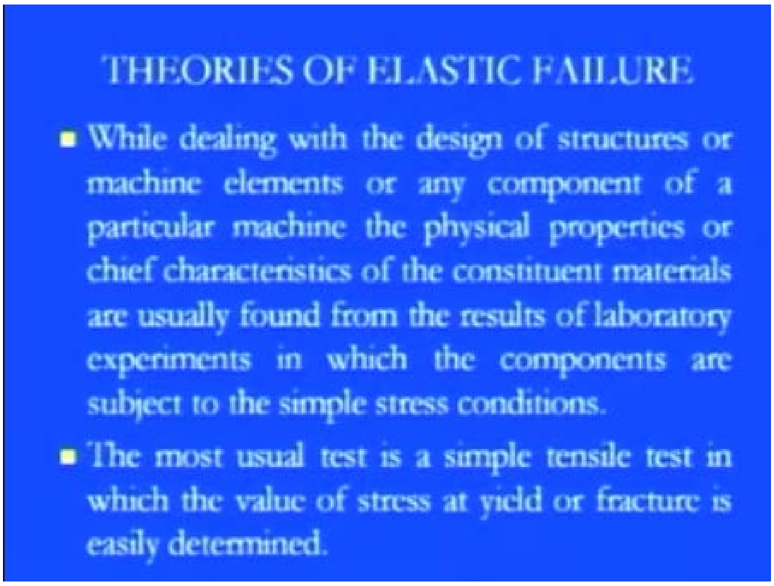

So, while dealing with the design of the structure, or we can say the machine element or any component of the machine part, or we can say you know like this kind we are talking about the bearings or the shaft or gears, so any kind of the part of this particular machine. The physical property of the chief characteristic of these constituent material means whatever the materials are there which is associated with that particular machine, this component are usually found from the results of laboratory experiments in which the components are subjected to a simple stress conditions. You will find that this is a pretty common machine, this UTM, this ultimate tensile stress machine.

So, UTM is a pretty common machine through which you see we can simply get that if a material is subjected by a tensile or compression or bending or twisting, whatever you see here and based on these laboratory experiments, we can simply segregate the limits of yield limit or we can say the proportional limit or you know like the tensile limit. The maximum tensile limit or the elastic and plastic deformation can be easily as separated out from these things. So, when we are talking about the elastic failure, our main focus is that actually when you know the load application is there and you know in the laboratory when we are like this material are being subjected by these kind of loadings, then what the theories are there or what this associated forces are there which simply you know like which are the basic cause of the failure.

So, the most usual test is simple tensile test as I told you in which the value of the stress at yield. That means, you see the elastic, our plastic limit, this is the main you know like the region is there through which we can simply segregate that. Below that we have the elastic limit and beyond that we have the plastic limit, or we can say the fracture that is a last part means after that ultimate tensile strength, where the fracture is easily determined. So, that is the basic you know that is why we are always trying to conduct, or we are always trying to equate you know like whatever the materials are there with these particular laboratory tested.

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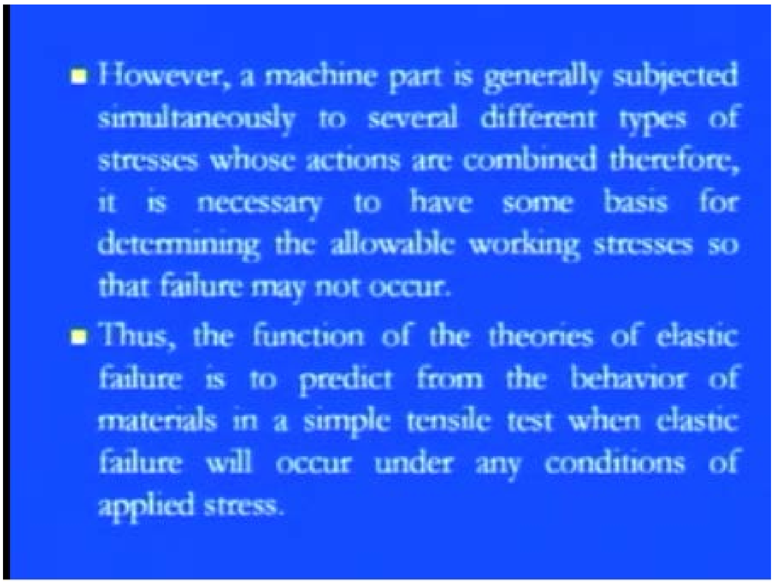

However, a machine part is generally subjected simultaneously to several different types of stresses because you see when a machine is vibrating or when machine is running, we know that there are various interactive forces, which are transmitting from the various objects when the machine is running. That means, you see we have a shaft bearing arrangement through which you see the rotation is there, and when we are saying that this machine is running, the forces are being transmitted through the shaft bearing to gears. That means, you see there are various components, which are closely associated.

So, we can say that all these dynamic forces are being transmitted and we can say that due to this several types of forces, we have several types of stresses in this particular machine, and if we are saying that the different types of stresses are there whose actions are combined. Therefore, it is necessary to have some basis for determining the allowable working stresses, so that the failure may not occur. So, you see you know it is not a simple test that the bending, the simple tensile action is there or the compression action is there or only a bending action is there or the shearing action is there. It is you know like when we are saying that the machine is running under these action of these dynamic forces, we have variety of the stresses are being inducing in all of these components and then, we need to focus on the individual dynamic forces within those machine part.

So, the meaning is that simple. When we are discussing about a component, then we need to determine the allowable working stresses, so that we can simply you know like get the real feeling about a failure that the basic cause of failure is this rather than to go with the simplicity simplified theories. Thus, the function of theories of elastic failure is to predict from the behavior of materials in a simple tensile test when this is important, and when the elastic failure will occur under any condition of the applied stresses. So, we equate those things in the similar kind of boundary conditions, and we say that if this go up to the yield point or beyond yield point, this would happen.

So, that is what you see you know like this is we can say a kind of continuum mechanics phenomena in which we can say that if this is there in this kind of machines, we can simply replicate with the simple tensile test with the application of these boundary conditions. So, this is kind of you know like this is pretty simplified theories are there, not only in the solid mechanics, but also in the fluid mechanics as well as you see in thermodynamic processes, we are doing all these kind of simplified applications. So, prior to go into the theories of failure, actually again we would like to define the basic component in that is strain energy.

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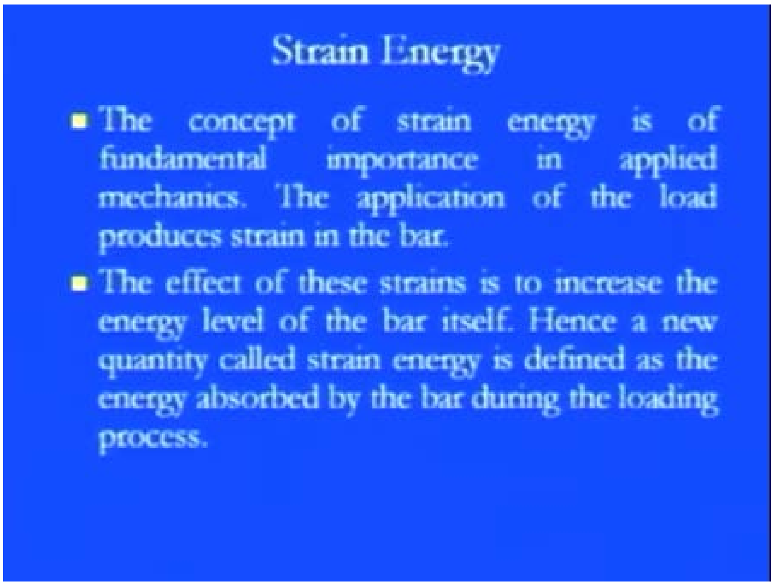

So, the concept of the strain energy is of fundamental importance in applied mechanics because you see if we are talking about the deformation and the load application and the stresses, we know that there is some part of a material or we can say the basic property of the material is there through, which there is some energy being absorbed, though we are talking about the elastic part. That means, you see you know like when you apply the load, this energy is being input and this energy is being restored by the material, and the application of the load produce the strain is the key part in any of the bar or the component. So, the defect of these stress means the deformation is to increase the energy level of the bar itself because you see it is absorbing the energy because you know like when you apply the load, the kind of deformation is there and there is a change in the stiffness. That means, it is you know like absorbing stiffness you see it is a kind of spring part.

So, they are absorbing some sort of the energy. Hence, a new quantity which is called the strain energy is defined as the energy absorbed by the bar, or any element during the loading process, but important thing is that this is under the elastic deformation only because you see whenever we are talking about the contact mechanics, always our main focus is on that whenever two objects or two parts or any two components are in contact, either the point contact or line contact or surface contact we say that there is a kind of deformation, and this deformation produce a kind of stiffness variation. That is why you see some sort of excitation is there and that is a different phenomena altogether, but here our main focus is that whenever the change of the deformation is there means whenever the change of stiffness is there, there is you know like the energy is being introduced or added and that is what you see this energy is known as the strain energy because of the deformation under the load application. So, this is one phenomena of this strain energy.

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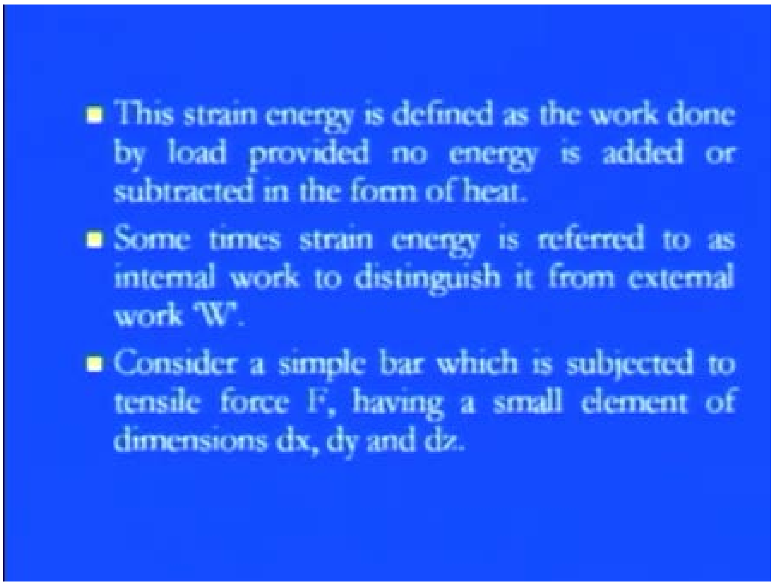

These strain energies defined as the work done by the load, obviously you see the load is simply applied on the worked piece. So, work done by the load provided, no energy is added or subtracted from this particular in form of heats. That means you see here we are always trying to detect these particular part in terms of like heat that there is no heat generation is there. As we can see generally in the frictional part, the heat generation is there, but this strain energy is always induced because of the load application and the deformation is occurring. Sometimes strain energy is referred to as internal work to distinguish it from the external work W because you see you know like that what the stress definition is. It is nothing but the internal resistance internal intensity of the resistive forces. That means, you see when the load application is there, the stresses are being induced.

So, sometimes we can also refer because you see you know like when we know that the deformation is there, there is an internal micro structure of the material is disturbed altogether. So, sometimes we can refer this strain energy is also then internal work within the micro structure of the material. Consider a simple bar which I am going to show you in the next slide which is subjected to tensile force F, you know like means the pulling part is there having the small element of dimension dx, dy and dz. So, you see rather to focus on the whole element, we have just taken a small element segment part which you can see the dimension in dx, dy and dz.

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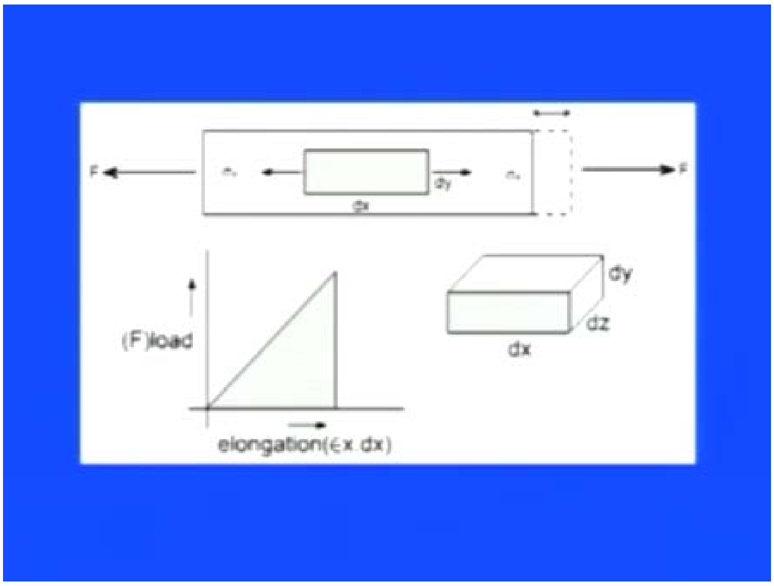

So, you see here this particular diagram in which the straight bar is there, you see here and the force is applied tensile. So, we have the tensile stresses altogether within this particular structure, and you see here the sigma x is simply, this is nothing but the normal stress component is there along this particular force application. So, the load application is towards outward direction. So, we have the sigma x towards the outward direction of the tensile stresses, and you see the dimensions are dy and dx is there along with particular, and dz is you know like this one is the dz.

So, if we are talking about these particular element and this element is subjected by the force, then we can simply draw because of the load application, we have the elongation. And if we are saying that this elongation or the deformation is this delta, so we can simply draw this load per deformation graph, and we will find that under the elastic again that we are applying the load under the elastic deformation only. So, under the elastic deformation, the load verses this elongation or deformation is coming as the straight line. So, the area under this particular thing will give you that how much energy is being absorbed and that is nothing but the strain energy under the elastic deformation.

So, you see here this elongation is nothing but epsilon x into dx, because you see you know like it is in the x direction epsilon the deformation. So, the strain into this distance and then, you see you know like we have x into dx is the area. So, we have the total you know like what the stress component is there along this particular way. So, you see here we have straight relation within this particular force elongation and under this particular area, we have the strain energy and that is what we know that whenever the load application is there on any of the element and the deformation is there corresponding to the load application, whatever energy is being consumed by this micro structure of the element is always referred to strain energy of the element under the elastic deformation only. So, you see this is the perfect example for that.

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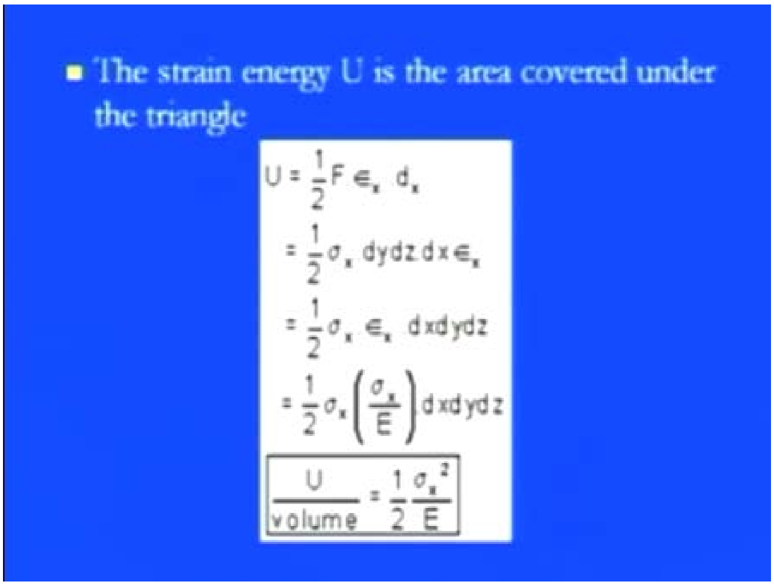

You see now if you want to compute those things, the strain energy is nothing but equals to half into you know like this half into F into epsilon x into dx because you see here only we are referring to the force which is acting in the x direction, and due to that we have you know like the stress part in sigma x, and we have the strain part epsilon x in these x direction only. So, you see here half the F is the applied load, this epsilon x into the epsilon x into dx is nothing but the corresponding dimensions of the deformation and this object part. So, you see here simply you know like just replace this F by the stress part. So, what we have F is nothing but the force. So, stress is nothing but equals to force into force is nothing but equals to strain into the stress into area.

So, the stress into the area is dy by d, this dy into dz. So, F is now being replaced here by sigma x into dy, dz, dx is dx was already there, epsilon x was there. So, now if you compute these things, what we have? We have simply you know like this dx, dy, dz is nothing but the volume of that particular element. So, half into dx into dy dz. So, this is the volume sigma, x is the strain epsilon, x is the strain part is there. So, if you rearrange those parts, what we have? We have half of sigma x epsilon x into this dv. We can say it is dv because it is a volume part in that or rather we can say that epsilon x is nothing but you know like we know that this load application is there. On that particular part, this element is under the elastic deformation only. So, Hooke's law, the generalized Hooke's law is very much valid.

So, under the Hooke's law, what we have? We have the strain stress is proportional to strain. So, if we equate those things, then we have the stress sigma x equals to a modulus of elasticity E into epsilon x. So, you see here the epsilon x, that is the strain part is being easily replaced by sigma x by E. So, you see within this elastic deformation or we can say under the subjected condition of the Hooke's law, we can simply replace that part. So, we have half of sigma x into sigma x by E into this dv. So, you see here the strain energy per unit volume for an object is nothing but equals to sigma x square by 2 E. This is one formula, either you see or you can also replace this. Sigma x is nothing but equals to epsilon x into E.

So, what we have is either we have sigma x square by 2 E, or we can say that if you replace this sigma x, this epsilon x into E, then we have this half of epsilon x square into E. So, you see here both of the manipulation of these parameters only. So, we can also play with the parameters. Simply replace here sigma x epsilon x or E in terms of those things under the elastic deformation. So, here you see you know like we can simply define or we can simply calculate. If we know the load conditions that what the kind of load conditions are there, and what is the Young's modulus of elasticity. That means what is the property of material is there. Based on that we can simply get that how much energy can be absorbed by this particular kind of material, and that is why you see the strain energy is also the function of material that what kind of material which we are using.

So, you see here you know like in these phenomena, we just got now that yeah that is one part when the load application is there. We have the strain energy, but if we are talking about the three-dimensional state of stress, generally you see you know like we refer the stress as the tensile part. So, we know that at least we need nine independent parameters to define the stress. So, if we are talking about the three-dimensional state of the stress, then that means, you see the three-dimensional space is there for defining the stress.

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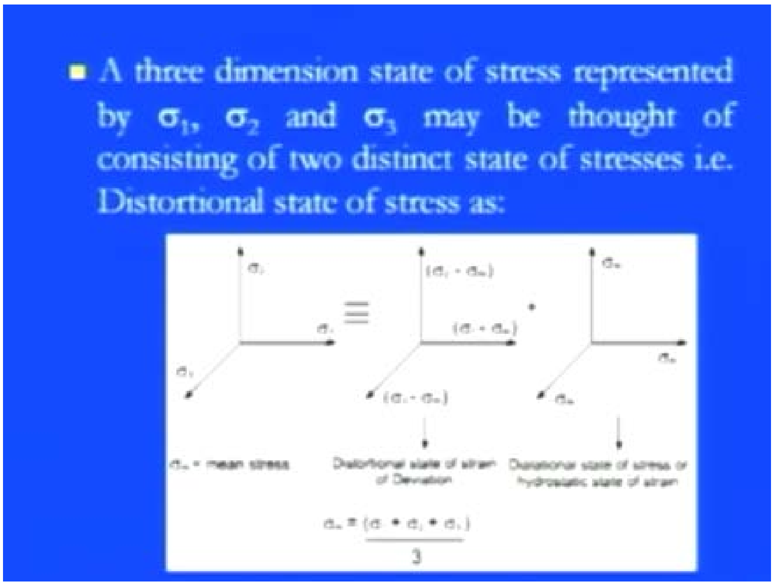

Then, you see we have you know like altogether we have the three types of stresses, sigma 1, sigma 2 and sigma 3, and that may be you see you know like thought of consisting to you know like the distinct we can say the state of stresses. That means you see the distortion state of stresses. That means, you see when the load application is there, they may be defined in the two different phenomena. How you see? We can simply see this particular diagram in which there are three you know like the mutually perpendicular stresses are there, sigma 1, sigma 2 and sigma 3, and you see here now if I am saying that if there is a load application is there, so now there are two main things. One is you see here that sigma 1 minus sigma m sigma 2 minus sigma m and sigma 3 minus sigma m in which sigma m is the mean stress and this is nothing but you see the torsional state of stress of deviation. That means, you see if any you know like the distortional part is there. That means when the load application is there, when some sort of distortion is there and dilation is there, there are two different phenomena. One is the hydrostatic state of stress and one is the distortional part.

So, if only there is a distortion, then you see you know like the strain energy can be defined in some other way, but if you see it is combined part is there, then that means you see some sort of the distortion is there and some sort of you see we can say the hydrostatic state of stress is there. Then, we need to define you know like both of the terms in different way. So, in the first you see you know like if I am talking about the three-dimensional state of stress in which sigma 1, sigma 2 and sigma 3 is there, then it can be simply segregated in two components. One is sigma 1 minus sigma m sigma 2 minus sigma m sigma 3 minus sigma m. In that you see you know like sigma m is nothing but equals to it is a linear or we can say the algebraic combination of that divided by 3. So, sigma 1 plus sigma 2 plus sigma 3 by 2 and you see the sigma m is responsible for hydrostatic state of stress.

So, you see here sigma m which is you see you know like uniform in all three directions because of the hydrostatic nature of the intensity, so you see here we have you know like sigma m, sigma m, sigma m in all three directions. So, you see here this is a kind of distortion, this is the dilation is there or we can say it is hydrostatic state of stress is there in that.

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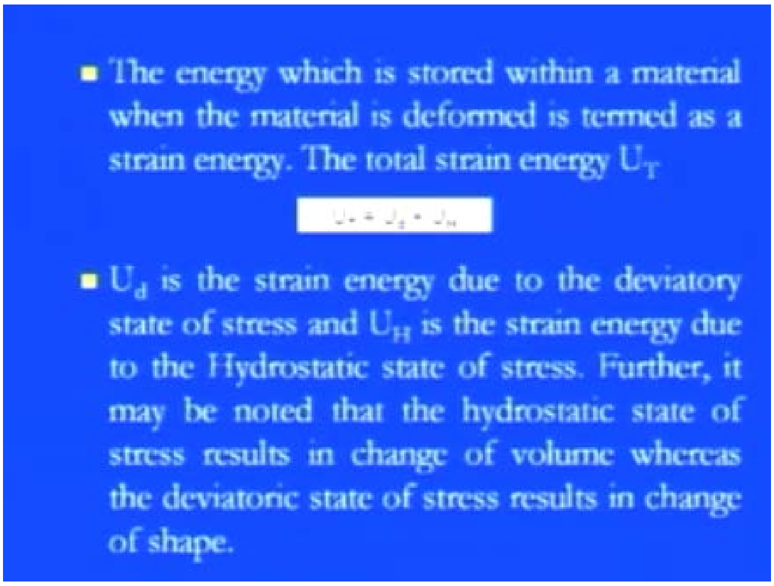

So, the energy which is stored within the material when you know like the material is being deformed is termed as the strain energy as usual, and the total energy of that is nothing but equals to U d plus U h. As I told you U d is nothing but it is the strain energy due to the deviate twist state of stress or we can say it is you know like the distortional part is there due to that which sigma 1 minus sigma m sigma 2 minus sigma m sigma 3 minus sigma m was showing in the previous diagram, and the strain energy due to the dilation or we can say the hydrostatic state of stress. Further it may be noted that the hydrostatic state of stress results in a change of volume.

So, you see here if we are talking about a domain of volume and if there is any change in the volume domain, then we need to focus on the hydrostatic phenomena of the stresses, whereas you see you know like if there is deviatory part is there, then that means if any change in the shape is there, it will straightway come in the deviation part of which size is. It means you see if we are talking about the plane, then this is an important phenomena U d and if we are talking about the whole volume, then U h is an important aspect of the distortion. So, you see here if we are talking about the whole strain energy, then this is the perfect combination of U d and U h is there based on which domain you are providing. If you are providing the shape means the plane, then obviously it is U d and if we are providing the volume domain, then U h is there.

So, these two you see you know like the distortion as well as the dilation, or we can say the hydrostatic phenomena are there as far as the strain energy is concerned. So, thus that was the key phenomena was there when the load application is there, and how much energy is being observed by a material.

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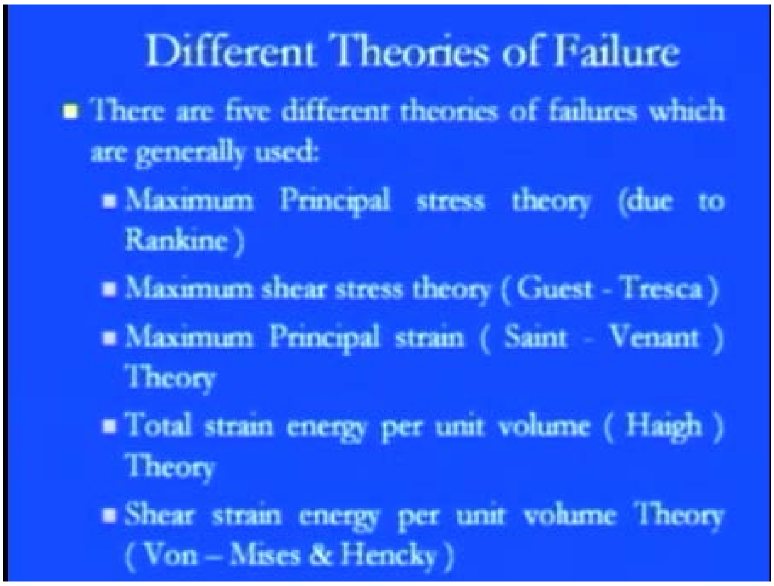

Now, come to the main part of this chapter that is different theories of failures and here, you see we are going to discuss the five different theories of failures in which you see you know like generally as I told you that they are simply name by the inventor or the originator of these kind of theories.

The first theory is you see based on the maximum principle stress theory. That means, you see what we are going to do here is, we are simply you know like find out that now when the principle stress are there, which are having the maximum you know like value on the principle planes. We just try to equate with the simple test under the tensile load, and we will find it out that actually if it is like that, then what will be this yield stress is there under these conditions, and since you see it is simply based on the Rankine theory. So, that is why it is also known as the Rankine theory of failure, or we can say that the maximum principle stress theory based on the Rankine.

Second stress, the second theory is the maximum shear stress theory in which you see you know like we just trying to find it that if you see shear stresses are there only in the object. Then what is the potential area or what is the potential region in which the maximum stresses are being occurring, and we will try to equate to the normal stress theory you know like with the maximum tensile stress. Then, we will try to find it out that what will be the yield stresses there that we can simply define the regions of the elastic and plastic for that and since, you see in this we are basically using you know like the Guest and the Tresca's formula since they have derived first time this particular kind of theory. So, that is why some time this theory is also known as Guest and stress Tresca's theories of failure which is based on the maximum shear stresses.

The third theory is the maximum principle strain theory. Again you see you know like the first theory was the maximum principle stresses. So, here you see you know like rather to focus on the stress part, principle stress part, here we are discussing about the main principle strain theory which is basically based on the same Saint Venant principle. Saint Venant is very common you know like if we are talking about this strength of material part of you know like some lower level, then you will find that Saint Venant has given many theories about bar when it is subject to you know like simple normal stress or shear stresses part. So, based on those theories you see here you know like this theory of failure is known as Saint Venant theories of failure which is based on the maximum principle strain theory.

Then, the fourth one is nothing but you know like it is the total strain energy per unit volume theory. That means, you see here how much you know like the energy being absorbed per unit volume and since, it is based on the Haigh formula, so that is what sometime it is known as the Haigh theory of the failure which is absolutely based on the totally strain energy per unit volume.

The last one is there which is you know like very common theory is the one Mises and you know like later it is also being derived by the Hencky theory. So, you know like both have given similar kind of the phenomena. So, this phenomenon is based on the shear strain energy per unit volume. So, you see straight way what we have now the strain energy is there which is total in the previous one Haigh, but here our main focus is on the shear strain energy per unit volume you know like the theories there, and we just try to equate those things with the generalized theory of the tensile part. And then, we will try to find it out what is the yield point is there and what is the criteria for the failure theory, and that is why you see it is also known as the Von-Mises and the Hencky theory.

So, these are the five theories as I told you like simply based on what the basic phenomena is there right from principle stress to shear stress, the shear strain energy. So, all these theories are simply based on the unique feature of the stress or strain under the specific load conditions per unit area or per unit volume, and they are named by their inventors.

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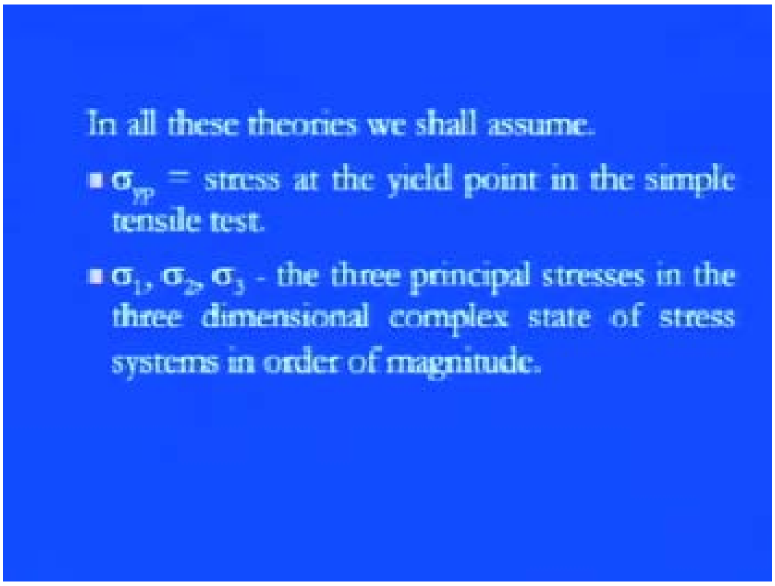

So, in all cases or in all theories, we simply assume that the sigma yP as I told you because you see we are just trying to find out the yield part. So, sigma yP is nothing but the stresses yield point in a simple tensile test, because we are trying to equate in the simple tensile test. And then you see these are some of the sigma 1, sigma 2, sigma 3 are nothing but the three principle stresses because generally, you will find that the principle stresses are pretty common and they are coming in most of the theories. So, these sigma 1, sigma 2, sigma 3 are noting, but the principle stresses in these three-dimensional complex state of stress in order to their magnitude.

So, what exactly you see you know like they are appearing and what is the physical significance is there in the particular material, accordingly we will consider in our theories. So, come to the basic first theory you know like that is based on the maximum principle stress theory.

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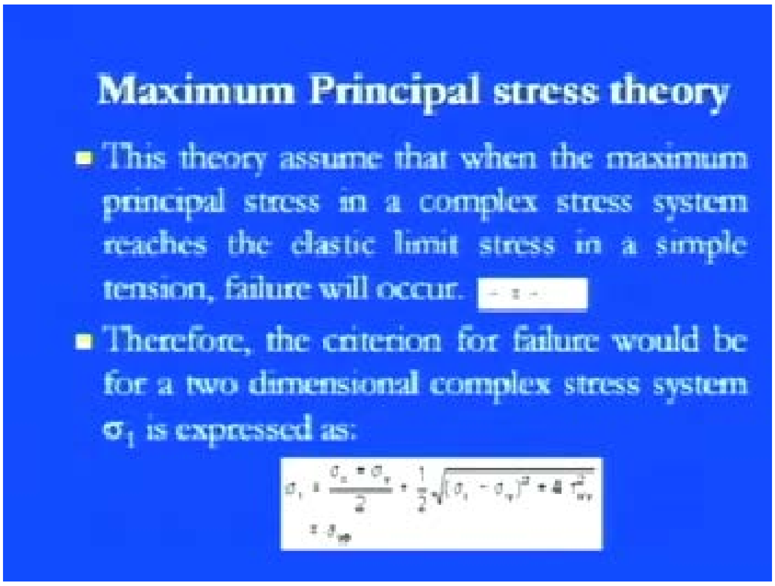

So, we read the statement and then, you will find that the feeling of this particular theory, the statement of this theory assumes that when the maximum principle stress in a complex stress system reaches to the elastic limit, stress in a simple test failure will occur. That means, you see the failure will occur when sigma 1 is equal to sigma yP. Therefore, the criteria of the failure would be for two-dimensional complex stress system sigma 1. That means, the principle stress in the first plane, the first principle plane and you see since we are talking about the two-dimensional complex stress system, you see 2D, so we are assuming that we have sigma and sigma y. So, when you see the sigma y sigma 1 is approaching up to the sigma yP, the failure may occur and this sigma 1 can be easily evaluated based on the plane stress theory. That sigma 1 is nothing but equals to since the sigma 1 is a principle stress and principle stresses are being induced due to the influence of sigma x and sigma y in these two different planes.

So, sigma 1 is nothing but equals to sigma x plus sigma y plus half of square root of sigma x minus sigma y whole square plus 4 times of tau x square tau square xy, and it equals to sigma yP. So, you see here if in a complex stress system, if you see the principle stresses are approaching up to the maximum value under the elastic limit. Then we can say that the material will fail and in that case, sigma 1 will be equal to sigma yP which is equal to this sigma x plus sigma y by 2 plus square root of these things.

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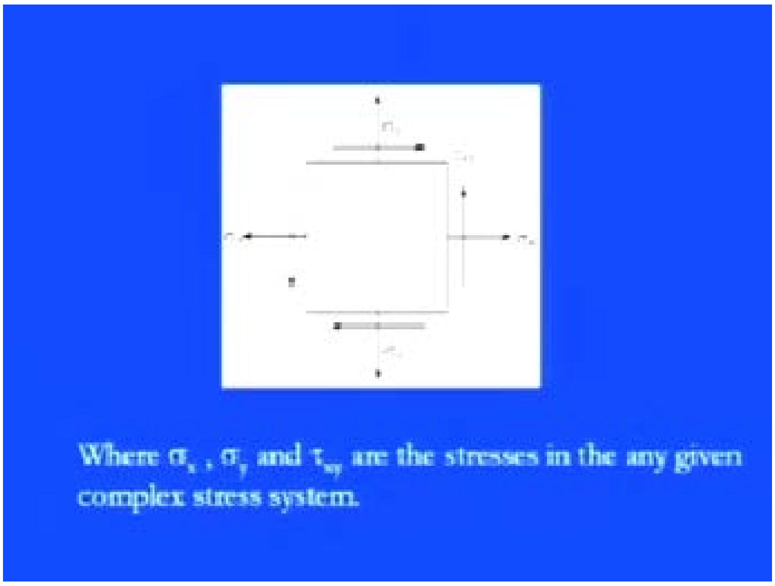

So, you see here in that case as I told you like what we are trying to deal with. We are trying to deal with two different types of phenomena. One when you see we are saying that the material if being subjected by the sigma x and sigma y with tau xy. That means you see altogether the normal and shear stresses are being applied to a system and then, you see under the action of you know like under the combined action of sigma x, sigma y and tau x y. We have you see sigma 1 which equals to sigma x plus sigma y by 2 plus of these things which we discussed, and when this sigma 1 which is coming due to the combined action of these all force, these all stress components. When it reaches to sigma yP, then there is a chance of the failure.

So, you see here in under all these you know combinations of this stress components, we can easily evaluate sigma 1 which equals to sigma yP which is equal to you know like this sigma x sigma y by 2 plus half of square root of sigma x minus sigma y whole square plus 4 times of tau square xy.

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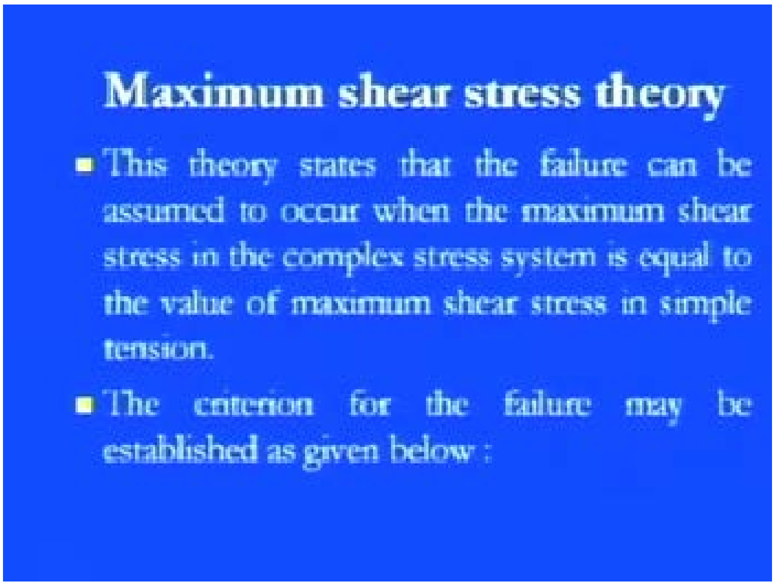

So, this is you see the first theory and since based on the Rankine part, so that is why it is also known as the Rankine theory of failure. Second theory is the maximum shear stress theory. This theory states that the failure can be assumed to occur when maximum shear stress in a complex stress system is equal to the value of the maximum shear stress in a simple test. So, you see here when a simple tensile test is there and you see the maximum shear stresses are occurring, when we equate that part to the complex system of that, then there is a chance of failure. In this statement we assume this.

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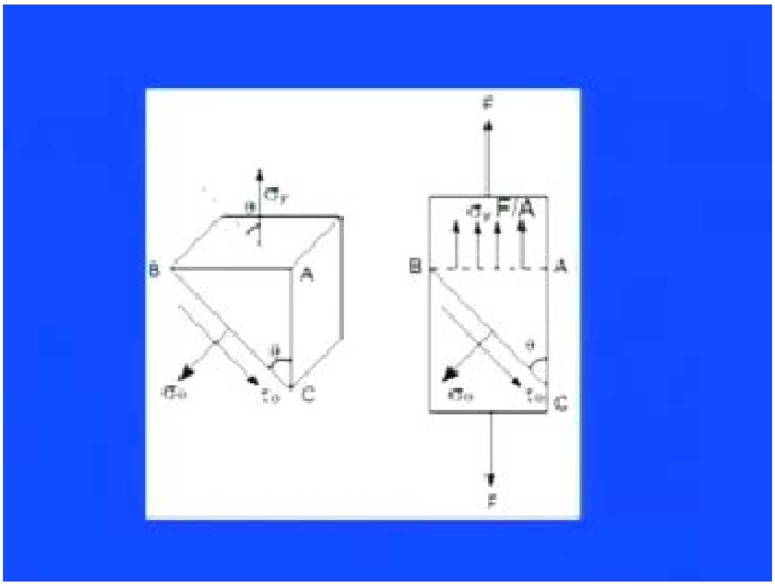

So, the criteria for the failure may be established like this you see here what we have? We have a simple tensile test. As I told you like in that what we have? We have sigma y into this you know like this is force per unit area. So, this is sigma. So, this is sigma y because it is in the y direction, and now if we are taken you know like the inclined plane and at that particular inclined plane theta, what we have? We have the sigma theta, which is the normal component and the oblique plane, and we have tau theta that is shear stress at this particular part. So, if we simply take out this particular part A you see here this is ABC, and then we can simply say that within this inclined plane, you know like what will be the maximum shear stress is coming under the complex system. If we equate this complex system to this simple system, we will find that this tau 1 is equal to tau yP which can be easily calculated under the force of this normal tensile part.

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So, now you see within that part, we can simply say that from this particular diagram, you know like we have the simple calculation sigma theta which is nothing but equals to sigma y sin square theta. You know based on that trigonometrical relation in the previous figure, and we can also say that tau theta which is nothing but equals to sigma y sin 2 theta by 2. So, now what we have? We have the theta means at the inclined plane now we are simply trying to correlate because we know that the only force is there along the y axis. So, due to that what we have? We have the inclined stresses at this particular theta

So, we can simply correlate sigma theta which is equal to sigma y into sin square theta and tau theta which is you see along that particular plane is nothing but equals to sigma y sin 2 theta by 2 and also, we have some of the reasons where the maximum shear stress is there. So, maximum shear stress is nothing but equals to sigma y by 2, or we can say it is nothing but equals to sigma y because you see the failure occur at the yield point. So, sigma y is equal to sigma yP by 2 whereas, for you know like the two-dimensional complex stress, when we are talking about this we know that thus maximum shear stress is nothing but equals to when you know like the complex system is there. We know that it is equal to sigma 1 minus sigma y by 2. That means, you see it is nothing but the difference of two maximum principle stresses are there.

So, sigma 1 is the maximum principle stress along the principle plane 1, and sigma 2 is the minimum principle stress along the plane 2. So, you see when we are talking about the plane part within the complex stress system, we have the maximum shear stresses sigma 1 minus sigma y by sigma 2 by 2 or if we simply correlate with the generalized theory of you know like the plane, the strain theory, then sigma 1 by sigma 1 minus sigma 2 by 2 is equal to half of square root of sigma x minus y by 2 whole square plus 4 types of tau square xy. So, now if we are computing those things because we know that tau maximum which is the maximum shear stress is nothing but equals to tau yP by 2.

So, now sigma 1 minus sigma y by sigma 1 minus 2 by 2 is equal to half of tau yP is equal to you know like this is the square root, that is square half of square root of sigma x minus y whole square plus 4 times of tau square xy. So, if we computing now, then we have the final one, tau yP which is you know like this stress L, the E at the yield point is equal to square root of sigma x minus sigma y whole square plus 4 times of tau square xy, and now this is you know like the criteria through which we can simply get the feeling of the failure that when we know that the failure is there due to the maximum shear stress, then we can simply get the real you know like the value of this stress part tau y yP which is nothing but equals to square root of sigma x minus sigma y whole square plus 4 times of tau square x tau square xy. So, this is you see the real combination of the normal stress and shear stress under the maximum shear stress theory.

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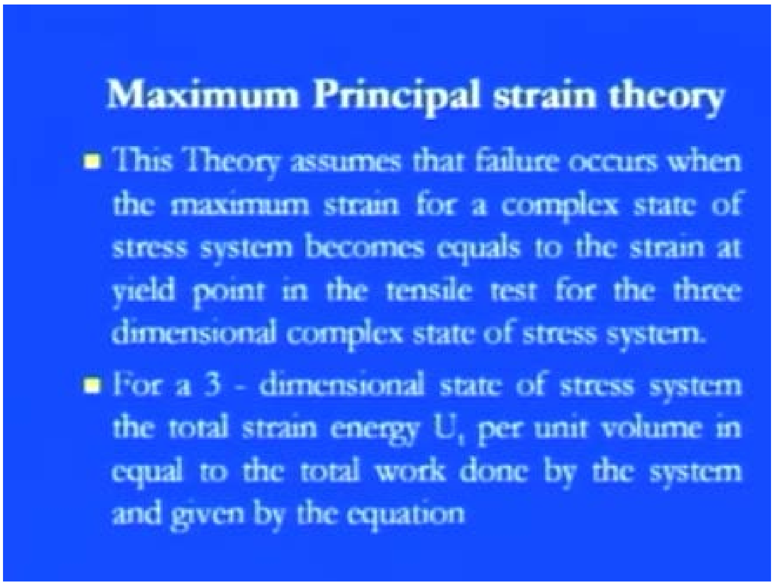

Then the third theory is the maximum principle strain theory. Now, you see here instead of talking about the principle stress and strain, we have you see instead of talking about the principle stress and shear stress, if we have the principle strain. That means, you see you know like the principle planes are there instead of the force intensity. Now, if we have you see the deformation altogether, then you see how we can correlate those things. So, this theory assumes that the failure occur when the maximum stains for a complex stress system becomes equal to the strain at yield point.

Again, you see this is very important that you see when we are talking about the maximum strain for any complex state of stress system. Then, we need to be very careful that it has to be equal of the strain at yield point in a simple tensile test for three-dimensional complex state of stress system. That means you see when we are talking about a simplest you know like the tensile test then what will be the principle strain is there. It is exactly you see equal to the complex stress system. This assumption is very well valid to check it out the theories of failure under the principle strain theory.

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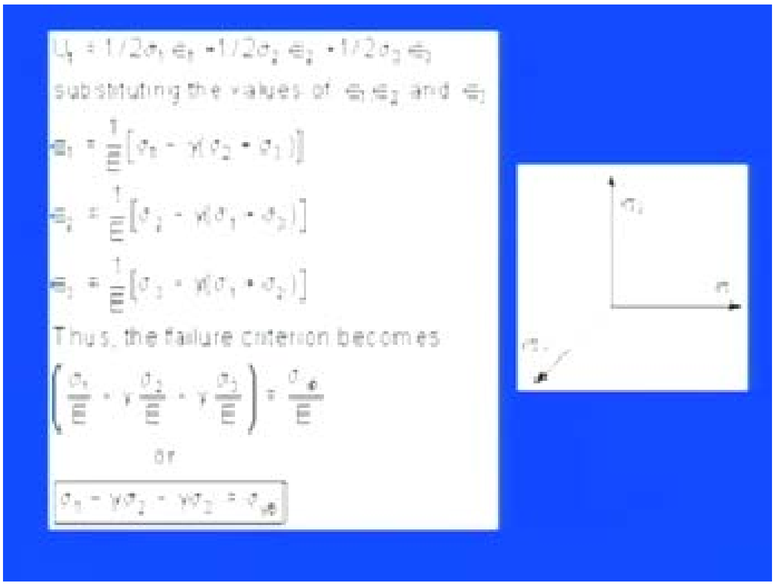

So, for a three-dimensional state of stress system, the total strain energy that is the U t per volume is equal to the total work done by the system, and under the total work done by the system which is simply you know like we can simply calculate because you see it is working in all three directions. And we are simply calculating that what will be the strain energy, and we know that the strain energy per unit volume was nothing but equals to sigma square by 2 E or you see I also told you that usually it is half of you know like the sigma and epsilon.

So, we can simply calculate in all three directions if we know that you see we can simply see the direction that these three-dimensional state of stress is there in which the sigma 1, sigma 2 and sigma 3 is there in all three directions sigma 1, sigma 2, sigma 3. So, along with these particular things, now if we are simply calculating the strain energy, then we have half of sigma 1 epsilon 1 plus half of sigma 2 epsilon 2 plus half of sigma 3 epsilon 3. So, we can simply you know like replace these values sigma 1, sigma 2, sigma 3 because we know that the stresses are being you know like the stresses are there in these directions, but the strain are not equal and we can simply compute this strain component with the use of this Poisson ratio.

So, with the inclusion of Poisson ratio, what we have? We have epsilon 1 that is the strain energy. So, if you substitute the value of the strain and in the x directions, this epsilon 1 which is nothing but equals to 1 of 1 by E into sigma 1 minus you see this is the Poisson ratio gamma into sigma 2 plus say a sigma 3. So, similarly you see we can also get the strain part in the other two directions, the epsilon 2 and epsilon 3 which can be also calculated in a similar pattern because the main influencing part is this you know like the corresponding direction, but the other direction also has the similar impact. So, we are computing in terms of as you know that we have discussed already this part in terms of the Poisson ratio. So, if we are talking about the epsilon 2 which is nothing but equals to 1 by E sigma 2 minus gamma into sigma 1 plus sigma 3, and if we are talking about the epsilon 3, then we have 1 by E into sigma 3 minus this gamma times sigma 1 plus sigma 2.

So, you see here you know like in comparison to all these things, what we are getting that like when you know in all three directions, we have the force conditions. Then, you see like the deformation is there, and the deformation in terms of the strain can be computed like here. Thus, the theory of failure of criteria becomes sigma 1 minus half if we are talking about in one direction. So, it is sigma 1 by E because this influencing part is sigma 1. So, sigma 1 by E minus you see in other terms you if the reduction is there in the size, so we have sigma, this gamma times sigma 2 by E minus gamma times sigma 3 by E equals to sigma times of yP by E. because you see the yield point is there, or if we are computing these things, then we have the yield stress within those you know like truly computing in all three directions.

Then, it is equal to sigma 1 minus gamma times of sigma 2 minus gamma times of sigma 3. That means, you see if you know like the elongation is there in one direction, then the contractions are there in other two directions which can be easily computed with the use of these Poisson ratios. So, this is one part.

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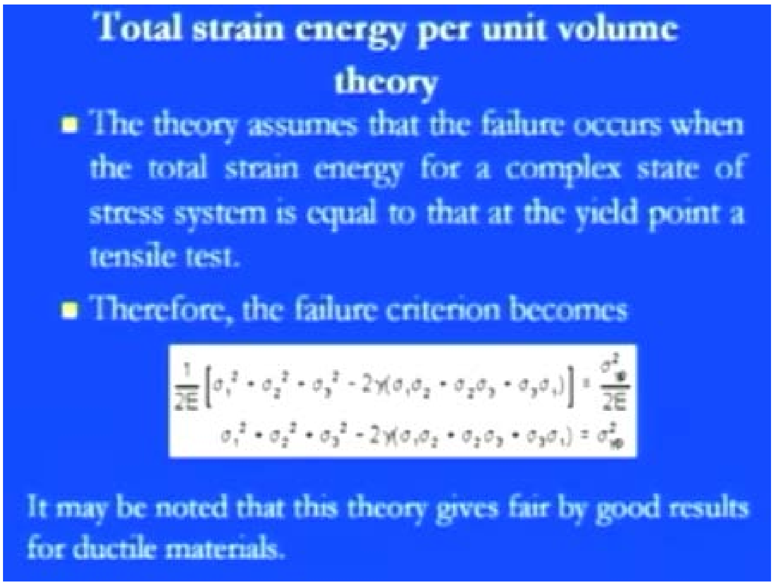

Now, you see here you know like go back again to one of the key features that the strain energy, and we would like to now focus on the theory of failure based on the strain energy which we discussed. So, the total strain energy per unit volume theory simply you know like assumes that the failure can occur when the total strain energy of the system you see in which the U d U h, everything if there you see due to the distortion, due to the hydrostatic part everything. So, the totally strain energy for a complex state of stress system is equal to that you know like that to the yield point at a tensile stress.

So, you see here if we are computing all these three here based on that particular failure theory, then what we have is sigma square yP divided by 2 E, because this is the strain energy sigma square yP divided by 2 E equals to 1 by 2 E. And now, these are the responsible parameter under the energy absorption is there. So, sigma 1 is square plus sigma 2 square plus sigma 3 square in all these directions you see in the mutual perpendicular directions plus whatever the distortion or you know like this dilation is there in other parameter. So, we have minus 2 times of gamma. That is a Poisson ratio into sigma 1, sigma 2 plus sigma 2, sigma 3 plus sigma 3 times of sigma 1.

So, if we compute all these things, then what we have? We have you see you know like the yield point based on the total strain energy per unit volume sigma square yP is nothing but equals to sigma 1 square plus sigma 2 square plus sigma 3 square minus 2 times of Poisson ratio sigma 1, sigma 2, sigma 2 sigma 3 plus sigma 3, sigma 1. So, you know like it is important thing is that when we are talking about material in which you see you know like this kind of the complex state of stress system is there, then it is truly you know like aspect is there on which we can simply get this as the yield point region, where you see you know like if it is below that then it is an elastic region. And you see if it is touching to this yield point, then there is a good chance of a failure. So, based on that, we can simply get that.

Now, if we have ductile material or if we have a different kind of you know like in the ductile material, we have the mild steel or this tungsten steel or any kind of high lid steel, high carbon steel, then we can get the failure region. If you see these 2 are exactly matching. So, it may be noted that this theory gives fairly good results based on you know like the ductile because you see how much deformation is there and based on that deformation, how much energy absorption is there. This is the true part and you see we can simply get that. Now, it can absorb up to this much you know like the energy or we can say this much load application can occur on this particular kind of structure without any failure, and if you go beyond that. That means, if you reach up to sigma yP, then there may be a chance of failure beyond that reason or at this particular reason. So, that is why you see sigma square yP which is equal to all the sigma 1 square or sigma 2 square or whatever, it will give you a clear reason for a failure theory. So, you see this is really good example based on the strain energy per unit volume.

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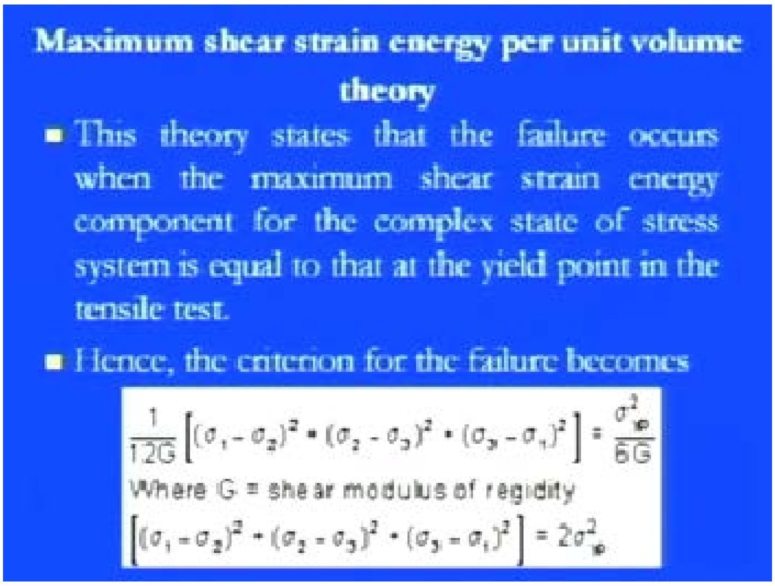

Lastly, you see what we have? We have the maximum shear strain energy per unit volume. That means you see here the shearing action is there and under the action of the shearing strain, what we have is the energy absorption in the material. So, this theory states that the failure occurs when the maximum shear strain energy component for complex state of strain is equal to the yield point in the simple tensile test. So, you see here if we are talking about the maximum shear, this energy always we know that whenever the tensional part is there or any shearing stress is there, always we have instead of E, we have the shear modulus of elasticity.

So, we are trying to compare this you know like instead of sigma yP square by 2 E, now we have for the failure criteria, we have sigma square yP by, please excuse me. So, you see G is the shear modules of elasticity. So, in that you see we know that sigma 1, sigma 2 and sigma 3 are nothing but they are in the corresponding axes, the stress components are there, but we know that this shear stress is nothing but it is a plane theory. So, we need to talk about a plane. So, if we are talking about the xy plane, then in the xy plane we have sigma 1, sigma 2. In yz plane, we have sigma 2, sigma 3 and xz plane we have sigma 3, sigma 1. So, in these particular planes whatever the kind of distortion is there, this distortion can be computed in terms of the shear strain energy.

So, in shear strain energy per unit volume will exactly execute the similar kind of results in terms of the relative phenomena. So, you see here this sigma square yP by 6 G will be equal to 1 by 12 G into sigma 1 minus sigma 2 whole square. So, instead of the sigma 1 square in the previous parts, straight this strain energy we have in shear strain energy, the relative terms are there because of the shearing action. So, 1 by 12 G sigma 1 minus sigma 2 whole square plus sigma 2 minus sigma 3 whole square plus sigma 3 minus sigma 1 whole square which is equal to sigma square yP by 6 G, where the G is shear modulus of rigidity or we can say that we can simply get the reason of the failure or we can say the clear segmentation is there about the reason of you know like the elastic and the plastic part.

So, for that you see two times of sigma square yP will be equal to sigma 1 minus sigma 2 whole square plus sigma 2 minus sigma 3 whole square plus sigma 3 minus sigma 1 whole square. So, you see here this is you know like based on the relative coefficient or relative interaction of these stresses, we could know that how much you know like the shearing action will be occurred in those kinds of things.

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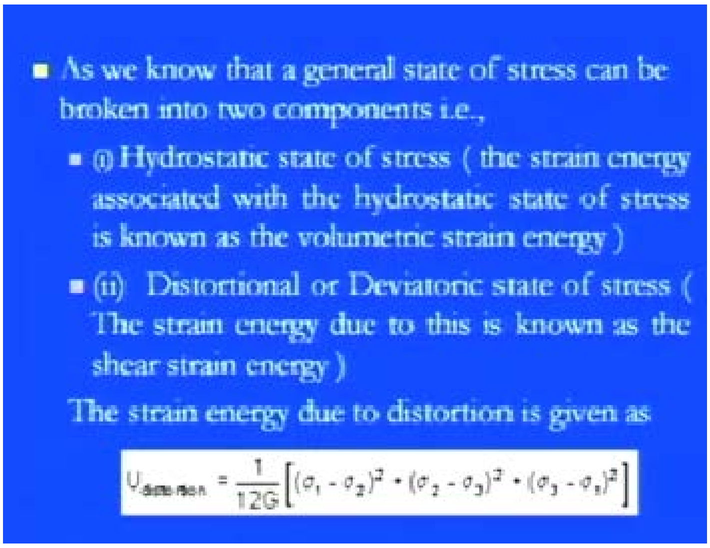

So, as we know that general state of stress can be you know like broken into two main components. As I told you one this is the strain energy is there. So, one is the hydrostatic state of stress or dilation is there that the strain energy associated with the hydrostatic state of stress is known as the volumetric strain energy as we discussed already, and the second was the distortional or deviatory state of stress. That means, whenever this shape change is there of an element or shape distortion is there, then this kind of phenomena is coming or we can say that the strain energy due to this is known as the shear strain energy because you see here in the domain is the shape and the hydrostatic state of stress, the domain is the volume.

So, you see here with all those things, the strain energy due to the distortion will be given as U distortion equals to 1 by 12 G into sigma 1 minus sigma 2 whole square plus sigma 2 minus sigma 3 whole square plus sigma 3 minus sigma 1 whole square, and in these you see U distortion, both components the hydrostatic as well as the distortional part is easily computed, and they are equally responsible for having U distortion.

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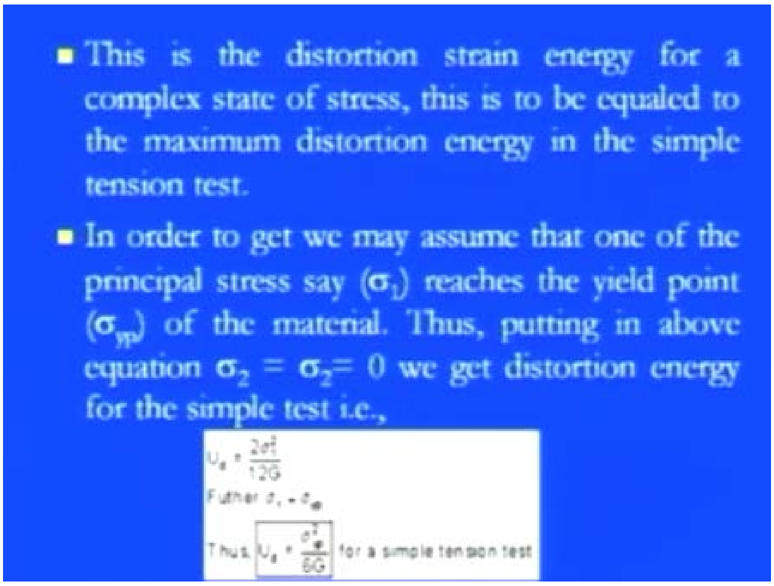

So, with that this is the distortion energy like the distortion is strain energy for a complex state of stress and this is to be equalated to the maximum distortion energy in a simple tensile test as we discussed. So, in order to get we may assume that one of the principle stresses sigma, either sigma 1 reaches to the yield point sigma yP of the material and that you see that we need to keep the sigma 2 and sigma 3 because the other stresses on the planes have to be equal to 0 to get the distortion energy under the simple tensile test.

So, we have this U d which equals 2 sigma 1 because only the responsible stress in this particular way is a principle stress 1. So, sigma 1 square by this 12 G or we can say that the sigma 1 which is equals to sigma yP for a failure criteria. So, we have the distortion energy U d is nothing but equals to sigma yP square divided by 6 G for a simple tensile test, and this sigma yP square by 6 G is equalated by 1 by 12 G into sigma 1 minus sigma 2 whole square plus sigma 2 minus sigma 3 whole square plus sigma 3 minus sigma 1 whole square. So, you see here you know like in these criteria, we could easily figure out that now two times of you know like the sigma yP square is equal to differences of the relative stresses. That means sigma 1 minus sigma 2 whole square plus sigma 2 minus sigma 3 whole square plus sigma 3 minus sigma 1 whole square.

So, through these you know like stress components, we could easily figure out that what will be the criteria or what are the reasons through which you know like this material can be failed on the basis of the maximum shear stress, shear strain energy concept per unit volume. So, these were you see like the five different criteria which we discussed, and we found that you see all criteria simply you know like based on their own types load. That is kind of load applications are then which are responsible you know like in the failure type. That means pretty simple that what kind of applications are there of this particular material, so that we can simply figure out that now these load applications are there, and under these load applications, these kind of stresses can be occurred on these particular material, and then by equating you know like these conditions to a tensile test, we can simply figure out that what are the reasons or we can say what are the potential reasons where you see the failure criteria can become.

So, you see here in these particular all these theories, the key part was there that the principle stresses, the maximum shear stresses, the strain energy maximum shear strain energy and you see you know like combinations of all. So, that is why you see like we need to figure out that actually what are the responsible forces in these particular elements, and due to that how the distribution is there and what are the key points along with particular materials are the real feature to get the failure of this particular material.

So, in this chapter we basically discuss about the theories of failure under variety of conditions, and now you see here you know like based on these theories of failure, in the next lecture we are going to discuss about the columns and the strut. That means, you see what are the elastic stability is there of the column. That means you see if we have a column and you see the variety of the conditions are there in the column and you see we have the strut.

So, first of all you know like we will discuss about what the basic difference between this column and the strut and then, you see we will discuss about that if the variety of the boundary conditions are there in the column. With the simple load application, then you see you know like what will be along with this particular, what will be the failure you know like the criteria is there in that, and what is the stresses are there and how the stresses are being set up within the load application of this column. So, these you see the column theory and in that you see the Euler theories are there.

So, all these part you see based on the Euler assumptions and all these things, we will discuss many things about the column and the strut and we will try to differentiate that out. That means how the stresses are being setup when you know like both end of the column is fixed, or one end is pin joint and one end is fixed, or both ends are pin joint, or the load is you know where the load applications are there at any of the end or the middle part. So, based on you see like all variety of these particular parameters, we will try to see the stress component and the strain component within the column, and that is why you see the next chapter is absolutely based on the elastic stability of the column.

Thank you.