Strength of Materials
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Lecture – 34

Hi, this is Dr S P Harsha, Assistant Professor from mechanical and industrial department, IIT Roorkee. I am going to deliver my lecture 34 on the course of the strength of the materials, and this course is developed under the national program on technological enhanced learning.

Prior to start of this lecture I would like to briefly discuss that what we have discussed in the previous lecture. In previous lecture we discussed about one of the different methods to evaluate the bending moments as well as you see to get the deflection as well as the slope if a beam is subjected by a point load, as well as this UDL the uniformly distributed load. And we found that if the distribution is not proper the load distribution is not proper on the beam, then how we can get you see for the different segments of the bending moment as well as you see how this boundary conditions are affecting to the you know like the slope as well as the deflection for that.

So, again you see briefly we will just want to you know like revise those things. So, in the Macaulay method that was the third method, because you see we started from the direct integration method which was pretty simple you see, and we derived the equation for that the moment equation. And then you see you know like there was an integration method is there straightway by keeping those boundary conditions we can get that slope as well as the deflection.

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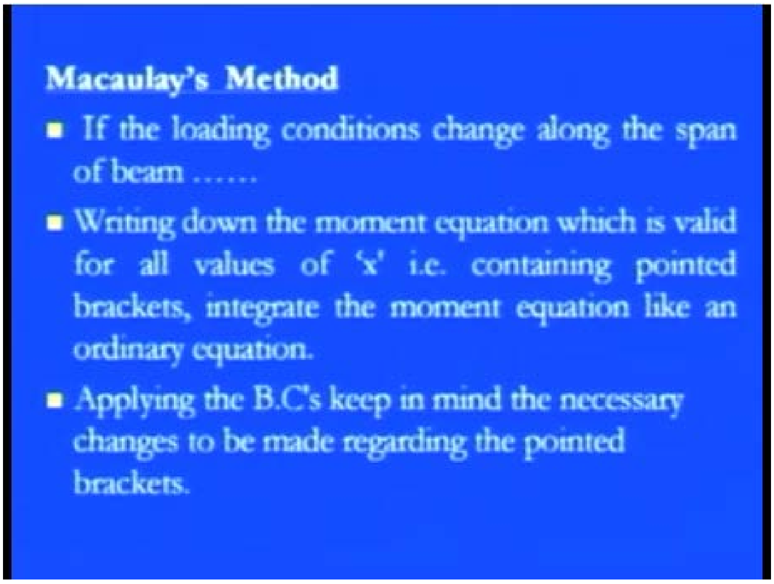

And the second method was the moment area method in which you see you know like based on what we are getting in the bending moment diagram; under that you see whatever area is coming from that we can get easily the slope. And once you multiply that area with that you know like the distance of the centroid from you know like the main reference point, then we can get you see the deflection just by dividing y E I; that is the flexible rigidity of this. And you know like E is the young's modulus of elasticity, and I is the moment of inertial based on what the cross section of beam is.

So, these two methods we discussed in the previous two lectures, and the last lecture you see it was the Macaulay method; this is quite different. And this method is basically you see we discussed that if the loading condition is changing along the span of the beam; that means you see for the different different segments, if the different loads are there and boundary conditions are very sensitive towards that, then how we can get those things. So, in that there were you know like certain steps are there. So, again I briefly revise those things because you see in this lecture we are going to analyze some of the numerical problem also.

So, first of all we need to right the moment equation which is quite valid for all those values of x along the path of or we can say the span of the beam; that is you see the containing pointed brackets integrating the moment equations like the ordinary equations. Because you see here if within the bracket you know like if it is showing the negative values then we need to ignore that part, but you see if it is a positive then we need to consider in the moment equation.

So, first what we need to do here? We need to describe you know like the moment equation for entire span of the beam that how this loads are applying or how this combined UDL is coming on that, and what the interaction is there in between the point load and the UDL. Based on that you see we have the moment equation, and once you apply the boundary conditions then you are getting you see. So, you need to apply the boundary conditions keeping in mind. This is a very important point you see while you apply the Macaulay method which is a pretty simple method, but you see you have to be very very conscious when you when you using this Macaulay method, because the key point is that whenever the change is to be made regarding the pointed bracket.

That means you see when you are keeping the value of x within the bracket. And if you see the overall value of this parenthesis or the bracket is coming negative you need to neglect that; that is an important part. Because you see this divergence is coming if you are adding that or subtracting straightway you know like misleading concept is there, and it will give you a false diagnosis about the moment. So, this is the key point and we solve some of the numerical problem also in the previous lecture about you know like when we have the simply supported beam.

And there are you see supports are there at the two extreme ends, and even if we apply the load you see the point load at the extreme end or if it is in between you see these two loads or the UDL is there. Then how we can resolve these moment equation as well as the forces along the path of this beam and then how we can get you see the deflection as well as the slope for that. So, you see here you know like even for the extreme end supports or you see the support is there in between some of their end our hanging part is there. So, this kind of you know like the discussion which we made in the previous lecture; again we would like to continue some of you know like the numerical problems in that.

And then in this chapter our main focus will be there that if a beam is there and if it is subjected by a combined loading; that means you see on the beam if the loading is like that due to a we have a bending action along with that you see we have a twisting moment, then how we can calculate you see the principle stresses. Because you see now we have you see you know like the bending; that means the normal stress component is there, and we have the shear stress component. So obviously, you see we need to get the principle stresses of those things. And what the combined effect is there because you see if you want to design the beam which you see we are going to discuss about the failure theories in the coming chapters.

Then these kind of analysis is very, very important, because this will provide you the basic input that what are the you know like the basic stress components are there and which of the part of the beam is you know like influencing by the maximum or we can say the critical bending part., so that you see if you want to design those things with the combined loading then we could easily figure out those problem and we can simply put the more factor of safety towards those points so that you see our design will be safe. So, this kind of analysis you see which we are going to discuss about that. So, prior to that you see again the two numerical problems are there for you guys. So, just you can see those that how Macaulay method will provide you the basic information about the moment and how the moment distribution is there along the span of the beam.

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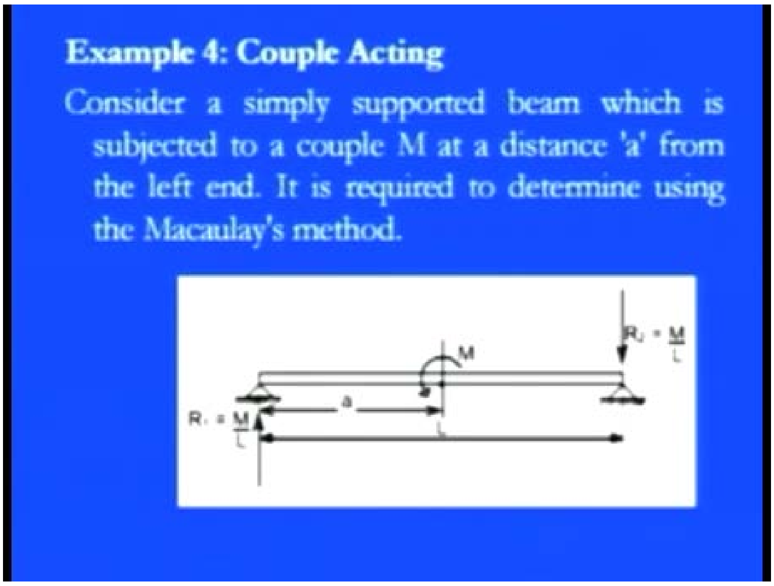

So, the first thing is that you know like if the couple acting is there; that means you see previously whatever the three problems which we discussed only the point load and the uniformly distributed load was there. But now if the couple is acting on the beam then how we could figure out those things means what the impact is there on this couple on the beam design. So, first of all the problem statement is like that; consider simply supported beam you see you know like the two supports are there, what is the R 1 and R 2 you see and you can simply figure out those R 1 R 2 by resolving those force balance as well as the moment balance. So, it is pretty simply to calculate those reaction forces at these two extreme ends.

So, this simply supported beam is subjected to a couple M, and you can see the location. The M is simply acted at a point A from the left hand point corner; that means you see apart from you know like the left hand corner if A distance is there we have you know like the couple is there. And on that you see if we are saying that the total span of this beam is L, then at point A our main you know like the couple is acting and you see there is no other force is there on that. So, you see at distance A form the left end we have a couple; it is required to determine using this Macaulay method that what exactly the deflection as well as the slope is there.

So, again you see this straight procedure is there for that. You need to first write the equation of this what the moment equation is there along with what the boundary conditions are there. And again just keeping this thing in mind that actually this whatever the bracket sign is there, if it is the negative we need to ignore; otherwise, you need to consider in the moment equation.

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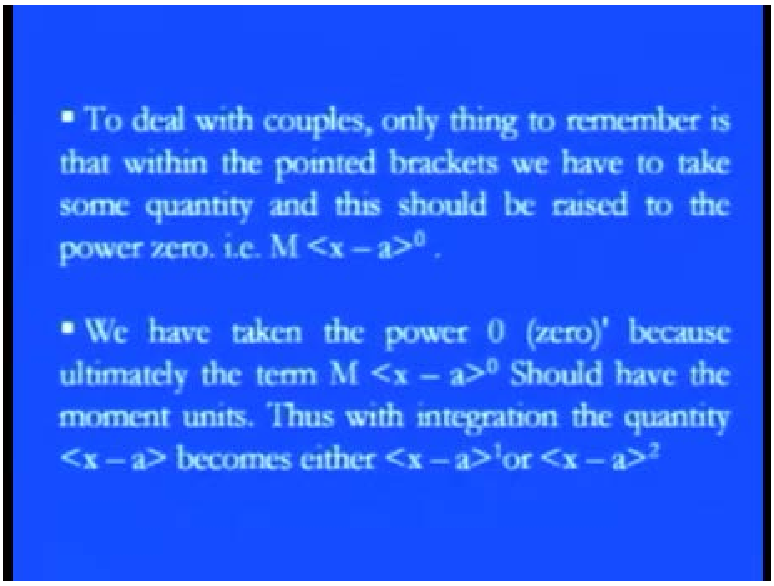

So, to deal with the couple only thing is to remember is that within the pointed brackets as I told you we have to take some quantity you know like and this should be raised to the power zero; that means you see M into x minus A to the power zero. So, this is the general you know like a unit function approximation is there to describe the moment equation which is very very important. Because you see in the previous cases we found that whatever the force component is coming and along you see what the distances are there we were simply writing.

But now you see we need to write the moment equation in this unit function formation which is quite you know like Lagrangian's function is there in that way. So, we need to describe this one as M into parenthesis of that x minus A to the power zero. So, this is one point. And you know like we have taken the power zero because ultimately you know like the term x M into x minus A should have the moment unit. Because you see, obviously, if you multiply moment by distance, then probably you see you are going in a different unit case. But you see here our main intention is to write the moment equation and all those components which are coming within the equations, they should have you know like this units as in the moment Newton meter like that.

So, here obviously, you know whatever the things will come is that A distance is there, and we are simply taking the cross section at the x distance So, whatever the moment action is there on the beam it should be M into parenthesis x minus A to the power zero. So, zero should come because of the moment equation. Thus the integration quantity x minus A becomes either x minus A to the 1 or x minus A square. So, you see when you integrate those things then the different formation will come, but once you write the moment equation then probably you see you need to write with M into x minus A to the power zero.

So, this is somewhat different than what we have discussed in the previous cases when the point load and the UDL were there. So, now you see here. So, this was a pretty you know like simple equation was there because only a moment was acting, and it was acting at a point A distance from the left end. So, probably you see the two reaction forces were there, and they are equally responsible for carrying out this applied moment and then you see our beam is to be in a statistical equilibrium manner. And then probably you know like you can get it those moment equation, and once you have the moment equation you can get all those you see you know like E I into d 2 y by dx square equals to whatever the moment equation M into x minus A.

And then you see you can easily get the dy by dx by integrating you have some you know like the integrate constant C 1, and then again if you integrate that then you have the deflection equation E I into y which is nothing but equals to you see you know like the double integration of that. So, you have the two constants C 1 and C 2; by applying the boundary conditions you can easily get those a constant value C 1 and C 2. And by keeping those values in the main equation; you have the generalized equation for this kind of loading for a beam when it is simply you know like exerted by a bending moment at just distance from A.

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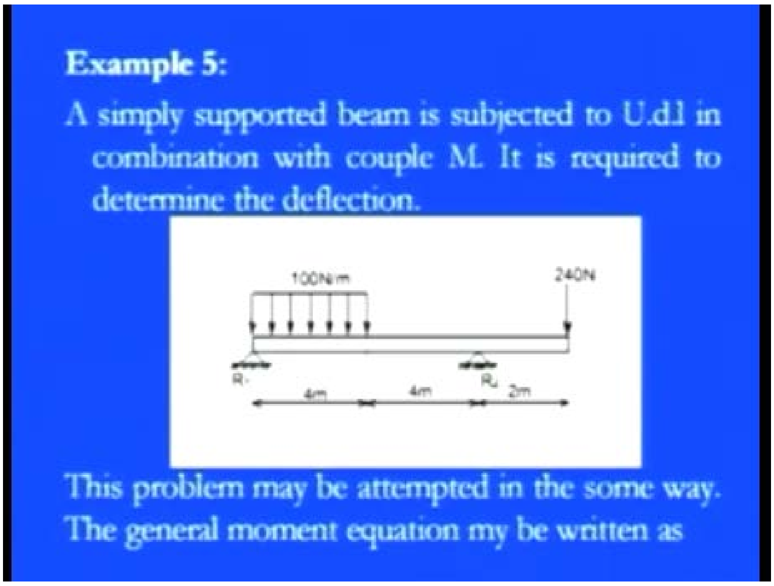

So, this is you see you know like a simple procedure which we have discussed many times you see that from the direct integration to this Macaulay method. Now come to you see one more example. Again you see this example is simply based on if we have a simply supported beam and it is subjected to UDL, but in the UDL you see you know like it is starting from the main reference point So, this is you see the place where the load application is there where the point of application of this UDL is quite different than what we have discussed.

So, in this case the important thing is that we are you see you know like we have the two reaction forces and these reaction forces; that means the supporter the pin joint supporters are one at the extreme corner of left end and one at somewhere you see in between the beam; that means it is not exactly at the extreme corner of right So, what has happened due to that? We have when we apply the force at the extreme free end corner of the right end you see; on the screen you can find that we have the 240 Newton due to that we have a couple, because you see it will just try to act in this way.

So, if you look at this point then you will find that whatever the deflection will come, you just see the arrow of this. It is in between these and then it will go like that. So, you see here we have a kind of deflection at this point. So, it is just lying in this way; in this way it is lying and then you know like this moment will be coming due to that. So, this problem may be you know like again attempted in some way somewhat you see you know like we need to write the moment equation generalized moment equation by keeping in this mind that there is a couple within you know like the combination of this UDL.

And we see here if you want to design those things again first of all we need to get the values of the reaction forces R 1 and R 2 simply by you know like the force balance and the moments balance equations. And then you see we have the applied two main things; one is we have the 100 Newton per millimeter UDL intensity is there, and second you see we have the point load which has the magnitude of 240 Newton.

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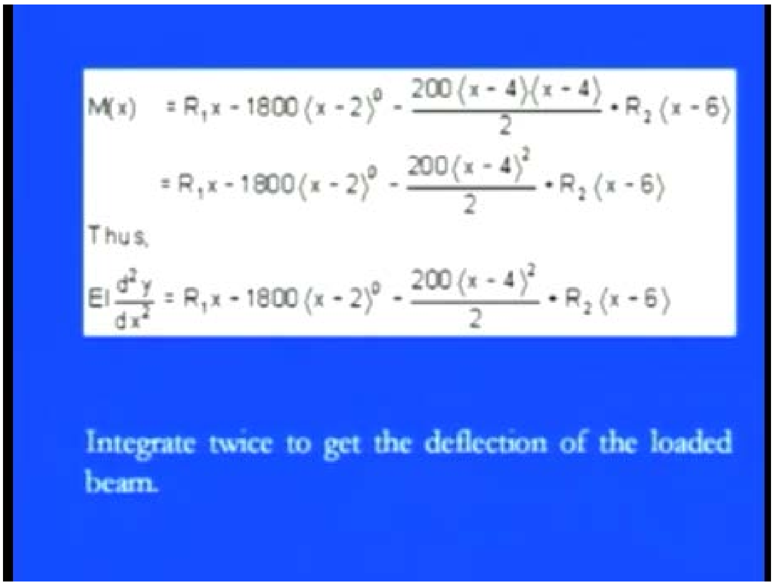

So, with those things now you know like I would like to discuss some things that what is the generalized moment equation. So, you see generalized moment equation is M of x which is nothing but equals to R 1 into x, because you see the x section is there; starting from the left end the reaction is R 1. So, R 1 into x minus; now you see we have you know like the 1800 because you see you know like we have the UDL. UDL is having intensity of 400, and then you see you know like it has the distance. So, when you multiply that distance and divide it. So, you see you have you know like 400 into 3 divided by you see the 2.

So, we have 1800 into x minus 2 zero, and then again you see because of the moment application of that load application, what we have? We have 200 into x minus 4 at the extreme corner you see at the full distance into x minus 4 divided by 2 plus the extreme right hand corner we have the reaction forces from you know like the bottom side. So, we have R 2 into since you see the distance is there from the left hand corner is 6 meter. So, we have x minus 6. So, if we combine that whole equation then probably we have R 1 into x minus 1800 x minus 2 zero minus 200 x minus 4 square divided by 2 plus R 2 x minus 6.

So, you see with the simple equation that we have E I into d 2 y by dx square is nothing but equals to the bending moment. So, simply keeping that equation in this form we have E I into d 2 y by dx square is nothing but equals to R 1 x minus 1800 x minus 2 zero minus 200 x minus 4 square divided by 2 plus R 2 into x minus 6. So, we have you see the common equation for the bending moment we can say it is a generalized moment equation for the kind of loading on the particular beam. Then you see now simply by integration we have the slope value E I into dy by dx which is nothing but equals to R 1 x square by 2 minus 1800.

You know like divided by 2 x minus 1 x minus 2 into 1 minus 200 x minus 4 to the power 3 divided by 6 plus R 2 into x minus 6 whole square divided by 2. And then again you see and then plus C 1 is there as a constant. And then if you again apply you see the integration method then you have E I into y which is you see somewhat C 1 x plus C 2 and all these bunch is there by the integration. So, you have the two same coefficients and by applying the boundary condition because you see now we have the boundary conditions that you see the UDL is applied exactly at the midpoint.

So, at point x equals to zero you have the deflection zero, and at point x equals to 6 where another reaction force is there you have the deflection zero. So, by keeping x equals to 0 we have y equals to 0 by keeping x equals to 6 y equals to 0. So, you see you need to keep those two boundary conditions for that, and you can they have the C 1 and C 2 values. So, by keeping those C 1 C 2 values you have the slope and you have the deflection, and then you can simply get also by keeping those things in your mind that where the maximum deflection can come.

So, just take that segment, analyze that segment you know like and then you can have a maximum you know like the deflection value at certain segments. So, this is a common or we can say a generalized method to get the slope as well as the deflection and also to get the maximum deflection this is pretty simple method; for different different segments you can check and verify those values. So, this is you see you know like the numerical problems for these things, and this is all over you see about the Macaulay method So, Macaulay method just provides you the basic flexibility to check it out the bending moment at the different different segments. So, now you see here you know like if you want to calculate a continuous part you can simply go for a direct integration equation. So, whatever the load application is there simply you know like put all those segments with the dynamics that where they are applying what you see you know like the distances are there.

So, you know like by keeping all those things in your mind just write the simple equation and then do the integration that is the simple integration method. Once you have you see you know like the bending moment diagram for that, calculate the area under that and simply you see once you have the area you can get the slope, and once you multiply the distance from the centroid you have the deflection for that. So, as we discussed in you know like the area moment method. And third was the Macaulay method which is quite flexible and you see once you know the unit function for the kind of load applying, pretty easy to you know like write straightaway write no need to you know like go for that.

Straight write the moment equation apply the boundary condition and keep this thing in your mind that whatever the bracket is there it should not show the negative value and you are done with your problem. So, this is you see you know like the generalized procedure for getting the slope as well as the deflection and even to get the maximum deflection when the load application is there on the beam irrespective whether the beam is simply supported or a cantilever.

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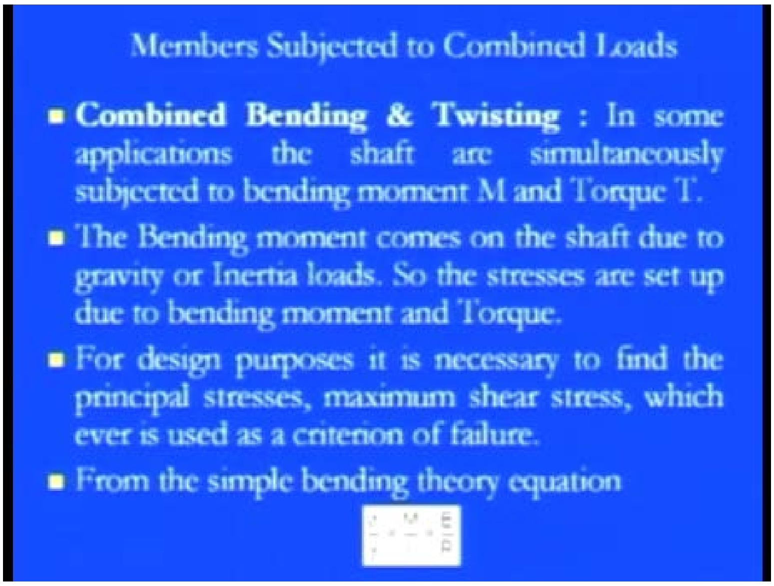

Now you see we have a different topic altogether that when a member or a beam or a bar is subjected to a combined loads, and combined loads is simply you see we have a bending as well as the twisting. Means in some was the application this is pretty common application you see; in industrial application where the shaft is rotating always we have and if you see certain unbalance is there in that. So, not only when it is you know like moving at certain speed and if unbalance is there, so certain bending action is there because of the load application plus if due to unbalance is there we have a torque. Because you see it is a rotating shaft and the shear formation is there all along the circumference of this particular shaft.

So, that means you see whenever shaft which is rotating in a machine in all the industries where the power transmission is there where any kind of action is there; these two you know like the action is always there. And they are you know like bending as well as the twisting you know like acting are combinedly. This is a realistic problem. So, we need to analyze this kind of beam when you see it is simply acted by bending action as well as the twisting action. So, bending moment comes on the shaft due to the gravity or inertia allowed.

Because you see if any mass or disc is you know like attached to this particular shaft; obviously, it will provide you know like the inertia forces on that and due to that we have a bending action. So, the stresses are set up due to the bending moment as well as the torque when it is rotating. And for the design purpose it is necessary to find the principle stresses, because you see it is very very important maximum shear stresses whichever is used you know as criteria of the failure. Because you see this principle stresses maximum shear stresses they are providing the basic inputs for designing of the shaft.

Because we know that when you see you know like all the material of the shaft has just you know experienced by this bending as well as the twisting action. Then how these bending as well as this you know like the shear stresses as you know like spreading all across the beam and which portion of the beam is highly influenced by these combination of these you know like these stresses; that means you see where the principle stresses are maximum, and then accordingly we need to design. So, that is what you see I am keeping on telling that these are whatever principle stresses and the maximum shear stresses; these are the basic inputs for a design engineer to get the real feeling that, okay, these stresses are there and once somebody wants to design those things.

Then he can even keep those design value for that and also they can keep their limiting that, okay, now if you go beyond the speed then probably you have a permanent sort of failure or permanent sort of deformation even in the shaft. So, you know like once we have the real feeling that we have a bending and twisting then again you see we need to go back for you see because we have already discussed about the bending theory as well as the twisting theory; that means the torsional stresses. So, come to that point; when we have the bending stresses then from the simple bending theory or the beam bending theory we have simple expression that sigma b by y.

That means you see the sigma b is nothing but the bending stresses which is a normal stress component divided by y which is the distance from the neutral axis to the fiber where these stresses are being evaluated equals to M by I; M is the bending moment is always there because of the bending action always bending moment is applicable. And divided by I is the moment of inertia based on what the cross section is there generally you see we have the cross section of the rectangular or square or we have the circular or any kind of cross section. Based on that we can simply calculate the I moment of inertia is equals to E by R. R is the radius and E is the modulus of elasticity or generally we are writing this curvature radius is there for that because you see when the curved beam is there then it has you see you know like curvature. So, probably we need to go accordingly.

But you see if it is a simple beam is there then probably we need to go with the R whatever the distance is there from that. And then E is the young’s modulus of elasticity because whatever the deformation whatever the stresses are coming, either the principle stresses, maximum shear stresses, whatever kind of stresses are there; we are always assuming this is the basic assumption in the strength of material that whatever the stress formation is there, they are under the elastic deformation only. And that is what you see the generalized Hooke’s law is valid; that is what we can say that we can define within that stress and strain limit within that elastic region only, and for that the young’s modulus of elasticity is defined.

So, this again you see we have discussed thoroughly that simply you know like certain assumptions are there for this generalized equation for bending, and those you know like assumptions are again valid for this kind of analysis. So, this is you see a simple equation for a simple bending theory, then again you know like if you want to go for the maximum bending stresses.

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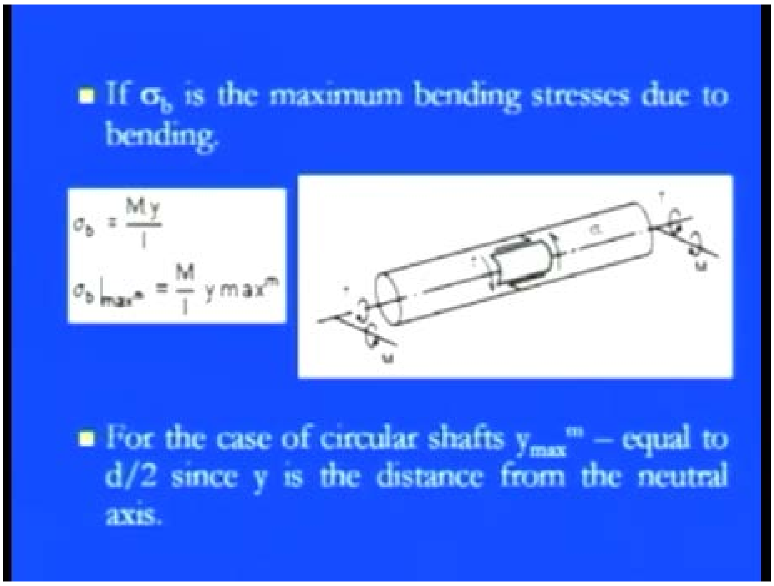

Then again you see you know like we discussed already that just the maximum bending stresses are nothing but equals to you know like what the distance is there from the neutral axis to the fiber. That means you see if you go the top of that and whatever you see the neutral axis there, what is the y maximum distance is there corresponding the stresses are maximum. So, if you look at the figure which is there in one screen, then you will find that we have a simple shaft, and this shaft is rotating. So, we have a torque at the straight you see the neutral axis, this axis which is passing form this.

So, this is the torque T is simply applied, and then we have a moment. So, you see here the moment is just on the normal to the surfaces. So, we have the moment. So, due to this bending moment we have you know like the corresponding stress that is the sigma b which is the normal stress component is there. So, the sigma b is there which is all along the longitudinal axis. So, you know like it is a normal stress component; sigma b is the bending stresses, and we can get the maximum bending stresses. First of all bending stresses can be calculated is equals to M into y divided I. So, M is nothing but this applied torque, the applied moment is there; this is the bending moment applied you see on your screen. This is clear; you know like this vision is there of this particular bending moment.

Then you have the y which is the distance as I told you and i is the moment of inertia as you can calculate this bending stress. And you can also calculate where is the maximum bending stresses are there just by keeping this you know like y maximum. So, you have sigma b maximum is nothing but equals to M by I Y maximum. And further you know like because as I told you the cross section is very very important for you know like calculating the i and which is also important you see to see the feasibility of the stress distribution that how bending stresses are being distributed along this particular beam.

So, if we are using the circular shaft as you see on your screen you can find that we have y maximum is equals to d by 2. That means it is exactly you know neutral axis is passing from the central axis, and we have you know like the two separate parts are there from top and bottom. So, for a circular case we have y maximum which is nothing but equals to d by 2, and if we are keeping those things you have and you know like corresponding value is coming in the moment of inertia i also.

So, for that case you can have the maximum stresses on the circumference of that, because you see this is you know like the central axis and this is the y, this B by 2. So, the total distance is this. So, you have all those maximum stresses on the circumference of this circular shaft where the bending stresses are forming. So, this is all about the bending stresses, and now you come to the torsional part.

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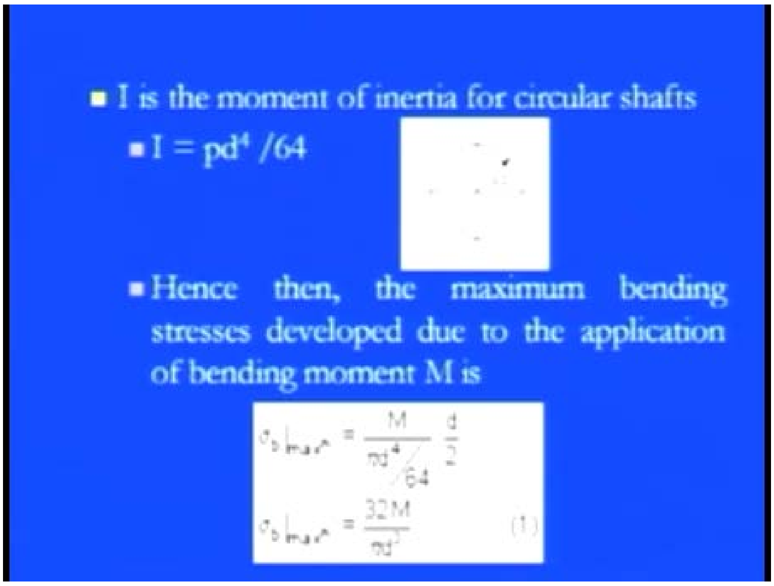

That means you see if you have you know like the moment of inertia of as I told you this circular shaft it is nothing but equals to this sigma d 4 divided by 64. So, pretty simple you have you see this d by 2 and this is the total you know like the diameter is there. So, if you are considering those things you have the moment of inertia for this thing. Then you see now you know like the maximum bending stresses as I told you simply M by i into y maximum. So, by keeping those values you have the bending moment M which is applied part.

So, we know the real figure of those things and you know also since it is circular cross section. So, we have pi d 4 divided by 64, and then also you have the d 2 because this is the y maximum is d by 2. So, by keeping those value pretty simple we can calculate sigma b maximum; this is the maximum bending stresses is nothing but equals to 32 M divided by pi d cube. So, you have this; the maximum bending stresses when you know like the bending action has happened due to the applied conditions.

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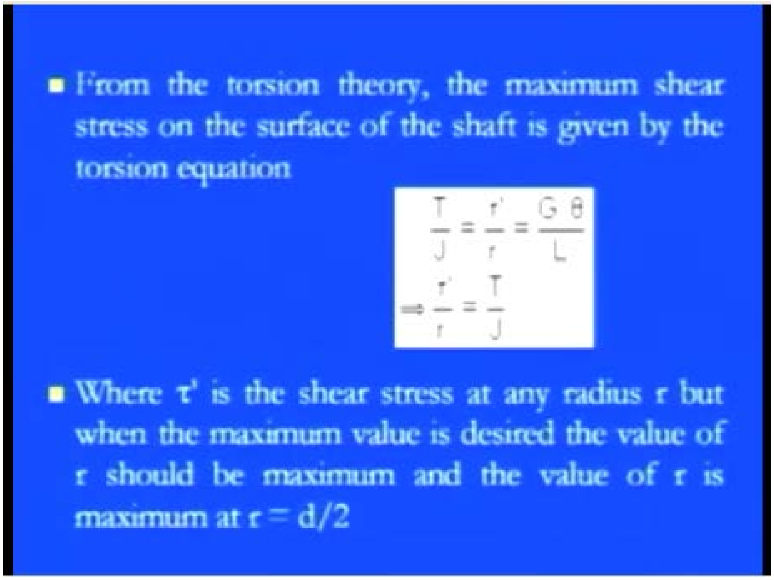

Now come to the main torsion theory because the torsion stresses also there along with the bending stresses. So, the maximum shear stresses this is you see as I told you the torsional stresses are one kind of shear stresses because you see it is in the rotation form. So, all the whatever the moment is coming it is due to the shearing action, and that is why you see we have all along the circumference of the circular shaft. So, the maximum shear stresses on the surfaces of the shaft is given by simple equation which we have discussed many times you see T by J; T by J is nothing but equals to tau dash by r.

So, T is the applied torque is there, J is this section modulus of elasticity is there which is equals to tau dash by r or we can say that zhi theta by L where zhi is you know like since the Young’s modulus of elasticity is there, and here you see the shear modulus of elasticity is zhi. And then we have theta which is the angular twist is there because of the applied torque and then we have the L is the total length. So, from these equations and this equation also has you know like some of the certain assumptions are there that whatever the cross section is there it has to be uniform and it has to be according to the inner circumference of that. So, all those assumptions are valid here also for this particular equation.

So, based on those assumptions and this theory that torsion theory we have in a final equation tau dash which is the shear stresses at r distance. Tau dash is you know like the subjective shear stress at r distance; tau dash by r is nothing but equals to T by J. Tau dash is the shear stress as I told you at the radius r, but when the maximum value is described; obviously, you see r should be having the maximum part. That means, you see if we are talking about a circular cross section as in the bending case we have r equals to d by 2. So, again you see the same you know like we can get the tau maximum or we can say tau dash maximum is nothing but equals to this r is T by 2, so T by J into d by 2.

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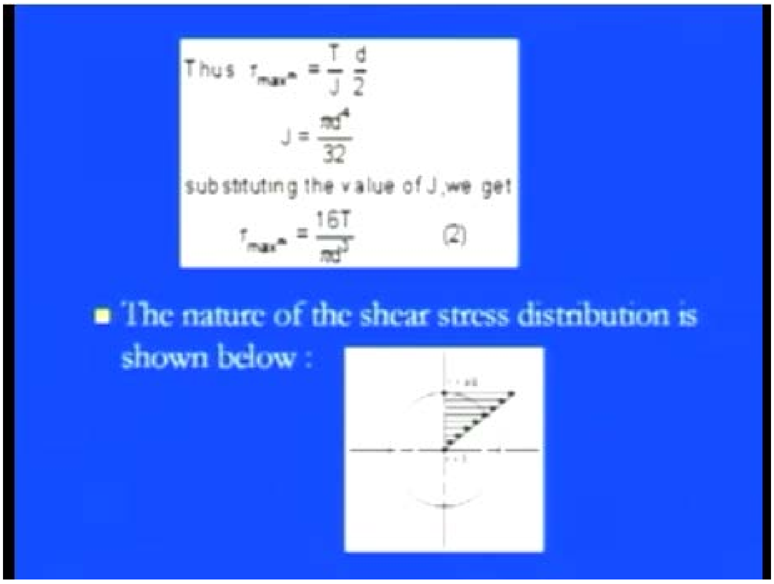

So, by taking that part you see here we have you know like tau maximum as T by J d by 2 or we can say the J which is you see you know like the section modulus which is just showing that what the cross section of you know like the beam is there where is the shearing actions are happening. So, we have pi d 4 divided by 32, and you know like by substituting those values we have tau maximum is 16 T by pi d cube So, now you see we have the maximum bending stresses, we have the maximum shearing stresses, and again you see if we want to see the shear stress distribution all along you know like this circumference of the beam. Then you will find that starting from the zero of the neutral axis because always neutral axis is simply free from whatever the stress development is there in the beam.

So, from that and it will go and the maximum you see on the circumference of the beam. So, you see here this kind of distribution is there on the shear stresses starting from zero to maximum and we have you see this kind of distribution So, it is at zero at r equals to 0 we have shear stress zero tau maximum. And we have you see the maximum whatever the 16 T by pi d cube at this particular circumference of the beam.

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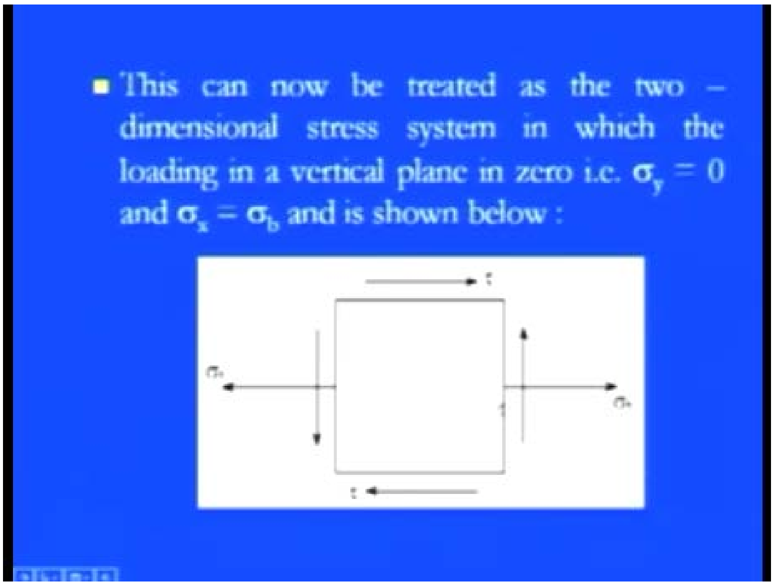

So, now you see what we have? We have a normal stress component that is sigma b; we have the shear stress component that is the tau maximum. So, based on that we can simply figure out that what will be you know like if we have a simple element then how they are acting. So, this can be now treated as a two dimensional stress problem, because at one point you see we have a normal part sigma b on longitudinal part and on latitude part or in the circumferential part we have a shear stresses So, with this 2D problem you know like simply we need to take sigma y, because there is no stress component in the vertical direction. And you see there is no you know like the z direction is there.

So, only we have sigma b which is equals to sigma x, and you see you know like with those particular conditions we can simply you know like show on a simple cubical form. So, we have the square term is there on the cube; we have sigma b on this particular action, and we have the tau on the section So, now under the action of these combined stresses, the bending stresses and the torsional stresses, our element is well stabilized or in equilibrium position. So, now we can simply calculate the principle stresses for that.

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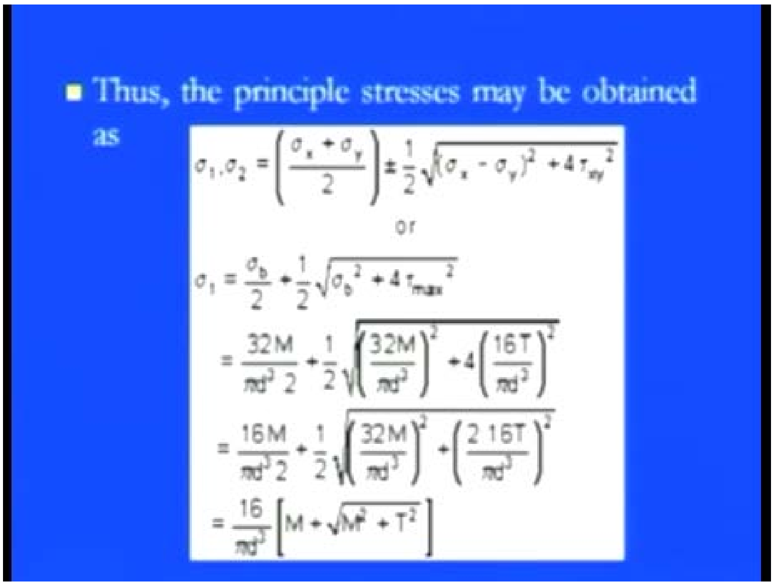

So, the principle stresses can be easily found out by the simple formula sigma 1 comma sigma 2; these are the two principle stresses for that which is due to you see a generalized form that if you see the two main you know like the stresses are there sigma x and sigma y and tau xy is there along that particular plane. So, we have you know like the sigma 1 comma sigma 2 is sigma x plus sigma y by 2 plus minus half of square root of sigma x minus sigma y you know like whole square plus four times of tau square xy.

So, this is a generalized form, but here as I told you there is no normal stress component is there in the vertical direction; only on the horizontal direction we have the bending stress component. So, sigma x is equals to sigma b; sigma y is equals to zero. And whatever the tau xy is there that is you see the tau maximum or the shear stress maximum. So, by keeping those values in the equation we have the two principle stresses. So, if we take the plus sign we have the first principle stress sigma one which is equals to sigma b by 2 plus half of you see the square root of sigma b square plus four times of tau maximum square. And now you see you know the value of sigma b that is 32 M by pi d cube, and you know the value of tau maximum that is the 16 T by pi d cube.

So, by keeping those values you see we can simply have you know like 32 M by pi d square into half plus half of square root of 32 M by pi d cube whole square plus four times of 16 T by pi d cube whole square. So, by keeping you see you know like simple manipulations we have the final principle stress; one is a 16 by pi d cube into M plus square root of M square plus T square. That means you see here your principle stress is consisting you know like both of the component and the main component if you look at sixteen is, okay, the numerical values, the pi and d you know like these are all the dimensional part.

The main thing is the moment M plus again moment square plus this torsional part, or we can say this shear stress component. So, meaning is pretty simple that in combination of these two, the bending is real a good action because you see it is happening all around you see you know like the entire span of the beam, and torsional is somewhat you know like they are influencing. So, again we have to be very very careful that how much bending action is happening while you know like during the combined action. So, based on that, again it is very sensitive to design according to the bending moment. So, we have sigma 1 which is 16 by pi d cube M plus square root of M square plus T square.

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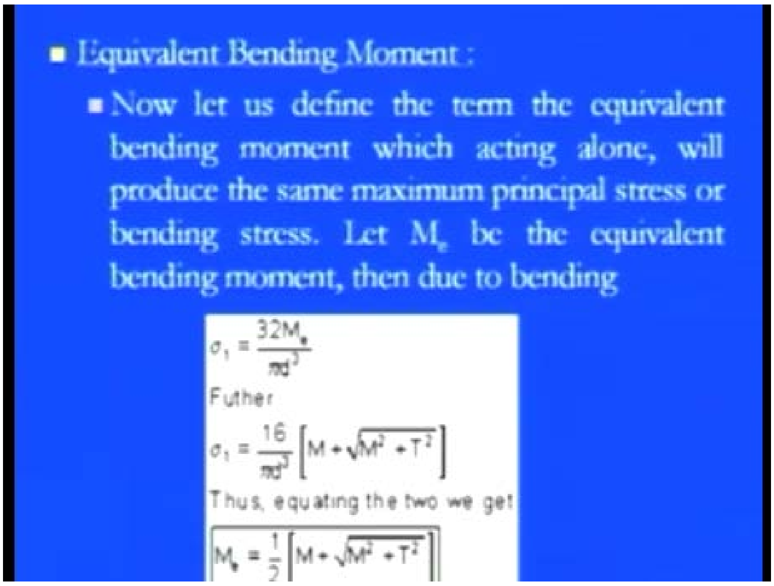

So, you have now sigma 1. So, you can also calculate sigma 2 which is nothing but equals to 16 by pi d cube into M minus square root of M square plus T square. So, you see you have sigma 1 and sigma 2, then also you can calculate now the equivalent bending moment. Because you see you know like we have bending moment altogether you see it is spreading in the entire span So, now let us define the term equivalent bending moment which is acting all alone you see you know like produce the same maximum principle stresses or we can say the bending stresses. Because you see if we know that this is producing the similar kind of bending stresses, because as I told you that in those equations we have the maximum portion of the bending moment.

So, what is the equivalent bending moment is there which can equally produce similar kind of principle stresses as you know like simply applied by these particular combination of bending moment as well as the shear stresses. So, for that now if I am saying that M e is nothing but the equivalent bending moment. So, by keeping those things, what we have? The principle stresses sigma 1 which we calculate 16 by pi d cube into M plus square root of M square plus T square. And now if I am saying that the similar kind of this principle stresses are being produced by this equivalent bending moment. So, we have the sigma 1 which is equals to 32 M e divided by pi d cube.

So, now by equating those things we have an equivalent bending moment which is equals to half of M plus you see M square plus T square. So, you look at this picture; you will find that the real feature of the bending moment. We have you see you know like the bending moment is there which is exactly whatever the equivalent amount is coming, it has you see the real contribution of this whatever the bending moment which is actually happening over there and the combination of this torque.

So, by that way you see we can simply spread that how the distribution of this bending moment is there all across the span of the beam when you see it is there with the bending moment as well as the shear stresses. So, this is you see the equivalent bending moment we can simply figure out, and now you see what the equivalent torque is there.

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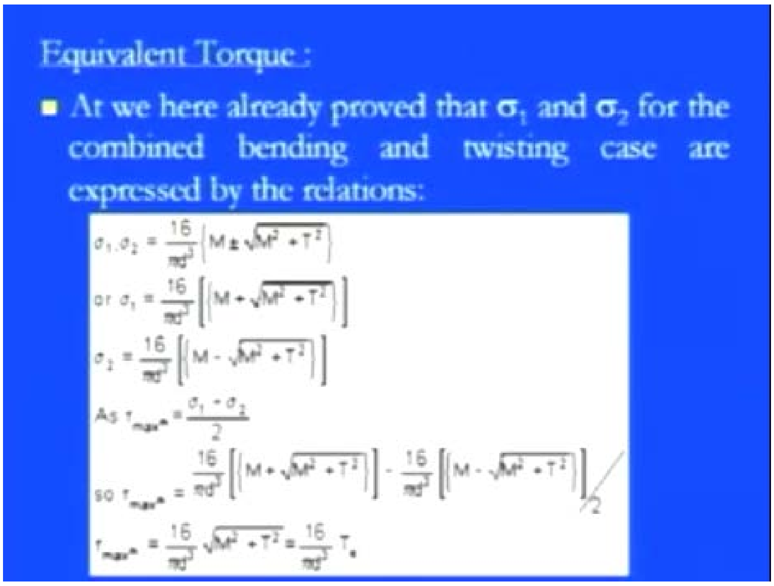

So, again we need to go back to those things that what you know like these principle stresses are there. So, we have a principle stresses sigma 1 or sigma 2 which has you see 16 by pi d cube M plus minus square root of M square plus T square So, we have both the component sigma 1 and sigma 2 just by adding and subtracting, and we know that when there is a principle stress which is acting all along this particular plane. So, how we can get those maximum shear stresses? Maximum shear stresses are nothing but equals to the principle stresses sigma 1 minus sigma 2 by 2.

So, now you see we have sigma 1 which is 16 by pi d cube into M plus this square root of M square plus T square and sigma 2 which is 16 by pi d cube M minus square root of M square plus T square. So, now you have sigma 1 and sigma 2. So, just by keeping those sigma 1 and sigma 2 values in the tau maximum; that means the maximum shear stresses. We have the total combination of you see the 16 divided by pi d cube into M plus square root of M square plus T square minus 16 divided by pi d cube, the minus square root of M square plus T square divided by 2.

So, just by evaluating those things we have the tau maximum the shear stress maximum is nothing but equals to 16 by pi d cube square root of M square plus T square or we can say that the equivalent amount of you see 16 by pi d cube T e. So, what we have now? We have the torque equivalent; that means you see what is the equivalent torque is there which is nothing but equals to square root of M square plus T square. And we had also the moment equivalent is there; that was you see half of M plus square root of M square plus T square So, with that you see here now we just want to you know like conclude that part where you see you know like the square root of M square and T square is defined as the equivalent torque as we discussed.

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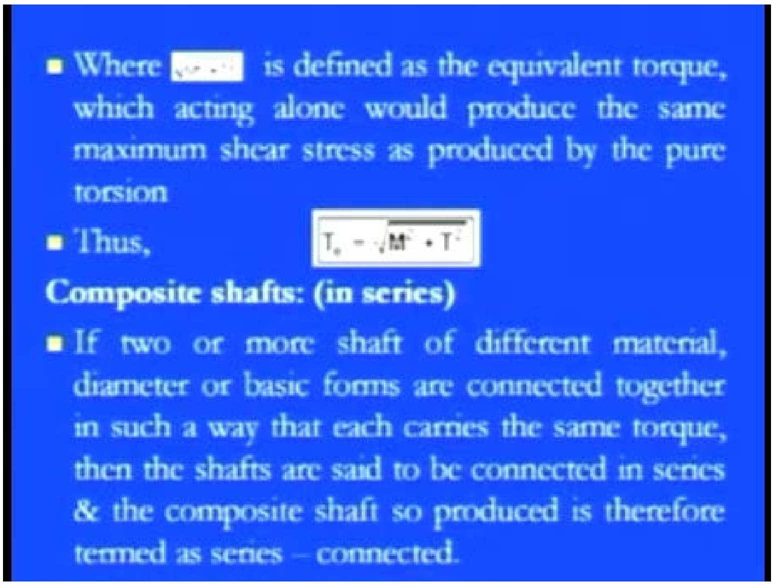

So, T equivalent is nothing but equals to you know like the M square plus T square with the meaning is that this equivalent torque is producing the similar kind of maximum shear stresses as we produced by the combinations. So, the meaning is pretty simple that now we can simply replace you know like the individual contribution of this twisting moment and the bending moment by equivalent amount of bending this moment and equivalent amount of this torque moment So, you see with the equivalent torque and equivalent bending moment, we have you see the real figure you know like the combination of both bending action as well as the shearing action.

So, this is you see all about this combination. Now you see we are going for if we have you know like the different set of the shafts, then how we can figure out those things? That means you see if we have like you see the composite shaft and the first case if the shaft is there in the series how we can say the shafts are there in the series that if the two or more shafts of the different material you see. And even you can say the different diameter or you can say that if they have a basic form and if they are connected together;

basic forms means if they have different diameter altogether different area is there, different material is there.

And you see if they are you know like connected together in such a way that each you know like carries the same torque, then we can say the shafts are set to be connected in the series. That means you see there is no change in the torque application or whatever the torque transmission is there; there is no change in that irrespective of whether there are different material or the diameter. Means you see if they are carrying the similar kind of torque they are in the series formation and you see the composite shaft. So, produce is you know like known as the series composite shaft in which you see whatever the two three components are there they are treated to be a series component.

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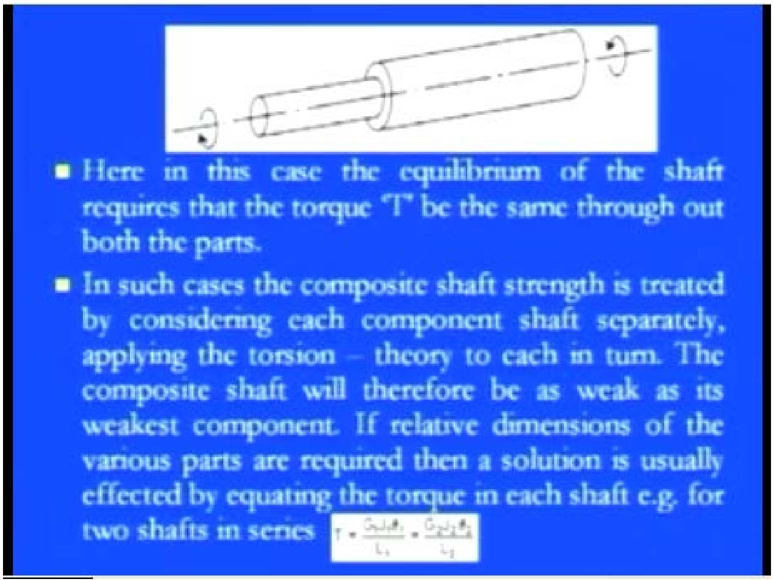

So, for that as you can see on this particular figure we have you see you know like these two different diameters are there on this shaft. And they are well connected such that you see whatever the torque transmission is there from here and here it is just T; there is no change in the torque transmission or the torque applied. So, here in this case you see the equilibrium of the shaft which requires you see you know like the torque T is the same you know like throughout both of the part. So, whatever the T input is there, exactly we have the T output; that means you see there is no this transmission, ratios are there in between those torque transmission, so that we have a torque reduction or torque enhancement.

So, pretty simple assumption for the series is that if the torque input is equals to torque output or the torque transmission is same we have the shaft is in the series So, this is the either you can say the assumptions or this is very good input for solving the numerical problem that whatever the torque is there, it has to be equal for all the segments of the shaft irrespective of what the different material is there or different diameter of the shaft is. In such cases the composites shaft you know like the strength of that kind of shaft is treated by applying the torsional theory you know like in each of the separate term. That means you see you know like what we are doing here if the different different segments are there as you can see on a screen you have the two different segments.

So, we need to apply the torsion theory on individual you know like the component there and the composite shaft which is you see you know like the combination of these two shaft will be therefore as weak as its weakest component is there. And if the relative dimensions with the various parts are required then solution is usually you know like effected by equating the torque in each shaft. Because you see you know like somewhere you see the diameter is very small or some of the material which you see which are very good in this twisting moment which some of you know like are not good.

So, when we know that you see the torque applying is exactly you know like coming similar, then we need to go according to you know like the series theory. So, for that you know like when the two shafts are in the series; obviously, the torque which is applied is equal then you see T 1 equals to T 2. And we can say it is equals to T which is equals to zhi 1 J 1 theta 1 by L 1 which is applicable to the first shaft means whatever irrespective of its diameter material because zhi 1 and zhi 2 are nothing but the shear modulus of the rigidity, and it is absolutely based on what kind of material which we are taking.

And you see now if we are saying that the combination is equal then we need to check it out that what the ratios are there of their zhi 1 and zhi 2. Then you see the J 1 and J 2 these are you see the section modulus of then section modulus is always based on what kind of cross section is there of these beams. And then we have the angular twisting is there theta 1 and theta 2. So, if we know that let us say if their material is same. So, we can say zhi 1 and zhi 2 are same; if we know the cross section is same then we can say that J 1 and J 2 same, but if we are saying that the ratio of this particular length this segment length L 1 and L 2 if they are different. Then obviously, you see the corresponding changes are coming in the theta 1 and theta 2.

So, there are various things which are associated together if we are saying that if they are in the series. Then obviously, the corresponding ratios of their zhi 1 or zhi 2 or we can say J 1 J 2 or theta 1 theta 2 or L 1 L 2; they have to be you know like the proportionate manner is there, so that we can say that the T 1 equals to T 2. So, this is the basic theory about the series.

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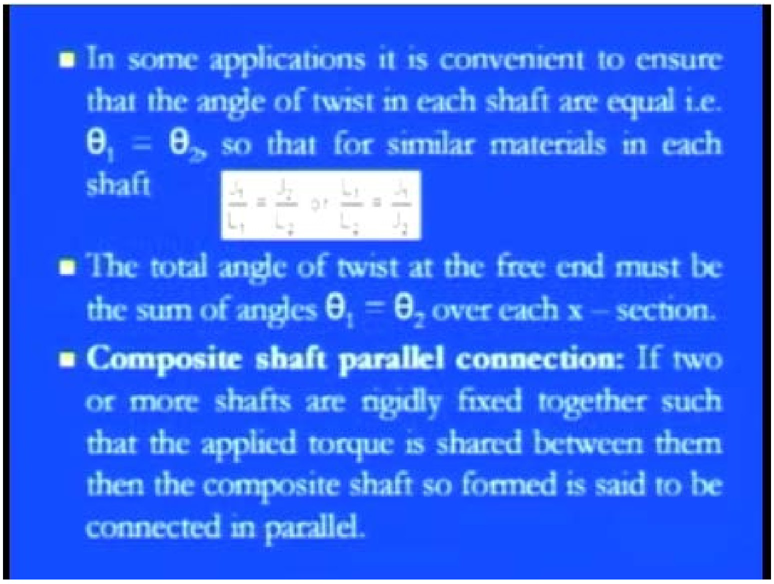

And in some of the application it is convenient to assure that the angle of twisting in each shaft is equal. So, if we are saying that the theta 1 is equals to theta 2; that means you see they have this kind of ratio then only it is possible; that means if the J 1 by J 2 is equals to L 1 by L 2 or we can say you see whatever the corresponding cross section of the section modulus is equal to the corresponding length is there on the segments. Then we can say that whatever the angular twist is there on these two shafts, they have to be equal, and the total angle of twist is as you see at the free end or we can say they have to be you know like when we are saying that is equal. They have to be algebraic sum of these two this theta 1 and theta 2 is there over the each cross section.

So, now you see this is the brief theory about when we are saying that you know like the combination of the two separate segments are there in the series way or we can say it is a composite shaft. Second part if the composite shaft is having the connection in the parallel way. So, if two or more shaft are rigidly fixed together, okay, important thing they are rigidly fixed; in that case it wasn’t like that such that the applied torque is shared between them. The composites shaft is formed you know like whatever this kind of shearing action is there in terms of the torque, they have to be treated as the parallel shaft.

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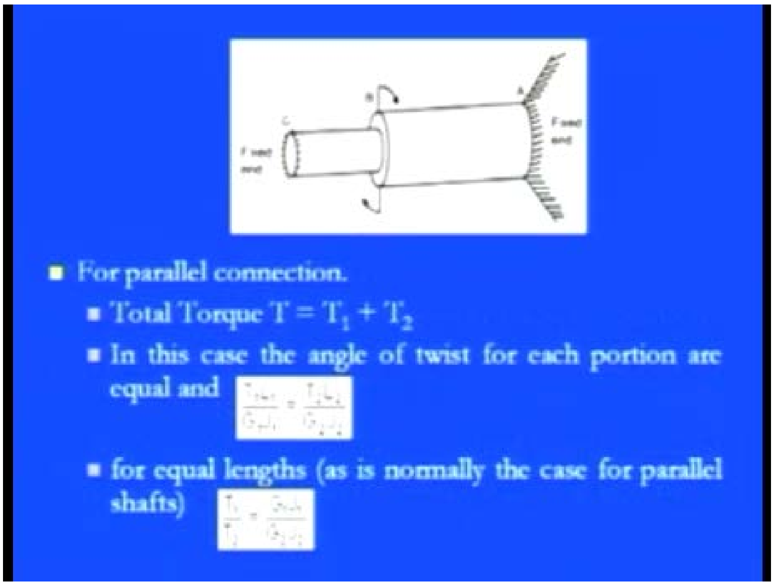

You just see the figure here. You have the fixed end at both the part, and you see here they are equally connected; you see here this is simple connection only what we have done here? This is the fixed form at the extreme and this is the extreme end and left end and you see this is the fixed form at extreme right hand. So, both ends are rigidly fixed to each other, and then you see if both part; this part as well as this part is sharing the torque. So, if I am saying that the torque is shared by this portion is T 1, the torque is shared by this portion is T 2. So, the total torque shared if I am saying the total torque is T.

So, it is equals to T 1 plus T 2; that means you see in the series formation the torque was same, here the torque is now shared by these two components. So obviously, this kind of connection of the shaft is known as the parallel connection of the rotating shaft. For that now the total torque T is equals to T 1 plus T 2, and in this case you see the angle of twist for each portion is equal. Now this is a very important conclusion out of it that whatever the theta 1 is coming it is exactly equals to theta 2 because they are rigidly connected.

So, whatever the angle of twist is forming and that too you see what we have? We have both extreme ends are rigidly fixed up. So, when you apply the torque here let us say T is applied. It has to be shared by both of the component in equal manner of their angle of twist, so that we can say that the torque is now different in that terms because theta 1 is equals to theta 2. So, we have T 1 L 1 divided by zhi 1 J 1 is equals to T 2 L 2 divided by zhi 2 J 2. And if I am saying that both the segments are equal in the length then obviously, L 1 equals to L 2.

So, what we have? We have you see for the parallel shaft the torque ratio whatever you see the torque is sharing by these two the T 1 by T 2 is absolutely based on what this shear modulus of elasticity is there; that means zhi 1 by zhi 2 ratio and J 1 by J 2 ratio. So, you see its absolutely depend on that what the kind of material which we are taken and what kind of cross section is there; corresponding changes or corresponding shearing is to be taken by these two component for the shear action.

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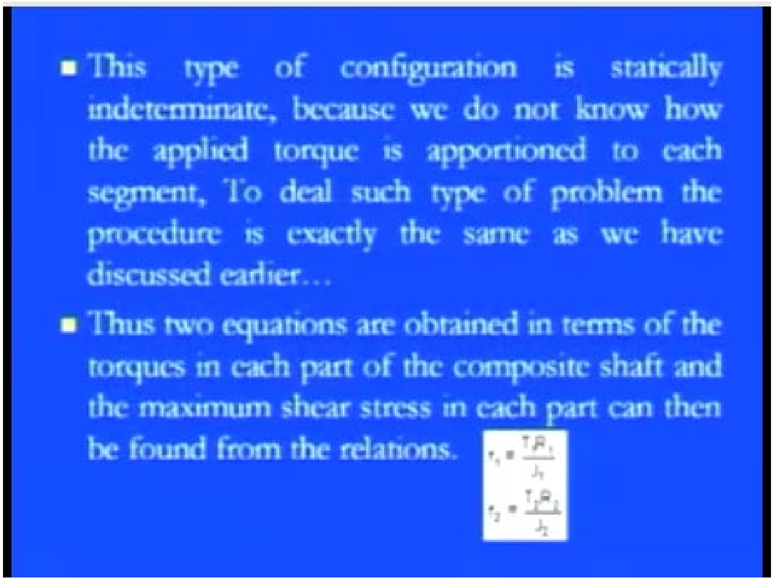

So, this is you see a kind of you know like the good information as far as you see we have the series connection or the parallel connections. So, this type of configuration is statically indeterminate, because we do not know how the applied torque is to be you know like apportioned to each of the segment means you see how they are just coming towards the segment. We know the ratio, but what the contribution is to be shared by the small or bigger section that has to be you know like again a big question. So, to deal this kind of problem the procedure is exactly same which we have discussed in the previous case.

So, again you see we need to check it out the different different segment that actually, okay, now this is we need to take the separate segment, go for you know like the calculation and check it out; yeah, this is the safer side or not and then go for the another section. So, this is the similar procedure which you know like discussed in the series connection; that the two equations are obtained in terms of the torque in each of you know like the part of the composite shaft for parallel connection as well as this series connection. And in those connections you see again you have to be very careful that when we are talking about the parallel connection, then you see the angle of twist has to be same.

And then you see this is you know like the big key part is there to resolving any kind of you know like the issues which is quite related to the parallel action. And then you see the corresponding torque ratio or the section modulus ratios or you see this shear modulus of rigidity ratios are coming correspondingly. And then you see if you are talking about this composite shaft which is to be in the series connection, then you have to be very very careful that whatever the torque transmission is there that has to equal. So, T 1 and T 2 for the two components are to be equal, and then you see the corresponding you know like the angle of twist theta 1 and theta 2 is coming over based on what the length ratio is there, what this shear modulus ratio is there or section modulus ratio is there or what the shear modulus of rigidity ratio is.

So, you see these are the key parameters for solving this kind of action. And you see if you want to calculate the maximum shear stresses in each of the part irrespective of what the combination is there. Then you see this relation is there the tau 1 which is nothing but equals to T 1 R 1 by J 1, and tau to which is nothing but equals to T 2 R 2 by J 2. So, again you see individual sections we can you know like figure out that what will be the maximum you know like the shear stresses are there in these kind of things.

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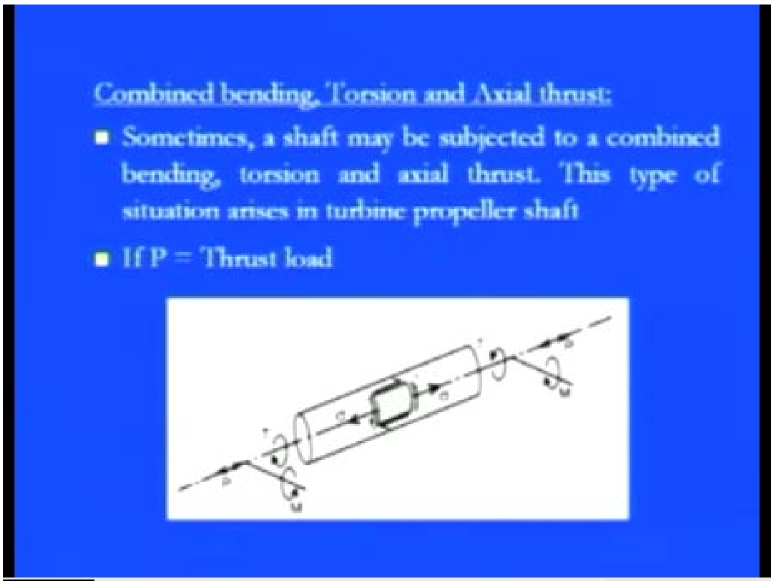

Now come to you see that if we have a combined combination of not only the bending and torsion but also there is an axial thrust is there you see because of the interconnection of the different this you know like component in the machine. We have you see certain kind of force influenced by different you know like the segment on the shaft. So, we can say that there is an axial thrust may happen while in the rotation. So, sometimes you see a shaft may be subjected to this kind of situation where you see we have the bending action, we have the torsion action, and we have the axial thrust.

So, this kind of you know like the pretty common problem as I told you in the turbine propeller shaft where you see the interconnections are there, and here you see we are simply showing the torque T applied here on the you know like extreme ends of these things we have the bending moment. So, due to that the bending action is there, and we have the axial thrust is there on the P.

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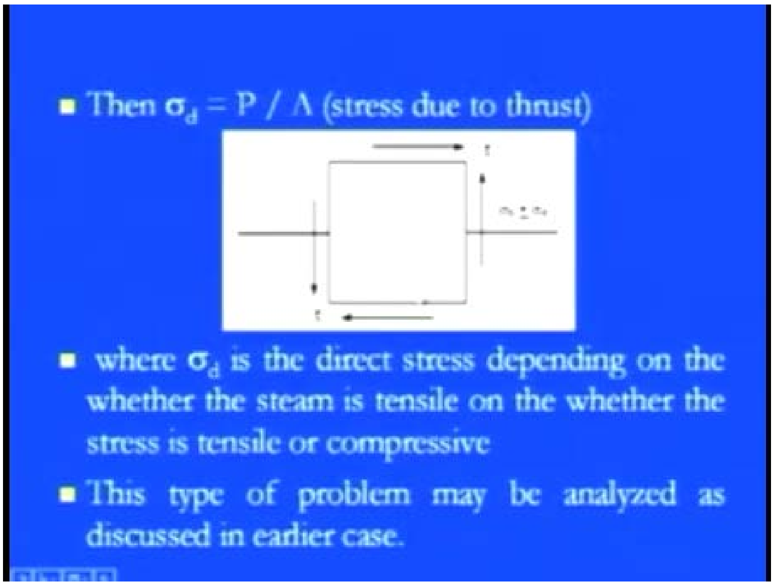

So, you see this kind of you know like the problem again, what we need to do here? We have a bending action which is a kind of normal stress component, and we have the axial thrust which is also a form of this normal stress component. So, now you see in the normal action we have you see the addition of sigma b and sigma d; sigma d is nothing but equals to P by A which is you see the direct stress component, sigma b is the bending stress component. And now you see in addition to you know like this we have the tau which is you know like the shear stresses.

So, sigma b is the direct stress as I told you depending on what kind of you know like this steam is there; that means it is a tensile or the compressiveness is there. Then you see you know like the sigma b; if it is a tensile part we have sigma b plus sigma d, if it is a compressive part then we have sigma b minus sigma d. And you see you know like the corresponding analysis is very simple because you see now you have the combined direct stress. So, we need to you know like make the algebraic sum of these direct stresses and you can simply make you know like the shear stresses as per are the simple way. And then corresponding you see the principle stresses are coming sigma 1 and sigma 2, and then corresponding you see the tau maximum and you see this bending moment maximum we can get it.

So, you see you know like the procedure is pretty simple; only there is an addition of sigma d is there in terms of sigma b. So, you see in the longitudinal part we have an additional you know like this stress component is there that is the bending; apart from this bending moment we have the direct stress is there in the normal stress.

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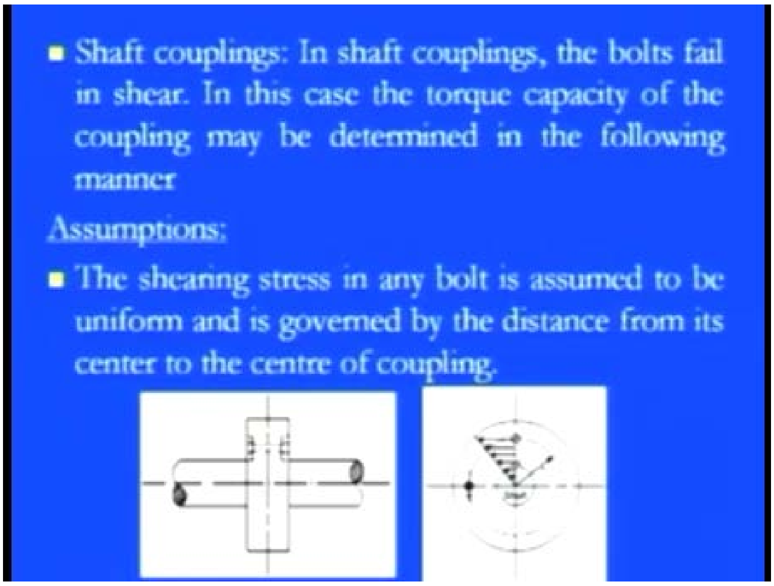

So, when you see again if the shaft couplings, there generally you see we will find that always we have in a coupling. So, in the shaft coupling the bolt all, whatever the bolts are there in the coupling part they all always fail due to shear. And in this case you see the torque capacity for you know like of this particular coupling it may be determined by the following manner. First you see you know like as you can see here we have this kind of you know like the phenomena. So, if we look at this particular this side view then you will find that we have the circular action here. This is the shaft circular action, and this is you see the outer part of that you see.

And at this section starting from this zero because you see it is at the neutral axis; this is the neutral axis at which there is no this stress action is there. Starting from that and we have the maximum you know like the shear stresses are there, and that is why you see these bolts are always they have to be failed due to the shear because here the shear stresses are maximum. The assumption is that the shear stresses in any of the bolt is assumed to be uniform, because you see if there is certain jerk is there or impulse action is there then it is not like you see some times it is maximum, sometimes it is minimum.

It has to be uniformly distributed all along that, and that is what you see you can see here it is uniformly distributed starting from zero to the maximum at this extreme corner and is governed by the distance from its center to the center of the coupling. So obviously, you see it has to be governed by zero to the maximum as you can see in this diagram. So, irrespective whether the bolt is here or here they are always failed due to the shear action and which is quite significant phenomena.

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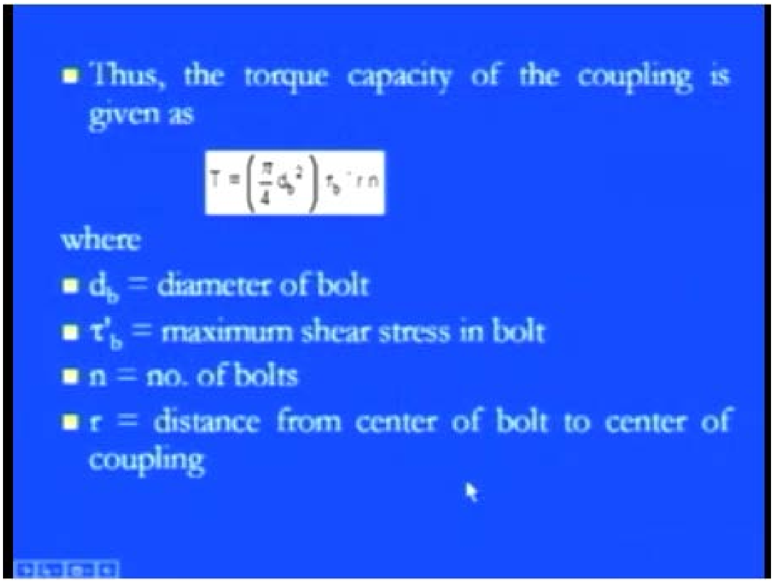

Thus the torque capacity of the coupling can be easily found by that because you see it is only the shear action is there. So, T which is the torque applied you see is equals to pi by 4 d b square into tau b into r into n where you see the d b which is because the failing action is there at the bolt only. So, we have the diameter of bolt; tau dot b is nothing but the maximum shear stress which are coming due to this shear action at this particular bolt part right from zero to the maximum at the circumference, and n is the number of bolt that how many number of bolts you are applying. so that the corresponding you know stresses are coming on those things or the distribution of the stresses are there. And r is the main distance from the center of bolt to the center of the coupling.

So, you see you know like from that one can easily figure out that how much maximum shear stresses can come in the bolt, and how we can make the safe part of the bolt due to the shear action. So, once you know the torque applied you can simply get the tau B and then you see you can simply get you know like you can simply apply the factor of safety based on that, or you can simply increase or reduce the number of bolt n corresponding to safer design of our coupling.

So, this is all about you see this chapter you know like starting from some numerical problems about these Macaulay method, you see if the couple is there or the different action of the couple and the applied load part is there. Then how we can figure out those moment equation and how we can get the slope as well as the deflection for this kind of combination that part we discussed. And then the middle part is very important you see which we discussed that if you know like the combination of the bending stress as well as the shear stresses is there.

That means you see the bending and the torsion actions together which is quite feasible phenomena in the all industrial part. Then how we can resolve those issues, how we can say that you know like which part is bearing this kind of thing, or what will be the principle stresses are there, and what will be the this maximum shear stresses are there, and what are the values of that? And then in the later part we discussed that if the shaft is connected parallely then what is the basic theory behind it, how we can analyze the torque, and then if you see the shaft is connected; that means the composite shaft is having a parallel connection then how you see the issues can be resolved in this way.

So, this is all about you see you know like this chapter, and you see in this chapter we simply you know like figure out that what will be you know like if the different kind of combination of these bending as well as this torsion is there. Then how we can get those values of this basic principle stresses as well as the shear stresses. And then you see based on that now in the next chapter we are going to analyze those you see the failure theories that you see if you know like if this kind of actions are happening on the beam as well as the coupling or the bolt. Then how you see what kind of failure theories are there and based on that how we can make the safer design for that, so where are various theories and we will take individually all those things in the next chapter.

Thank you.