Strength of Materials
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Lecture – 33

Hi, this is Dr. S. P. Harsha, Assistant Professor, Mechanical and Industrial Engineering Department, IIT Roorkee. I am going to deliver my lecture 33, on the course of Strength of Materials, and this course is developed under the National Programme on Technology Enhanced Learning (NPTEL). Prior to start this lecture I would like to briefly discuss about the previous lectures. And, you know like, we discussed about the deflection theories of the cantilever beam or simply supported beam. In that, you see, here the first theory was discussed as the direct integration methods. That if, you see, the beam is loaded by various kind of loads as well as, you see, the combined loads, point loads, and you see, the UDL; then, how we can get the slope as well as we get this deflection?

So, the first method, which is pretty simple method, is the direct integration method in which, you see, we need to derive the equation as E I into d2y by dx square which is equals to M. So, through that, you see, you know like, first of all we would like to know about the shear force diagram and bending moment and based on that - the bending moment - we can easily get the dy by dx - that is the slope as well as the y that is the deflection.

And then, the second method which we discussed about the area moment method. In the area moment method, simply, you see, we do not have to calculate individual deflection, and individual, you know like, the integration points at the boundary conditions. In that we discussed about, that if you know the bending moment, because you see, in that all these cases we know - we need to know - the bending moment. So, once you have the bending moment diagram, only we need to get the area under the bending moment. And once you know the area under the bending moment, then you have the slope, and then once you again, you see, multiply this area with the distance, then you have the deflection.

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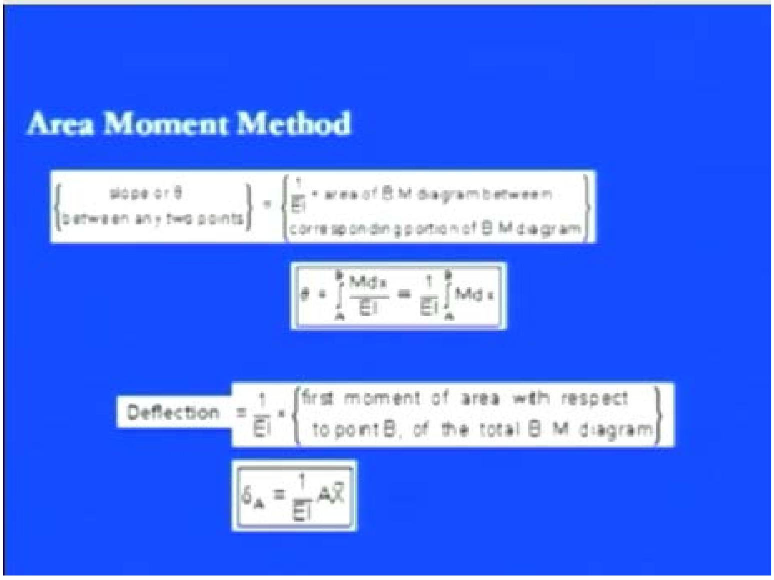

As you can see on your screen that, these two formula in the area moment method. So, first of all, you see, the slope, which is nothing but the theta between the two different points on the beam as well as, you see, on any of the two conditions, we want to know the slope. Then it is pretty easy to calculate, because it has to be multiplied by 1 by EI, and EI is the flexural rigidity of the beam, depends on what kind of a beam material is there, because E is the Young's modulus of the elasticity and I is the moment of inertia for that. So, 1 by EI into the area of bending moment diagram between those two corresponding points where the bending moment diagram is to be drawn.

Or we can see that, you see, theta is nothing but equals to integration of A to B, because you see, these are the two corresponding points where we want to calculate the bending moment. So, you see, for that integration A to B M dx by EI and M dx integration of A to B M dx is nothing but the area under the bending moment diagram, because you see, this bending moment diagram is coming within these two reference points.

So, this is your slope and once you see, you have the slope, then it is pretty easy to calculate the deflection, because it is nothing but equals to, again you need to integrate that, you know like, the theta. So, once you integrate the theta within these two points, you have the deflection; that is the delta which is equals to 1 by EI; again the same flexural rigidity of the beam into the first moment of the area with respect to point B. Because you see, you know like, this is - the A - is the reference point and B is the second point through which we need to calculate the first moment of area and the then, you see, you need to multiply with the distance X bar.

So, if you see, you know like, the formula delta A which is equals to 1 by EI A into X bar; X bar is nothing but the distance of centroid from point A and, you see, A is the area of the bending moment diagram and EI is the flexural rigidity. So, these you see, you know like, these are the two key formulas to calculate the deflection as well as the slope for a beam irrespective of whether the beam is a cantilever beam or beam is a simply supported beam; or a cantilever beam is, you know like, subjected by a point load at the extreme end - the free end I should say - or we can say the various combined loads are there, only we need to get the bending moment at the individual point or we need to get the bending moment diagram. And once you have the diagram, at individual points you can easily calculate the slope, as I told you, as well as the deflection.

And, you see, we discussed the various numerical problems also in the previous lectures about the cantilever beam, simply supported beam, even, you see, it has a point load UDL - the uniformly distributed load - or a triangular load. So, if, you see, the regular or irregular kind of loading is there, then also it is pretty easy for us to calculate the deflection as well as the slope for that. And that is why, you see, it is preferable to go for, you know like, if you have a different kind of loading area moment method. Then, you see, we would like to now discuss one more method as I told you that is the Macaulay's method.

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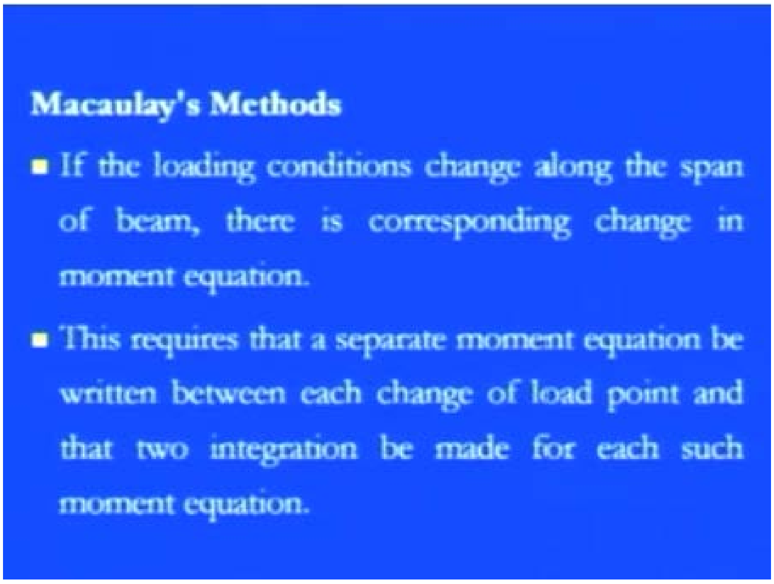

And this is, you see, always preferable to use this method if the loading condition changes, you know like, which is along with the span of beam. That means, you see if the load condition is frequently changing, then it is very hard to, you know like, check the interaction of these bending moment and it is really hard to draw these things. So, that is what, you see, for this kind of loading always it is better to drop some sort of function for a kind of deflection. Or we can say the slope, and then, you see, whatever the boundary conditions are changing it can be easily incorporated in those kind of functions.

So, that is what, you see, Macaulay says that if you see that the loading condition is frequently changing along the span or along the path of the beam then always, you see, you need to, you know like, get the corresponding whatever the changes are there in the bending equation. So, you see, we need to incorporate those functions in the bending moment equations. And this requires that a separate moment equation be written between the each change of load point and thus, you see, the flexibility is there in this Macaulay method; so, that you can easily incorporate those changes by writing a simple separate bending moment function for individual load segments, for each change of the load point and the two integration be made for each such bending equation. So, that is what you see, you know like, we need to simply write the separate functions for the separate loading and then simply incorporate whatever the boundary conditions are there correspondingly.

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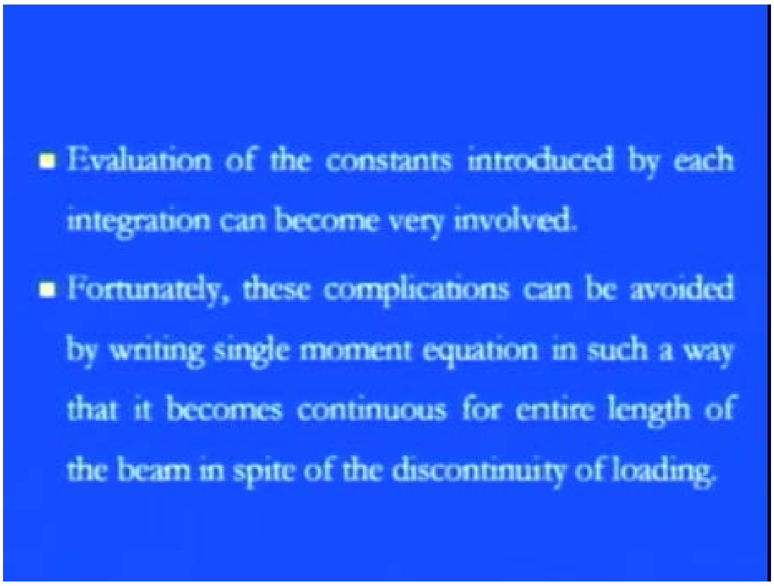

Evaluation of the constraints or the boundary conditions introduced by each integration can be, can become very involved, because you see, you know like, you can simply involve all those kinds of changes in those equations. Unfortunately, you see, these complications can be avoided by writing single moment equation in such a way that it becomes continuous for entire length of the beam in spite of, you see, discontinuity of a loading. So, you see, here what we are doing here, simply you know like, after taking the different, different segments of those functions, you need to write a simple equation by incorporating all those different functions of the loading within that segments, and then incorporate the boundary conditions, because you see, if you are simply taking the boundary conditions of outer part, some of the functions can be easily ignored. So, this is the key feature of this Macaulay method.

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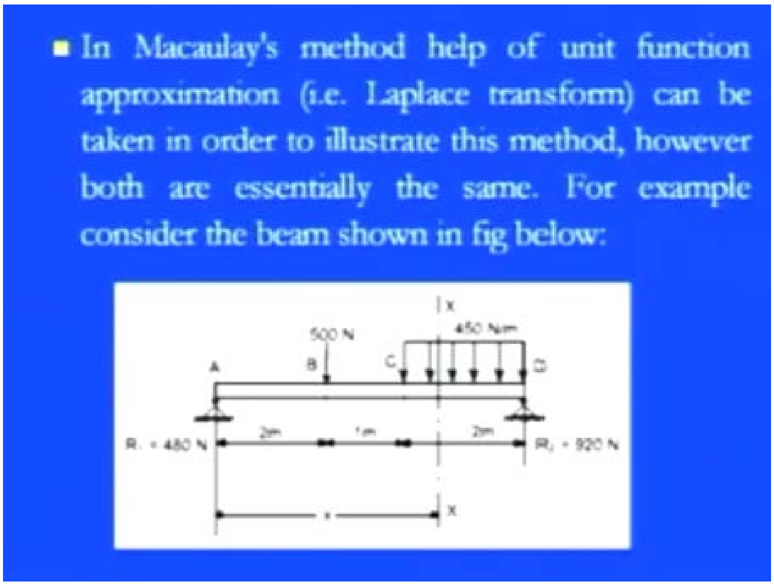

And now, you see, would like to discuss briefly about that how the Macaulay method is functioning. In Macaulay method, you know like, we need to take the help of a unit function approximation as I told you or we can say the Laplace transformation which can be easily taken in order to illustrate this method. However, both are essential; means, you see, we need to take this unit function as this approximation, as well as what the direct loading is there; because, whenever the direct loading is there or we can say the initial load is there, there is no need to describe this kind of loading by the function.

As you can see in this diagram you can easily, you know like…These are A and B, C, D; these are the four different points at different kind of loadings are there. At point A and point D there are two reaction forces Ra and Rd. So, you see, if you want to write, you know like, this if you want to incorporate that what the loading is there at point A, then there is no need to take the unit function approximation or I should say the Laplace transformation. Only straight away you can write the direction loading condition at this point.

And then you see, if let us say if you want to take the impact of load at point B, then again you see, we do not have to write the function, but now if you are changing the, you know like, the loading condition from segment B to C. Then we need to describe that how this, you know like, the loading function will be, you know like, approximated and how this unit function will come in this particular way.

So, you see here, you know like, by taking the point load at B 500 Newton and from C to D we have UDL which has the intensity of 450 Newton per meter, then this distances are given to us. Now, what we are doing here? We are simply taking the section XX at this position, you see, UDL because we just want to incorporate whatever the changes are there right from A to X. So, you see, here the X distance is there at this XX section. So, just keep this figure in our mind now.

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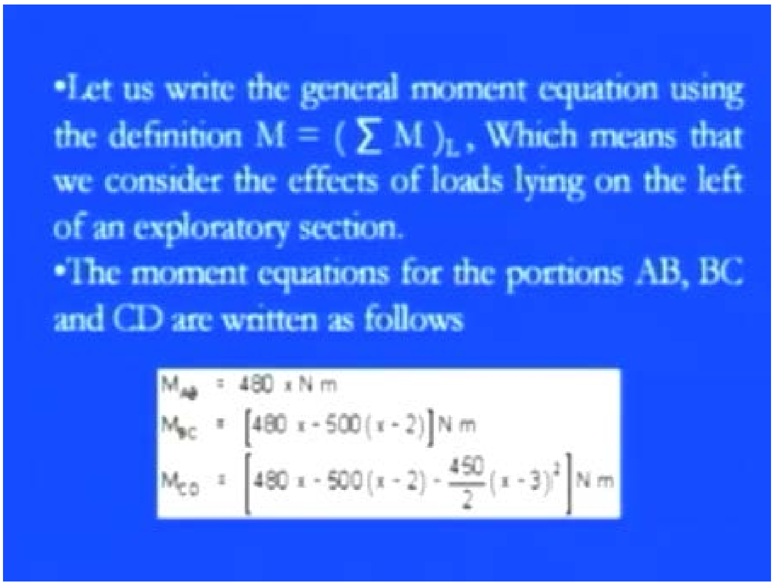

We just need to write the general moment equation using the deflection M, which is nothing but the combination of all those, you know like, the moment at the different, different load conditions. So, you see, here the moment equation M is nothing but equals to summation of M for all those loading conditions which means that we consider the effect of all the loads lying on the left of the exploratory section; that means, you see, now we are starting from point A and then, you see, we are going towards the point D.

And whatever the load conditions are there, we need to write the moment equation for all those segments and we can see that we have the different segments at this or we can say different portions are there where the loading conditions are changing. Like, you see, for point A to B at point B the point load is there; from point B to C at point B we have point load - means the unit load is there; and point C we have the UDL and from point C to D we have an effect of UDL of 450 Newton per meter.

So, with the consideration of all those things, now we can simply write the moment equation as M of AB is nothing but equals to 480 Newton meter. Then we have you see, you know like, because only the reaction forces are coming at point D. So, it has only the impact within this particular segment AB. So, you can straight away write this moment as 480 Newton meter. Then, you see, here now we are incorporating the point load at point B which is 500 Newton meter, 500 Newton.

So, you see, here with incorporation of that now, since, you see, we are considering the X section at UDL portions. So, now we need to expand our moment equation M of BC is nothing but equals to 480 into x minus… Now, you see, we need to incorporate that what, you see, because, you see, 480 into the reference point is there. So, we need to multiply with the X distance, minus because the load into distance is the moment minus 500 into, now this is a kind of, you know like, because x minus 2, because now 480 we considered already for AB. So, we need to neglect that part and we need to write this 500 just for this BC section. So, that is 500 into x minus 2 Newton meter.

And now if you are going for MCD, then you see, it has, you know like, the indirect impacts are there from RCD or point load at point B and then, you see, we need to… when we need to write, you see, the equation then it has, you know like, all those components like, you see, 400 into x, 480 into x because of the reaction force at point D. We have 500 which is your acting, you know like, just downward direction; so, minus sign is there. So, 500 into x minus 2 and minus your UDL is also going towards the downward direction as you see, you know like, this reaction forces are going upward. So, it has a positive reference point.

So, now we have the combination of all these three moments all together, if we calculate the moment at CD which is nothing but equals to 480 x minus 500 x minus 2 minus 450 by 2x minus 3 whole squared Newton meter. So, which incorporates all the, you know like, the combined effects of the load.

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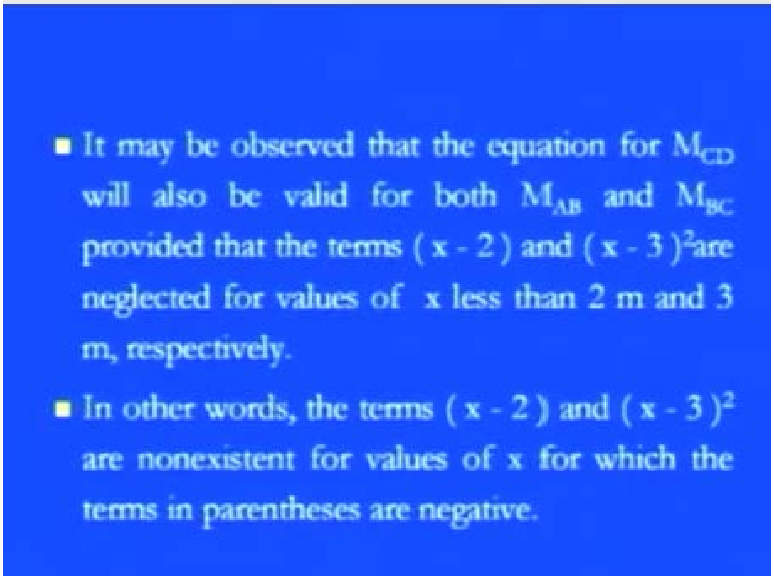

So, it may be observed that the equation for MCD will also be valid for both MAB and MBC, because you see, it is just providing all the terms x minus 2 and x minus 3 square which are, you know like, neglected for the values of x which is less than 2 meter or 3 meter respectively. Because, you see, we cannot go for the negative terms. In other words, we can say that the terms x minus 2 and x minus 3 square are nonexistent for the values of x for which the terms in parentheses are negative. So, it is pretty clear that if you are writing the different functions of M then, you see, we need to calculate all those, you know like, the parameter with the separate constraints. But if you write the combined equation, then you will find that some of the values are just going, you know like, in the nonexistent form because of the negative terms. So, this is quite invalid certain things.

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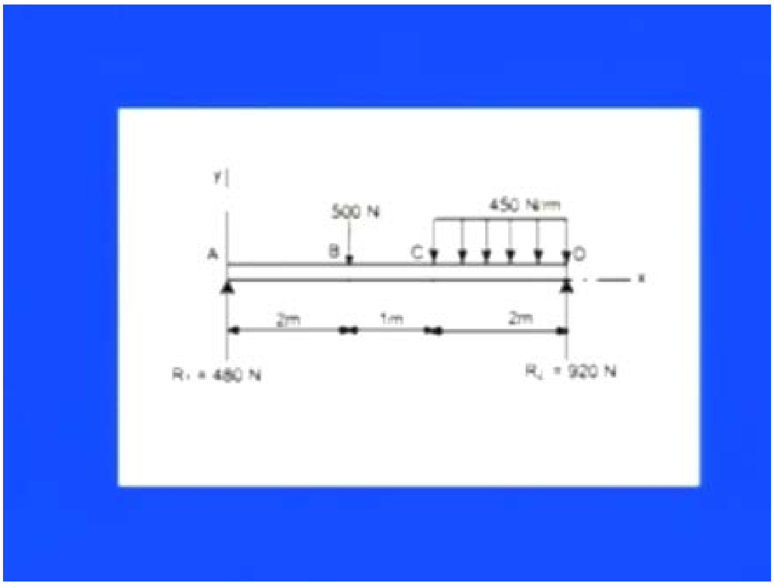

And now, you see, again if you would like to see those things, then we found that its again the same diagram is there, with reference to, you know like this, the same loading condition that at the point A we have the same - these Ra and Rd - these two reaction forces are there which has 480 and 920 Newton. And then 500 Newton of the point load is there at 2 meter from point A and, you see, for CD it is 450 Newton per meter is there of the UDL.

So, with the consideration of the same thing, now what we are doing here? Straight away, we need to take the reference point of y, as you can see in this diagram, exactly at the reference point - this point A. So, now, you see, instead of taking x at this, you know like, those you see, sometimes as we can see that if we wrote the MCD which has you see, you know like, inclusion of all those components at point ABCD, but certain values when you are going for x less than 2 or less than 3, then certain, you know like, the components are simply neglected, because of their nonexistent form of the negative values.

But now, if you are, you know like, chasing the… if you want to include those things and if you are chasing those values, then we need to consider those, you know like, the reference point at A like you see here, this is point A where the reference section is y. So, that now if you want to calculate all those things now it can be easily incorporated here.

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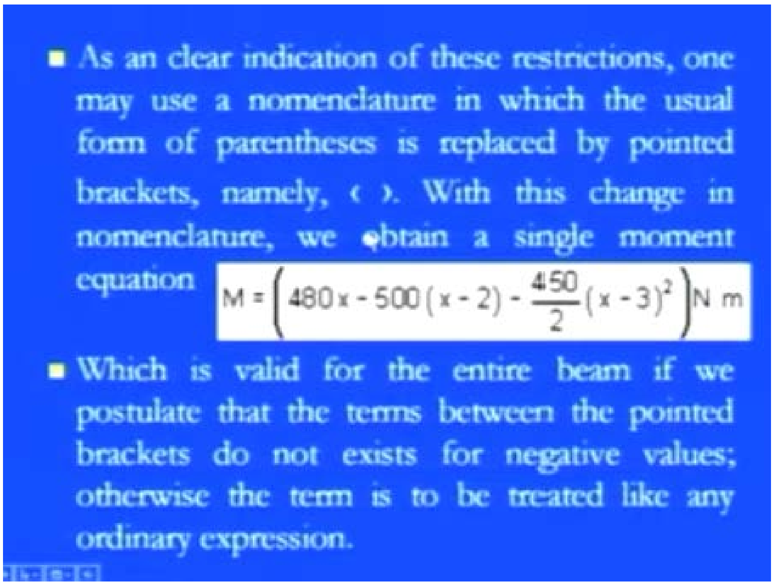

So, as an clear indication of these restrictions one may use, you know like, a nomenclature in which the usual forms of parentheses is replaced by the pointed brackets. And these are namely, you see, this pointed brackets are there. With this change in the nomenclature, now we need, we have you see, you know like, single moment equation which Macaulay methods, you know like, simply gave is M in the simple parentheses 480 x, because of the initial this reaction force moment, 500 into x minus 2 because of this point load moment at point B, and minus 450 by 2 x minus 3 whole square. So, this is the combination of these, now in the parentheses, which is valid for the entire beam if we postulate that the terms between the pointed brackets do not exist for the negative values. Otherwise the term is to be treated like an ordinary expression because, you see, if it is a common expression is there, then we cannot, you know like, go for the negative and the positive values for these kind of expressions. So, as an under example consider now the beam as, you know like, I would, shown in this particular figure.

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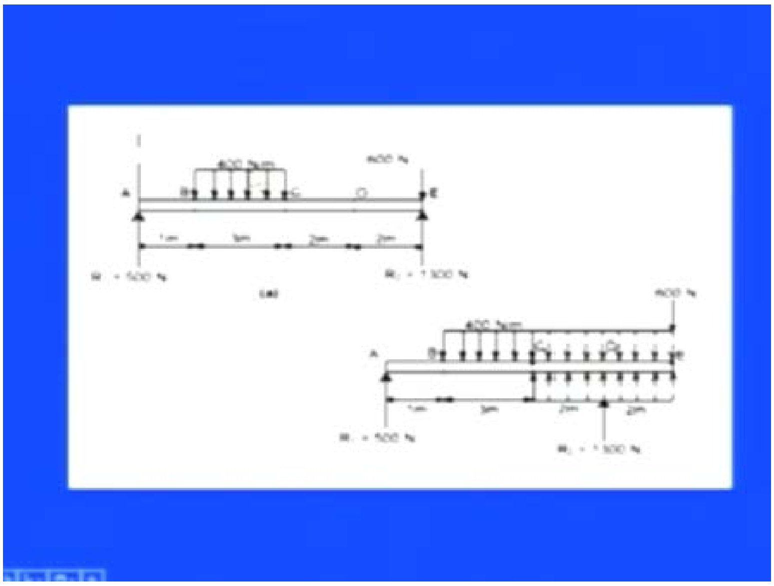

You can see here now, in this figure, we have simply shifted the loading condition in spite of the extreme end to middle one. So, if we can see this figure, initially now at point A and point E, the two, you know like, the reaction forces are there as Ra and R 1 and R 2. So, R1 is nothing but equals to 500 Newton and R 2 is nothing but equals to 1300 Newton.

And now, this UDL is in between point B and point C and point B is just 1 meter apart from point A. And, you see, if this UDL having the length of 3 meter and it has the intensity is 400 Newton per meter. So, now, you see here, what we have? We have the two things: one is the point load and one point load, which is exactly at point E of the capacity of 600 Newton, and we have the UDL which has the intensity of 400 Newton per meter which is spreading in the span of 3 meter.

Now, you know like, what we need to do here? We need to ignore those negative values and for that we simply put the idealistic condition of the UDL on both of this free parts. So, what we done here? We simply put the UDL of the same capacity on the top of part and now, you see, to balance this condition, we simply put again the similar condition there in the lower part, as you can see in the another figure, this is 400 Newton per meter. And now, you see, this is dotted part is the UDL of this kind and to balance this UDL now we have the same intensity of this UDL on the bottom of the part and other factors have remained same. So, by viewing this thing now, this is the another, you know like, the example to just view that how we can incorporate those expression in that. So, here the distribution load extend only over the segment BC as you can see the Right hand part was there and we can create the continuity.

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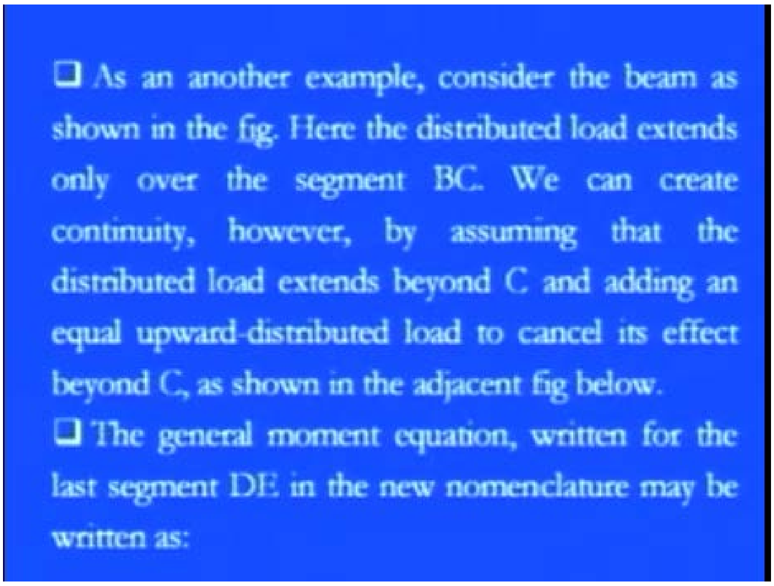

However, you see, by assuming that the distribution of this particular loads extend beyond the C and adding an equal and upward distributed load to be cancel with the, you know like, just by adding the lower portion as, you know like, you have seen in the previous diagram, the general moment equation written for the last segment DE in which, you see, the upper and the lower portion are to be added which has, you see, the new nomenclature.

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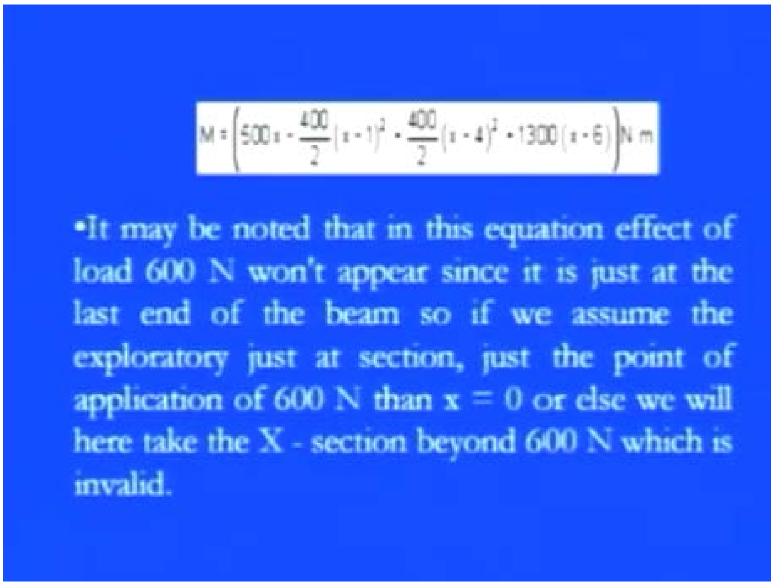

In that, you see, now we can see this M is nothing but equals to 500 x that is due to this reaction support, and then we have 400 by 2 x minus 1 square is there, because of now the UDL is coming, and then if you go beyond, you see, this is the point where the B point is there at which this 400 by 2 x minus 1 whole square is the moment part. And now, if you go another point, means at the C point 3 meter apart from that, now we need to include that the impact of the UDL.

So, we have the intensity of UDL which is 400. So, 400 by 2 into x minus 4 whole square because the total distance from point A is now the force, that is what you see, we need to include those function, you know like, with the distance of x minus 4 whole square. Now, if you move to point C to D we have, you see, one more, you know like, the reaction force is there right from upward direction; so obviously, we have the positive sign. So, 1300. 1300 into x minus 6, because the total, the length of beam is 6 meter. So, we need to include that part. So, it is x minus 6 Newton meter.

But, you see, it may be noted that in this equation the effect of 600 Newton load won't appear, since it is just at the last end of the beam that is point E. So, if we assume that the exploratory, you know like, section at exactly at this particular point, then we need to include, you know like, that point otherwise, you see, it can be easily ignored, because at the application of 600 Newton, we have been there, the distance of x equals to zero at this point or else we will have to take the x cross section of this particular, you know like, the beam beyond the point of E which is means, you see, beyond the length of 6 meter ,and then, you see, we can simply include the 600 Newton of the load impact

So, you see, here the 600 Newton load in this method is simply invalid because of our beam is ending at this point and, you see, our reference point is also at this particular section. So, now, you know like, with that particular reference point, we are starting from point A and then, you see, we are moving from point A to B, B to C, C to D, and then, you see, at point E where the junction is there, or we can say the reference point is there whatever the load conditions are there it is simply ignored. And that is why, you see, the 600 Newton is simply ignored at this particular point.

So, now, you see, we have the total moment equations in which, you see, all those functions are to be evaluated or included in the condition that we have whatever you see, 400 Newton, the intensity of this UDL is there at point B, which is the starting point or at point C which is the ending point, you see, it has been, you see, simply carried out with the x minus 1 square at point B and point C x minus 4 square. And then, you see, it has been, you know, simply incorporate at the point load at point A as well as, you see, the reaction forces at point A and last point.

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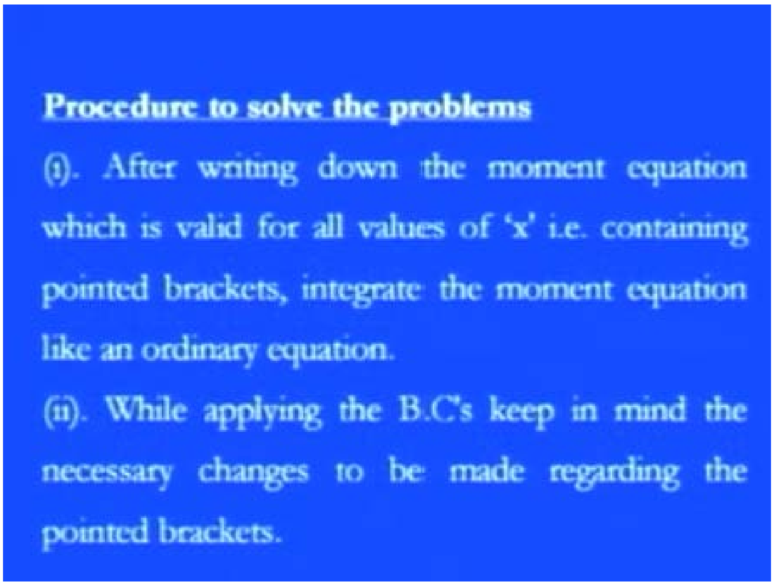

So, you see, here the simple procedure to solve these kind of problems we have the two main points under this category - the first point is there: After writing down the moment moment equation because, you see, for individual sections only we need to consider that what are the, you know like, the loading conditions are there. Because, you see, we are saying that the point loads are there and with the combination of that, if you have the UDL, so what are the interactive effects are there on that particular beam accordingly, you see, we need to write the moment equations.

So, after writing down the moment equations for the beam with the loading condition, which is simply valid for all the values of x, because you see, we need to consider the x cross section where, you see, means where it is a line that is the, you know like, it containing all those pointed brackets. We can say that what are the these particular brackets are there, or what are the point loads are there with that, and what are the impacts are there of these brackets, as well as point loads in that equations, then integrate the moment equations like an ordinary equations.

So, you see here, this is the first thing that… first you need to write the moment equation and then, you see, in this equations it has to be, you know like, incorporated all those brackets as well as the individual points, then we need to those, integrate those simple, you know like, the moment equations just like an ordinary equation. While applying the boundary conditions, now this is the important thing here - keeping mind that the necessary changes to be made regarding the pointed brackets. So, you see, whatever the necessary changes are coming, it has to become within this, you know like, the pointed brackets. So, these two, with these two points, you see, it is pretty simple procedure to evaluate the deflection as well as the slope. So, now, you see, you have some of the numerical problems to simply visualize those things.

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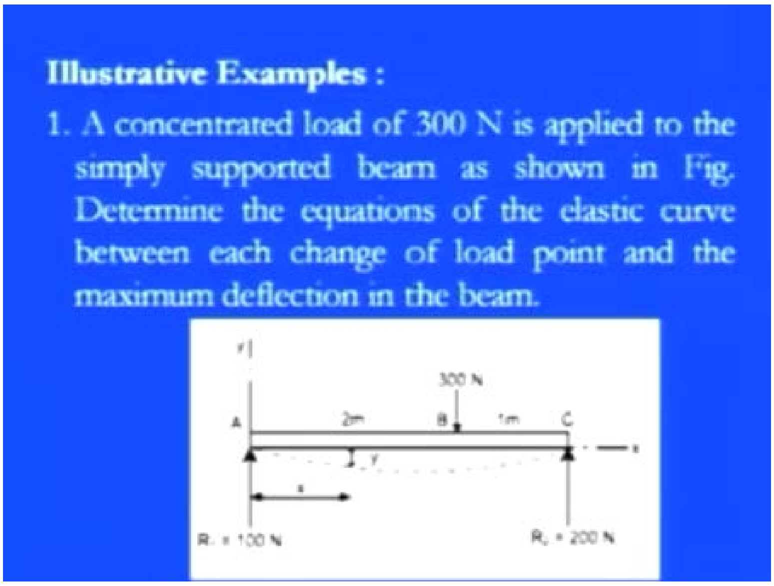

The first example is we have a simple, you know like, the UDL and in the UDL you see, you know like, the concentrated load is there - 300 Newton - which is to be applied at simply supported beam, as you can see that, and we need to, you know like, determine the equations of the elastic curve between the each change of load point and, you see, the maximum deflection in the beam. So, you see here, we need to calculate.

So, if you see the figure, you will find that we have these two A… these three points are there A, B, C. At A and C we have the two reaction forces which are coming on the top of that. So, we have, you see, R 1 which is the reaction force at point A is equals to 100 Newton and the reaction force at point C which is equals to 200 Newton. We can simply evaluated those things just with those force balance and moment balance condition. Then, you see, we have the point load as it is given 300 Newton at point B.

So, you see, we have the distance of 2 meter from point A and 1 meter from point C. So, the total length of beam is the 3 meter. So, what we need to here in this, first we need to take the XX section. So,, you see, here this is the XX cross section from, you see, the point A, it has a distance A. And now, if you look at this particular the deflection curve, which is the idealistic curve is there, we can see that it is simply because of the point load of this, we have this dotted portion of the deflection curve.

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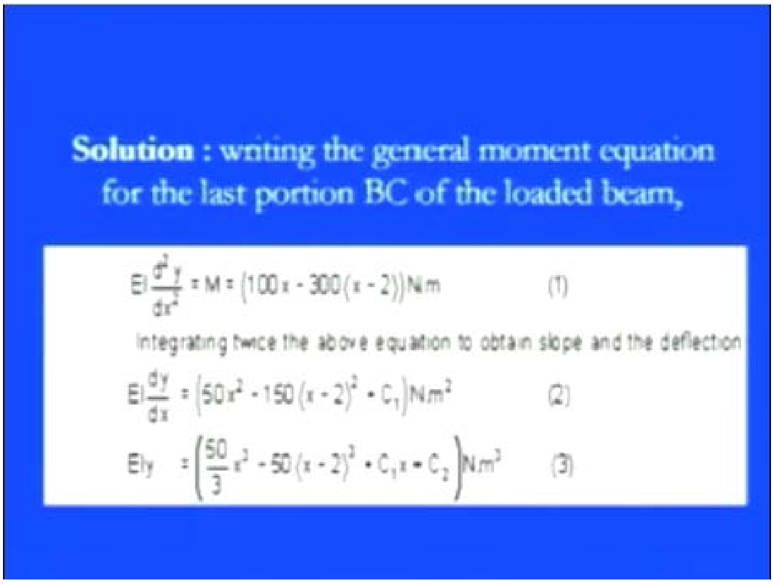

So, now, you see, here first would like to write, as I told you that in the procedure, first of all the beam equation, which is important here. So, first of all, write the general moment equation for the last portion BC of the loaded beam.

So, now what we have? We have EI into d 2 y by dx square is equals to moment which is pretty common equation - the basic equation for moment - is equals to 100 into x, you see, for that reaction force minus 300 which is the point load is acting at 2 meter apart from point A. So, we have 300 into x minus 2 Newton meter. So, this is the first equation for the portion BC because of the, you know like the, loaded beam is there, we have EI into d 2 y by dx square is equals to 100 x minus 300 into x minus 2 Newton meter.

So, now you see, we need to integrate twice to get the deflection. So, first, you see, we need to, you know like, integrate first, so we have the slope equation which is equals to EI into dy by dx is equals to simply integrate. So, you know like, 100 x is nothing but equals to 50 x square minus, you see, we have 300 into x minus 2. So, we have 150 x minus 2 whole square and this parenthesis is, you see, of a special shape because of it is simply showing the unit function approximation for, you know like, the Macaulay's method. So, 150 x minus 2 whole square plus, you see, the integrating, you know like, the constant is there C 1 Newton meter square.

And then, again if we integrate, then we have the deflection part EI into y EI. As I told you it is a flexural rigidity is there, just depends on what kind of material, which we have taken and what kind of the shape of the beam is there. So, I is there. So, EI into y is nothing but equals to 50 by 3 x cube which is pretty simple, you see, as x square. So, x cube by 3. So, 50 by 3 x cube minus, you see, you know like, x minus 2 whole cube by 3. So, 150 by 3 is evaluated. So, it it has minus 50 into x minus 2 whole cube plus C 1 x plus C 2.

And these are, you see, the two constants, two integrating constants are there because of the integration of these main moment equations. So, we have now the deflection equation, we have the slope equations, and we have, you see, all those boundary conditions along with us. So, now the to evaluate the two, you know like, the constants C 1 x and C 1 and C 2 of the integration.

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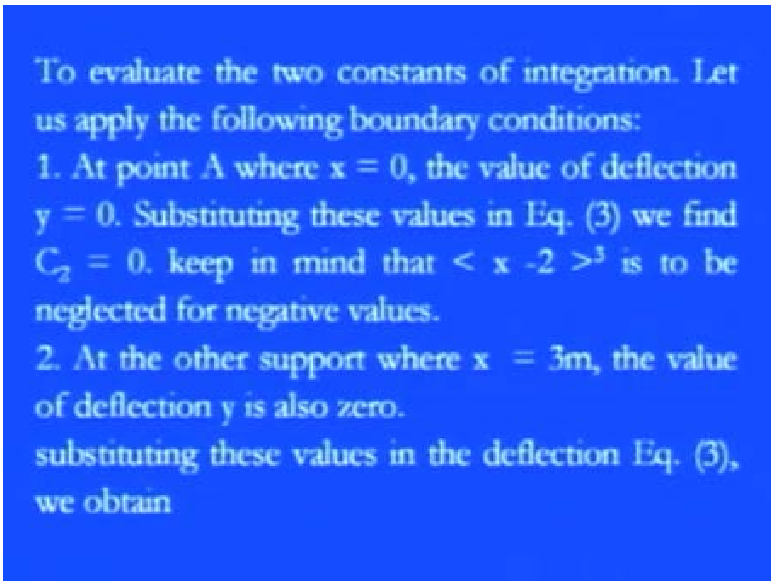

Now, we need to apply the boundary conditions which is valid for the, you know like, those equations, those figure which we have seen in that, you know like, those point loads are… point load is there at exactly, you know like, 2 meter apart from point A and we have the reaction forces at point A and C, you know like.

So, at point A, where x equals to zero, because this is the starting point, we have the deflection zero, obviously there is no deflection at the point A because at the simply supported beam is there; so, hinge joints are there; so, y equals to zero. And now, if you substitute x equals to zero corresponds to y equals to zero in the deflection equation EI into y. So, now what we have? We found that C 2 equals to zero. So, one of the constant is gone out, keep in the mind that x minus 2 whole cube is to be neglected for the negative values. So, you see, if you are keeping x equals to zero, obviously it has minus 2 whole cube. So, it has the minus value. So, if this parentheses is just giving you the negative values, we are simply ignore that part; so obviously, we need to neglect that part.

On the other hand, you see, we have the, you know like, just the point load is there, the reaction forces are there at point C, exactly at the x equals to 3 meter. So obviously, since again this is a hinge joint, the point joint is there; so, there is no deflection part is there because of the simply supported beam. So, again the deflection is zero. So now, we have the two main condition at x equals to zero, y equals to zero; at x equals to 3 meter, y equals to zero. So, obviously again, we need to put those things and now we have the different conditions of C 1 also.

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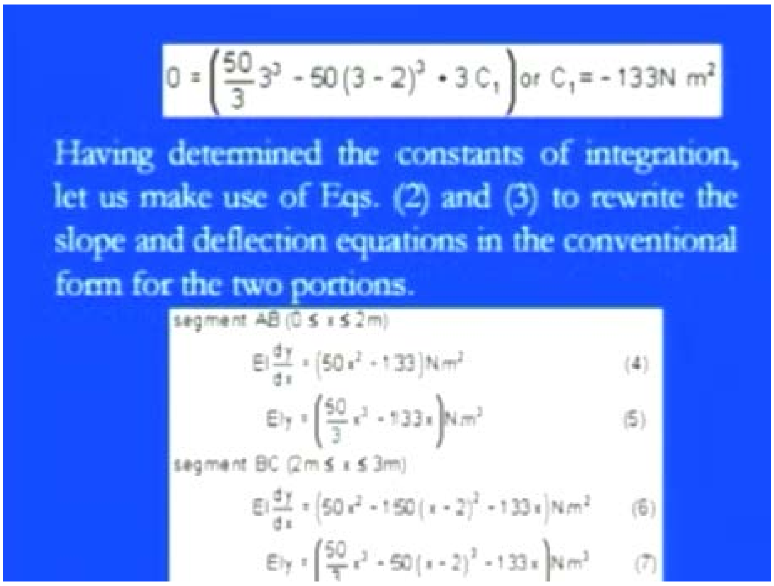

So, after having you see, you know like, y equals to zero, 50 by 3, now we need to keep the x equals to 3. So, 3 cube minus 50 into 3 minus 2. So, x minus 2 whole cube. So, 3 minus 2 whole cube plus 3 1, you see, this x into 3. So, 3 into C 1 plus C 2 has already gone zero. So, we have C 1 which is nothing but equals to minus 133 Newton meter square. So, after keeping the value C 1, we have the total EI into y is nothing but equals to 50 by 3 x cube minus 50 into x minus 2 whole cube plus this minus 133 this is. So, this is now the moment equation which is valid for the applied condition of the beam.

And the, you know like, with those constant of integration C 1 and C 2, now we can simply write the differential equations or we can say the moment equation for the different segment. So, if you are writing this moment equation for segment AB. So, AB is just valid for the x is in between zero to 2 meter. So, for that, you see, we have the reaction force A at point A, you see, on just going upward direction and we have, you see, the load condition which is 300 Newton which is going downward direction. So, for that, you see, we can simply write the moment equation EI into dy by dx which is nothing but equals to 50 into x square minus 133 Newton per meter square. So, because of that, you see, only we are going up to 2 meters. So you see, here the beyond 2 we are just going to neglect.

So, now, you see, the slope equation is this and if we integrate those things, then we have the deflection equation EI into y is nothing but equals to 50 by 3 x cube minus 133 x Newton meter cube. So, you see, here you have the deflection, you have the slope for the AB segment; similarly we can find for the BC segment the same deflection as well as the slope and it is valid just for x equals to 2 meter to x equals to 3 meter. So, you see, here now x which is greater than equals to 2 meter and less than equals to 3 meter, we have the equation is EI into dy by dx is nothing but equals to 50 x square minus 150 x minus 2 whole square minus 133x.

This is the whole equation as you see, previously derived the equation for BC segment. And again, if we can calculate, you see, you know like, by integrating that - that particular equation - we have the deflection equation EI into y is nothing by equals to 50 by 3 x cube minus 50 into x minus 2 whole cube minus 133 x Newton meter cube.

So, you see, here for the different segments, you have slope equation, you have the deflection equations, and from that by keeping the boundary conditions we simply got to know that where is the maximum slope and where is the maximum deflection is there. So, you see here, just continuing the solution we simply assume that the maximum deflection will now starting that assumption because now you have the two different segments, so simply by putting the boundary conditions and with the assumption that now for in the first segment, you see, you have the maximum deflection. So, just keep those boundary conditions.

So with that assumption, the maximum deflection will occur in the segment AB, its location may be found by differentiating, you see, the equation 5 as I have shown in the previous case, in the first segment AB EI into dy by dx. So, for that with the respect to x and the setting the derivative to be equal to zero, because, you see, we are calculating the maximum deflection. So, for that, you see, we need to differentiate those things.

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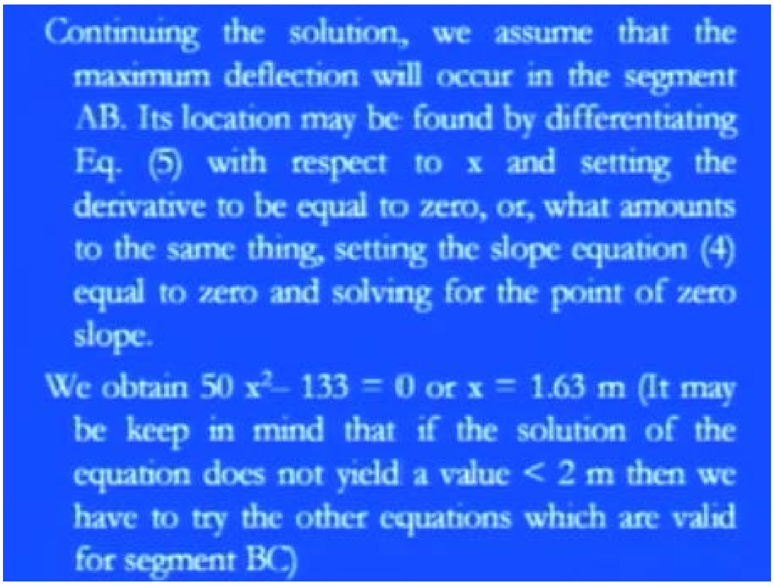

Or we can say what amount the same you see, you know like, the same thing will coming in the deflection part, setting the slope equation means, you see, the EI into dy by dx 4 is equals to zero solving for those things, we have, you see, 50 into x square minus 33 equals to zero or we can say that when the x is equals to 1.63 meter . So, means you see, you know like, what we are taking, we have a reference point A and, you see, x is just going towards the right hand direction. So, where you see the x equals to 1 point and total length is, you see, the 2 meter. So, within that at 1.63 meter we have the maximum deflection is there.

So, we it may be, you know like, keep in mind that if the solution of the equation does not yield, you see, the value of less than 2 meter, then we have to try for the another equation of that side; that means, for the segment BC. But fortunately, you see, we got that the solution is, you know like, the positive sign is there 1.63 which is less than 2, you know like, the 2 meter; that means, the maximum deflection is coming in the segment of AB exactly, you know like from the point A 1.63 meter we have the maximum, you know like, the deflection x is there, and for that you see, we can simply get the value also by just keeping x equals to 1.63.

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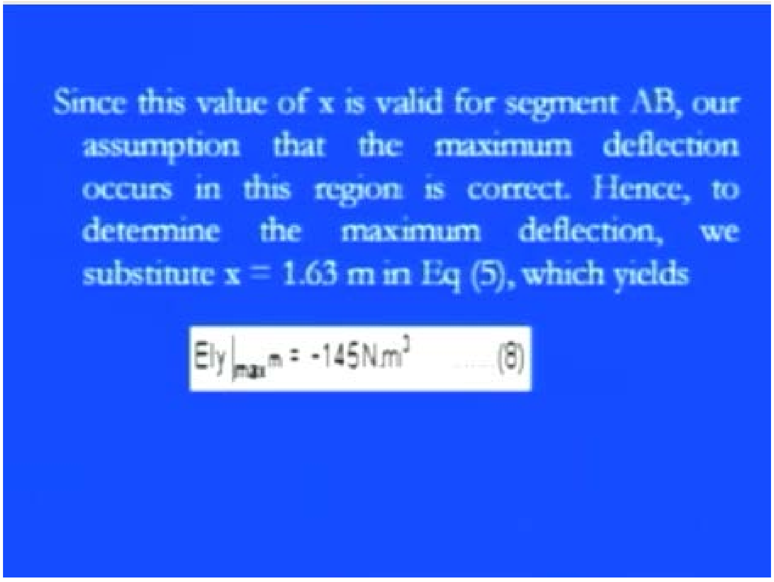

Since the value of the x is valid for the segment AB, as I told you or whatever the assumption which we have made, you know like, for the maximum deflection occurs in this region is exactly correct. And hence you see, you know like, we can simply determine the maximum deflection just by keeping as I told you x equals to 1.63 in the main equation, where EI into y is equals to the same equation was there and by keeping those things, we have EI into y maximum is minus 145 per Newton meter cube; minus sign is there because deflection is coming just on the lower portion; so obviously, it has the minus sign.

So, you see, here this is the real procedure to evaluate, you see, you know like, the deflection at the different, different segments and the slope at the different segments and also, you see, by keeping the boundary conditions, we can simply get those values also. And to find out the maximum deflection, again we need to assume that in which section, by simply our visualization, we can simply assume that in this section it may happened to be there as a maximum deflection. So, with that assumption we can again incorporate that part, and by keeping that assumption we have the value of the maximum deflection by just keeping the value of x that where is the maximum deflection is there, what is the point of location.

So, this is, you see, the first example, for that and as I told you the negative value, you know like, which we obtained here, it just shows that because the deflection is going in the downward direction as the x is quite usual, you see.

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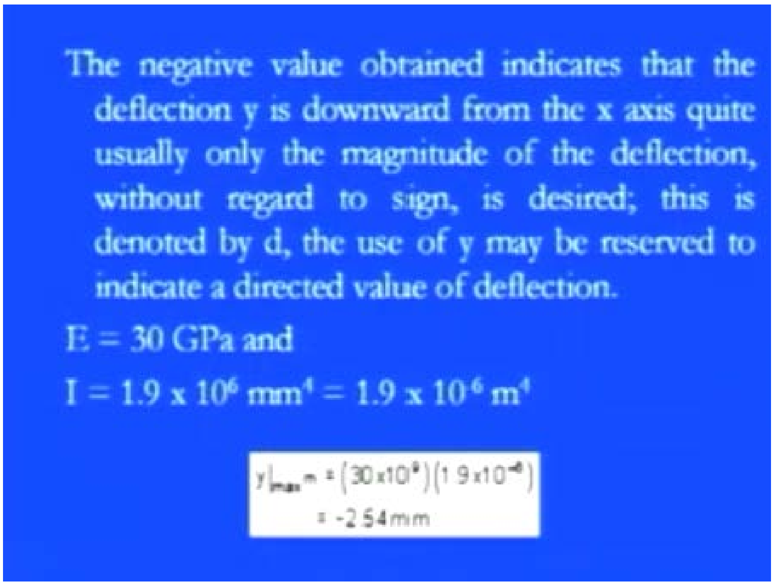

Only the magnitude is just, you know like, in the lower direction with regard of the sign is as usually, you know like, the desired part; this is denoted by d and the use of, you know like, the y may be reserved to indicate the direction of this deflection towards the downward direction.

And now, you see, if we take, you know like, as I told the E and I are the nothing but the property of materials. So, if we are taking the beam material which has the Young's modulus of elasticity as 300 giga Pascal and the cross section of the b this, you know like, based on that, we have the moment of inertia I is nothing but equals to 1.9 into 10 to the power 6 millimeter 4. Or we can say that 1.9 into 10 to the power of 6 meter 4; whatever, you know like, the arrangement is there we can simply keep these values in the EI into, you know like, minus whatever the figure was there, by keeping those values now we have the maximum deflection at the 1.63 meters from A is equals to minus 2.54 millimeter. So this is the correct value of the deflection. And we can get, you see, that what is the location is there, and what is the value of this maximum deflection is there.

So, this is, you see, the simple procedure to calculate the maximum deflection, as well as the deflection and the slope value at different segments of the beam, where the loading conditions are abruptly changing.

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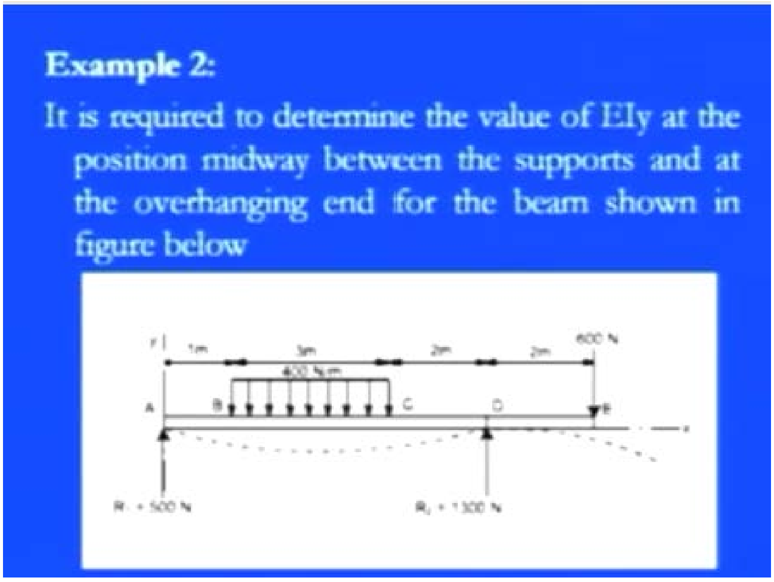

Example two, now we have going to take. Now, you see, in this particular example, again the similar kind of, you know like, the simply supported beam is there, but the support is there within the beam structure; means, you see, we have the free, you know like, the span is there in which there is no load condition is there within that part, there is no support is there from the bottom side. So, it is required to determine the value of EI into y; that means, you see, the deflection part at the position midway between the support and at the overhanging end; that means, you see, you know like, we have a portion in which there is no bottom support is there, the overhanging part is there of the beam as shown in this particular figure.

So, in this figure, you see here what we have? We have A and D. So, in this figure, you see, we have the two main reaction supports are there at point A and point D. And, you know like, simply by a force balance, we can simply calculate this reaction forces at R 1 as it is, you see, the 500 Newton and R 2 which is 1300 Newton. And what we have in this figure is just the UDL, which has the intensity just say the same thing, you see, which we have taken that 400 Newton per meter. And the span of this 3 meter for the total, you know like, the UDL spreading. And, you see, the UDL is starting from same - 1 meter apart from the point A and it has a 2 meter from point D.

But the key feature is that, whatever the point load is there, which was in the previous case exactly matching with this reaction force; now it is hanging; it is just going beyond the point D and it is overhanging portion is there, which is just, you see, the 2 meter apart from this reaction force, this D and it has the 600 Newton at point D. So, that means you see, whatever the slope or, you see, the deflection will come, it will come you see, you know like, in this way, as you can on your screen the diagram, just this is the slopey equation for the A to D portion. Then, you see, we have the deflection portion at this because of this point load is there at the free end.

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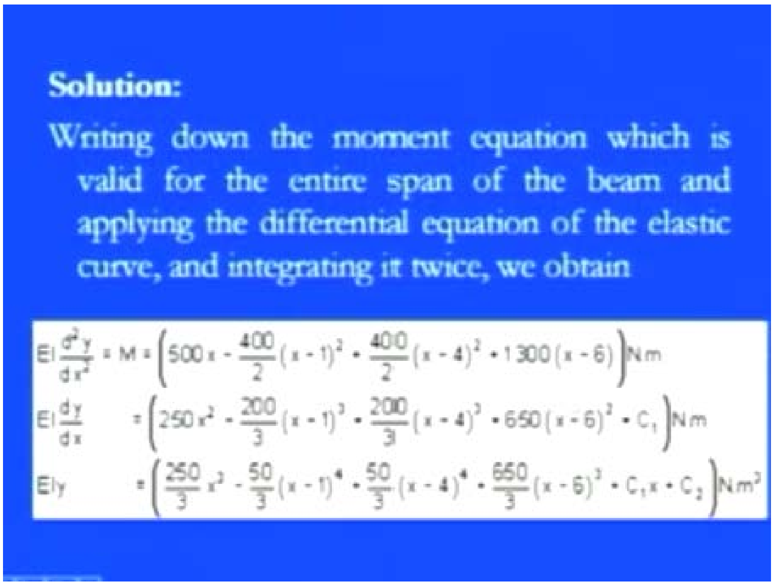

So, we need to evaluate those things. So, first exactly, you know like, the same procedure is there. We need to write the moment equation for, you know like, the entire span of the beam which is just valid, you see, right from point A to D to E and then what we need to do here, we need to simply apply that, you know like, the different boundary conditions with those things incorporating, you know like, all those what the kind of the elastic curve is coming and what you see, you know like, the kind of the brackets or the parenthesis is coming in the way.

So, starting from the first thing, a simple EI into d2y by dx square is nothing but equals to the bending moment which is equals to, you know like, from point A starting, you know keep this thing that, you see, at point A… point A is nothing but the reaction forces is there. we just going upward direction, so we have 500 into x minus 400 by 2 into x minus 1 whole square because UDL is starting from point 1 apart from, you know like, this point A. So, we have, you see, starting point of UDL which has the intensity of 400 Newton per meter. So, 400 by 2 x minus 1 whole square plus now, you see, this UDL has a total span of 3 meter. So, the total distance from point A is 4 meter. So, we have and the same intensity of the UDL is there - 400 Newton - 400 Newton per meter.

So, 400 by 2 x minus 4 whole square plus now at point D, the reaction force is just going above the, you know like, the top up portion. So, we have 1300 into x minus 6 Newton meter. And you see, you know like, just by integrating that, we have first the deflection part, first the slope part EI into dy by dx, which is nothing but equals to, you know like, just by integrating that part we have 250 x square minus 200 by 3 x minus 1 whole cube plus 200 by 3 x minus 4 whole cube plus 650 x minus 6 whole square plus C 1, that is the first integration constant, and then again, you see, by integrating that, we have the deflection part. So, EI into y which is the deflection point is there is nothing but equals to 250 by 3 x cube minus 50 by 3 x minus 1 to the power whole 4 plus 50 by 3 x minus 4 to the power whole 4 plus 650 by 3 x minus 6 whole cube plus C 1 x plus C 2. So, now, you see, by keeping the boundary conditions of the loading conditions at different, different, you know like, the segments, we need to put those values and we need to get the integration constant C 1 and C 2.

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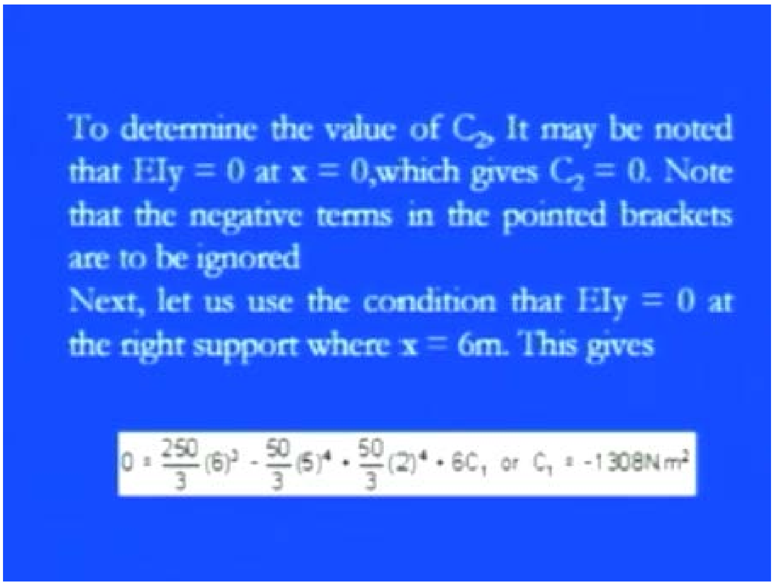

So, to determine those, you know like, the C 2 value again you see, you know like, we know that at the starting point - at point A - it is a hinge joint; so, there is no deflection point is there. So, at x equals to zero, we have this y equals to zero or we can say that E into I y is equals to zero. So obviously, we have the C 2 equals to zero for the same thing as we have, you know like, justified the previous case; note that the negative terms in the pointed brackets to be ignored or to be neglected.

So, again, you see, this is the basic phenomena is there in the Macaulay method that whatever the negative values are coming in the parenthesis you need to ignore, and then, you see, again whatever the positive values are there, we need to consider and then evaluated the total impact of this moment.

Now, next you see, let us use the condition of E into y equals to zero at the right portion; that means, at x equals to 6 meter, because here, you see, the reaction force is coming from the bottom part we have a hinge joint; so, there is no deflection point is there. So, EI into y at equals to 6 is zero.

So, by keeping that value what we have? We have zero is equals to this 250 by 36 cube minus 50 by 4. Now 6 minus 1 was there or x minus 1 was there. So, 6 minus 1 that is 5 to power 4 plus 50 by 3 2 to the power 4 plus 6 C 1, because you see, C 1 x was there. So, now we have the value of C 1 is minus 1308 Newton meter square. So, you see, by keeping the value of C in this main equation what we have? We have the total, you know like, the phenomena is there that, at what the kind of, you know like, the loading conditions are coming and what the moment is there by incorporating C 1, this minus 130 8 and C 2 zero we have total equation for that.

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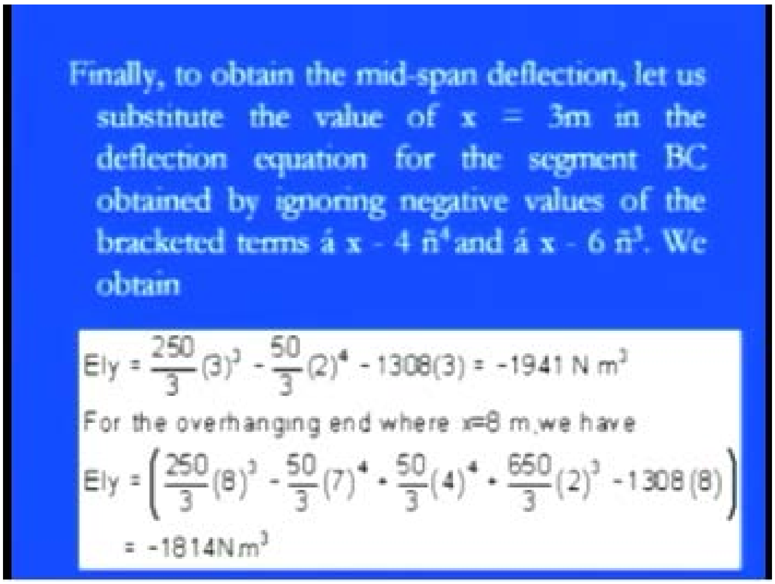

Finally, you see, our main intention to obtain the mid span deflection. So, for that let us, you know like, substitute the value of x equals to 3 meter in the deflection equation for, you know like, the segment BC because in the segment BC this x equals to 3 meter existing because, you see, this UDL starting from point this x equals to 1 meter, x equals to 4 meter. So, the total, you see, x equals to 3 will come in the segment BC.

So, you see here, just to update by ignoring the negative values of the bracket in terms of x minus 6, obviously you see, you know like, or x minus 6, whatever the things are coming in terms of minus 4 or minus cube we have the negative values. So, this bracketed are to be neglected. So, now what we have? We have the EI into y for calculating the maximum deflection at x equals to 3, because if we keep x equals to 3 in x minus 4 we have the negative value; if you x equals to 3 in x minus 6 we have the negative values; so, these two parenthesis has to be ignored.

So, what we have? We have EI into y which is equals to 250 by 3 into 3 to the power cube minus 50 by 3 2 to the power 4 minus 13; this is C 1 value 1308 into 3. So, we have the total deflection value EI into y is nothing but equals to minus 1941 Newton meter cube. So, this is the deflection at x equals to 3 meter.

So far you see, overhanging portion because, you see, still what we have done? We have simply calculated in between x equals to, you know like, at x equals at A point D point. So, now you see, but we have again one loading condition which has an impact on the deflection portion and, you see, for that we just want to check that because of that whether the deflection is more than this point or what? So, for that, you see, what we need to do? We need put the x equals to 8 meter, where this 600 Newton point load is acting.

So, for that now we have this equation - deflection equation - EI into y is nothing but equals to 250 by 3 8 cube minus 50 by 3 7 to the 2 power plus 50 by 3 4 to the power 4 and, you see, we have see, you know like, just by keeping x equals to 8, all those parenthesis is positive. So obviously, you see, we have to consider all those bracket values for the segment of the loads. So, 650 by 3 2 to the power cube minus 1308 into 8 because x equals to 8. So, what we have? We have EI into y is nothing but equals to minus 1814 Newton meter cube.

So, meaning is pretty simple, that if you check it out both the things we have the maximum deflection exactly in between the point A and D, and we have the another deflection at point the E where, you see, the 600 Newton is there, but the maximum deflection is coming in between the segment BC.

So, this was the example, you see, only if we have the overhanging condition then how to calculate the deflection for different different segments? So, obviously, we need to consider the moment, you know like, the equation for the overhanging conditions is separately and then we need to calculate the deflection as well as slope for that particular segment.

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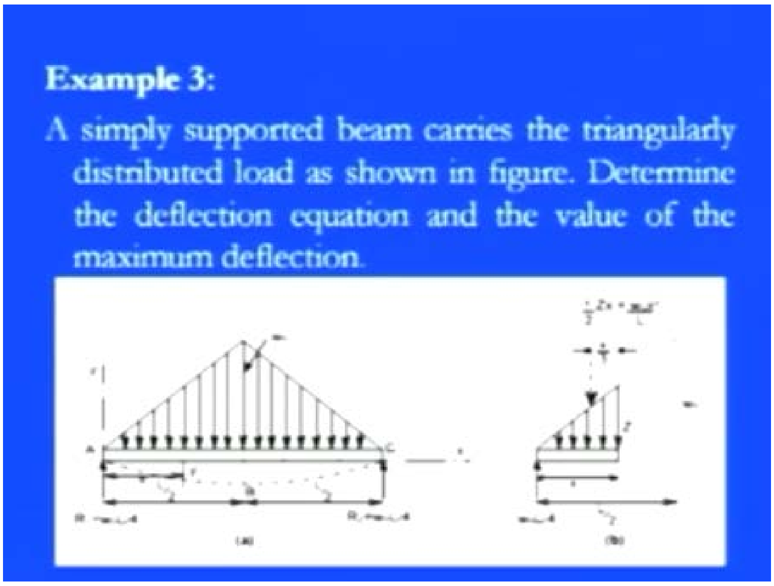

Come to the last example. We have again the same simply supported beam, which carries a triangular distributed load. So, you see here now, the load itself is you know triangular; that means, you see, it is in the triangular shape. So, at the two extreme corners we have the minimum load, but exactly at the centre portion we have the maximum which has the intensity of w zero Newton per meter; just we need to calculate, you see, the maximum deflection equation and the value of maximum deflection for that.

So, you see here, what we have? We have in the equation, in this particular figure, at point A to C this deflection curve is there, on the bottom of that you can see here and for that, you see, we have the reaction at point A and point D is the same, that is w zero L by 4, w zero L by 4. So, if we cut the portion, just take this particular portion out, and if, you see, if you analyze those things what we have, we have simply the loading condition and we know that the centroid is exactly acting at the one-third of, you know like, this from left hand portion or two-third of the right hand portion.

So, from that, simply taking the total distance, as you see, L is there. So, L by 2, L by 2 for this L by 2 portion we have this, you know like, we have you see, the kind of you know like, the loading condition w zero x square by L for that and it is simply carried out from this x by 3 distance at this way. So, we have this load, at this particular way, this is w zero L by 4, this is, you know like, the reaction force is there on the top of that and on the bottom of side, you see, we need to consider the regularity of that and, you see, since it is a varying load, so obviously, we need to take, you know like the what the kind of variation is there and incorporating that variation zx by 2 equals to w 0 x square by L which is to be acted at this one-third distance of this.

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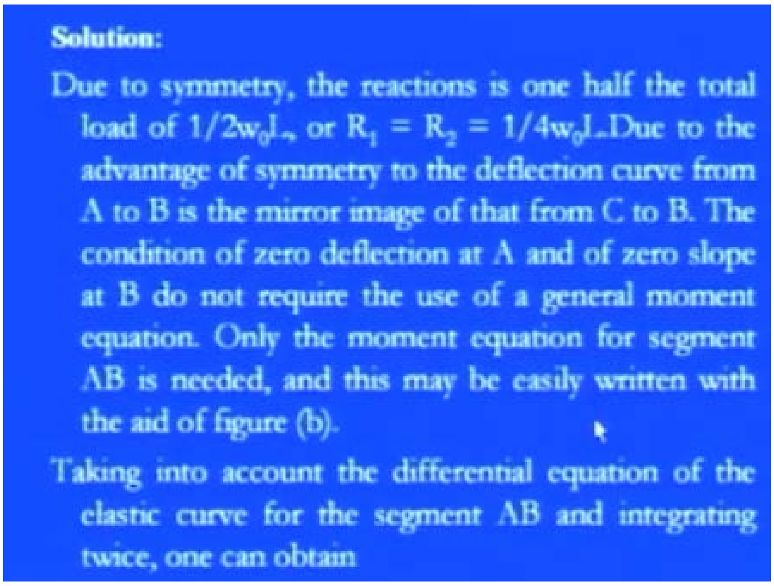

So, you see, here with those configuration, we can simply calculate, you see, what will be the maximum or what will be the deflection is there at different, different segments. So, due to the symmetricity again, you see, this is pretty important thing here that we have the symmetricity in this loading; means it is just, you see, at the extreme maximum at this particular joint section and we just, you see, going downward, you see, and going to be the minimum at the extreme to hinge joint. So, the reactions at is one-half of the total load; so, obviously, you see, it is 1 by 2 w zero into L as, you see, R 1 and R 2 which we have already seen that, w zero L by 4 is there. And due to the advantage of the symmetricity, the deflection from A to B is a mirror image of obviously, C to B.

So, simply we can cut these two portion and if one portion showing the same deflection as well as the slope obviously, the another triangular is showing the similar kind of thing, because it has the mirror image of that and the condition for zero deflection at the point - at point A, obviously you see, the zero slope is there at point B; do not require to use the general moment equation for entire span of the beam, because one part is valid to the another part; equal segments are exactly symmetric. So, only the moment equation for the segment AB is just required as I told you because of the symmetricity and this may, you know like, just make our analysis is simple. And you need to write only for, you know like, as I shown in the previous figure, all we need to show the just one figure of half of the portion, and then whatever the analysis there, which is pretty, you know like, similar to the another thing. So, taking in to the account of the differential equation of the elastic for segment AB, now integrating twice just, you know like, for the entire beam.

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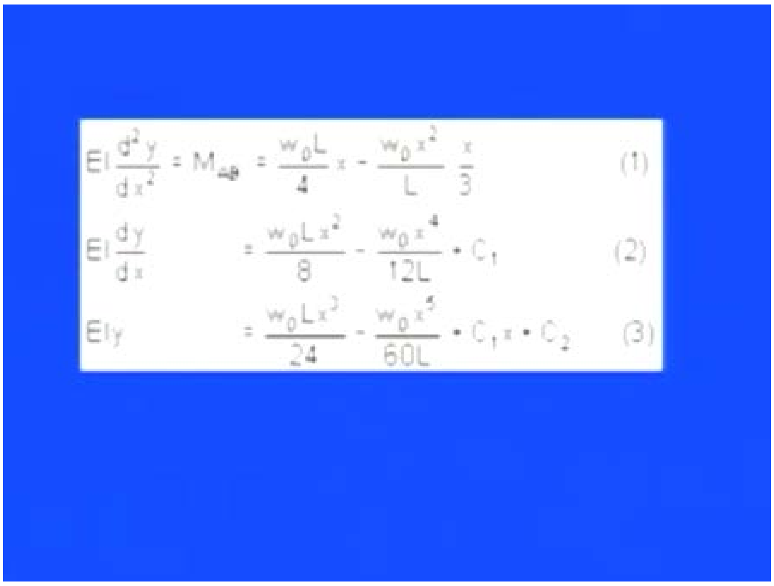

So, what we have? We have EI into d 2 y by dx square which is the moment equation for AB is equals to w zero, you know like, L by 4 because of, you know like, the point this reaction forces are there on the top of that.

So, this R into A you can say, or w zero L by 4 into x minus w zero x square by L, which you see, you know like, the combined load is there, which is acted x equals to, you know like, at one-third distance x by 3 or we can say by integrating that we have the slope as well as the deflection. So, slope is EI into dy by dx is nothing but equals to w zero L x square by 8 minus w zero x 4 by 12 L 12 into L plus C 1, or we can say EI into… EI is nothing but equals to the deflection equation is w zero L x cube by 24 minus w zero x times 4 divided by 60 L plus C 1 x C 2. So, C 1 x C 2 are nothing but you see, you know like, this integrating constants are there.

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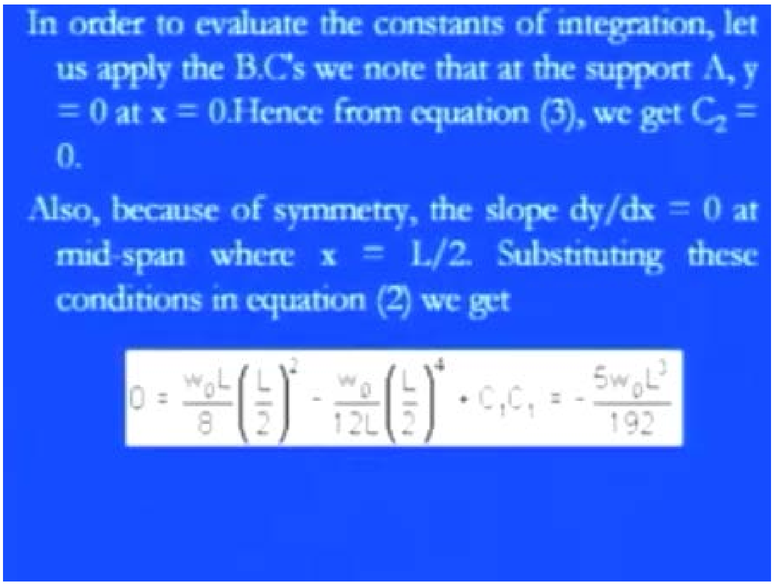

So, again by keeping those boundary conditions we can simply get the C 1 and C 2 value. So, again the similar kind of things are there, because this is the simply supported beams. So, both ends A and C are nothing but, you see, has the zero deflection. So, at x equals to zero the starting point A, we have the deflection y equals to zero; so obviously, C 2 equals to zero and then, you see, because of the symmetricity dy by dx; that means, the slope at mid of the span; that means, at x equals to L by 2 is zero always. So, keeping those conditions what we have dy by dx… EI into dy by dx just keep zero we have w zero L divided by 8 and x square was there; since x equals to L by 2. So, L by 2 square minus w zero divided by 12 L L by 4 to the power 4 plus C 1 into C, you know like, whatever you know these conditions are there. So, we have L by 2. So, C 1 is nothing but equals to minus 5 times w zero L cube by 192. So, keeping this C 1 value in the main equation we have the entire, you see, the coefficients with those boundary conditions.

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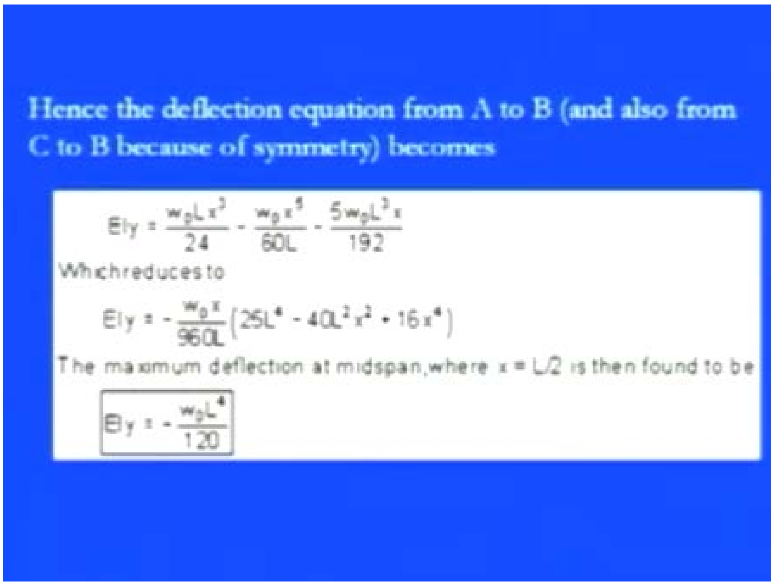

So, you see here the deflection equations from A to B, also from C to B, because it is a symmetricity is there, we have EI into y is nothing but equals to w zero L x cube divided by 24 minus w zero. Because it is the intensity of that triangular load x 5 to power 60 L minus 5 w zero L cube x by divided by 192. So, this is the total equation of the entire beam of there, and now, you see, we can simply reduce this equation by taking all those w zero L x cube and just taking out.

So, we have EI into y is nothing but equals to minus w zero x by 960 L, if you taken then we have 25 L 4 minus 40 L square x square plus 16, 16 into x 4. So, you see here what we have? We have an algebraic in terms of L and x. So, we can simply calculate, you see, the maximum deflection because we know that at exactly mid span at x equals to L by 2 we have the maximum deflection is there. So, we have EI into y the maximum is nothing but equals to minus w zero L 4 divided by 120 for a triangular load, and you see, the simply supported beam is to be supported.

So, you know like, and by keeping, you see, the value of E and I it is pretty simple to calculate what will be the total value of the deflection is there for these terms. So, in these you see, you know like, the lecture we mainly discussed about the Macaulay method and Macaulay method is very much suitable for, you know like, when the changing of the load is there and if you want to calculate the moment for a different, different segments.

So, you can pretty easily you see, you know like, describe those segment by a unit function - approximate function - or we can say, you see, the Lagrangian's function and then only by keeping the thing in your mind that if the negative values are there of the parenthesis you need to ignore that part. Only you need to consider the positive value and by integrating all those things, we can simply calculate moment, deflection, as well as the slope of those conditions.

And then, you see, if you keep those, you know like, the boundary conditions you have all those coefficients - integrating coefficients - and you can evaluate also where the maximum this slope or this deflection is there and what is the value of this maximum deflection is.

So, in this, you see, only we discussed about when we have a point load and we have the UDL, but if we have a combined load altogether, that means, you see, if they are combinedly acted on a beam and if the beam is having itself, you see, a different cross section then, you see, how we can calculate the deflection. And how we can calculate the slope, you know like, we are going to discuss in the next lecture that how we can evaluate those things. So, for this lecture I think these, you know like, the Macaulay method is sufficient and you just try all those numerical problems again, then you can again clearly, you know like, see the feasibility of this parenthesis. Then how write this, you know like, the moment equation for that. And once you write the moment equation here, half of the question is solved and then only you need to put those boundary conditions to get the value of deflection as well as the slope.

Thank you.