Strength of Materials
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Lecture - 32

Hi, this is Dr. S. P. Harsha from mechanical and industrial department, IIT Roorkee. I am going to deliver my lecture 32 on the course of the strength of materials, and this course is developed under the national programs on technological enhanced learning. In this lecture we basically are going to discuss about the alternate deflection method, because in the previous lecture you see we discussed about the main method of the deflection that if the beam is you know like subjected under the various kind of loadings. And due to that if the deflection is there then first we discuss in the previous lecture about the beam deflection theory that under certain assumptions like you see you know like whatever the deflection is there it will follow the Hooke’s law and you know like the stress is proportional to strain.

And the second assumption which we made that whatever the deflection curve, which we are getting it has to be very, very small as compared to you know like the main deflection curve. So that we can say that whatever the stream lined or the central axes of the beam, it has to be remain you know like in a particular limit, then only we can say that this theory whatever we developed is applicable. And the third thing actually we assume that whatever you see you know like the deflection is there mainly it is due to the bending, the shearing effect we are always trying to ignoring.

So, with those three assumptions we derived one equation for a basic deflection theory in the previous lecture that was equals to M you know like d 2 y by dx square is equals to this M or M divided by EI. So, with that you know like we found that this all moment with this rigidity - flexural rigidity is always be compared. Now with that you see you know like if you have this bending moment M which is equals to you know like EI into d 2 y by dx square, we can also find that we can easily get the shear forces or we can get the rate of loading or else also we can get the slope.

So, the slope as well as the bending moment as well as the shear forces, and the rate of loading, they are pretty symmetrical with those differentiations. So, we found that they are all associated with like if we are talking about the slope then it was d 2 y by dx square, if we are talking about the bending moment, then it is a second derivative that d 2 y by dx square, and if we are talking about the of the shear force that was nothing but equals to d m by dx, it is the cubic form. And then the rate of loading which is nothing but equals to this w which we have taken generally in our notation which is equals to you know like df by dx. So, we have the forth order of the derivative.

So, in all of the sequence we found that they are you know like somewhat multiples of the differentiation of these deflection because you see here we have taken the deflection in the y axis and on x axis we have the longitudinal axis of a beam. So, with that consideration we found that if you have an integration method then it is pretty easy to find all those you know like quantities in a sequential way. So, with that concept you see the first method which we applied for the deflection theory of beam is the direct integration method. And then we found that you know like it depends on the boundary condition we can get easily you know like the deflection for a beam.

So, you see here in previous lecture if you focused then we discussed four different cases with respect to you know like deflection. So, first case, which we discussed that if we have a cantilever beam and the point load is acting at a free end, then what has happened? Then you see you know like we applied that the maximum deflection and maximum slope will come at x equals to zero; that means you see at the free end where the point load is acting. So, with those boundary conditions you see and there is no slope and there is no deflection is there at the rigid end where you see you know like the rigid joint is there between the beam and the rigid support.

So, with those boundary conditions we found you know like the generalized deflection and generalized slope expression and then you see you know like after applying those conditions for maximum and minimum deflection we get the maximum deflection as well as the maximum slope. So, that was you see you know like we discussed for the first case and similar you see the conditions we got for you know like when this cantilever beam is there and UDL is there instead of point load at the extreme end free end we simply applied this time the UDL. And then you see we observed the similar kind of expression for the slope as well as the deflection criteria.

And then you see again we applied the similar you know like the boundary conditions for the cantilever you know like because certain constraints are there in those things and then you see we observed the maximum slope and maximum deflection at the free end also. So, these two cases simply we discussed about the cantilever beam. Then you see they followed these 2 cases which we discuss later part that was you see simply based on this simply supported beam in which you see in the first case we discussed about when a UDL is there which is spread all across you know like the span of this cantilever beam. And simply the supports are at the extreme two corners of that particular beam, then how you can you like put those boundary conditions on that particular beam, and how we can get the deflection as well as the slope expression.

So, you see that part we discussed and we found that you know like at the central point because in the cantilever beam we got you know like the maximum slope as well as the maximum deflection at the extreme corner where the free end is there its quite obvious. But here in this case when UDL is there, there is no other loading on a simply supported beam. Then we found that the maximum deflection is coming at the center point of a beam and at these two extreme corners where the support reactions are there; there is no deflection at this point.

So, with that condition you see we know like we observed the maximum deflection as well as the maximum slope this value as well as you know like we observed the expressions for the slope as well as the deflection one. And then last case which we discussed that actually if we have this simply supported beam and the point load is there which is not exactly at the midpoint which is somewhere you see you know like at this distance from you know like we assumed the previous case that we have A distance from this left hand reaction. And we have a B distance and A and B are not equal; they have you see you know like a big difference in terms of the distances, but A plus B is the total length of the beam.

So, with those cases you see you know like we need to apply and we have you know like some sort of bulky these equations for this kind of this simply supported beam with this kind of loading. Then we apply the similar conditions you see for one portion AB and one portion BC. Then you see you know like we obtained and we found that actually wherever the point load is there, at those points you see the deflection as well as the slope is equal.

So, by putting that condition by assuming that condition you see you know like we obtain certain you know like the value and then also we got that what will be the value of this maximum value of the deflection as well as the slope, and also we describe you know like those expressions for that. So, these four cases which were very important as far as the deflection of beam is concerned we discussed but the same method the direct integration method. But we found that you see you know like in some way it is very bulky.

So, we observed that actually we need some alternative methods to describe this kind of condition, because sometimes you see you know like the loading is not simple. It is not because of you know like the point load or it is because of the UDL; if we have a combination of load or if we have you know like along with the loading, if we have you see the additional couple or twisting moment is there, then how we could find out those things. So, here you see in this lecture we are going to discuss about the alternative methods for you know like to calculate the deflection for this kind of beams.

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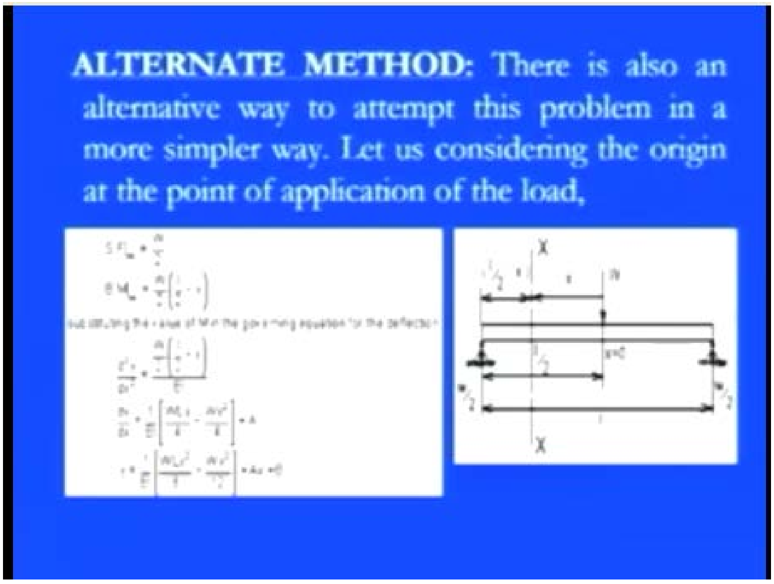

So, first you see you know like in the four cases which we discussed yesterday again you see you know like if a point load is there at the center position, then how we can go for the alternative method. So, you see here the alternative method is there is also you know like alternative way to attempt this problem in a more simplified way, and let us consider the origin at a point of application where the load is there. So, that is what you see if you see on you know like the diagram on your screen, then you will find that we have a simply supported beam and these two reactions are there, because you see here the point load is acting exactly at the center point.

So, due to that we have the reaction forces you know like at the two extreme corners equally and they have the magnitude of w by 2. So, with that w by 2 you see at both of the corner w by 2 w by 2 here and the w at the center position, now what we are going to do here? We are not going for the similar pattern which you know like observed in the previous way. What we are doing here we are simply taking the XX this you know like cutting this beam at XX section. So, here you see this is our XX section. And then this is the total length of beam is there and since this load is applied at L by 2 at the midpoint.

So, this is L by 2 and if I am saying that this is X by X section. So obviously, you see from this distance we have the X instead of you see in the previous cases we have taken the x distance from the reaction part; here we have taken you know like the X distance from this load application. Because you see the load application is the main criteria for the deflection, because these are the originating for any kind of deflection in the beam. So, we need to focus that actually where this point of load is there and then how you see you know like we just want to see that how the variation is there.

So, for that actually we need to put the section with respect to this point load. So, with that consideration you see you know like we simply cut this beam at XX section, and it has a distance of x from this load. And then obviously, you know like since this is L by 2. So, this distance is quite obvious L by 2 minus x. So, with that now you see here we are going for the similar pattern that first of all we need to find it out the shear forces and for that you see the shear force is nothing but equals to w by 2. And again you see as far as the sign conventions are concerned; since, you see I just focused on the left hand portion of this particular you see on this screen this portion.

So, here you see the reaction forces are going on upwards side; the load is there on downward side. So, with this particular loading condition shear forcing we have a positive sign. So, shear force at XX section is, obviously, w by 2 due to the reaction. And then you see you know like we can also calculate the bending moment and again if you are going for the bending moment the sign convention, then it is you see you know like the point load is there on the beam. So, it is just going like that. So, this is kind of a sagging position.

So, for sagging position again you see we have a positive this bending moment is there So, bending moment at XX section also it is mainly due to this reaction forces. So, we have w by 2 into L by 2 minus x this is the distance. So, for this you see you know like we can simply consider the shear forces as well as the bending moment. So, now with the consideration of you know like the shear forces and bending moment go with you know like the main equation of the deflection that is M is equals to EI into d 2 y by dx square. Put those values here; in that equation we have d 2 y by dx square is equals to this is bending moment M w by 2 into L by 2 minus x divided by you know like these whatever the reactions forces are there.

So E I, E I is the flex on rigidity. It is a material property as I told you first that what is the Young's modulus of elasticity is there, because here whatever the deflection is there as we already assumed that it is under the elastic deflection. So, you see whatever the stress is there is proportional to strain. So, E is valid for these things, and then I is based on you know like the mass moment of inertia or we can say the second moment of area which absolutely depends on what the cross section of the beam is. So, we can calculate if we have you know like the rectangular cross section then we already discussed that it is b d cube by 12.

So, with those things you see you know like we can simply get the d 2 y by dx square with those things. Now if we integrate those things we have dy by dx that is the slope. Slope is nothing but equals to you know like 1 by E I w into L by 4 w into L into x divided by 4 minus wx square by 4 plus one constant A; that is the integration constant. And then again if we integrate then we have the deflection term that is equals to 1 by E I w into L into x cube divided by 8 minus wx square by 12 plus Ax plus B.

These are you know like the constants are there for the integration and by keeping you know like this is simpler method, because what we are doing here instead of going from the left hand side we simply do like taking the distances from the load, so that if any variation is there in the load the distances can be easily calculated and incorporated in this particular expression.

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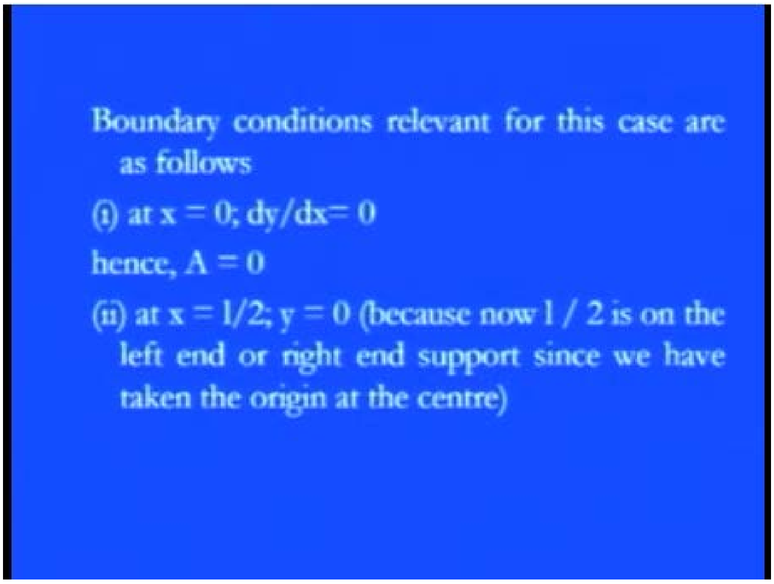

So, now you see here with those things again we would like to put certain boundary conditions to get the value of the deflection as well as the slope, so that you see you know like A and B can be easily calculated. So, with that you see first of all at x equals to 0 there is no deflection; there is no slope. So, with that because you see it is a simply supported beam. So, wherever the reaction forces are there they would not allow the beam to have any kind of slope because they are providing a rigid support at those points

So, with that particular physical concept here again we are applying here that at x equals to 0 or at x equals to l we have dy by dx equals to 0; that means the slope is 0. So, if you keep that x equals to 0 in a first expression where dy by dx we calculated only the coefficient A was there and it becomes 0. Then we have you know like one coefficient A 0; now at x equals to L by 2 the middle portion where you see you know like we are putting the point load

So, when we are putting the point load; obviously, the deflection will be 0. So, we have y equals to 0. Then you know like because now L by 2 is on the left hand side or right hand side of the support, it is exactly at the middle of the portion. So, we can simply say that this is the origin where you know like the deflection and this is the origin where the deflection will start. So, at this particular point where at x equals to L by 2 is there; that means where the point load is there we can simply put y equals to 0.

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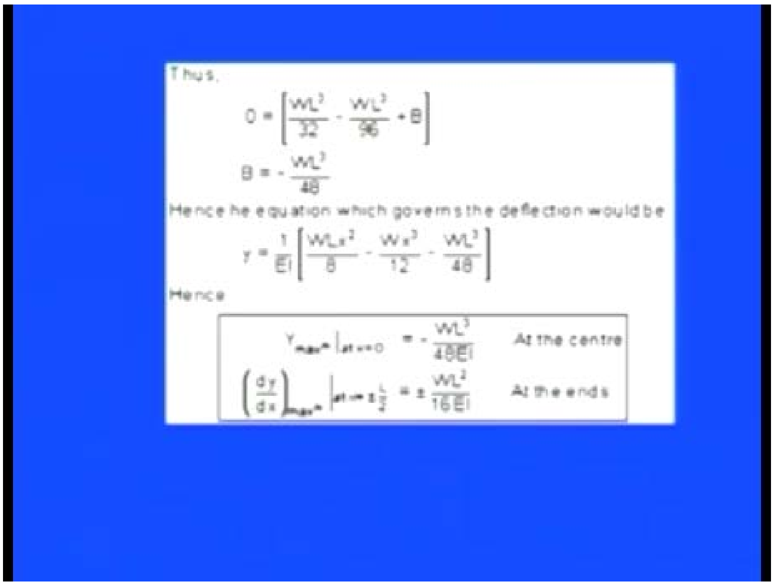

And then we can calculate those B values. So, you see here by keeping that at x equals to L by 2 y equals to 0. So, the y is equals to 0 and you know like 0 is equals to because it was the y expression deflection equation. So, y equals to 0, zero is equals to W L cube by you know like 32 because here we simply put x equals to L by 2. So, you see now x is totally replaced by L by 2. So, we have W L cube by 32 minus W L cube by 96 plus you know like the B coefficient because we want to calculate this B. So, we have minus W L cube by 48.

So, now you see we have the A value, we have the B value. So, we can put those A which is 0, B which is minus W L cube by 48. We can keep those values in the final expression of the deflection, and we can get the real feeling about the deflection that how the variation of the deflection is there in the beam when a point load is applied on a simply supported beam exactly at the midpoint. So, you see here by keeping those values A and B we have y equals to 1 by E I W x square by 8 minus W x cube by 12 minus W L cube by 48.

So, now you see here we have a clear feeling that this deflection is varying if those you know like due to the point load is in a cubic way, because you see you can see that the total dimension in all of the expression is x square into 1. So, it is total cubic form; this is cubic form, and this is cubic form. So, the variation is there in the cubic form irrespective you see you can put the value of x in terms of the L and you can get the different different deflection at different points.

But now our main intention is to find out the maximum deflection, and always you see when we are you know like putting those point loads at the center we have the maximum deflection. So, by keeping at x equals to L by 2 we have the maximum deflection value. So, you see here at x equals to 0 you know like if we are keeping because now we are reducing the x; whatever you know like the distances were there in between the x we are simply going up to the L by 2. So, here you see if by keeping those things what we have minus W L cube by 48 E I.

So, you see by in this x has gone, this x has gone. So, we have only this value. So, this value is nothing but equals to minus 48, this minus W L cube by 48. So, we have and into this E I is there. So, this is the maximum deflection is there. And then you see if you want to find it out the maximum slope; obviously, you know like they will be there at this point particular. So, you see and if we are keeping that x equals to L by 2 plus minus you see where the maximum slopes are there the center point. So, by keeping those values we have you see at x equals to L by 2 L by 2 here we are keeping. And then if we manipulate those things we have the final values plus minus W L square by this 16 E I.

So, meaning is pretty simple that one can easily get. This is a very simple method as compared to the previous one, because here in the previous cases you know like what we are doing here; if we do not know you know like those distances that actually at what point these loading conditions are there. Then what we need to do here? We need to you know like take the x distance from the left hand portion; that means you see from the reaction forces.

But instead of that you see here it is pretty simpler to take the distance from the loading conditions so that if any variation is there in the point load, it can be easily calculated. So, this is you see the alternative method, but again you see in both of the method the direct integration method you see in one form or other there are certain you know like discrepancies are there.

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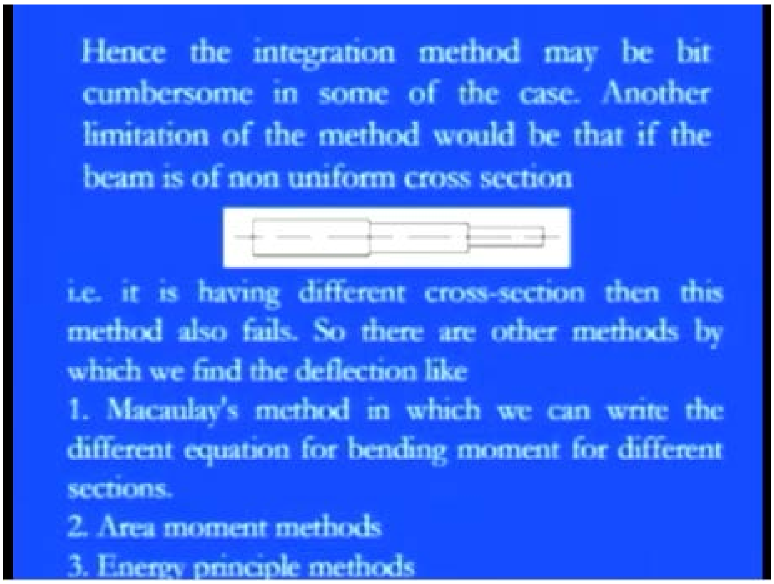

So, you see here hence the integration method may be a bit cumbersome or bit bulky because you see again you need to describe you see for all the expressions for even the slope or for deflection or for you know like shear forces kind of that. So, in some way you see you will find that we have a bulky equation, the cumbersome equations are there. And then you know like if we have you know like the kind of this stepped bar you can see on your screen; if you have the non uniform cross section of the bar this stepped bars are there you see. Then it is you know like pretty horrible to analyze all those things for different different sections because you see for individual sections we have a different equation, and for that you see then again you need to sum up.

You have very cumbersome equations or we can say it is very bulky or lengthy equations are there. Then again you need to put the boundary conditions for those things which is very very you know like complicated. And if you see if any other kind of loading is there on this kind of beam, then it is really disaster is there to you know like calculate those deflections as well as the slope value for this. So, what we need to do here? We need to go for alternative methods. So, there are you see you know like the three methods are available you know like in the solid mechanics theory for undergraduate courses which we can simply apply for this kind of beams or even for simpler beams also.

So, one method is the Macaulay's method in which we can simply write these different equations whatever you see you know like those equations are coming for bending moment for a different section. So, what we are doing here? Instead of going for entire beam here we are simply taking individual sections and then you see you know like just write this equation, apply the this boundary conditions and get those values. So, Macaulay's method is very much suitable for this kind of sections where you see the different particular total beam if we have a different different stepped sections are there or non uniform sections are there, then this method is quite applicable,

But now that does not means that itself we cannot apply in a simpler beam also; we can also apply the simpler beam to get the value of those deflection as well as the slope. Second method is there, the area moment method. So, in this chapter basically our focus will be there on the area moment method that you see you know like we have a bending moment and you see under the bending moment diagram we have certain you know like the diagram is there. So, if you know like because the loading conditions are there and whatever the cross section of the beam is there, they will definitely go with the kind of specific bending moment diagram.

So, our focus will be there that actually what will be the area under that diagram, and then based on that you see we can simply find it out that, okay now this much deflection is there or this much slope is there on those things. So, it is basically based on the area with the moment method. So, whatever you see the bending moment is there and whatever the area is coming under those things based on that we can calculate. So, we will discuss this particular method in this lecture. And third one is there, the energy principle method. That you see you know like when you apply the load definitely you see some sort of the energy exchange is there in terms of the potential to kinetic energy.

So, if you compute those potential and the kinetic energy in some sort with using Lagrangian's method on any other method, we can simply know that you know like here you see the kind of variations of the energy is there. So, we just want to relate that energy with the deflection and then we can go for the slope also, and we can also find out that actually at what positions or what are the potential you know areas are there where the maximum deflection or maximum slopes are coming and what will be the value of those things.

So, these three methods are there which will provide some sort of a different methodology as compared to the direct integration. Though you see you know like for academia purpose the direct integration method is quite simpler because you see once you know that okay this d 2 y by dx or d cube y by dx cube or d 4 y by dx 4. So, we can calculate simply all those values like the slope or deflection or you know like this bending moment shear forces or you know like the rate of loading. So, that is why you see this method was in academia this method the first method which we discussed is quite popular.

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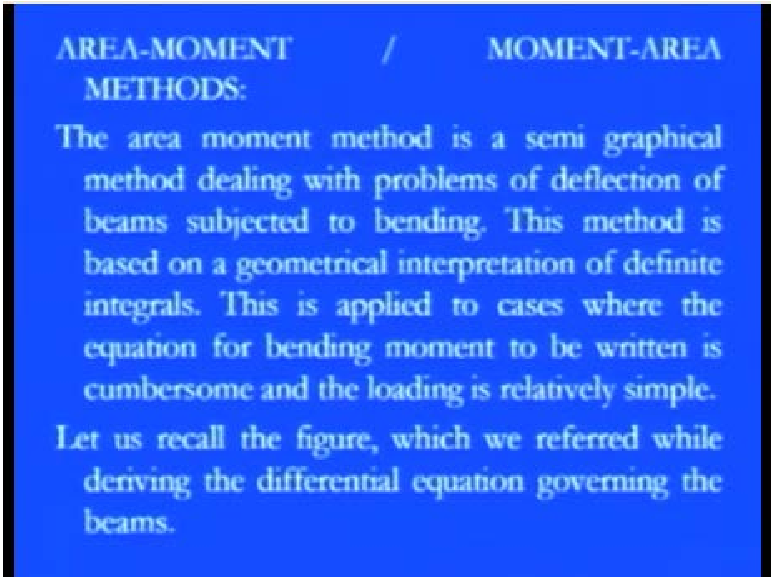

But here you see you know like the second method which we discussed that area moment or you know like the moment area method anyway you seem like it is considered the similar kind of manner. The area moment method is a semi graphical method because as I told you we have simply focused on the bending moment diagram. So, it is the kind of you see first we need to focus on the diagram and then we need to calculate. So, it is not purely graphical, but it has some sort of feeling about the graphical interaction with the mathematical way.

So, it is a semi graphical method dealing with the problems of deflection of being subjected to bending. So, you see here you know like if any kind of beam is there simply supported or cantilever beam and when any loading is there, sort of bending is there and then you see we need to focus with the kind of reactions and the load application that what the boundary conditions are there. And then once you draw the bending moment diagram you have a clear feeling that, okay, now you see you know like these are the portions where the maximum bending moments are there; these are the portion where the minimum bending moments are there, and based on the area under that particular diagram we can calculate the deflection.

So, this method is based on the geometrical interpretation of definition this definite integrals as I told you, and this is applied to the case where the equation for bending moment is to be written in cumbersome and we can say; that means if the bending moment equation as we discussed you see M is equals to you know like E I into d 2 y by dx square. If you go for a lengthy equation or for longer equation this method is quite popular in those cases and loading is relatively simple here. Let us recall the figure you see you know like which we discussed in the previous case which while deriving you know like the differential equation for governing equation like M is equals to E I into d 2 y by dx square.

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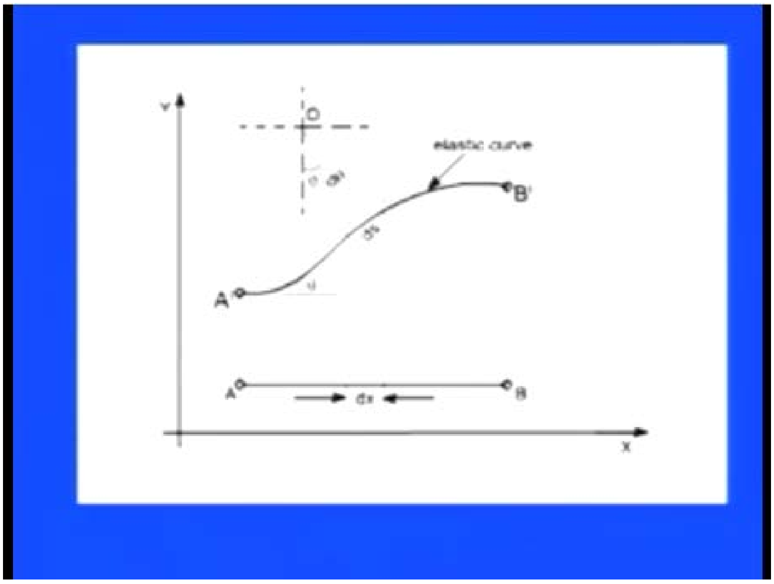

So, again if you go back to that equation you see here what we had? We had you know like under any kind of loading kind of you know like this the central axis or longitudinal axis of the beam is having certain kind of deflection. So, as I told you when there is no loading condition was there then we have a straight you know like the central axis that is the A to B, but if we have the loading condition then you see this A to B is you know like changing into the A dash B dash, and we have a kind of curvature shape is there.

So, in that curvature shape again we describe that was the pretty you know like the assumption was there that whatever the curve is coming that curvature this either deflection curve or the elastic curve, they have to follow the elastic deformation or elastic deflection terms. So, that is why you see the deflection curve is also known as the elastic curve as it is written here. So, A dash and B dash is the deflected curve; in that you see the deflection is absolutely on the d theta. So, here you see though like the d theta is there, and if you take any origin which is the reference point from you know like this curvature point then you know like these kind of that how much change is there when a straight portion and when a deflected portion is there. So, d theta can be easily measurable.

So, if I am talking about a small segment we have the ds in the deflection part; we have the ds is the deflection part and dx is the straight part. So, if I am talking about a small angle like you see you know like this tan theta like which we are taking the slope; this d theta by dx or ds by dx whatever you see. So, always you see this slope will be you know like make the clear difference in between the ds and dx, because you see the slope will come for dx; there is no slope here. But if the slope is very very small as we are generally assuming that you know like the deflection curve is you know like always these theories are applicable for a small deflection curve.

So, for that kind of section you see the ds is almost equal to dx. We will discuss this part you know later it will come, and then you see you know like we have the theta that how much deflection is there from the main axis in the longitudinal way as compared to that. So, you see here this is the longitudinal axis; we have the theta or we can measure in this way also. So, this is the clear feeling about the deflection curve which we discussed in the first method direct integration method and here also we are using the similar kind of concept here.

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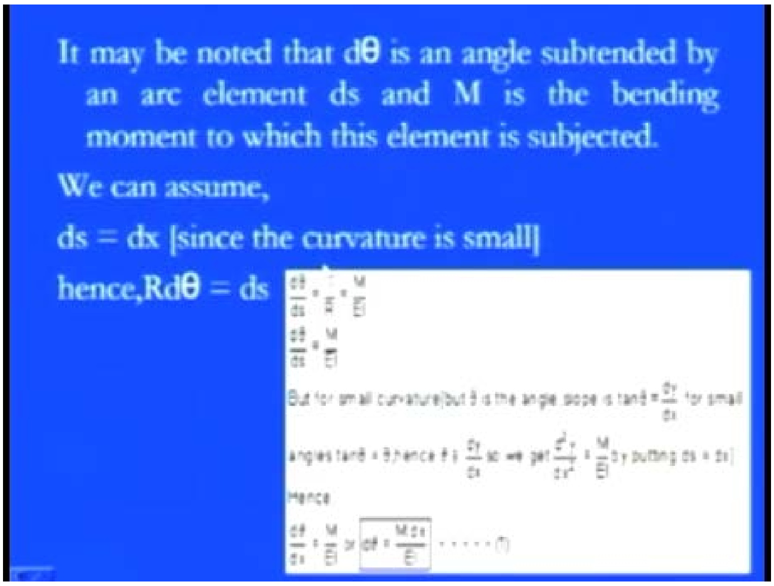

So, here in that we just noted that d theta is the angle subtended by you know like the arc element with the ds and M is the bending moment to which this element is subjected, and due to that bending moment you see we have the deflection in that particular beam. So, you know like just go to the equation. We have the arc equation that R theta is equals to ds. So, d theta by ds from this equation because you know like this is a pretty simple curvature theory is there that if we have a radius of curvature R and any angle is there d theta then it is exactly equals to R into theta will give you the ds.

So, d theta by ds is equals to 1 by R, and if you go to the main bending equation then we have this M by I is equals to sigma by y is equals to this R by E. So, you see here with those you see if you manipulate those equation we have M by I is equals to E by R. So, 1 by R will be equals to M by E I. So, now we can simply replace this 1 by R is equals to this M by E I. So, we have d theta by ds is equals to M by E I. So, now with that if we are going for the slope that is you know like the slope is there in terms of the dy by dx which is equals to tan theta, and if I am saying that the theta is very very small. So, tan theta is equals to theta. So, we can say that theta is equals to dy by dx.

And now if we are going for the main this deflection equation which is d 2 y by dx square which it is giving that M by E I. Now if I am putting that those values then we have you know like, since, as I told in the previous figure that the ds is the deflected part and dx is the straight part. But if the theta is very very small we can go for the ds is equals to dx, because it is a curvature part, and this is the straight part. And if I am saying that this curvature a slope is very small it is exactly equals to not exactly but it is somewhat equal to the dx value.

So, ds is equals to dx; if I am keeping those values what I have now you see the dx will be coming here. So, d theta by dx is equals to M by E I and by keeping those values we have d theta is equals to M into dx divided by E I. So, what it concludes? We have a basic equation of the d theta; that means you see how much deflection is there of any small element like this one you know like in the deflection part. So, this deflection which can be computed with respect to this t theta which is equals to M into dx divided by E I.

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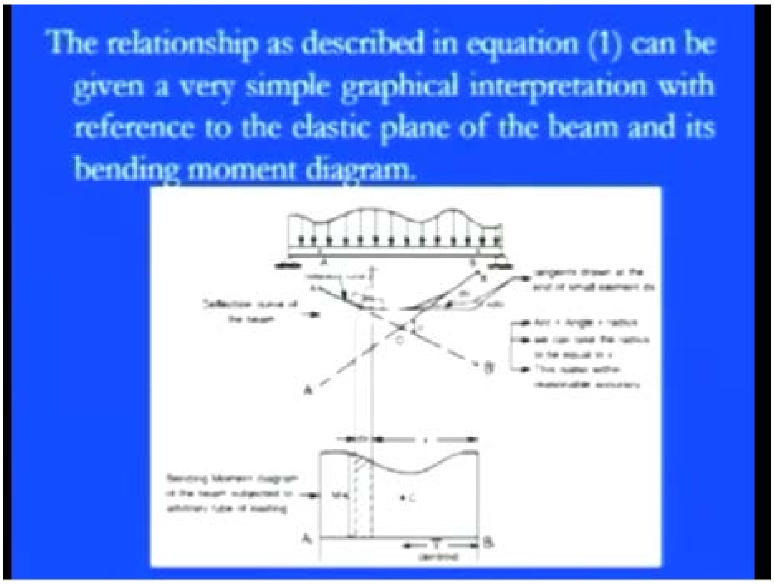

So, with that condition you see you know like again we would like to focus that, okay, this condition is there in which we have a d theta which is very much valid for M into you know like dx divided by E I. But with that relation which you know like we just setup in the previous equation it can be given in a very simpler graphical interpretation with the reference to the elastic plane, because you see these are all the elastic curves or the deflection curves are there of a beam and its bending moment diagram is pretty simpler, because now our main focus is that once you have a deflection then how you know like incorporated in terms of the bending moment diagram.

So, here again we have a simple case that we have a simply supported beam at the extreme two corners, and if we have you see this kind of non uniform loading is there, okay. This is UDL sort of you can say that we have this distributed load is there on that. So, with that you see we have a deflection curve. So, this deflection curve is like that you see here. So, this A and B points are the real deflection points are there; we can say these are the initial point of the deflection, and then you see if we put the tangent from A and from B. So, these tangents are meeting at some point that is you know like the point where the centroid is locating.

So, if you just focus on the bending moment diagram just lower portion we have you see you know like this centroid position, and always you see we are measuring the centroid from one of the reaction forces in terms of x bar or y bar. So, here we are taking the x bar distance from this thing. So, we have the centroid, and with the kind of variation you see here we have the bending moment diagram in this way. So, this is a kind of deflection is there in this particular beam. So, as you can see here this is the deflection curve. So, it has a maximum value at the center position, and you see you know like starting from the 0 deflection at these two supports they are coming in this way. And if you put the tangent then you can simply locate the origin of those things, and you know like this always will go for the arc that actually what is the arc length is there in these particular kind of deflection curve.

So, with those things if you projected you know like on the bending moment diagram, what we have? This is you see you know like this particular portion where the bending moment is there this is the M, and if we are talking about generally you know like we have taken the small segment. So, if we are talking about the small segment dx from the x distance of this particular portion, then we can simply show that our main focus of the study is on this one. Once you get the real feeling of the bending moment as well as the shear forces for these kinds of things, then you can integrate those things and you can get the real feeling of the bending moment or we can say in other this kind of stresses in this kind of bending.

So, here you see you have the deflection curve and you have the bending moment diagram and you know like they are pretty you know like simpler and they are giving you main kind of information that actually where the centroid is located and how these deflections are taken place if this kind of loading is there.

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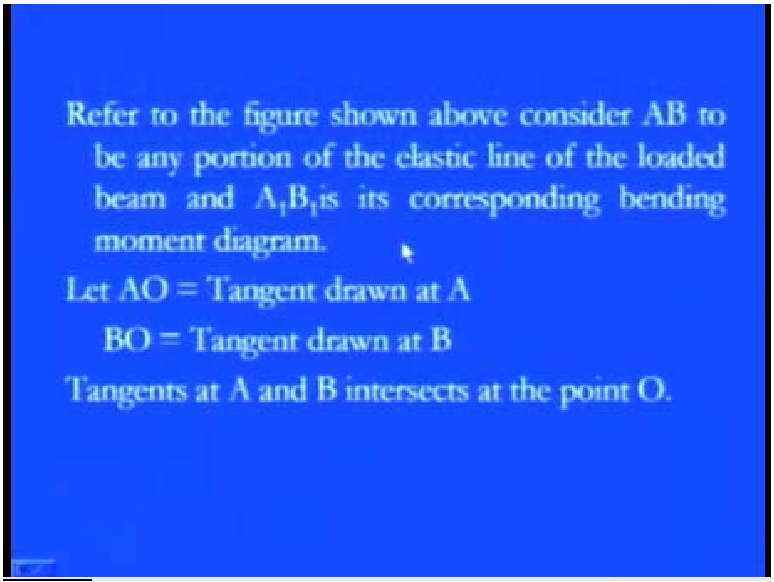

So, just focus on that particular figure we have AB; as I told you these two corner points were there of this portion of you know like and they are pretty simpler portion, because you see this is the starting point. And if we are projecting those you know like the tangented then we have you know like the points are there A dash B dash in the previous figure and wherever you see the A 2 A dash and B 2 B dash are meeting you know like there is a point and that point where the intersection is there of these tangents it is the origin point.

So, you see here we have AO that is the tangent drawn at A and BO is the tangent drawn at B. So, with those things you see you know like this O point was come, and once you plotted that O point in this bending moment diagram we simply got that the centroid point.

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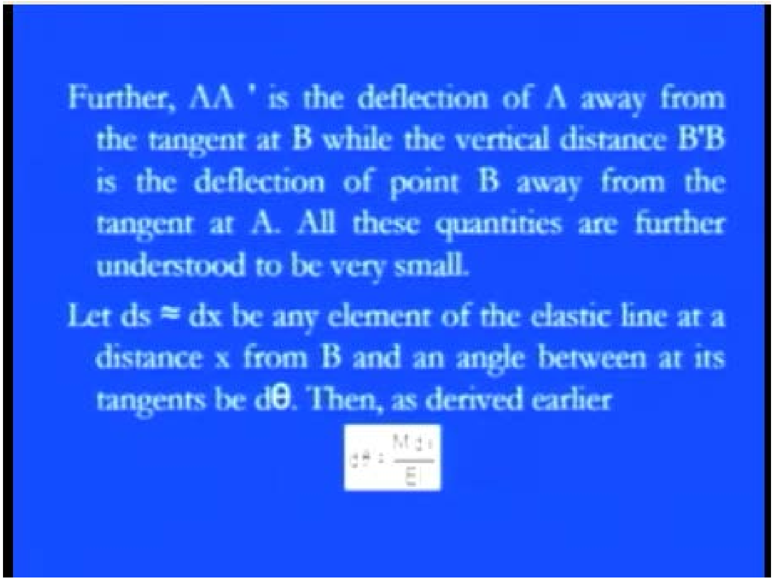

So, AA dash is the deflection at A as you know like I told you that whatever the deflection is there we are measuring with the tangent away from the tangent at B while the vertical distance BB dash you know like is the deflection at point B away from the tangent at A. So, both are you see you know like we just want to find it out real feeling that how the deflection will take place under this kind of loading. So, that is what you see we are going for the tangent, and then you see wherever these tangents are meeting as I told you the O point is coming.

All these you know like the quantities are further understood to be very very small because you know like if it has a very large then always it is going beyond that limit then you see we have a permanent set of deflections, and then you see these theories are not at all applicable. So, just you see keep this thing in your mind that whatever the deflection you know like they are coming in those beams particular under these loadings, they are very very small and they are just up to the limit of the elastic region, so that whenever you remove the load it comes to its original state.

Now you see we assume that the theta is very very small. So, ds is almost equal to dx for any element in the elastic line at a distance x from B and angle between its tangent will be definitely d theta is there. So, d theta can be derived as derived in the previous one is equals to M into dx divided by E I.

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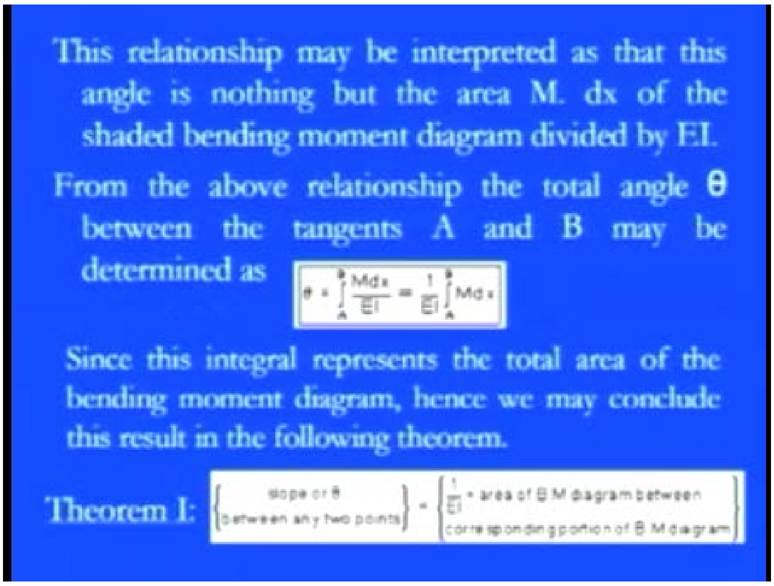

So, with that you see you know like this relation we can simply interpretate you know like that angle is nothing but the area you know like into dx. So, you see what we have? We have the area under that. So, if we integrate those parts like we have d theta which is M into dx divided by E I. So, now you see if you integrate that we have the theta value. Theta is nothing but equals to 0 you know like right from let us say we have the limiting conditions for that integration into M into dx divided by E I. E I is a constant one for any kind of cross section of the beam with the material part.

So, we have just taken this one out, what we have? We have integration M dx. So, now you see this M dx is nothing but it will always give you that actually what we will be the area under that particular portion. So, you see here these deflections are coming due to the bending moment and these deflections are varying with the dx distance. So, M into dx will give you you know like the area under that curve. Just like you see, you just focus on the work done. What the work done is there if you focused on any thermodynamic processes we have the work done is equals to PDV, okay.

And you see if you are talking about any process like isochoric process, isomeric process, any process in the thermodynamic processes, what we are doing here? We are simply drawing the PV curve, and you see you know like with that the constant value process or constant pressure process; we simply draw the curve and whatever the area is coming under that curve which will give you the work done, because work done is nothing but equals to integration of P into dV. So, same concept is here. We have the theta which is integration of M into dx; the M is varying with the dx.

So, we can get you see you know like the real feeling about those at the bending moment if you know this area. And then you see you know like this; whatever the area is coming if it is divided by E I you have the clear deflection. So, you see here the total angle theta between the tangent A and B let us say we are talking about point A and B generally in in that particular figure. So, from A to B you see we have you know like M dx divided by E I or we can say that 1 by EI is a constant one. So, it is just taken out and then it is A to B M into dx.

Since the integral represents that total area, obviously, you see we initially we need to talk about the small segment and then integrate for the entire section. So, the total area of the bending moment diagram will give you the real feeling of theta, and hence we may conclude that the slope or the theta between the two points will be equals to 1 by E I into the area of bending moment diagram between the corresponding you know like portion of bending moment diagram, but you see you know like and that is what you see it is very much valid for the stepped bar.

If we are talking about this portion means the non uniform section then another different section for another section, just go with those you know like the portions draw the bending moment diagram and multiply with 1 by E I for that particular section, then you see you have the slope or we can say the theta for in between that particular section. So, that is why you see this is the basic principle of the area moment method that you can easily calculate the theta by equals to 1 by E I into integration between those point M into dx. So, you here once first of all you need to calculate the shear force, based on that you need to calculate the bending moment. Once you have the bending moment put that equation, take the integration and get the value of theta.

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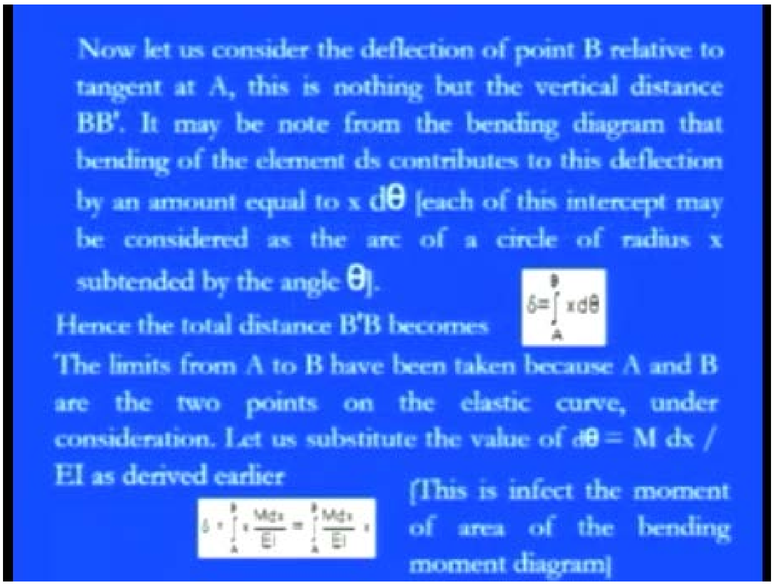

Now let us consider the deflection at point B related to the tangent A. So, you see here you know like we simply put the tangent you know like at point B with respect to A. This is nothing but the vertical distance between the BB dash; obviously, you see which we noted in the previous section where the two deflection points are there. It may be noted that from the bending moment diagram, the bending of element ds contributes a bit this deflection by an amount of you know like x d theta; that means each intercept you know like may be considered at the arc circle you see. Because you see at each of the segment there is an a small arc, and for that you can simply go for the d theta and whatever the deflection will come that is equals to xd theta.

So, you see here with that consideration you simply go that we have a dx small segment where the small deflection is there of delta. So, with that you see you know like if we are talking about a total distance BB dash then we have the delta is equals to integration of A to B x into theta. Because you see what we are doing here, we are going for a small segment where that d theta this segment is there and the distance is x. So, now once you have this particular segment you know like x into d theta just take the small you know like the deformation for this, integrate for entire section, you have the total deformation for that.

So, with that you see you know like what we can do here simply put those limits A and B between you know like those particular portion, and we can simply substitute that value into the main equation d theta which is M dx by this E I. So, if you are keeping those conditions there, what we have? We have the deflection term; this delta is equals to A to B x times you see you know like this one is there M dx divided by E I or we have you know like this deflection delta is nothing but equals to A to M dx divided by E I into x.

So, now you see here you have two main equations; one is the theta equation. So, theta is equals to integration of M dx divided by E I and you have the delta equation that is nothing but equals to integration of M dx into E I into x. So, you see here now you know like just based on that bending moment diagram is coming due to whatever the load application is there or whatever the moment is coming; Based on that first of all we need to draw the bending moment diagram. Once you have the bending moment diagram just calculate that you know like whatever the area is there for that, and based on that you can simply get the value of either the theta or we can say that we can get the value of deflection delta.

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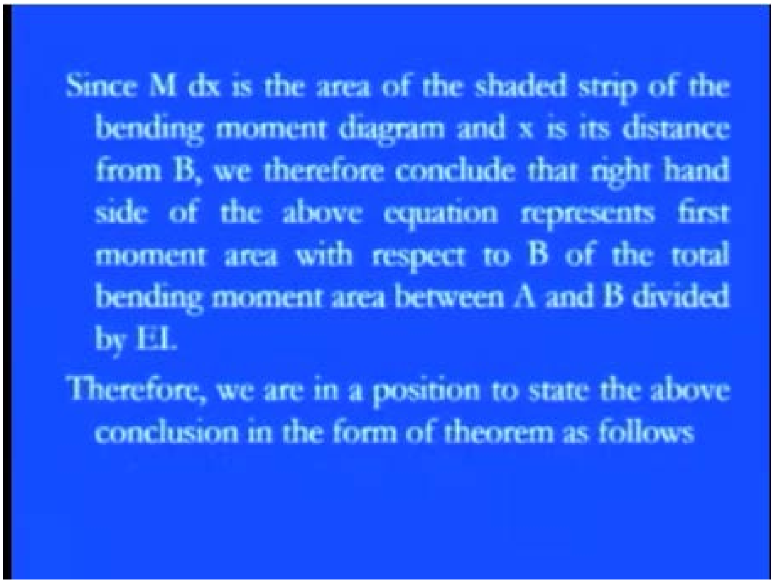

Since M into dx as I told you know like the area of the shaded strip of the bending moment diagram and x is the distance from B you know like the this particular distance the x is the distance of that particular thing, and we have the bending moment diagram. Therefore, we can conclude that to right hand side of the above equation from this one side you see here just in corresponding way; right hand side of the above equation represent the first moment area that you know like with respect to B and the total bending moment area between A and B is always divided by E I, so that whatever the flexural rigidity is coming due to this load application, it can be easily incorporated in those deflection term.

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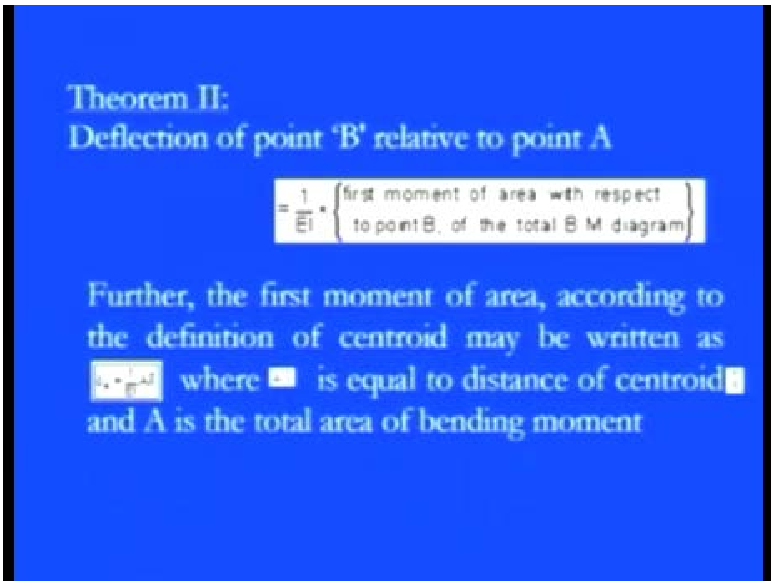

Therefore, we are in a position to state above condition in form of the theorem which is nothing but is equals to this that the deflection of any point; at any point we are taking here the B point because you see we are taking the distance from point B in relation to the point A is nothing but equals to 1 by E I. The first moment area you see with respect to the point B of a total bending moment. So, what we are doing here in the first case where you see we want to take the slope is nothing but equals to 1 by E I; the total bending moment area you see only just whatever the area is there, but here you see if you want to calculate the deflection because you see in that case what we have done you see the delta was nothing but equals to M into you know like this M into dx divided by E I into the x was there.

So, now you see it converted into the first moment of area. So, you see here the second theorem says that if you want to calculate the deflection for any point, you can simply go within those limit region, and then you can keep those values there. So, we have 1 by E I into first moment of area with respect to the point B of the total bending moment diagram. Further the first moment area according to the definition of a centroid because you see where the O point was there and we simply located the O point in this bending moment diagram which gives you the centroid and which has a distance of x bar.

So, with the definition of the centroid it can be written as delta is nothing but equals to 1 by E I into A into x bar. So, x bar is you see you know like it is the location from you see the one region like you see if we are talking about this right hand region or the left hand region from one region where you see all of the center of mass is concentrating. So, with that you see you know like always it is coming in terms of the x bar because we are flowing with the longitudinal axis. So, x bar is there where Ax bar in this you see where the delta A if you want to calculate at certain position of the deflection.

So, if we are talking about the deflection at point A which concludes that it is Ax bar divided by E I and this Ax bar is nothing but you know like the distance of the centroid x bar, and you see A is the total you know like area of a bending moment. So, we need to calculate that what will be the total area of the bending moment is after keeping those bending total bending moment area in that. Again you see we need to locate that where this centroid position is. Once you have the centroid position once you have the area once you know that which material of the beam is there and what is the cross sectional area is there, you can simply get the deflection of any kind of beam or whatever the kind of loading is.

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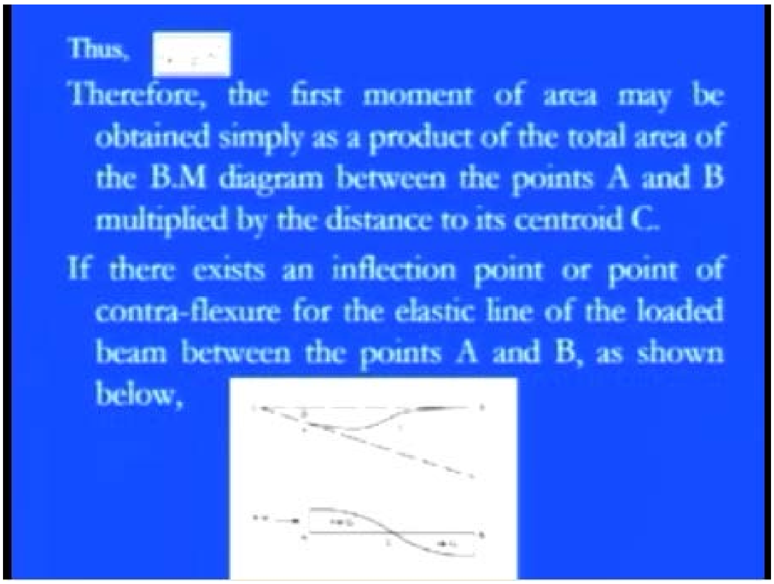

So, with the deflection term you see which we described you know like in terms of the centroidal part that if we have a centroid in this particular beam you know like the bending moment, we can simply calculate this you know like the delta A which we discussed is nothing but equals to Ax bar divided by this E I. Therefore, the first moment of area may be you know like obtained simply as a product of the total area of the bending moment between you know like at certain point, and then we need to multiply with the distance to its centroid x bar.

So, once you have you know like this kind of the first moment of area and we have the x bar distance, we can easily calculate the slope at any you know like or we can say the deflection at any point. And if there exist any inflection point or point of contra flexure; that means you see if that is a special case is there that if bending moment is changing from sagging to hogging position or if there is you know like the change is there from plus to minus, in that case we have a point of contra flexure for an elastic line of loaded beam between any point. Then you see we need to go for a different section like you can see here on your screen there is you know like we have a section that O to B is there. And between you see if these kinds of you know like the deflection curve is there and then you see if we are putting the tangent on this particular part and if we have you know like this kind of abrupt change is there in this.

So, for these things if we are going for the bending moment then you have you know like for this section we have the bending moment positive and for this section we have the bending moment is negative. So, with that you see we have a point of contra flexure.

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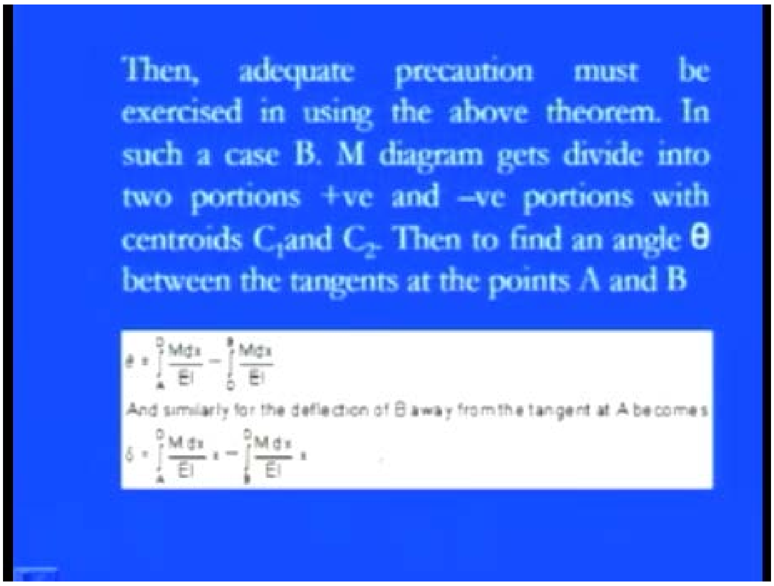

So, if this condition is there then what we need to do then? You know like the adequate precautions must be exercised in using of you know like the above theorem, because here you see there is a change in the area abruptly you see from positive to negative. And in such a case the bending moment diagram gets divided into the two portions positive and the negative as we shown in the previous diagram. And then you see we need to take the corresponding centroidal part from the different references like C 1 for positive, C 2 for positive, and then you see we need to write you know like the slope equation for .different different segments

So, here you see you know like as I told you see you know like we are dividing into two different sections for that; we need to describe you know like for A to 0 like the O position is there and if it is going for A. So, for that section we have a positive bending moment. So, for that we have A to O M dx by E I, and then you see for you know like from O to B where you see you know like abrupt change was there, the hogging part is there So, for that O to B M dx by E I and then you know like again the same processor in which we need to follow for the deflection part that you know like once we have the slope even we can go for the similar way that A to O M dx by EI into X minus this B 2 O you see M dx E I into X.

So, the relative changes are there as far as the slope and the deflections are concerned. We need to incorporate those changes in those equations, and we can get real feeling about that how the bending moment and how you know like the shearing will occur in those things under that deflection part.

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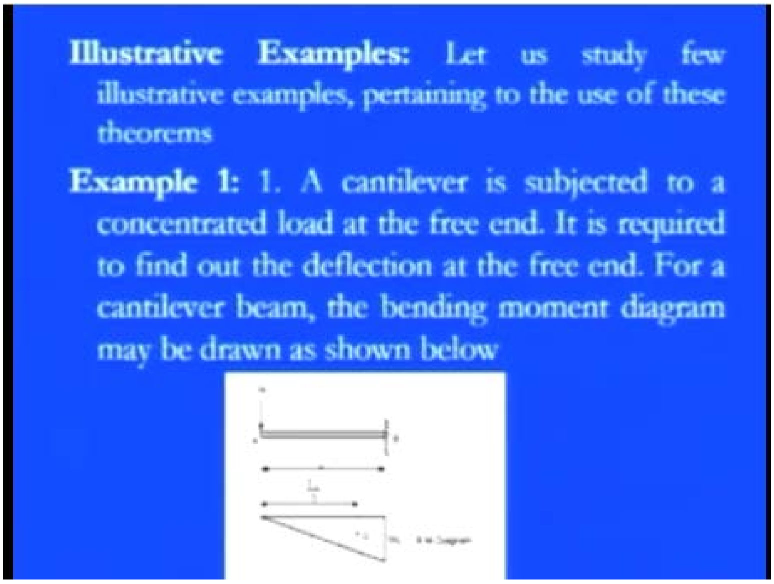

So, you see here this was a pretty special case when the point of contra flexure was there, and in that case you see we are dividing the beam into two different segments in the positive as well as the negative you know like the bending moment, because you see we are simply focusing on the bending moment diagram. So, you see here when we have a positive bending moment diagram when we have this negative bending moment diagram, then we need to consider accordingly and we need to incorporate those sign convention in calculation of the slope as well as the deflection.

So, now you see here whatever we discussed the area moment method now we would like to apply on certain you know like the example this conditions and it is coming in terms of the example. So, here you see the example one is we have a cantilever beam which is subjected to a concentrated load as we discussed in the first case. At free end it is pretty easy you know like it is required to find it out the deflection at the free end for a cantilever beam with using of the area moment method. So, you see here because we have already discussed about these cantilever beam you know like at free end we have this point load is there or concentrated load is there, and we calculated with using of direct integration method.

But here you see we need to calculate the deflection with the using of area moment methods. So, for that again as I told you you see all the focus is there on a bending moment diagram because it is a semi graphical method. So, first we need to calculate the bending moment diagram for that. So, here on your screen you see you have straight this cantilever beam, point load is there, and the total length is L. So, for that we have a bending moment diagram here. So, W L is the maximum bending the maximum bending moment which is there at this, and here you see you know like we have a zero. So, starting from that since a point load is there. So, there is no you know like the curvature path or parabolic path straight line is there.

So, if you go with this particular section we have a triangular shape is there of a bending moment diagram, and if you want to locate this centroid then it is always since it is a triangular part. So, two-third of the distance will give you the centroid. So, it is 2 L by 3 at which the location of the centroid is located in this particular diagram.

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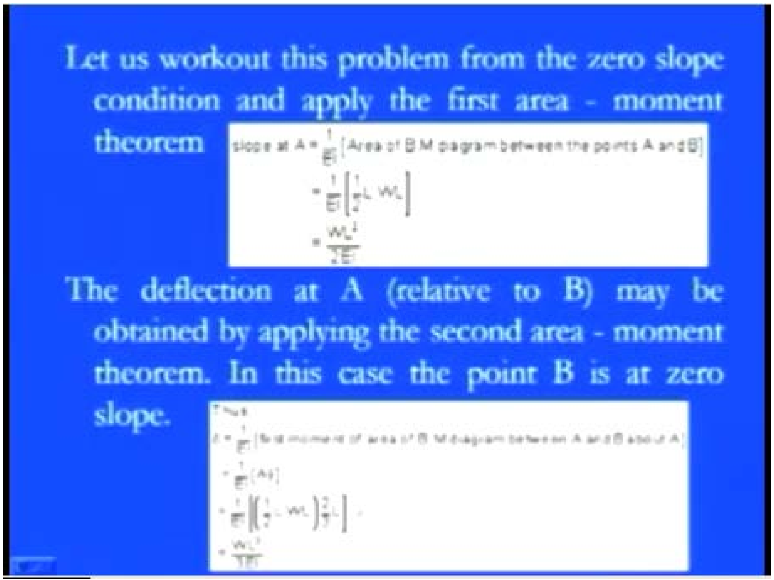

So, you see here we have the required information in that sense. Now you see we can simply work out the problem in terms of the zero condition slope to the maximum slope condition. So, for that slope at point A the initial point A is nothing but equals to 1 by E I which is the flexural rigidity into the area of bending moment between those points A and B. So, with that you see you know like what we have? We have you know like 1 by E I. Now what will be the area? It is a triangular area. So, half into the distance into the height; so, you see here half into L into W L which is a total height due to bending moment.

Keeping those conditions we have W L square divided by 2 E I. So, the deflection at point A which is related to point B may be you know like again obtained by applying the second area moment of theorem you see where we can simply get the deflection term. So, you see the deflection you know like is nothing but equals to 1 by E I into you know like the first moment of area of the bending moment diagram in between those points A and B. So, after keeping those conditions what we have? We have 1 by E I into Ay bar. So, you see here we have a triangular element; in that you see we can simply calculate the area as we calculated which is nothing but equals to 1 by 2 the distance this is L into the height that is WL. And then you see this y bar. Y bar is nothing but the location of the centroid from that point.

So, you see here since it is a triangular element and you see uniform triangular part is there. So, it is two-third of the distance. So, by keeping those values here, what we have? We have 1 by E I into half of L into W L into two-third of L. So, by keeping those values we have W L cube by 3 E I. So, you see here you know like it is in comparison to the pervious method it is a very simple method, no boundary condition; straightway once you have a feeling of the bending moment diagram, you can simply have the slope as well as the deflection. So, this was a pretty simple path

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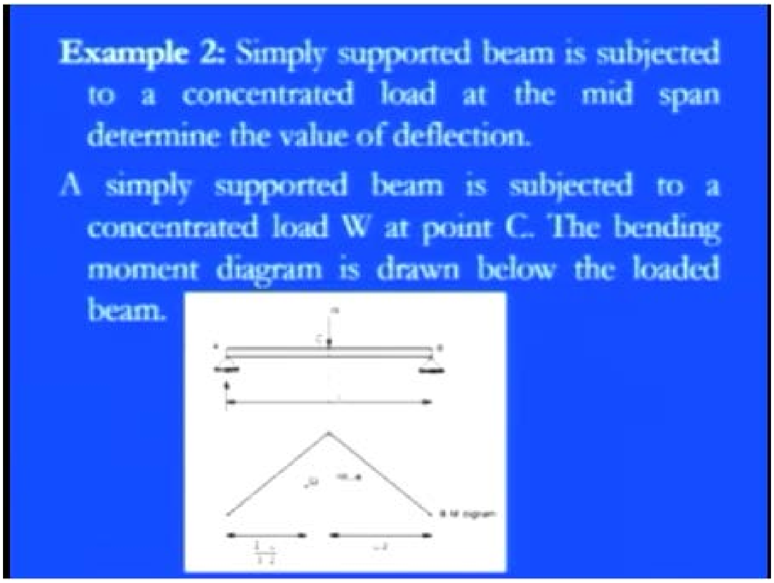

Then if you are going for the next you know like example in which a simply supported beam is there, and in the simply supported beam we have a point load at the mid span of this, and we just point to find out the deflection. So, here you see you know like at A and B these are you know like the rigid supports, and at this particular sorry this pinpoints are there. And at these pinpoints we have the reaction forces, and due to that you see we have you know like the point load is there due to that we have the deflection which is maximum at the center point.

So, now if we are drawing the bending moment diagram for this, we have you know like straightway at these two points there is no you know like the bending moment, here it is a maximum bending moment at this particular point. So, we have a clear triangular part is there and this height is nothing but equals to WL by 4, and then you see you know like now we want to find it out that where the location of this centroid is. So, centroid is located at since it is L by 2, this distance is L by 2 because this is the total L. So, in that you see the two-third distance. So, two-third of L by 2; so, we have you know like this L by 3 distance is there where the centroid is located.

So, now you see we have two different triangles in the sense that they have a symmetricity in terms of their height W L by 4 in terms of their base they have the symmetricity. And then you see since it is a symmetrical triangular part we can simply go for the location of the centroid which is two-third of the distance of this. Since, it is distance is L by 2. So, we have a two-third of L by 2; that means L by 3.

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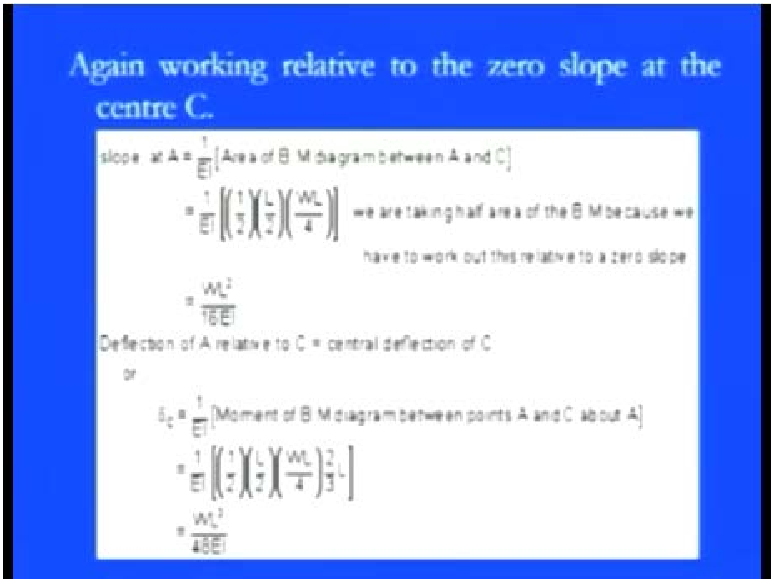

So, now with remembering all these things now we would like to calculate the slope and the deflection for this kind of thing. So, for that you see slope at point A is nothing but equals to 1 by E I area of the bending moment diagram between the A and point C. So, now by keeping those things what we have? We have 1 by E I into half into L by 2 into W L by 4, because area you see half into the total this length; this is L by 2, and this is height W L by 4. So, by taking those things you know like we can simply go for the half of the area you know because you see for half of the rectangular part we have this kind of thing.

So, you know like in between the point A and C will be equals to WL square by 16 E I. And similarly, we can also calculate the deflection part in between the point A and C and which can be easily calculated by you know like the same theorem II 1 by EI into the first moment of area of the bending moment between point A and C. So, by keeping those things again you see we know the area which is nothing but equals to half L by 2 into W L by 4. So, this is one area, and then we know the location of the centroid that you that was nothing but equals to two-third of distance that is L by 2.

So, you see here and if we are going for the entire you know like this triangle, then you see it is the two-third of the total distance L. So, now you see by keeping those values here what we have? We have you know like this delta C is nothing but equals to 1 by E I half L by 2 W L by 4. This is the total; all those elements are calculating for the area and then you see this y bar is there which is nothing but equal to two-third of length. So, you see for the entire regions of this beam where you see the bending moment diagram is like that you see. Here the deflection is nothing but equals to W L cube by 48 E I.

And also you see we calculated you know like by using that direct integration method we calculate the same thing. But you see if you remember that part it was a real you know like the bulky equations was there, then you see we put the certain conditions to get those values. But here you see it is pretty easy to calculate by simply going with this bending moment area and the first moment of area for this particular bending moment. So, you see here you know like in these two examples the loading was pretty simple; that is the main constraint for this kind of thing that loading must be simple, so that we can get a straight feeling about the bending moment diagram and then we can calculate.

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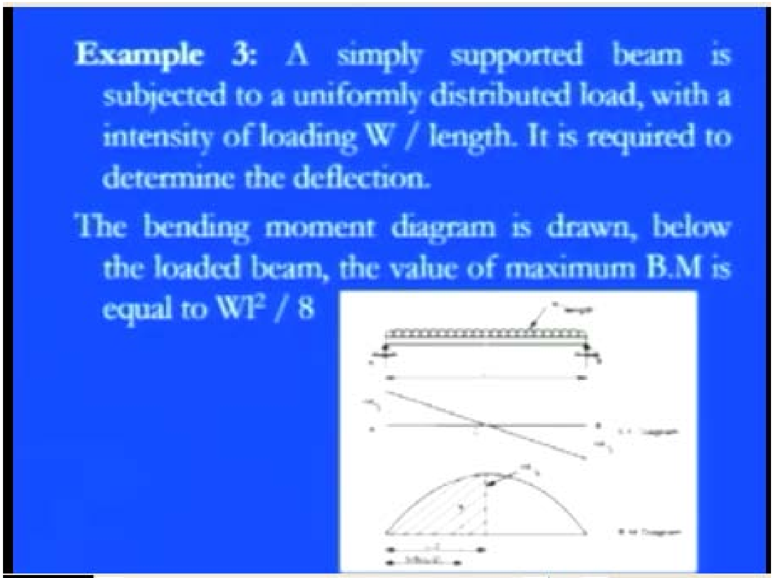

So, here in the third example also which is the last example again we have a simply supported beam. Instead of point load at the mid span here we have a UDL. So, with that you see we have this shear forces, and we have the bending moment diagram. So, in that if you are calculating this you see if you focus on the bending moment, then we have you know like the bending moment maximum is nothing but equals to W L square by 8 which is exactly at the center position. And then you see this distance is nothing but equals to you know like this L by 2, because this is the total length here.

So, for that you see if always you see when it is a curvature the parabolic path is there the centroid is different, because you see in the you know like the triangular element we had a centroid at two-third location. But here you see we have you know like the centroidal part exactly at the 5 by 8 into whatever the distance is there where the centroid is locating. So, now you see we have this parabolic element is there in this way bending moment diagram, and we have you know like the distances through which we can calculate the centroid.

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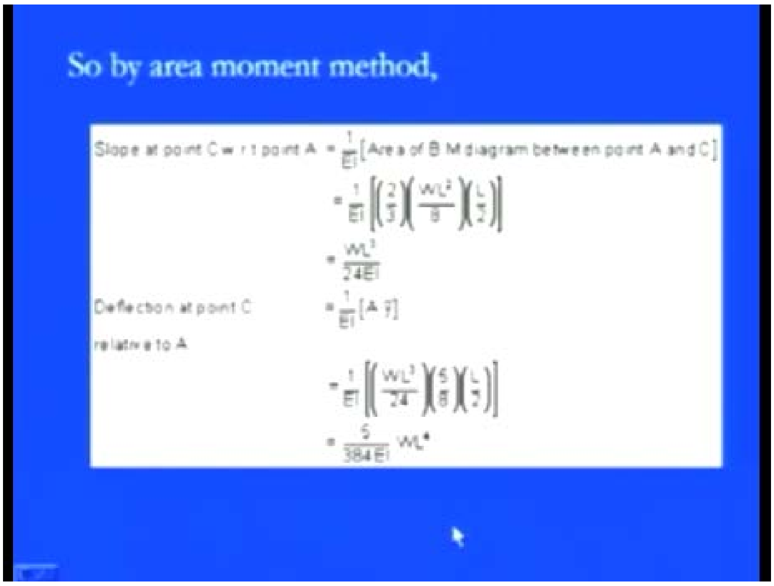

So, by keeping those things here we can simply go for you know like the slope as well as the deflection. So, slope is nothing but equals to 1 by E I into the area. So, again you see it is the area where the two-third into the total height is there which is W L square by 8 into you see the L by 2 the total distance. So, this and this once you know you can simply go for this area. So, once you have this area, once you know like have the structure properly you can calculate the slope. So, here theslope for this simply supported beam under the UDL is W L cube by 24 E I.

And similarly you see we can calculate the deflection which is nothing but equals to 1 by E I into Ay bar; y bar you see we already put those that 5 by 8 into L by 2 is there. So, by keeping those values we have 5 by 3 84 E I into W L 4. So, you see here the same conditions you know like this for deflection at point C the maximum deflection or maximum slope we observed by you know like the direct integration method, but that that method was very complicated, the bulky equations was there.

So, it is better that if we have a simple loading condition with you know like the simple bending moment diagram, it is always go for you know like the area moment method instead of direct integration method. Because it always reduces the computational time as well as you know like gives you a semi graphical method. So, you have a clear feeling about the distribution of this bending or the deflection in entire beam under the loading condition. So, in this you know like the chapter we discussed about the main method the alternative method that is the area moment method you know like, and then you see we found that you know like it is pretty easy to calculate the slope as well as the deflection based on the bending moment diagram. And you see the first moment of area once you have the bending moment.

So, in the next lecture you see we are going to focus on the different method like Macaulay's method and the energy method. So, that you see we just want to a have a feeling that instead of these methods which method is applicable in which conditions. So, that you see if we have a different kind of problems we can straightly go with the different kind of methods which are suitable method for solution.

Thank you.