Strength of Materials

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Lecture – 31

Hi, this is Dr. S. P. Harsha from Mechanical and Industrial Department, IIT Roorkee. I am going to deliver my lecture 31 on the course of the Strength of Materials and this course is developed under the NPTEL program National Programs on Technology Enhanced Learning.

Prior to start this lecture, I would like to refresh those prior concepts, which we discussed in the previous lecture. The previous lecture you see first we discussed about the main the formula, which we derived for the simple bending stresses and the shear stresses, when it is under the loading condition of point load are UDL or whatever like that with the simply supported beam.

So, that numerical problem which we discussed and we also found this the principle stresses for those things when a beam is subjected by the bending as well as the shearing stresses, how we can calculate the principle stresses and then how we can calculate the principle planes also. So, this problem which we discussed you know like the ((Refer Time: 01:18)) problem and then discuss the real feasible problem that if we have the composite beam. Because right now you see in the recent time you will find that mainly it is say some fibers and we have different kind of the composites are there are.

So, because they provide the different kind of unique properties in the kind of beams, so we are not going for the conventional beam design, we are always going for the composite beam. So, but you see if we want to analyze the composite beam, then it is not exactly as straight as the normal beam is, because in that we have a two different kind of materials and we need to check it out that actually how the fibers are reacting each other, when a composite is there.

So, even from the scientific point of view or from the research point of view again, we have to be very, very careful about the compatibility of these two different kind of materials. That, just we discussed about that if we have a wooden plate two wooden plates and in between if we have a steel plate, then how we can analyze.

So, if you go for the basics, then we discussed that you see it is not of valid thing just to apply straight way the same bending theories in that. Because, you see in that in the general bending theory or the shearing part in this beams, we found that we have a different this Young's modulus are there and we have a homogeneous material, but here you see it is not the same.

So, for that what we need to do here we found that there is a alternate way is there that just replace the middle portion or the different material portion, by an equivalent section based on their theories like the stresses with the thickness or the Young's modulus with the thickness. So, we found that if they are valid means you see if the ratio of their thickness is exactly equals to the reciprocal ratios of their, these Young's modulus or we can say that if they are going up to the elastic deformation. Then the stress into thickness is equals to in the one section, it is equals to the stress into thickness into the another section, then we can say that this theories are applicable.

Then, you see know like we absorbed that if we put the equivalent section, then sometimes the equivalent section is much more much larger than the previous section, based on their Young's modulus ratio. Or you see if like in the other case, which we discussed that if we have the two section and between the concrete section is there; that means, you see the 2 different ductile material and between there is a brittle material. Obliviously, it has a less Young's modulus of elastic the property then it is a small portion is there.

So, how the fibers are the just lined up or how the stream lines are there of the settlement of this fibers, which is very, very important. So, all this discussion which we made up in the composite beam in the previous section, and then you see in the last section, we discussed about the deflection part that actually if a beam is under the deflection, then what you see, which of the theories are applicable.

Then, you see we put certain assumptions in those things that actually if the deflection are there, then again you see it has to follow the Hooke's law means the elastic deflections are there. And, you see if we want to measure the deflection this x axis, is going along with the longitudinal axis, because or we can say the horizontal the fibers are there and the y axis, which is always measuring the deflection.

And then, we describe the how the deflections are coming in those things, and then again the another assumption was very important, than you see whatever the curvature path is coming in the due to the deflection is it has to be very, very small in order to ensure that this is the elastic reason is there within those component.

So, with those certain assumptions, you see we found that and then third assumption was very important that, the shearing is to be neglected. So, whatever the deflection is coming or the deformation is coming due to the shearing action it as to be neglected, because they are coming in a very small way. But, if you are minutely observed those things, then we found that it is very, very complicate to analyze those things along with the bending part, so it has to be neglected.

So, these three assumptions we made for the deflection and with those assumptions see we concluded that actually, the bending moment is equals to E into I, E is the Young's modulus of elasticity, I is the mass moment of inertia, into d 2 y by d x square. So, with that theory now again in this lecture, we would like to continue the same deflection theory and then you see we will see that, if we have different combination of the beam, then how we can analyze those things with that particular way the method of analysis.

So, here you see again with the same equation we have this M is equals to E I into d y by d 2 y by d x square. So, now we find differentiating the basic equation of the deflection, then we have dm by d x is equals to E into y d cube y by d x square, so if you know that we have the basic relation between the bending moment and the shear force F is equals to d M by d x. So, here if I am replacing this shear force by d M by d x, then we have F is equals to E into I, d cube y by d x cube.

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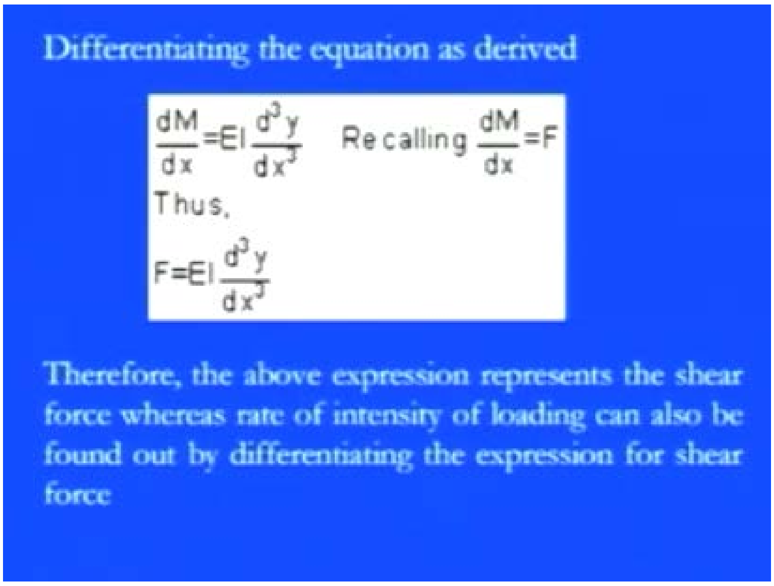

So, above expression represent the shear force, whereas you see the rate of intensity of loading can also be found out by differentiating the expression of the shear force. So, if you remember what we have we have d f by d x is nothing but equals to the rate of loading q or W whatever you see, so you see here by again differentiating this F by d F by E d x you will have the rate loading.

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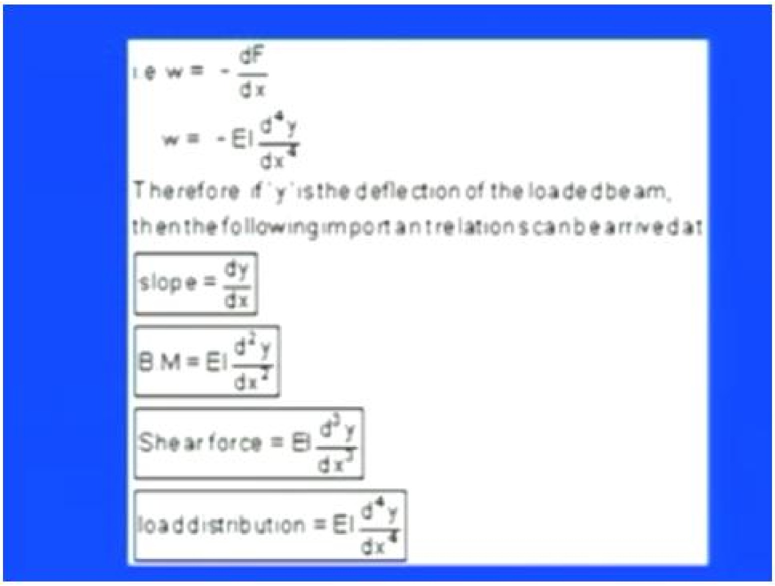

So, the rate of loading in nothing but equals minus d F by d x or we have w is equals to minus E into I, d 4 y by d x 4. So, concluding those things what we have we have first of all the slope I is equals to d y by d x, so you see if a beam is under any kind of loading we can get easily the slope by simply differentiating their domains x and y. So, d y by d x will give you the slope that at what slope the deflection is occurs and then you see once, you have the slope. Then, by simply the bending moment can be easily calculated which is equals to E into I, which is the flexural rigidity that is a very good term in that.

So, bending moment is equals to E into I d 2 y by d x square and then corresponding shear forces are nothing but equals to E into I, d cube y by d x cube and then the rate of loading or the load distribution can be easily calculated which is equals to E into I, d 4 y d x 4. So, now you see we have all those things, but one has to be very, very careful to check it out that how one can get easily the bending moment shear force rate of loading or slope with those they derivative component with the single, second derivative, third derivative or the fourth derivative.

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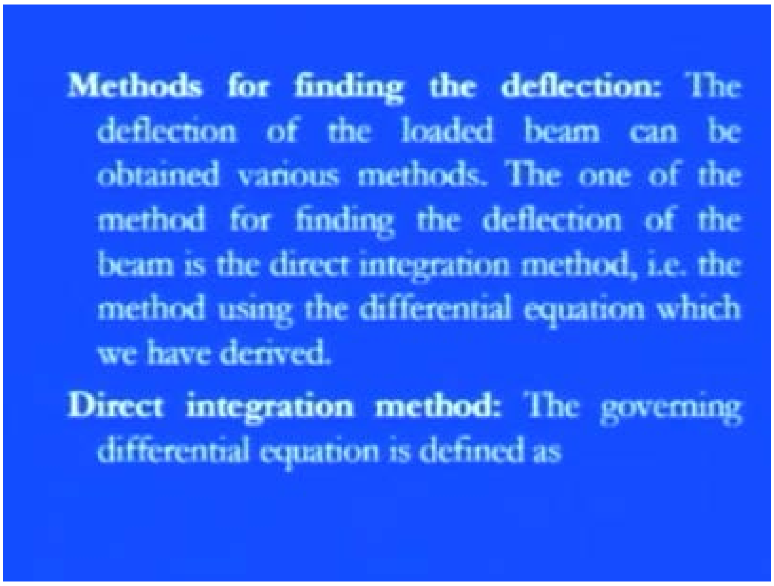

So, with those components now you see, we just want to find just want to found out the deflection, so there are various methods available for that. So, the deflection of loaded beam can be easily obtained by using various methods, and one of the method for finding out the deflection of the beam is direct integration method, that is the method using differential equation, which we have derived.

Previously, that for the slope for the bending moment for the shear force and for the rate of loading, we found that the only the differential equations are there. So, if we put the integration method is straight way you can all those values, which we are looking for. So, first method is pretty common method, very standard method, is direct integration method, with the using of those governing equations.

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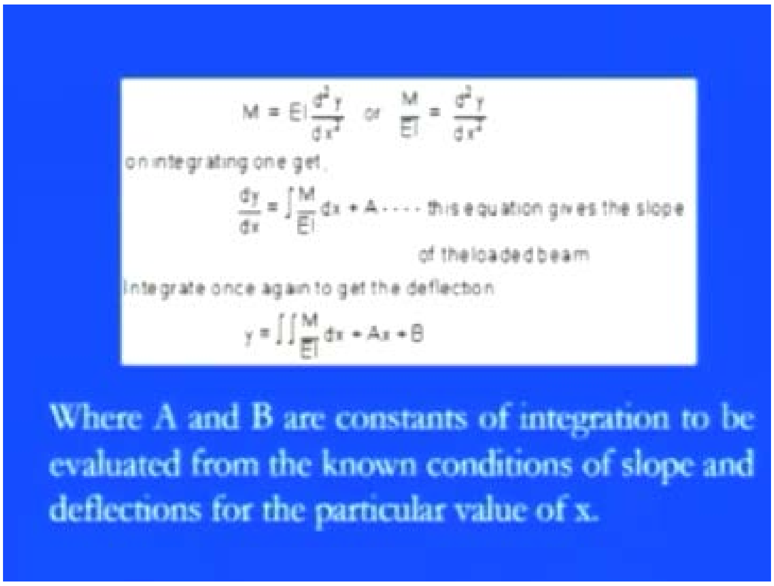

So, you see here the first basic equation for the deflection, which we derived is M is equals to E into I, d 2 y by d x square, so now by keeping M divided by E I will give you d 2 y by d x square. So, now you see here what we have we have d 2 y by d x square is equals to M divided by E I by keeping integration, because you see its direct integration method.

So, if we put the integration what we have d y by d x is equals to integration M E divided by E I into d x plus A, so this equation gives you the slope of that, because d y by d x is the slope of a loaded beam and then again if we put that integration here. Then, we have y, which is equals to double integration of M by E I into d x plus A x plus B.

So, in the deflection formula we have in the double integration of this M by I into d x, and we have the two main the constants A and B; which are the constants of integration to be evaluated from the known conditions of the slope that what kind of beam, which we are taking and what kind of loading conditions are there for that and the deflection for a particular value of x. So, with that you see now we would like to first of all, go for a different kind of combination, that if we have a cantilever beam with the, this a point load at the center or if we have the cantilever beam with the UDL, then how we can configure those things.

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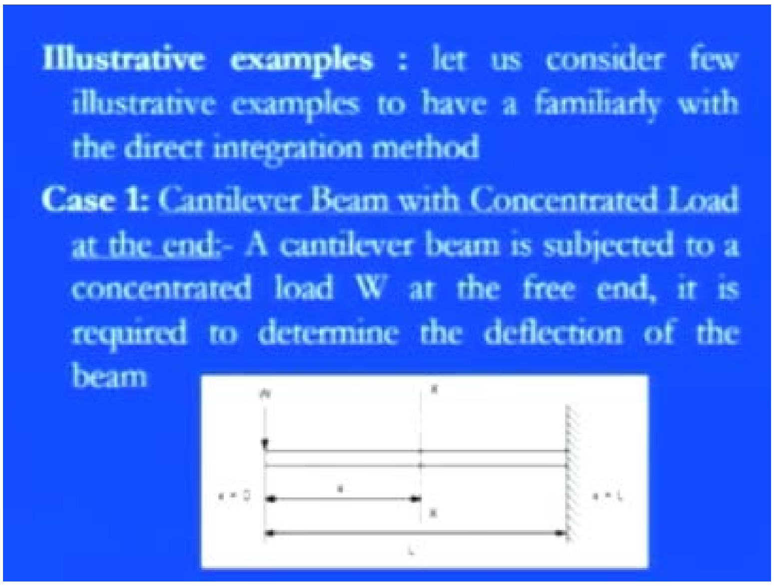

So, the first example let us considered the few illustrative examples to have a familiarity, with the direct integration method. So, the different cases, which we are going to discuss here the first case is, we have a cantilever beam with the concentrated load at the end; that means, you see we have a one free end, we have one fixed end. So, at the free end now we are keeping one the point load or concentrated load at the extreme end and a cantilever beam is subjected to a concentrated load W, you can see on this particular figure at the free end it is quite to determine the deflection at of this particular beam.

So, here you see this is the free end and this is this is the fixed end, this is the free end, and if we are going with the particular longitudinal way, then we have x is equals to 0 and x is equals to L. So, the total length of the beam is L and you see here this free end is subjected by the radial load W, which is going towards downward direction. So now when we keep those things here, we have a kind of deflection of this same, you just focus on this things here this is the maximum deflection going like these and at this particular end, we have a 0 deflection. So, now we would like to find it out that how much deflection is there of the beam, when it is subjected under the radial load of, at the extreme corner the free end. So, now we are taking the X x section at some portion, which has a distance of X from of a free end.

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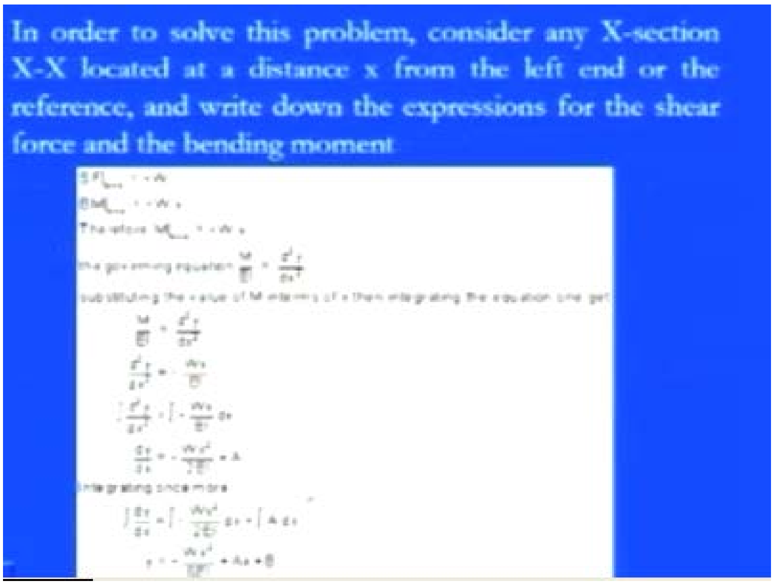

So with that consideration now, just to take the cross section of the beam, which is loaded at X x section from the distance, X from the left end corner. Are we can say that, this is the reference point for this and just, we need to write down the expressions for the shear force and the bending moment of those things. So first of all, the shear force it is going at the extreme corner it is going in this direction and at this particular right section is going in the up upward direction.

So, we have a negative as per the sign convention of the shear forces it is the negative once, so shear forces at X x section is nothing but equals to minus W or the bending moment. Since, it is going in the downward the hogging one just into the downward directions, so it is again you see in the negative directions are there as per the sign convention of the bending moment, so bending moment at X x section is minus W into X. So, therefore, you see the bending moment is nothing but equals to minus W into X for the X x section go for the first equation, which is M by E I is equals to d 2 y by d x square.

So, with that you see if you are keeping this M, which is equals to minus W into X we have d 2 d x square is equals to minus W into X divided by E I. So, now you see what we have, now we have the equation for d the generalized condition of the cantilever beam, which is loaded by on a point load W. So, W d 2 y by d x square is nothing, but equals to minus W into X, because we are just going with the x section, So minus W into X divided E into I.

Now, if we integrate, so integration will give d y by d x; that means, the slope is equals to minus W integration of the W into X divided by E I into d x. So, since we are going with different the integration of with the x, so we have minus W x square divided by 2 E I, because x integration will give x square by 2. So, we have d y by d x is equals to minus W x square divided by 2 E I plus constant A.

Again you see, we are integrating to get the value of deflection, so integration d y by d x is nothing but equals to integration of minus W x square by 2 E I plus A into d x. So, now you see we have the generalized the deflection equation for a cantilever beam, which is loaded at the extreme free end corner by radial load or we can say concentrated load w, which is equals to y equals to minus W x cube 6 E I plus Ax plus B.

So, this is the generalized condition generalized equation after putting the lots of constraints one or we can say the different the boundary values, we can get exactly that actually what we want for a generalized beam. So now you see with those A and B constant can be easily evaluated with the using of boundary condition as I told you.

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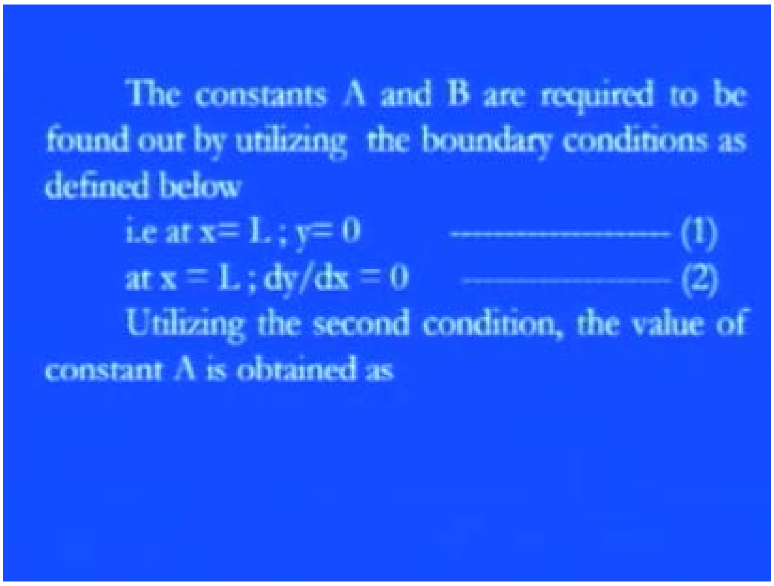

So, here you see what we have it is a cantilever beam at x equals to L; that means, you see the fix end there is no deflection as I told you, because it is a free at free. And we have the main deflection at the extreme corner of these there is no deflections, so at x equals L y equals to 0, so this is the first condition.

And, the second condition you see at x equals to L; that means, you see the fixed condition there is no slope. So, no slope no deflection condition at the rigid and, so you see at x equals to end we can again keep both the condition to get the value of this A and B. So, you see here if you are keeping that in the slope equation d y by d x where at x equals to L the y equal the d y by d x is equals to 0, and in the deflection condition x equals to L y equals to 0, if you are keeping those things, we can get the values of A and B.

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So, here you see here when we are keeping in this the slope condition d y by d x, which gives you A values. So, you see if we are keeping those values 0, then we have A equals to W L square divided by 2 E I, so you see here it is pretty simple that now we have these A condition is there. So, if we are keeping those A value in that we have the y equals to this minus W L square by 6 E I plus A into L plus B, because we are keeping x equals to L here.

So now we have the B value, because the slope is 0 at this particular condition where x equals to L, so if we are keeping those then we have this B is equals to W L cube divided by 6 into E I minus A into L. And now since, A is there already W L square divided by 2 E I by keeping those values, what we have W L cube by 6 E I minus W L cube by 2 E I, so by concluding those things we have the B value, which is minus W L cube by 3 E I. So, now, you have the value of A, which is W L cube W L square by 2 E I and you have the value of B, which is W L cube by 3 E I.

So by keeping those values here, we have the final generalized equation for cantilever beam in terms of the entire beam earlier, you see we have a equation in terms of the x section. Now, we have for entire length of beam entire length of beam that is capital L. So, y equals to 1 by E I, W x cube by 6 E I plus W L square by 2 E I into x minus W L cube by 3 I.

So, you see the slope also one can easily calculate the slope for that one also as the deflection would be the maximum at the free end and hence, you see if you are keeping x equals to 0, because we are keeping, we are simply putting the this radial load at the free end, so obliviously you see it will give the maximum slope at the x equals to 0. Because, the slope at rigid end is 0 the slope is maximum at the free end, so by keeping x equals to 0 one can get easily the maximum slope or we can say the maximum deflection.

So, y maximum is nothing but equals to if we are keeping x equals to 0 you have minus W L cube by 3 I, which is a good value; that means, you see now, if we have the radial load is there at the free end. We can easily get the maximum deflection at the free end just by knowing the W means, what how much what is the amount of these things are there, and then L cube.

So, if you have the longer beam cantilever beam the again you see here the conditions are the cantilever beam. Obliviously more and more deflection are there and if, we have a very small beam you see at has 0 see after certain distance, we have the rigid end, obliviously you see the with the whatever the resistance is coming is always maximum. So, you see the y maximum; that means, the deflection maximum is always once can get by putting more amount of load one thing by unit like, E choosing the more length of beam.

So, that you see we have the more deflection, but it has the reciprocal relation with the E and I means, you see if we are more the Young’s modulus is there; that means, if the more stiff stiffened properties are there definitely you see the less deflections are coming and you see the moment of inertia also that at what kind of cross section, you are using corresponding changes are there in the deflection terms.

So, these are the influencing parameters for calculating the maximum deflection, so again you see the maximum deflection is nothing but equals to minus W L cube by 3 E I or also we can calculate the slope maximum, which is nothing but equals to W L square by 2 E I. So, you see here one can easily get those values by keeping the value x equals to 0, because we now that the slope and deflection they are maximum at the initial point where the cantilever is there the free end is there the slope and deflection at 0, at extremely those rigid point, where you see the rigid bonding is there in between the beam as well as the support, so now you see this was the case where the free end is experience by a point load.

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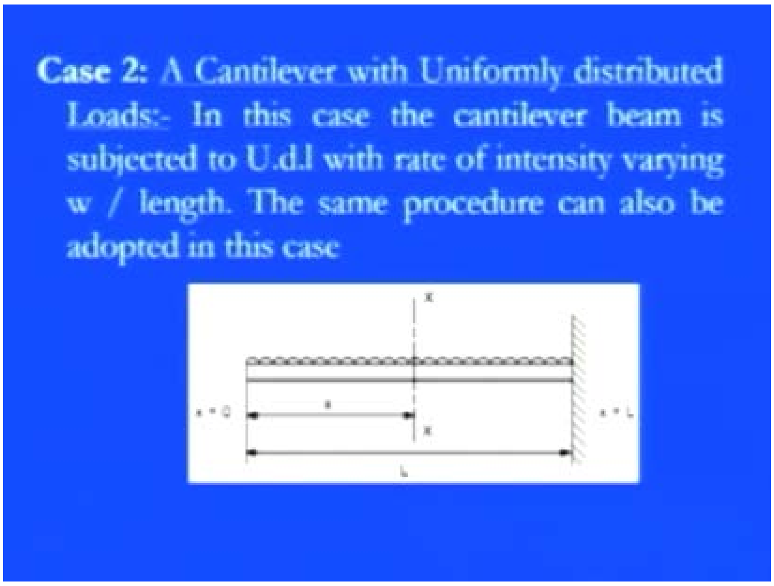

Now, you see the case 2 again you see we are going for a cantilever, which is experiencing by a uniformly distributed load. So, now you see here you can see on your screen the figure we have the UDL is there all across the entire span of the beam and then it is supported by one end, so in this case a cantilever beam is subjected by UDL, which has the rate of intensity of the loading W per unit length and the same process are again, we need to adopt for that again at x equals to 0, you see these all the conditions are there at x equals to L these conditions are there.

So, again we have to be very, very careful that at x equals to 0 the beginning point and at equals to L the end point, how the conditions are varying and then you see the total entire length is L again we need to cut the section at X x, which as a distance of x from the left end portion.

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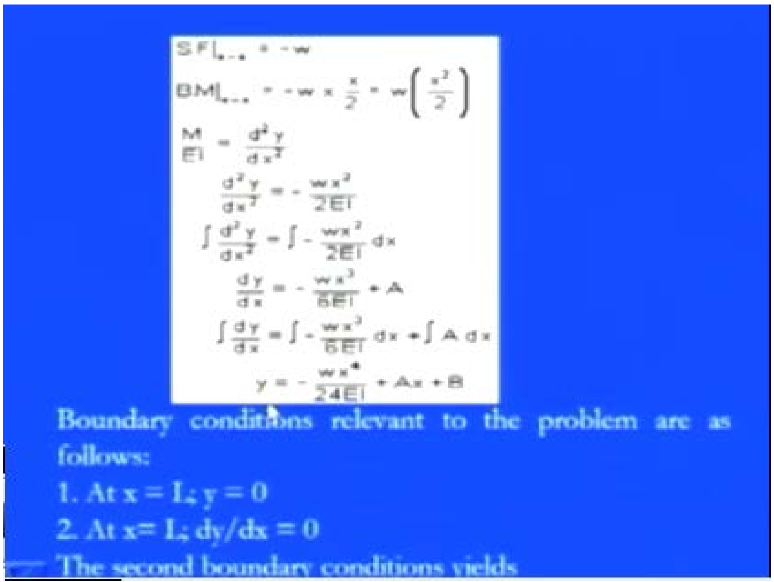

So, by taking those things again you see from this UDL, the loading condition is coming towards the down direction and you see at the support, which is exactly at the extreme corner of right the support reactions is coming on the top of direction, so with these direction again the shear force is negative and it is coming in this the hogging portion. So, again the bending moment is also the negative, as per the sign convention of shear force and the bending moment.

So, with that shear force is equals to minus W, bending moment is equals to minus W into x that is the load into x by 2, which is the pointing is there. So, we have minus W into x square by 2 by keeping those values in the basic equation of the deflection that is M by E I equals to d 2 y by d x square, we have d 2 y by d x square is equals to minus bending moment is W x square by 2, so minus W x square by 2 into E I.

So, again you see again the same processor, which we followed for the point load here for the UDL all UDL, also we have the slope d y by d x is nothing but equals to minus W x cube by 6 E I. Because, the square term is there, so x cube by 3 will come and says already to is there in the denominator side, so we have minus W x cube by 6 E I plus A.

So now, you see this is the one equation for the slope condition d y by d x is equals to minus W x cube by 6 E I plus A, again by integrating we have the slope this deflection equation. So, you see the integration d y by d x will give you integration minus W x cube by 6 six E I into d x plus A times of d x, so; obviously, we have the 2 it integrating coefficients A and B here. So, the final equation for a cantilever, which is experiencing by a load of UDL over the entire span of the beam and this cantilever beam is we have a free end we have a rigid end, so for that the deflection formula y equals to minus W x 4 by 24 E I plus A x plus B.

So know you see again, we need to follow the same condition here the slope and the deflection at the rigid point; that means, that x equals to L, which is always be equals to 0. So, know you see here the slope equation, which we have W x cube minus W x cube 6 E I plus A, if you are keeping at x equals to L, the value of A will be coming and you see we have the deflection term that is minus W x 4 by 24 E I plus Ax plus B. So, if we are keeping y equals to at x equals to L this d y by d x is also coming 0 and y equals to 0 will come. So, for those conditions we can get the value of A and B correspondingly.

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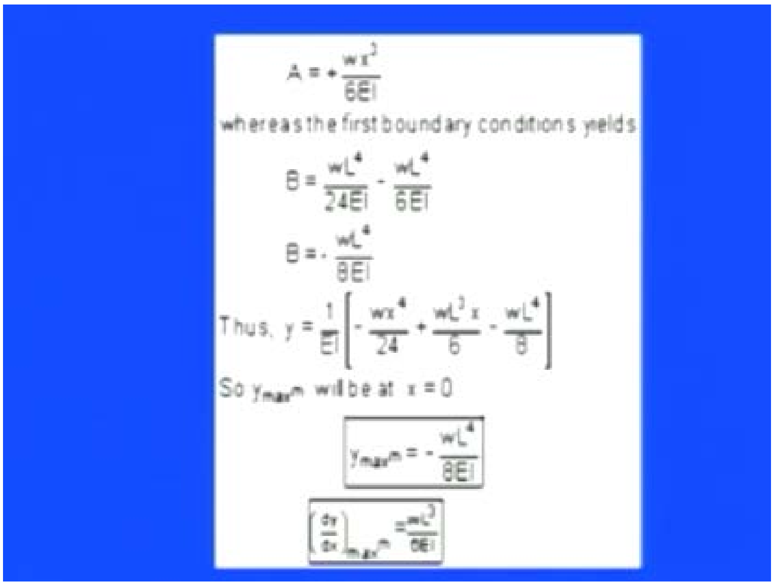

So you see here, we have for the slope condition where d y by d x is equals to 0 at x equals to L, if we keep those things here then we have the value of A, which is equals to W x cube by 6 E I. So, know you have the value of A by keeping the x equals to L, y equals to 0 by keeping those values in the deflection curve, then we have the value of B which is equals to W L 4 by 4 24 E I minus W L 4 by 6 E I from this A term.

So because, we are keeping x equals to L. So, you see here it will come as W L 4, so know you see B value will be minus W L 4 by it E I. So, know you have the value of A that is W x square by 6 E I, you have the value of B that is minus W L 4 by this 80 E I by keeping those values in the main equation of the slope, you have y equals to 1 by E I inside bracket minus W x 4 by 24 plus W L cube x by 6 plus minus W L 4 by 8.

So, now this is a generalized the equation for a cantilever beam, which is loaded by this UDL, which has the intensity of load, is W and you see the total length of the beam is L. So, for that you see we have the deflection term is this and as you know that the maximum deflection and the maximum slope is always occurs at the free and where x equals to 0, so we keep that value at x equals to 0.

We have the maximum deflection is y maximum is equals to minus W L 4 by 8 E I and we can also get the maximum slope where the x equals to 0; that means, the begging point is there which is also equals to W L cube by 6 E I. So, you see here for these cases also, we can easily evaluated that what will be the value, if the maximum and minimum values are there of the slope as well as the deflection for the cantilever beam when it is subjected by point load in the case 1 and when it is subjected by a cantilever this UDL in the case 2. So, now you see these 2 cases were discussed for the simple cases of the cantilever, so now you see the 3 case, if we now simply change the simply supported beam, so by taking a simply supported beam here the boundary conditions are altogether different.

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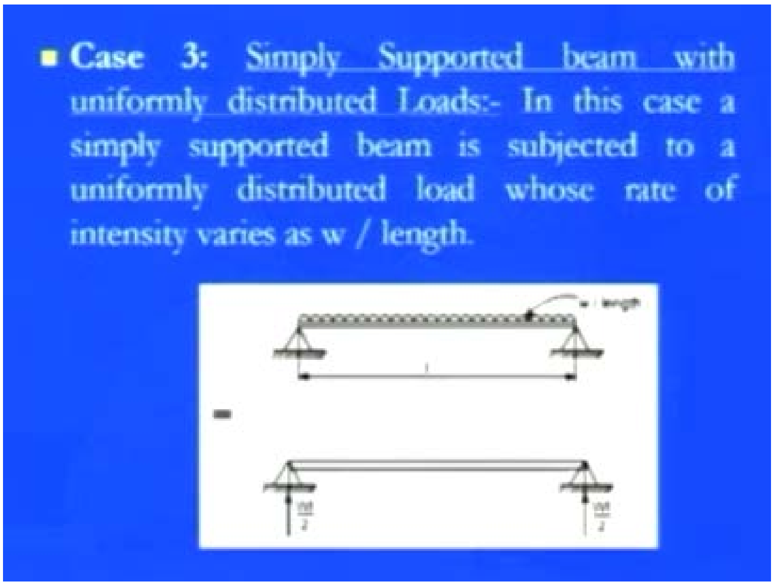

So, you see here just in the diagram in the front of you have a simply supported beam with the uniformly distributed load and in such a case simply supported beam is subject to UDL whose rate of intensity is W per unit length. So, here we have the total length is L and the intensity of this is W per unit length and you see the UDL is all across spread it out over the entire span of the beam and then you see we have the simply these supporting points are there. So, the reactions are coming all the way from these points, so know if we are replacing those things in the UDL is symmetrically distributed.

So, at the reaction are always be equal and they are coming from W by 2 W by 2, so you see we have W into L. So, this is you see w this is intensity is there, so if you multiply with the L we have the total load, so W L by 2 will be the reaction at this and W L by 2 will the reaction at these and they are coming throughout the load W into L and since, because the intensity is the W is there and the length is the L, so the total load is coming by the multiplication of these 2.

In order, to write down the expression for the bending moment consider any under any the cross section at the distance of x, again we need to go with the same procedure, which we followed for the cantilever beam cut the section by X x, which as the distance from 1 end the x. So, you see here this reaction force are W L by 2 you know and from this x distance, we are just our focused her for this kind of section is this 1 X x.

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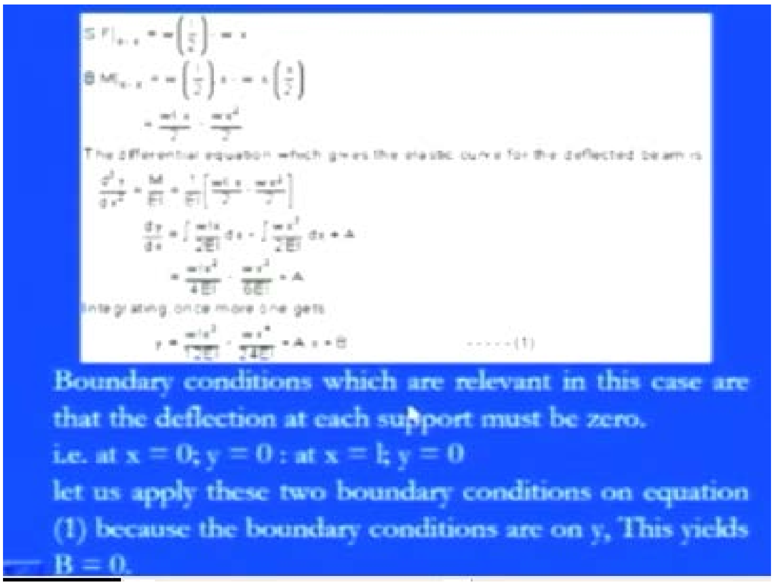

So, for these kind of sections again, we need to calculate the shear forces and the bending moment for these things, but here the sign conventions are different. Because, you see here the reactions from the simply supported beam at this extreme left corner is going in the top direction and the loading due to this UDL is coming in the downward direction, so for these kind of sign convention we have the positive the shear forces. So, shear forces at XX section is w into l by 2, because you see it is coming at the l by 2 minus W into x.

So since, it is the UDL is there, so obviously, it is always coming in terms of W into l by 2 minus W into x, because it is a intensity. So, now these forces are like that and then you see we have the bending moment while considering one end at the fix end, so you see the bending moment at XX is nothing but equals to W l by 2 into x minus W into x that is the load and it is concentrated at x by 2, so W into x into x by 2.

And since, you see a due to the simply supported beam it is again going into the sagging way like that, so again you see it is a as per the sign convention it comes as a positive way. So, either the shear shearing forces or the bending moment they are suppose to be positive as per the sign convention, which we have used, so bending moment at XX section is nothing but equals to W l into x divided by 2 minus W x square by 2. So, now, you see you have the bending moment go for the basic equation for the deflection that is M by E I equals to d 2 y d x square.

So, by keeping that value you have d y by d x square is equals to 1 by E I that is the constant 1 and M is nothing but equals to W l into x divided by 2 minus W x square by 2 and, now you see you can by integrating one can easily get the slope equation d y by d x, which is nothing but equals to integration of W x W into l into x divided 2 I 2 E I into d x minus integration of the W x square by 2 E I into d x plus A.

So, now we have you see since only the variation is the x is there and it is a integration of these things x into d x, so, d y by d x is nothing but equals to W l W into l into x square, divided by 4 E I minus W x cube by 6 E I plus A. So, now you see here we have a slope equation for that in the slope equation we have one constant term that is A, again by integrating we have the deflection equation, which carries the two different constants and one can easily get those constant, by putting the boundary conditions.

So, the by integrating again the slope equation, we have the deflection y for you see a simply supported beam carries the UDL W intensity and then you see for that the deflection y is nothing but equals to W into l x cube by 12 E I, because again by integrating these x square by 4 is there, so x cube by 3, so x cube into and since already 4 is then denominator. So, we have W l x cube by 12 E I minus you see W l minus W and x cube is there already, so W x 4 by 24 E I plus ax plus p.

So, this is you see the first equation, which gives you a clear cut description about the deflection that how it where is with the x. So, you see here now as we move write from x equals to 0 to x equals to L, we have a different boundary condition, so here boundary conditions which are relevant in this case for a simply supported beam and the two reaction forces are there at the extreme corner at x equals to 0 there is no slope and at x equals to l there is no slope.

As in the previous case, you see there was the maximum slope is there at the free and, but here you see both ends are equally supported and the reactions are also equal from for both of the end, so that is why you see there is no deflection is there at the extreme corners only the deflection comes in the middle portion, which will be the maximum part.

So, here you see by keeping by those things at x equals to 0, we have y equals 0 at x equals to l we have y equals to 0, now after applying those things we have if you just apply in the next equation, where x equals to 0 y equals to 0 you see here in all of the term in the deflection term, you see W l x W into x and A into x all those x terms are they are, so if you keep x equals to 0, we have B equals to 0. So, 1 co efficient is gone and then, you see by keeping at x equals to l y equals to 0 we have the value of A and this value of A is nothing but equals to W minus W L cube by 24 E I.

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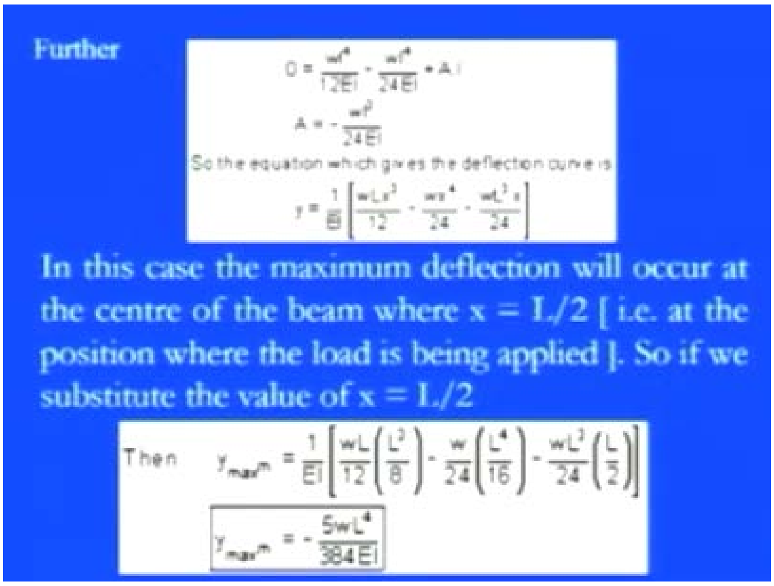

So, now you see you have the 2 co efficient value B, which is 0 A, which is minus w L cube 24 E I keeping those values in the main equation first, we have the final term equation for the deflection of a cantilever beam of a simply supported beam, which is carries a UDL, so y is the deflection for those kind of conditions, which is equals to 1 by E I W L x square by 12 minus W x 4 by 24 minus W L cube x by 24.

So, now you see you have a clear cut description that actually, if you go with the x right from x 2 L, then how what the variation is there in the deflection as well as the slope. So, in this case the maximum deflection will occur at the center of beam as I told you, so at if you are keeping at x equals to L by 2 you have y, which is equals to y maximum; that means, you see the position where the load is being applied at the middle portion.

So, if you substitute at x equals to L by 2, we have y equals to y maximum, so y maximum is nothing but equals to 1 by E I, now you have w L by 12, so x cube is L cube by yet minus W by 24 x to the power 4 is there, so L 4 by 16, because we are keeping at x equals to L by 2 and you see in the final term we have W L cube by 24. And since, x is there, so L by 2 is there, so by summing up all those things we have y maximum at the center point where x equals to L by 2 is there is nothing but equals to minus 5 times of w L 4 by 3 8 4 E I, which is very important formula that.

If we have a simply supported beam and it is carrying only UDL there is no additional force is there only UDL is there, which is spreader all across the span of the beam uniformly, then you see you have the deflection, which is maximum value at x equals to L and the value of these deflection is this 4 5 5 by 384 W L 4 divided by E I. So, one can get easily those values and you see one has to be very, very one has to be remember this formula, because this formula is very, very important for calculating the deflection for this kind of thing.

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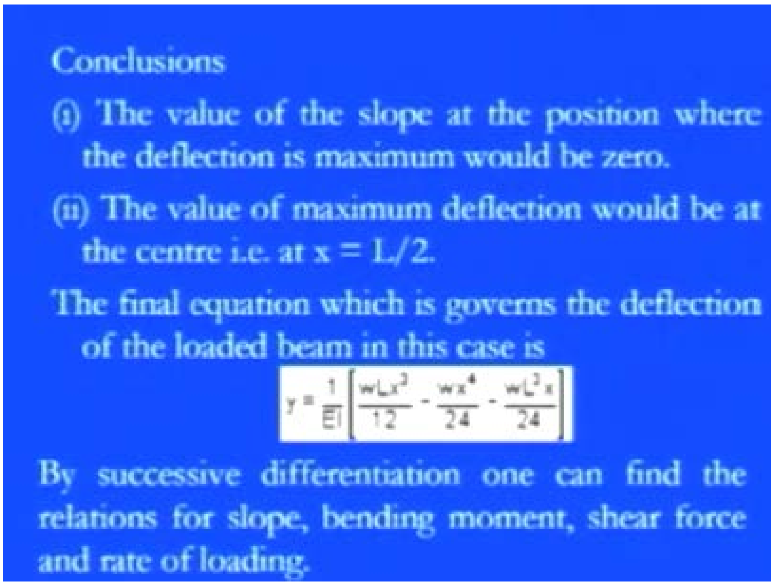

So, you see here know we would like to conclude those things the value of the slope at position where you see the deflection is maximum would be 0, means you see here. Now, if you want to calculate the value of slope, where you see the deflection is maximum always it gives you the 0 value; that means, you see at the center position where we have the maximum deflection the slope is 0, the value of the maximum deflection would be at always center as we discussed here that these things, so at x equals to L by 2.

The final equation, which governs the deflection of a loaded beam, just we discussed already that it is equals to 1 by E I W L x square x cube by 12 minus W x to the power of 4 divided by 24 minus W L square L cube into x divided by 24. By succession differentiation one can find the relation for the slope bending moment and the shear force and the rate of loading as we discussed in the first section of that that how to co relate those things, so here also for a simply supported beam under this UDL 1 can be 1 can easily evaluated those things for these cases.

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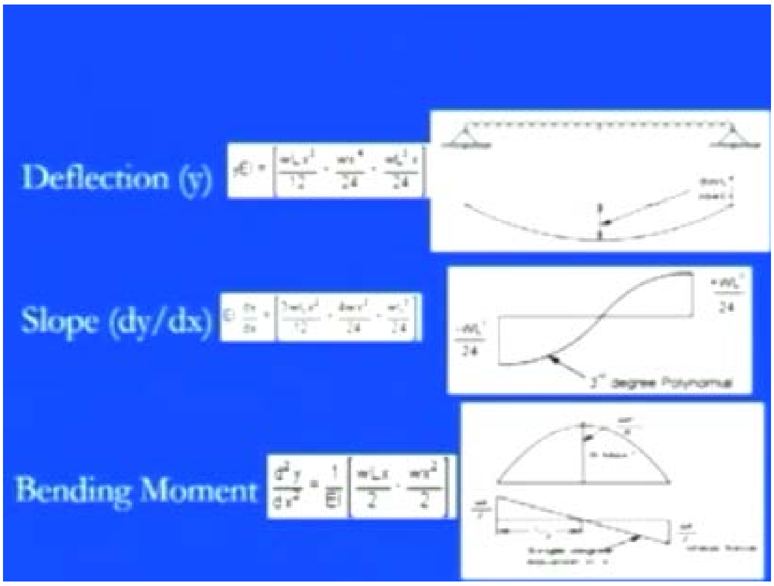

Now, you see here again for a different resents you can go for this here that, we have a simply supported beam again the same, which we discussed UDL is there, so the deflection term as we discussed here is nothing but equals to y into know, since it was it was 1 by E I, so multiply this, so y into E I is nothing but equals to W L x cube by 12 minus W x 4 by 24 minus W L cube x into divided by 24.

So, this is there, so here you can you can have a clear feeling that since it is a UDL and you see at these 2 points there is no the deflections, so deflection is 0 at the 2 extreme corners at x equals to 0 and x equals to L. But the maximum deflection is there at the center position where x equals to L by 2, so you see here and since it the deflection was minus this 5 by 84 W L 4 by E I. So, you can see here the deflection is going in this way and here you see since it is just like that, so we have you see these maximum deflection and it is the value of this.

And then, you see if you are going for the slope conditions d y by d x, then it is nothing but equals to E I into d y by d x is nothing but equals to 3 W into L x square by 12 minus 4 into W x cube by 24 and minus W L cube by 24. So, with those conditions in the sloping conditions, now you see if you are keeping those values there, then you see here what we have here in this particular situation the slopes are always as I told you the previous section, the slope is always 0 where of the maximum deflection is there and maximum deflection we are getting at x equals to L by 2.

So, you can see here this is the kind of these convention is there from positive to negative, so at this particular point where the maximum slope is maximum deflection is there the slope is 0. And since, you can see here in these terms what we have the square into length this is already cubic and this is cubic. So, we have a third degree polynomial the curvature path is there for this particular slope, so though this slope is very small, but it has the third degree of polynomial in this curvature.

So, we have you see at this particular corner we have the minus W L cube by 24 where the x equals to 0 and at x equals to value see we have plus W L cube by 24. So, you see here minus 2 positive see it very is in this particular way and you see it has a characteristic of the third degree of polynomial and it always passing from a 0 slope where the maximum deflection is there, so it is exactly these things and then you see again we can go for the bending moment and the shear forces, which we discussed many times.

So, you see here again by the this differentiating these things what we have d 2 y by d x square is nothing but equals to 1 by E I W L x by 2 minus W x square by 2. So, you can see that the bending moment variation is exactly similar to the deflection, because at these two extreme corners of the simply supported beam there is, now the bending moments are there, but it has a maximum value where you see, because of the UDL we already evaluated and you can see this particular the figure also corresponding to the equations, we have the maximum value of bending moment at the center point, which is equals to W L square by 8.

So, know you see with those bending moment diagram also you can again visualize that shear forces, which is in these condition it is positive in this reason and it is negative in the other reason and the right end direction. So, you see that we have the W L by 2 in the positive reason and it is minus W L y 2 in another resents, so it is passing from those things.

So, again we have a very clear cut reason that you see these equations are almost equal to those conditions; that means, you see when it is a simply supported beam is there and it is loaded by a uniformly distributed load, so we can easily evaluated those deflection where is the maximum deflection is there where is the minimum deflection is there with that, you see we can calculated the slope conditions, that actually how the slope varies with that kind of deflection, then we can also find it out the bending moment as well as shear force those things.

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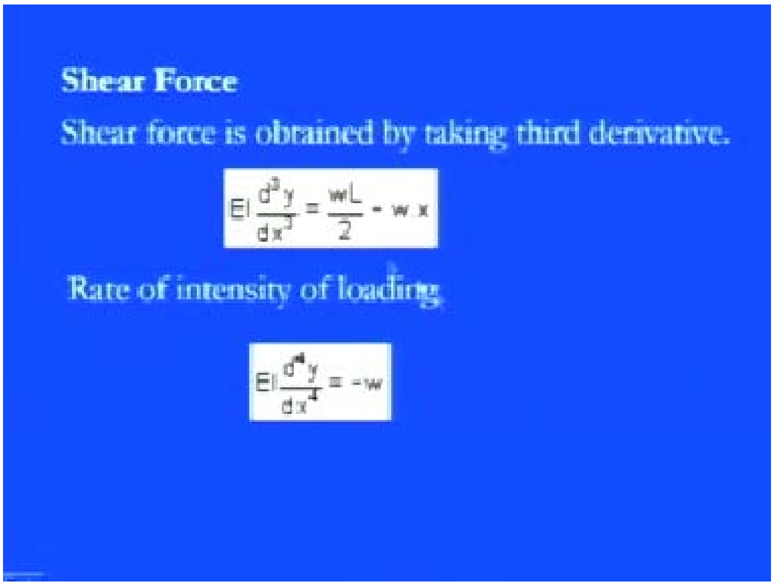

So, shear force is simply obtaining as we discussed that E I into d cube by d y d x cube will give you the F and we, which is equals to W L 2 minus W x. So, now you see again a d f by d x you will the rate of loading will discuss in that, so again the rate of intensity of loading is nothing but equals to E I into d 4 y by d x 4, which is equals to minus W. So, with that you see one can easily obtain those conditions, which are valid for either a simply supported beam with the UDL are what.

So, now you see here the last case will the last case, which we want to discuss that if we have cantilever the simply supported beam not UDL is there a point load is there, but the point load is not exactly at the mid of the a span of this particular beam. It is somewhere else, you see the eccentric loading in not eccentric it is somewhere else you see this kind of loading is there.

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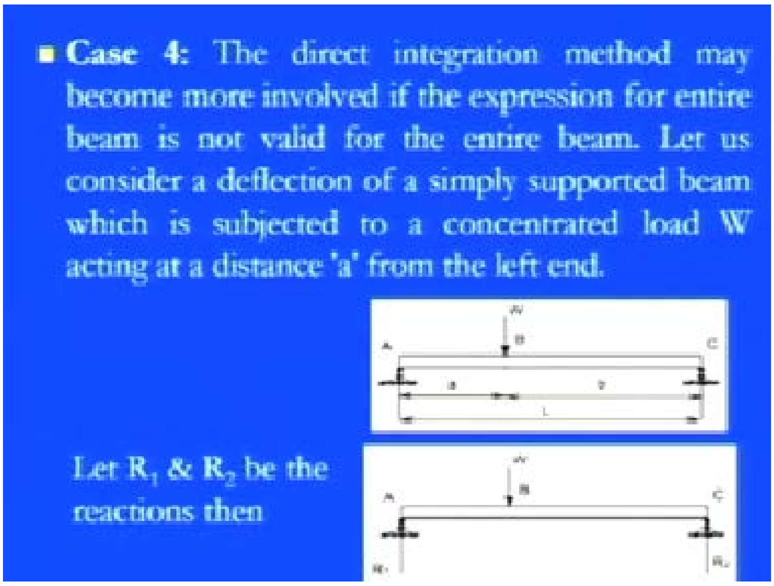

So, again we would like to apply the direct integration method, if the expression is entire of beam is not valid for the total beam; that means, you see whatever the expressions, which are coming it is not symmetrical just like you see in the previous 3 cases, we have seen that cut the x section and then go for the entire at x equals to 0 to x equal to L.

So, that part we discussed those things, but now here you see in this condition in, which you can see on your screen that we have simply supported beam, but the point load W is acting at the portion a; that means, you see portion A and portion B or not equal. So, you see here for this we would like to calculate the deflection for this kind of beam where this kind of loading is there at distance a from the left end.

So, you can see here first of all we would like to calculate the reaction forces at these to extreme corner and if we I am saying the that the reactions are R 1 R 2 at these 2 points and we have the point load at the point B, A and C are simply, you know the extreme corners and the a bearing these reaction forces on the top of direction and the some portion from the left end side at A, we have it is this point B, which is experiencing by you this concentrated load W.

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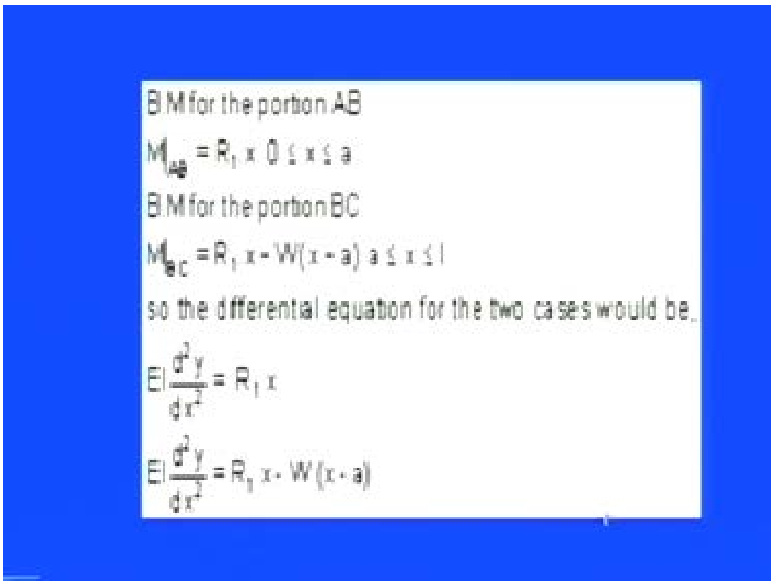

So, this is a complete configuration of this particular problem and now you see if you want to found it out the bending moment for the portion AB; that means, you see the begin portion you see right from left two this portion where the W is there M AB is nothing but equals to R 1 in to x for the reason 0, which is less then equals to x is less, then equals to A.

And know you see, if we are going for the another portion that is B to C the bending moment is somewhat different because you see here the both W is also putting some effort in that, because till now you see only the reaction forces are there and due to that the bending moment is coming that is why it is equals to R1 into x and for the this BC portion we have the bending moment is equals to R 1 into x minus W into x minus A. Because, you see now the middle portion this point load is there, so by keeping this remain, now we have this particular portion, which is B to C is there, so W into x minus a and this is valid for 0, which is less then equals to x is less than equals to l.

So, now you see you have bending moment for both of the reason, so now for we need to calculate again this deflection equation and the bending moment equations from though from those conditions. So, for that again we would like to go for the basic equation of the deflection, so we have the E I into d y by d x square, which is equals to R 1 x for AB portion and E I into d y by d x square, which is equals to R 1 x minuses W into x minus a for BC a reason. So, now you see you have the two different cases and the two different equations are there for the bending moment of these things.

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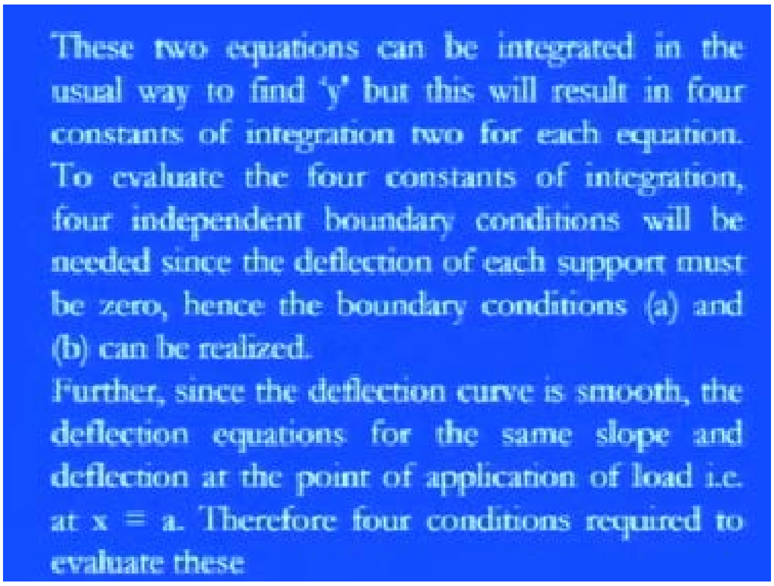

These 2 equations can be integrated in a usual way to finding out the deflection, but this will result in a 4 constant of the integration; obviously, you see in a first equation you have a and b in another equation, you have C and D. So, again you see the 4 constant of integrations have to be evaluated for these things to calculate the deflection curve, and to evaluate the 4 constant this the constants of the integration 4 independent boundary conditions are to be there actually which we always needed to be there at each support.

So, you see here for condition a or for condition b one has to be realize that actually there are 4 four different conditions and for that we need 4 different boundary conditions for this cases. So, which is not an easy task for this further since the deflection curve is smooth the deflection equation for us this same slope and the same deflection at a point of application of load; that means, at x equals to a it has to be same; that means, you see if we are talking about a smooth curve or a symmetric reason this deflection as well as the slope has to be having a constant value and they have to be same.

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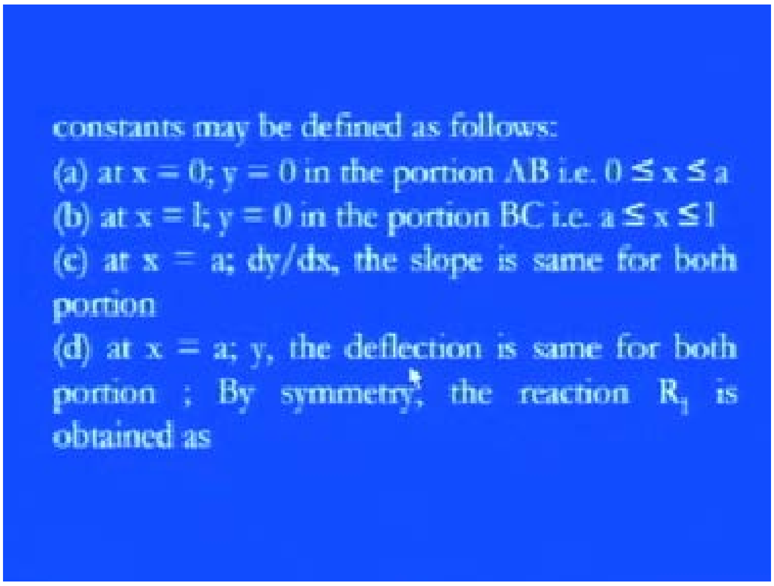

Therefore, the four condition required to evaluated this coefficients are to be derived one and then you see we can simply go for these a constant one is like that you see at x equals to 0; that means, the begin point we have no deflection, because it is a simply supported beam. So, there is no deflection is there, but this is for portion A or B; that means, you see for portion AB we have one boundary condition at x equals to 0 there is no deflection, so it is valid for 0, which is less than equals to x is less then equals to a portion.

At x equals to l, now we are going in the BC reason y equals to 0; that means, you see at the another reason there is no deflection for the portion BC; obviously, and which is a is less than equals to less is less than equals to x is less than equals to l. The fourth condition is at x equals to a where you see the point load is there the d y by d x is same for both of the portion, because you see we are keeping one point load at this particular portion.

So, slope must be equal 4 equal to the both of the portion; that means, for AB and for BC the slope must be equal and the final thing is that at x equals to a, the deflection is also same for both of the portion; that means, you see by symmetry city one can be easily get you see the slope as well as deflection part for both of the reason and then, it can be evaluated, because both are equal in the both portion.

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$$R_1 = \frac{Wb}{a+b}$$

Hence,

$$EI\frac{d^2y}{dx^2} = \frac{Wb}{(a+b)}x \qquad 0 \le x \le a \text{ --------(1)}$$

$$EI\frac{d^2y}{dx^2} = \frac{Wb}{(a+b)}x - W(x-a) \qquad a \le x \le l \text{ ---------(2)}$$

integrating (1) and (2) we get,

$$EI\frac{dy}{dx} = \frac{Wb}{2(a+b)}x^2 + k_1 \qquad 0 \le x \le a \text{ --------(3)}$$

$$EI\frac{dy}{dx} = \frac{Wb}{2(a+b)}x^2 - \frac{W(x-a)^2}{2} + k_2 \qquad a \le x \le l \text{ --------(4)}$$

So, now you see here first of all since you see the reactions are there of R 1 and R 2 and since it is coming from both of the this supporting points due to this the load conditions, so R 1 can be easily calculated by putting the moment equation and the force equation 0. So, you see here since because under the action of this load the beam is in the symmetrical way; that means, it is in the constant way.

So, R 1 can be easily calculated with that, so R 1 is equals to W b by a plus b and since you know the R1. So, you see here for a b portion means for the first portion E I into d y by d x square is equals to W b by a b that is this R 1 into x is there and for BC sections; that means, for the another section we have E I d 2 by d x square is W b by a plus b into x minus W into this x minus a; that means, you see unit to simply replace this are R 1 which is equals W b by a plus b.

So, now you see here the 2 equations integrate those things to calculate the slope, so we have the slop E I into d y by d x is nothing but equals to W b divided by 2, a plus b into x square plus K 1, know is we are taking K 1 is a constant and for K 2 is the different constant for the slop for BC reason. So, E I into d y by d x for BC reason is equals to W b by 2, a plus b x square minus W into x minus a whole square by 2 plus K 2.

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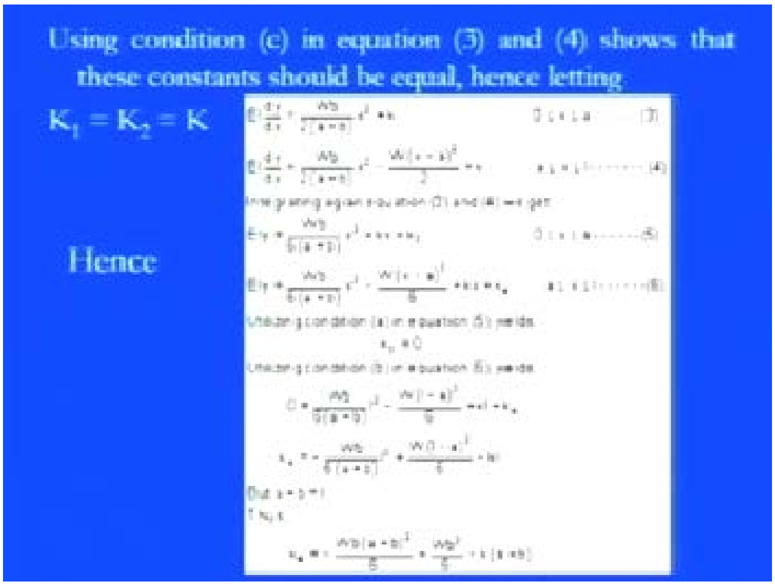

So, now you see here we have the two different constants K 1 and K 2 and now we would like to put the conditions here that the boundary conditions to evaluate the other things. So, for these things just we have K 1 and K 2, which is equals to a if I am saying that this constant, so now if we are keeping those things what we have E I into d y by d x is equals to W b divided by 2 into a plus b x square plus Kb.

Because you see we are simply replacing those the constant, so we have K is in place of K 1 we have the K and now for another reason. So, this equation first which is E I into d y by d x, which is equals to W b by 2 a plus b x square plus K is valid for AB reason, now for BC reason again we can go for the slope condition E I into d y by d x, which is equals to W b by 2 a plus b x square minus W into x minus a whole square divided by 2 plus K.

So, now you see here you have the two different conditions and you have the two different equation the differential equations are they are and know, you see by integrating those things we can get the slope conditions for a portion and for B portion, because you see as we just got to know that actually, if you apply the load at a particular position the slope will be equal for both of the sections the deflection is equal for both of the section. So, E I into y for AB portion; that means, the this deflection condition is W b by these 6 a plus b into x cube plus K x plus K 2, now you see here we have the new term that is K 3 is there, because of the integration term.

And then for BC portion we have the slope condition E I into y, which is equals to W b by 6 a plus b x cube plus this x cube is there, then minus w into x minus a whole cube divided by 6 plus K x plus K 4. So, know you see here we have the different conditions and now if we use the condition, which we have used for AB portion this is AB portion for this one you see and this is for BC portion, then you see with using those conditions in equation a, which we with using those conditions in equation a, which we have put those things we have K 2 is K 3 is equals to 0 and by keeping K 3 is equals to 0 in this condition here.

Because at what will be the deflection the deflection at the this reaction point is 0 as I told you that since it is the point load is there, so there is no deflection at the supporting reaction. So, at x equals to 0 we have there is no deflection, so you see K 3 will be 0, because if we are keeping x equals to 0 this x this x will be gone and this y will be gone, so K 3 is 0.

So, now we have the K 3 0, so by keeping those things what we have here in this equation, we have the W b divided by 6 times of a plus b L cube minus W times of L minus b whole cube this W minus a whole cube divided by 6 plus K times of L plus K 4. So, now you see here at x equals to initially we put at x equals to 0 y equals to 0 in the BC term we have x equals to L y equals to 0, so by keeping those things what we have we have K 4 value and this value is nothing but equals to minus W b divided by 6 times of a plus b L square plus W times of L minus a whole square divided by 6 minus K times of L.

So, now you see you have all the values of constant and now you see if we know that the this total a length of beam is L which is divided into two portion a and b, so a plus b equals to L. So, now by keeping here you see L minus a, so this a plus b equals to L, so L minus a will give you b, so here we can simply keep this b and you see we have what we have we have K 4 is equals to minus W b this one divided by 6 into a plus b whole square, because you see here L is nothing but equals to a plus b whole square.

So, it will be cutting out those things plus W into this L minus a, so L minus a will be coming as you see b square, so you see here the b cube divided by 6 minus K times of a plus b. So, what we have you see this cubic term will be gone by one way, so we have W b a plus b whole square by 6 plus you see W b square by this 6 minus K into a plus b, so all these you see in this particular co efficient all these terms are coming in terms of a and b, so we can simply segregate this beam in to the different sections of a and b as compared to the total length.

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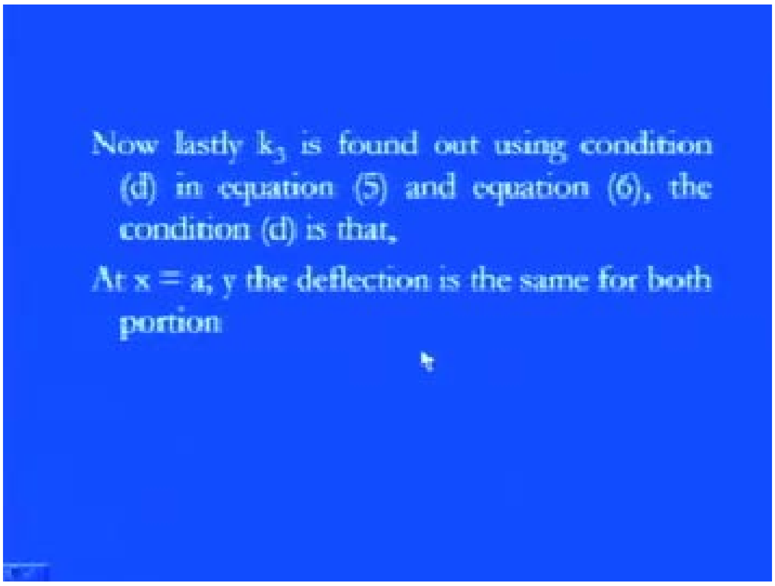

So, now you see here this K 3 can be easily found it out with the using of these conditions in the of this those, which we recently got if we keep this K 3 into the different equations. And then we can get the deflection which is same for both of the portion at x equals to a.

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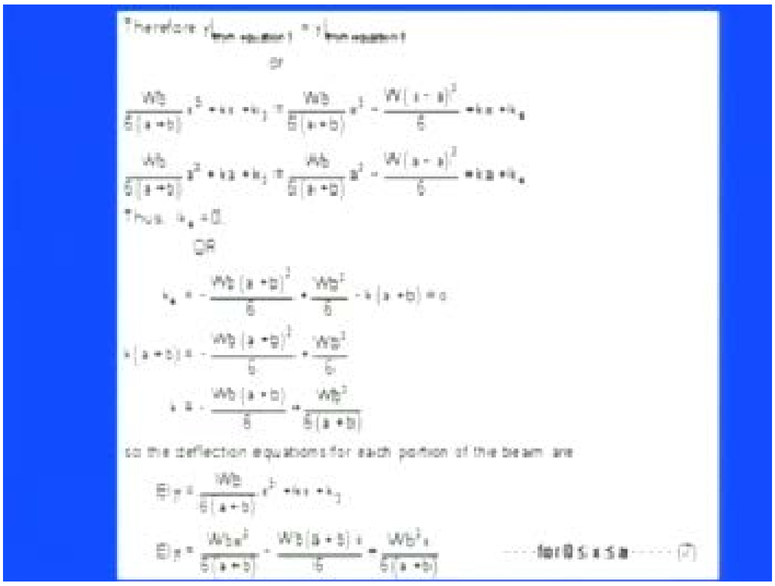

So, now if we are keeping those conditions what we have we have the deflection, which is same for the equation 5, which is exactly deflection is same for the equation 6; that means, for BC portion and for this a b portion we have the same deflection. So, now, by equating both of the this deflection term, which we obtain in the previous section you see W b by 6 a plus b into the x cube plus k x plus K 3, which is equals to W b divided by 6 into a plus b into x cube minus W x minus a whole cube divided by 6 plus k x plus K 4.

So, this is this deflection term is there for AB portion this deflection term is there for BC portion, now as per this simplicity we know that the deflection is same. So, by equating those things what we have K 4 is equals to 0 or we can say that K 4 if we are keeping those values there, then we have this is exactly equals to 0 W b plus a plus b whole square by 6 plus W b cube, which we obtained recently previous case by 6 minus K times of a plus b equals to 0.

So, we can get the value of K because K 4 we previously we observed those things from these equating we got that K 4 0. So, now you see here we can get the value by keeping those things equals to 0 we can get the value of K the K is equal to minus W b into a plus b by 6 plus W b cube divided by a 6 into a plus b, so now you see you have the constant value K, which we absorbed these things.

So, now, you have all those values for the constants by keeping those things what we have, you have the different equations for the deflection term E I into y for this first section W b divided by 6 into a plus b x cube plus k x plus K 3 and you see by keeping those values, what we have the final equation for AB portion deflection equation E I into y, which is equals to W b x cube divided by 6 into a plus b minus W b into a plus b into x divided by 6 plus W b cube x divided by 6 into a plus b.

So, this is a valid equation it is varying with the a and b and that, actually how the different loading conditions are there for a BC portion and AB portion for this kind of reason. And similar, you see we can get for a BC portion E I into y is equals to W b x cube divided by 6 into a plus b minus w into x minus a into whole cube divided by 6 minus W b into a plus b into x divided by 6 plus W b cube x into divided by this 6 times of a plus b, so this for BC portion. So, now, you see here we have a AB portion we have a BC portion and if we are keeping at x equals to a we have the maximum these deflection term is there and which is equals to W times of a square b square divided by 3 times of E I into a plus b.

So, you see here what we have a straight equation that if we do not exactly the this loading condition at the middle of the portion, then we can easily evaluate you see that the load conditions and due to the load condition you see how the deflections are varying and the slopes are varying you see we can easily get those values. So, here we have the maximum deflection where the x equals to a; that means, where the application is there and which is equals to this. Now, if we are keeping that if a and b are equal; that means, if the load application is there exactly at the middle portion; that means, at L by 2, then we have the W L cube by 480 I and which is the pretty simply case is there.

So, just keep in this keep in your mind that if we have a simply supported beam and the point load is acting if this point load is acting at exactly at the middle portion, then we have the deflection maximum deflection at the middle portion, which is equals to and it is going in the downward direction, so minus sign is there minus W L cube by 48 and if it is not exactly this load is acting on the middle portion if it is acting some where you see in this particular beam, let us say at a distance is there form this left end side as we discuss the previous case. So, we have the deflection at that point maximum deflection and that is equals to minus W a square b square divided by 3 times of E I into a plus b.

So, how these variations are there we can easily we could configrate those things. So, now, we would like to conclude these this chapter that we started from the main deflection formula that if you know the bending moment M, then you can calculate 1 by E I into d 2 y by d x square. So, with that you see we configured that we can easily get the slop as well as the bending moment shear force and the rate of loading and then we discussed the 4 different cases in that the 2 cases were discussed for the cantilever beam that if it is subjected by a point load and the UDL.

And then you see the 2 different cases were discusses for the simply supported beam that if it is you see the UDL is there and if it is the point load is there, but not exactly at the middle portion or somewhere else. Then, how we can get the deflection what are the boundary condition which has to be taken and then where we can get the maximum deflection as well as the maximum slope for this cases.

So, these you see of the four different cases which were very important and which are practical which have the very practical applications for this kind of the cases. So, now you see in the next this chapter, we would like to discuss something about the first the alternate method. Because you see sometimes you see here when there is an eccentric loading is there, then it is not simply the simple method the direct integration method. So, what are the alternative methods are there for this kind of analysis first and second you see we would like to analyze with the using of some examples. So, in the next lecture we are going to discuss all those uses.

Thank you.