Strength of Materials
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Lecture – 30

This is Dr. S. P. Harsha from Mechanical and Industrial Engineering Department, IIT Roorkee. I am going to deliver my lecture 30 of the course of the Strength of Materials, which is developed under the National Programmers on Technological Enhanced Learning.

Prior to start this course, because in this in this lecture, we are going to discuss about the ((Refer Time: 00:44)) problems based on the shearing stresses, I would like and we are also going to use the same formula which we derived in the previous lecture. So, we better you know like discuss first that what we discussed in previous lecture? We discussed mainly about you know like the shearing stresses, because you see in the previous lectures, we discussed mainly about the bending stresses pure bending is there.

And then we found that in this any beam which is under the kind of loading irrespective, whether it is a cantilever beam or a simply supported beam. Then it is also influencing by the shearing stresses, and then we derived some of the formulas for the shear stresses like tau equals to Fa y bar divided by IZ.

So in the previous lectures, we derived those things and the in previous lecture you see our main focused was on that if we have a different cross section of a beam. Then how we can you know like that the different, you know like the kind of shearing stresses at different sections particular. So, we started with that if we have a cross section of beam is rectangular, then we discussed that how we can get first of all the formula, that what the tau is there.

Then we found that the tau is where the tau means the shearing stresses variation is all was in among the parabolic distribution. And then we also concluded that the maximum the shear stresses occurs only at the neutral axis. And then they are just, when we are going approaching towards the extreme end, then it is 0.

So, we found that if we have a rectangular section, then the shearing stresses are starting from 0 from the extreme ends to the maximum at the middle end and this kind of slope was there. And then we also concluded that the tau maximum is always 3 by 2 times of tau average; that means, if you want to calculate the average stresses also, the average shear stresses. Then we can calculate, if the maximum stress or if this average stress, then you can calculate the maximum stresses by the 1 by 1.5 times tau average.

So, this was the first section which we discussed, then we took the different section about that, if we have a I section in which 2 flanges are there, and 1 web is there in between. Then we also derived the similar kind of formula and we found that maximum shearing stresses are always there on the neutral axis and 95 percent of the shearing stresses are being carried out by this web; that means only 5 percent shear stress distribution is there with this flanges.

So that is, what we concluded that, if we have the kind of beam is there, and if we are designing the beam, we have on our roofs and all that that kind of application. Then we always design the beam with the I section for bending stresses; that means, if we if our beam is influenced by maximum bending stresses, then it is always better to use I section, because it gives the mass moment of inertia minimum and that to also, they are always rigid with the flanges and they can easily bear out those bending stresses.

Because, the most of the shearing part is there with the web only means the middle portion. So, outer part is pretty with those kinds of application, and then the last section, we discussed about the cross section if we have the circular cross section. In that, we observed that it is a similar kind of the pattern is there, as far as the shear distribution is there.

Again, it showing the parabolic distribution, which has the maximum value at this neutral axis, and then at extreme corner; that means, at minus R to plus R, it has the 0 values. So, if we want to describe those shear stress values, then it is starting from this minus R to plus R 0 and it has a maximum value at this neutral axis.

And then again, the similar kind of pattern is there, that how we can describe those things about the bending and the shearing stresses, as far as this different cross sections are. So, this was pretty the important discussion in those things that if we have the different cross sections, then how we can get the shear distribution.

And, the other another thing is very important that actually how we can find it out that which area is potentially having the maximum shear stresses or minimum shear stresses. So that if, we want to design a beam for shear stresses, then we could easily get it those things that this area is influence by maximum shear stresses. So, just put more factors of safety or whatever as far as the design criteria's are concerned, so that we discussed.

So in this particular lecture, now we would like to put all those formula which we derived in the previous section, and uses we just want to solve some of the numerical problems based on those things. So, here it is the first illustrative examples here, we are taking some of the illustrative examples pertaining to determination of the principle stresses in that, because in the last section we described that if a section, if a beam is there which is under the action of bending stresses and shear stresses.

Because, bending stresses are always along the longitudinal 1 and those stresses are basically, these normal stresses or we can say the flexural stresses, so it is a just along these this X axis. And then we if the same time if the shear stresses are there; that means, we have a section in which this axial pulling is there or the kind of axial stresses are there.

And, then we have the shear stresses 2 we can go for the principle stresses and the principle stresses sigma 1 comma sigma 2 which we derived in the previous section. The last part of the previous section was nothing but equals to sigma X plus sigma y by 2 and since it is a sigma y 0. So, sigma X by 2 plus minus square root of sigma X plus sigma X minus sigma y by 2 whole square plus 4 times of tau square xy.

So, we know that a if we have a rectangular cross section or if we have a any kind of cross section we can easily get it those the bending stresses which is nothing but equals to M by y M by I into y. So, use this y is the distance is there in between the neutral axis and the other the flair or we can say that kind of whatever the layers are there on which we just want to find it out the bending stress.

So, through which and I is nothing but the cross section what cross section which we are using like if a for an example, if we are using that rectangular cross section then we can easily find it out the I by b d q by 12. So, with those this bending stresses and the tau is nothing but equals to F a y bar divided by IZ based on which the cross section which you are using rather we are using that rectangular 1 or the I section or the circular one.

So, based on that we can simply put those formula and we can get final the sigma 1 comma sigma 2 by with the using of this particular formula sigma X by 2 plus minus square root of sigma X. So, sigma square by 4 , sigma X square by 4 plus 4 times of tau square xy.

So, with the using of this formula 1 can getting easily the principle stresses and also the same time 1 can easily get the location that at what location these principle stresses are occurring; that means, what is the location of the principle planes? So, that was tan 2 theta X which is equals to the 2 times of tau xy divided by sigma X minus sigma y, so sigma y is 0. So, it is pretty easy by keeping sigma X; that means, the bending stresses.

So, with the configuration of bending stresses and shearing stresses, 1 can easily get the principle stresses as well as the principle planes all together. So now, whatever we discussed in the previous lecture about the different cross sections for shearing stresses and the bending stresses and for principle stresses for different cross sections.

We in this particular lecture, our first focus is to describe those things with the numerical problems with the certain this illustrative examples are there. So, here it the first example is again based on that we have we just want to find it out the principle stresses at a certain location at point A in this uniform rectangular beam.

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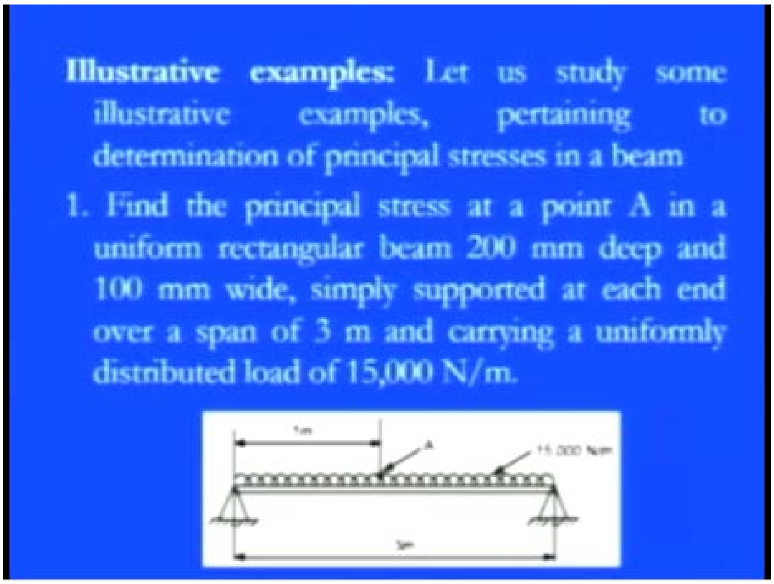

Here on your screen, we have an diagram which is a simply supported beam and or the entire span of this simply supported beam we have with the UDL the uniformly distributed load. So in that, we have the intensity of that load, the same time we just want to find it out the principle stresses at a particular location. And that point A, you can see on your diagram that at this particular point A, which is 1 meter apart from the left end this portion.

So, at this particular point we would like to find it out that what will be the principle stresses, and in that we have certain dimensional parameter like this is the uniformly rectangular beam. So, now our cross section is uniformly rectangular beam in which the 200 millimeter deep is there and 100 millimeter wide.

So, the depth and the the width is given for the rectangular beam, so from that cross section we can easily get the dimensional parameter that what we need particular for calculating the bending stresses as well as the shearing stresses.And, then it is a simply supported beam, so you can see in this particular figure that this simply supported beam of 3 meter length is there and it is carrying a uniformly distributed load with the intensity of 15000 Newton per Newton per meter.

So now, what we have we have the dimensional parameter of the cross section that is the cross section is first the rectangular cross section and the dimensional parameter are 200 the depth and 100 millimeter plus, we have the entire a span of this simply supported beam is 3 meter and all 3 meter this UDL is distributed which has the intensity of this is 15000 Newton per meter.

So now, we would like to find it out the principle stresses, since it is a simply supported beam and UDL is there. So, it carries both of the kind of stresses like bending stresses as well as the shearing stresses. So, first of all since it is a 15000 Newton per meter in and this 15000 Newton per meter intensity is spread it over the 3 meter, so total load is 45000 Newton. Because, we need to multiply with the 3 meter, so we have 45000 meter. And then it is simply just there at the middle of this span a middle of this particular beam. So, the total span is 3 meter at the middle section we have 45000 Newton load is there.

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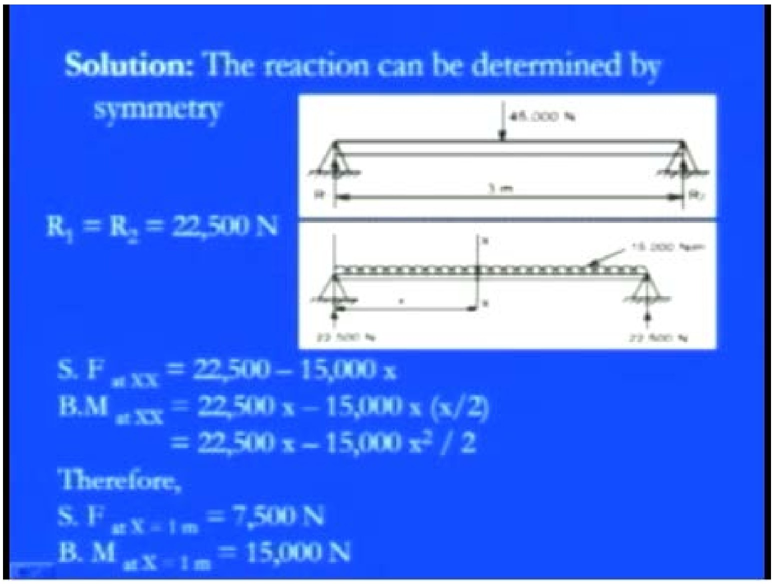

So, configure this particular problem in this way or we can say simple, just take the section first of all the first we need to balance the forces as well as the moment and with those things we can easily configured our reactions. And, the reactions are at the particular left hand section we have the reaction R 1 and the right hand section we have the reaction R 2.So, we can we can easily calculate the reaction by simply equating the forces as well as the moment because under these actions this beam is well balanced.

So, we have like the R 1 and R 2 values, because it is exactly at the mid span and the constant values there 45000 Newton, so both are equal, because both are resisting by this kind of load. So, we have the uniform load distribution and it is symmetry from both of the side because it is at the exactly middle. So, we have the similar value. So, R 1 and R 2 has to be equal value and it is equals to 22,500 Newton with that particular.

Now, our main intention is to calculate the shear force as well as the bending moment. So, shear force, we again we cut the portion of this beam form XX. So, once we cut the portion now, when we are starting from the left we know that from this the reaction force 22,500 Newton is just going on upward direction and it is upward and this is on downward.

So, we have the positive value, so in shear force, we better start from the positive direction, because it is going in this direction, so we have shear force at XX section is 22,500 minus. Now the UDL is there, so UDL intensity is 1500 like 15000 Newton per this meter. So, we need to multiply with the X, so the total configuration at XX section due to this UDL about the shear forces are 22,500 minus 15000 into X.

And now, if you focused on the bending moment, then bending moment is again means a kind of similar it is due to the sagging. So, it is a positive direction, so we have the 22000 into X, minus 15000 into X, this is force and it is located the middle of this X by 2. So, X by 2, so we have 22500, this is R 1 into X, minus 15000 X square by 2. So now here, we have a generalized the distribution of the shear force as well as the bending moment all across this entire span of the beam. Because, now you can vary the X right from X equals to 0 to X equals to 1 or X equals to 2.

So that, you can get the distribution of shear force as well as the bending moment in entire this span of the beam. So here, if and we would like to first calculate the principle stresses only at point A which is a 1 meter from the left end, so here we need to keep the value of X which is equals to 1 meter.

So, shear forces at X equals to 1 is if we are keeping those things at 22500 minus 15000 into 1, so we have 75000 Newton. Now, we have 75000 Newton is a shear force at X equals to 1; that means, that point A and the similar the bending moment for that e at X equals to 1. So, if we are keeping those things 22500 into 1 minus 15000 1 by 2, so here we have 15000 doubled of the shear forces. So, now, we have shear force as well as the bending moment at point A where we want to calculate the principle stresses.

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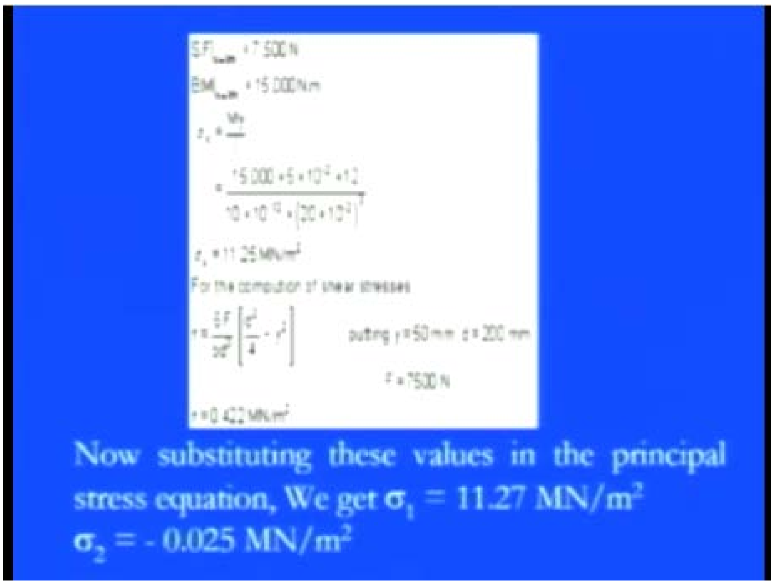

So now, once you have the shear force, once you have the bending moment and shear force, so that we can calculate the bending stresses. So, bending stresses nothing but equals to sigma B by y is equals to M by i, since it is a rectangular structure. So i is nothing but equals to bd cube by 12, so now by keeping those values here, what we have? We have M which is 15000, because it is we just calculated the bending moment, so bending moment 15000 into. Now, here what we have? We have y, so y is nothing but equals to 5 into 10.

We have already put those things here, so 5 into 10 raise to power minus 2, because we are calculating in the meter. And then if is bd cube by 12, so 12 is coming on top up side, so into 12 divided by 10 into 10 raise to the power minus 12 is there, because the dimensions are given here as in millimeter.

So, we need to change into in terms of the meter, so this is like that 10 into 10 to the power minus 12, minus 12 into. Now bd cube, so cube is 20 into 10 to the power minus 2 whole to the power cube. So, now if we manipulate those things then you have at the end that this 11.25 mega Newton per meter square. So now, we have the bending stresses in terms of mega Newton per meter square now for that we can easily calculate the shearing stresses, because it is a rectangular structure. So, pretty simple tau is nothing but equals to 6 F divided by bd cube into d square by 4 minus y square.

So here, y as just put this 50 a millimeter or 5 into 10 raise to the power minus 2 diameter we just this depth which we put 200 millimeter or we can say that it is in terms of 2 into 10 raise to the power minus 1 meter, and then F with the shear force which we calculated that is 7500 Newton. So, by keeping those values we have the shear is nothing but equals to 0.422 mega Newton per meter square.

So now, you have the bending stresses, now we have the shearing stresses, so by keeping those values means the bending stress is the longitudinal stress. So, it has 11.25 mega Newton per meter square and the shear stress tau which is 0.422 mega Newton per meter square.

So, by keeping those values, we can get easily the principle stresses like sigma 1 comma sigma 2 is equals to this bending stress sigma B by 2 plus minus square root of sigma square, whatever the bending stress divided by 4 plus 4 times of tau square xy; that is means, this shearing stresses.

So, by keeping those values in that formula, we can get sigma 1 and sigma 2. So, sigma 1 is 11.27 mega Newton per meter square and sigma 2 is minus 0.025 mega Newton per meter square. So now, it is pretty easy to calculate the principle stresses once the bending stress, once the tau this shearing stresses and for that we need to get first the shear force and bending moment to get those stress values. So, this is a pretty simple processor to get the principle stresses for that, so this was just to refresh your concept that numerical problem was there.

Now, we are dealing with the different problem, that if we have a kind of composite beam or the fletched beam is there; that means, if a beam is having the two different kind of material and it is mixed in such a way that it is it has a robust design. And for that, if you want to find it out the bending, that actually how the bending will be there in that different kind of structure.

Because till now, we have use the homogenous material; that means, homogenous isotropic material is there in which in all direction the stresses are pretty equal. So, for that all the bending theory was applicable. Now, we would like to check first the if you have a composite beam, then whether the same theory is applicable to this 1 or not.

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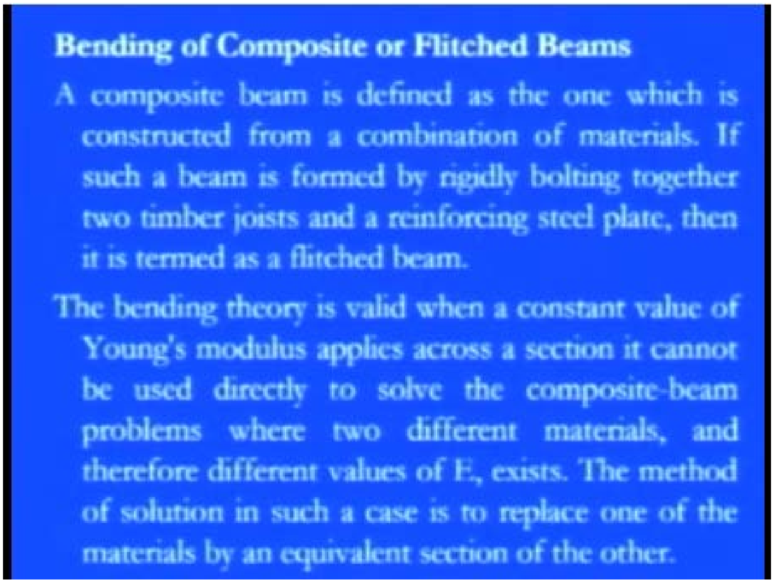

So, for that, now the analysis is that a composite beam is defined a as one of which constructed form the combination of materials, as the two different materials are there and for which that the main property of that which we are using to analyze the bending is the Young's modulus of elasticity.

So again here, if we have a composite beam, then obviously for the entire beam we have the two different values or if the two different materials are there then we have the two different values of Young's modulus of elasticity. So for that, we would like to check whether the same beam theories applicable or not.

So, since it is constructed from a combination of material and if such a beam is formed by rigidly bolting together 2 timber joints are this reinforcing steel plate, then it is termed as the fletched beam is if we are talking about a fletched beam. Then there are 2 the timber joints at the extreme end and we have reinforced steel plate is there in between.

So now, if we are talking about that or it is a common beam which has a two different material 1 is the wood and 1 is the steel, then obviously the Young's modulus of elasticity is entirely different. And, if you want to analyze those things that the bending is there or the shearing is there.

Then the entire the combination of these materials with the kind of theory is all together it is a different. So, we would like to check that, how we can analyze those things. So here, the first point the bending theory is valid, when a constant value of young's modulus applies across the section. And, it cannot be used directly to solve the composite beam problems as I told you where two different materials are there, because they have that two different values of modulus of elasticity.

And, the method of solution is such a case is to replace 1 material by equivalent section of the another, so which is a very important thing. Because here, the 2 different values of the modulus of elasticity is there, and when you apply the load definitely they will be a different kind of stresses first, different values of stresses second.

And then whatever the strain will come whatever the deformation is come it all together it has a different value. So now, when we are applying that sigma by y is equals to M by I is equals to E by R, this theory is almost invalid for this kind of section. So, the perfect replacement is that, what we need to do based on their the different value of the modulus of elasticity.

We just want to put the equivalent amount like in the previous case, as in the fletched beam is there. In which the two different the materials are there the 1 material is wooden plate, 1 material is the steel plate. So, we need to replace the steel plate by equivalent amount of the wooden plate. So that now, we have a same structure the wooden, and then whatever we want to apply straight the Young's modulus of elasticity can be easily applied. So, this is a perfect solution for this.

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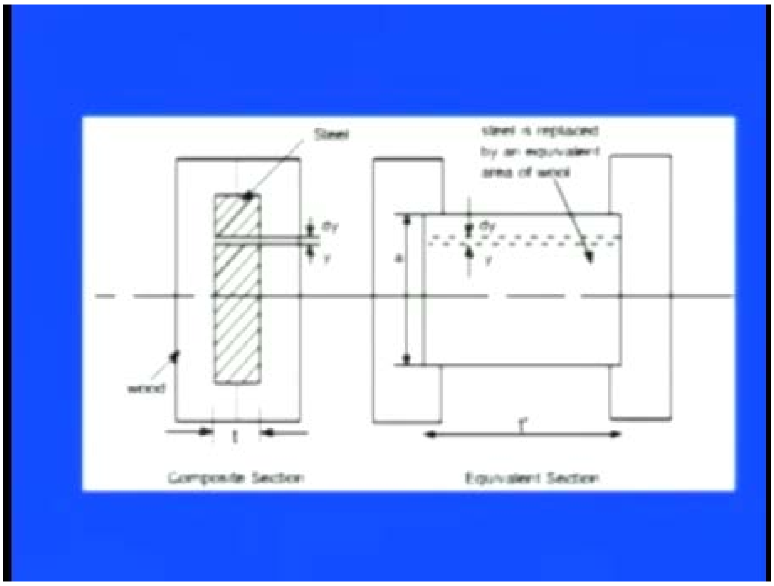

So here, again in on your screen, you can see your diagram that these two the outer parts of the wooden plate. And, this is the steel plate is there in between that and it has a thickness t. So, if we are going for the real configuration, then this is the actual configuration of a composite beam is in which this middle portion is showing the steel part end the outer portion is showing the wooden part. But now, if we want to apply let us say it the bending moment is there on these particular beam and it is under deflection or we can say the it just bearing the bending stresses as well as the shearing stresses.

But, if you want to apply the same theory which we applied for a common beam, it is not valid in this case as we discussed many times. So here, what we need to do here? We need to replace this steel beam by a equivalent amount of this wooden beam. So for that, what we need to do here, you can just see addition diagram that we have this is the wooden part, this wooden part which is exactly similar to this particular part.

But, this t dash this is the equivalent thickness of this 1 in terms of wood, so that we need to replace this, how we will discuss this part, but the physics says that we need to replace those things. And then you can apply those formula which we applied for a common beam. So, just go with the concept that we need to replace this in which the we have the new dimension of the equivalent amount of the wooden of the steel with the t dash and A. So here, you can simply configure those things and get the realistic way of dealing the problem, so this is one.

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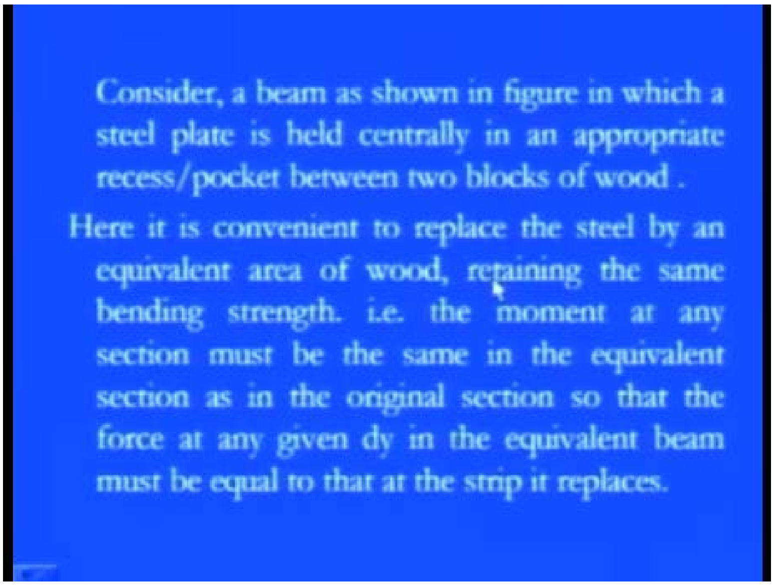

So, consider a beam as shown in the figure which I shown in the previous 1 in which the steel plate is held centrally in an appropriate recess oblique the pocket between the two blocks of wood in the previous 1 the first figure. And here, it is convenient to replace the steel by an equivalent strength or we can say whatever the main key features are there just equivalent area of the wood.

So, that we can say that it can be your it can remain the same bending strength; that means, the moment at any section must be same in the equivalent section and as in the original section. Then only, the theory are the analysis which we are doing it will give a realistic feeling about the bending moment as well as the shearing stresses.

Otherwise, if you are not dealing with the same kind of bending strength or the same kind of shearing strength, then probably whatever the design which we are whatever the design concept which we are applying to design the beam always it fails in a proper way.

So, for that the force at any given d the dy which we have taken the this small strip in an equivalent beam must be equal to the that to the strip at as it is replaces; that means, whatever the strength which we are carrying out for the analysis it has to be same exactly in the equivalent amount of the wood in terms of the same the steel strip. And then only, we can say that we are whatever the analysis which we are doing that has to be equal.

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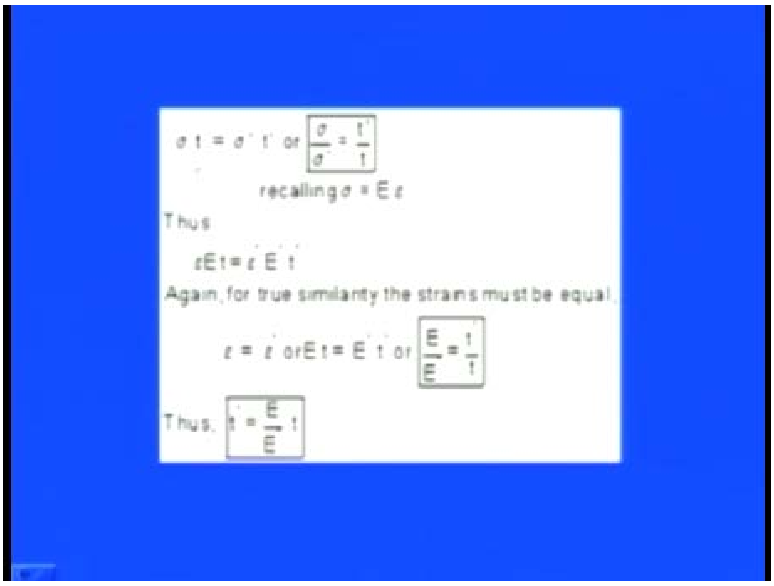

So here, as we discussed that actually the initially when we are talking about the this steel strip we have the bearing, this bending stresses is sigma into the real thickness t which will be equals to the equivalent amount of the wood which carries the bending stresses sigma dash into t dash which is the equivalent amount of thickness.

So; that means, the ratio sigma by sigma dash sigma is the stresses in the steel component, sigma dash is the stresses in the equivalent wooden component. So, the ratio must be the sigma by sigma dash must be equals to t dash by t; that means, t dash means the equivalent thickness and t is the real thickness of the steel. So, whatever the ratio they are coming it has to be equal to the thickness ratio.

So that, whatever the a bearing stresses or whatever we can say the shear stresses they are coming in the realistic section, they have to be gives you a clear feeling about the real section that. Now, these values are coming and the based on that we can simply calculate the whatever the stress components in those particular sections.

And now, again sigma as we discussed many times that you we are applying the load within the Hooke's law and this under the Hooke's law, the stress is proportional to strain. So, by keeping that thing in our mind sigma is equals to E into epsilon.

So now, see sigma if you replace the sigma in the first equation, what we have this epsilon into E that is sigma, sigma is replace by epsilon into E into t is equals to epsilon dash E dash into t dash. So now, this section epsilon is the strain component in the steel section and E is the Young's modulus of elasticity for steel and the t is the thickness.

So, now you can visualize easily that, what we have, whatever the strain component is coming forget about that part, this Young's modulus of elasticity for steel is much higher than the Young's modulus of elasticity for wood. So, now when we are comparing those things and definitely you can see the previous diagram that the first diagram in this the thickness was very small.

But, if the equivalent in a portion we have a very big thickness, why it has come because of the difference of the Young's modulus of elasticity value. So, based on that we can simply get that, whatever the different materials are there, once the Young's modulus of elasticity you can easily replace those things by equivalent amount of the section.

So here, based on that epsilon E t is equals to epsilon dash E dash t dash, and now if we are going for the similarity in the strain if we are saying that since it is a composite beam, so strain component or whatever the deformations are coming it is equal. So, if we are limiting that part means epsilon is equals to epsilon dash, then we have the realistic feeling about the thickness which is coming due to the difference of the Young's modulus of elasticity for the different material.

So here, E by E dash is equals to t dash by t or we can say that the really this equivalent thickness is equal to E by E dash means how much the difference is there. So, what will be the difference is there, it will come in terms of the ratio of E by E dash into t.

So let us say that, if we have some different value, it will multiply with this original thickness and with the, whatever that real thickness is there, it is much more than the previous thickness. So, that is what this concept is very much true if they are going with this kind of relations.

So hence, to replace steel strip by an equivalent amount of this wooden strip, the thickness must be multiplied by this ratio E by E dash and steel stress is nothing but equals to the modular ratio into stress in the equivalent wood. So here, we can simply get the stress in this either in the modular part or we can say in the steel to wooden things.

So, the equivalent section is then 1 of the same material throughout and the simple bending theory is applicable, if the previous the equations are valid up to certain sections. The stress in the wooden part of the original beam is found directly, and then in the steel first of all we are calculating those things. And then if you put the in the steel which is found from the value of the same point in an equivalent material which is always follow the same relation like sigma by sigma dash is equals to t dash by t or sigma by sigma dash is equals to E by E dash.

So, whatever the corresponding stresses are coming, they have to be follows the same relation then only we can say that if we are replacing one material by another material they are simply valid. And, whatever the equation which we are applying for calculating the stresses they are valid for this kind of sections.

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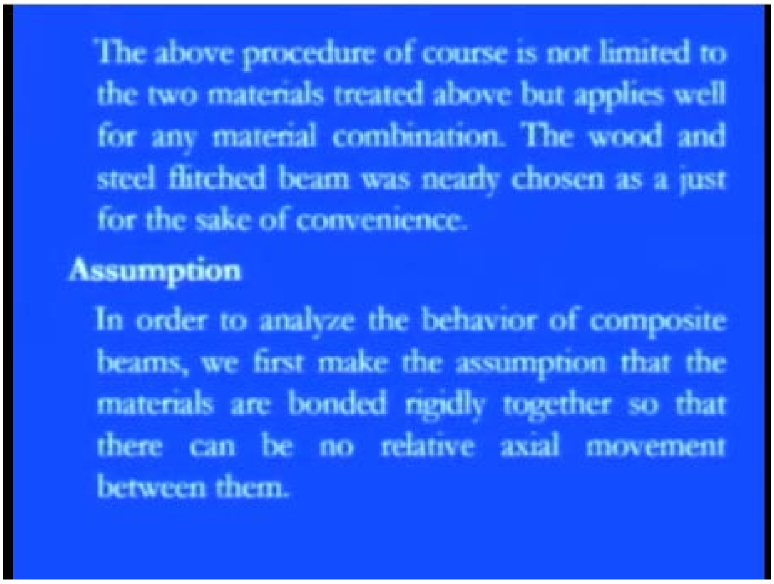

So the above processor of course is not the limited to the two different material and in the it can be treated above. But, it is applies well for any kind of material combination as I told you because, only what we are doing here basically, we are simply replacing there equivalent amount of the thickness from 1 part to another part.

And, the wood and the steel fletched beam was nearly chosen as just for the sake of convenience. So, that it is pretty easy to differentiate in between the Young's modulus of elasticity that. Now, this Young's modulus of elasticity for steel is much much higher than the Young's modulus of elasticity for wood. So obviously, if we have a small strip like let us say for 1 millimeter, then if we 1 millimeter of strip for a steel, then the equivalent amount of is always like 100 times more than this 1.

So that, we can easily visualize that, now if we are replacing those things replace in a proper way and it the replacement has to be follow the proper equations like sigma by sigma dash is equals to E dash by E or t dash by t like that. So, they have to follow this kind of relations then only these combinations are valid and whatever the equation which we are applying like sigma by y equals to this M by I equals to E by R or the tau equals to Fa y bar divided by IZ. They are absolutely valid for this kind of composite beam.

So, this is there but again prior to analyze those things there are certain assumptions in those composite beam. So, in order to analyze the behavior of composite beam, once you followed those things now certain assumptions are there, we first make the assumption that the material are bonded rigidly together. So that, there can cannot be any relative axial moment is there in between those.

Because once, you have any slip or any clearance is there, then definitely when we apply the load it gives a different kind of deformation because already there is a kind of clearance is there. So, it will end up with the different kind of deformation, and then the different kind of stresses are being come coming up due to the application of load or moment.

So, we have to be very much rigid that actually these bonded that is what I told in the very first my sentence was the composite beam must to be robust, robust means actually they have that there is no chance of any clearance in between these composite these 2 different materials there should be any gap. So, that there with they will not allow any kind of axial movement in between these two material, because if this kind of things are there, then definitely we are experiencing not only with the these two stresses. But also, the other type of stresses are there other type of deformations are there and whatever the theories which we are applying there it is not valid for those things. So, this is this was the first assumption.

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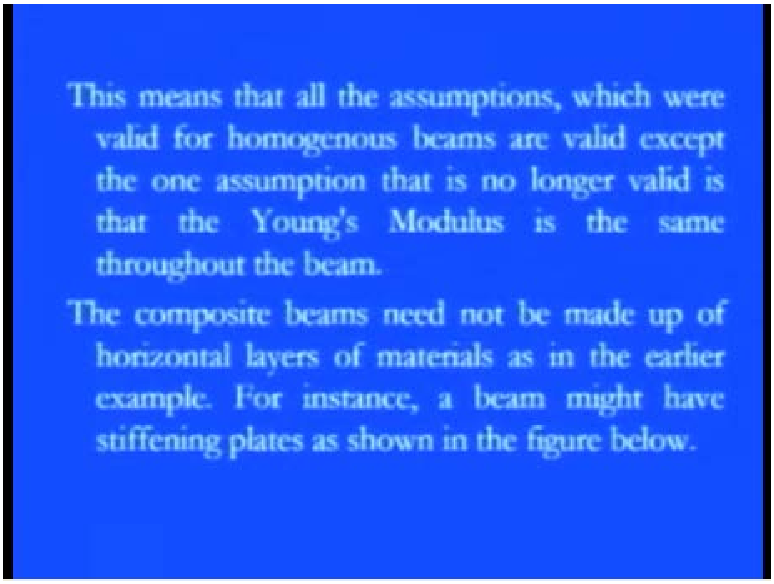

So this means that, all the assumptions which were valid for homogenous beam, because in that are valid except one assumption that there is no like longer valid in that sense that Young's modulus is same throughout the beam. So, that is like only the one critical part was there and that is what like for the composite beam need not to be made up of the horizontal layers of a material as in the earlier example that we are always assuming that this we have a beam. And they are just make a like made up of the horizontal layers or we can say like the fliers are there just we are checking those things. And that is why corresponding to that we are just taking the neutral axis along with that and we are taking all those centroidal or the another axis is along the horizontal layers.

So that to in the previous cases, we assume though that part, but here, in the since it is a composite beam two different materials are there and it is you now in the sort of the mixing part is there. So, in that this assumption is not at all valid for that and for instances a beam might have this stiffing plates as shown in this particular figure. So, you can see these thing we have it is not the kind of all the fibers are well displaced in a proper way it can be in this particular kind of structure.

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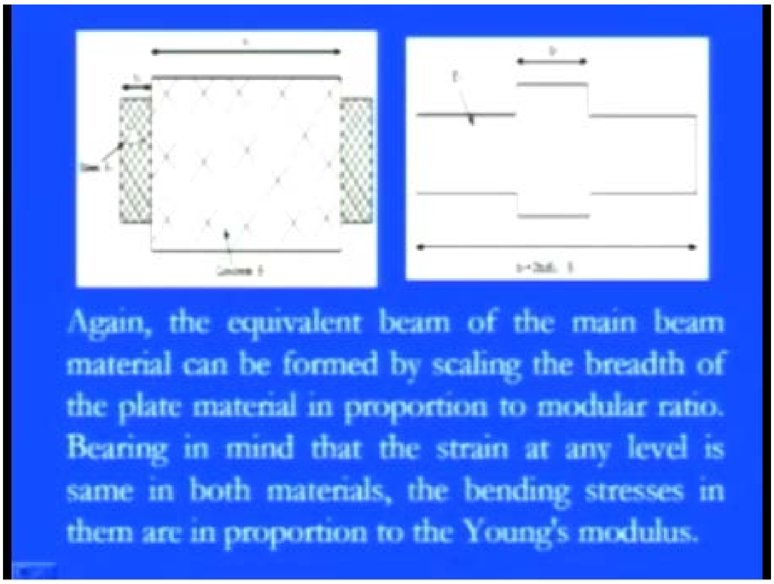

In which these are the steel plates and the different kind of sections are there and if I am saying that this is the concrete. So, you can see the real configuration of the material structure or we can say if we are going for the microstructure then how the configuration or how the distribution is there, the microstructure within those elements you can easily visualize with this particular diagram. So, if we are talking about the steel, then it is of this nature if we are talking about a concrete, then we have a this kind of structure in which the Young's modulus for concrete is E 1 and a Young's modulus of steel is this one.

So now, if you want to put an equivalent amount, then it is all together different here we have the steel structure of these we have the concrete structure is this and then this whatever the material fibers are coming they are all together different in that. So that is, what the in previous we discussed that if we have a common beam, then it has the fibers are well settle with the horizontal way.

But here, it is all together different, again the equivalent beam of the main beam material can be formed by scaling the breadth of the plate material in the proportion of a modular ratio and that is what in the previous case I told you that the stresses in steel is always equals to modular in the previous case, modular ratio into the stresses in the wooden part.

So, modular ratio is always gives the proportion that actually how it is to be distributed in the two different section, one section like in this case you can see on your on your screen that one case is the concrete one case is the steel. So, depends on that what the modular ratio is there corresponding changes are there in their strength as well as the bending part this bending stresses.

So bending in mind that, this whatever keeping those things in your mind that, whatever the things are coming in terms of just by scaling those things, it will come in terms of the proportional to the modular ratio. These thing keep this thing in your mind the strain at any level is same as the both of the material, because we are replacing those things.

And the bending stresses in then they are proportional to the Young’s modulus and that is what the corresponding changes are coming in the equivalent section of the thickness as well as the similar kind of the behavior is coming in the bending stresses. So, that is what the Young’s modulus is pretty important aspect as far as the composite beam is there.

Because here, we are replacing based on the Young’s modulus, because Young’s modulus is property of material, if we just focus on that,which will give you that, how much it can bear up to this elastic property. And, if it has a less value; that means, immediately once you apply the load it will go in to the deflection means the permanent set of deflection part. So, we do not want to go in the permanent deformation we just want to keep our the material within those elastic region, so that if we remove the load it comes to its original state.

So that is, what here if we are talking about a concrete which is a quite brittle material and if we are talking about a steel. So, altogether they have a different property and since we are replacing by equally equivalent amount, then definitely the bending stresses are coming into these kind of composite beam just depends on the modular ratio. And then corresponding changes are coming like just the stresses of bending.

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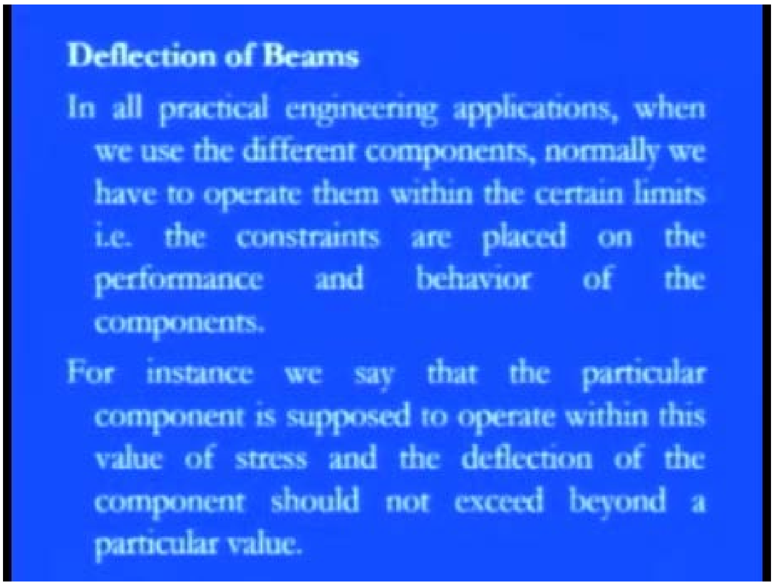

So, we need to design accordingly, now here we have the deflection of beams; that means, in all practically engineering application, when we use the different components, normally we have to operate then within certain limits that is the constraints are placed on the performance and the behavior of the component; that means, if we are talking about the deformation means actually how much deformed shapes are there under the load application. But the deflection means actually, all those small components they are replacing from one portion to another portion.

So, now we just want to see that, under the action of this bending moment or the shear forces, how the deflections are there, because it is just going in a various different kind of shapes, so how deflection will come into the beams under the action of these things.

So for the instances we say that, the particular component is suppose to operate within the values of the stresses the bending stresses and the shearing stresses. And, the deflection of the component should not be accept beyond a particular value, because once it goes beyond a certain value, then we have a permanent set of deflected part. And then we assume in the perfect bending theory that actually before end prior the plane should not be change. So, the plane of this horizontal beam must remain same before bending in after bending.

So, if we are saying that the deflection, after the under the application this bending moment the shearing forces, if this the value of the deflection is go beyond certain value;

that means, it is going in the permanent set of this kind of deflection. Then we have almost deform shape of these beam and whatever the theory which we applied is which is not valid at all.

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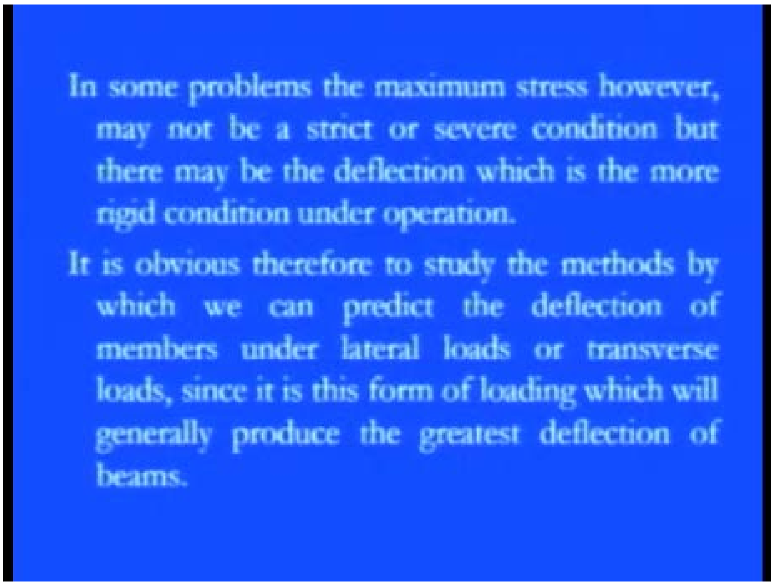

So, for that in some of the problems the maximum stresses; however, may not be strict or the severe this condition, but there may be the deflection which is more rigid conditions. Hence, when we found that the deflection which is the more rigid in this kind of condition operation.

So it is obvious therefore, to study the method by which we can predict the deflection of the members under lateral or transverse loads. Since, it is this form of this loading which will generally produce the greatest deflection of the beam we have to be very very careful that actually what kind of analysis which can give you exactly the kind of deflection. Because, when we are talking about a deflection then it is much more a severe problem than the kind of the other elastic deformation is so we need to be very very careful to design those things.

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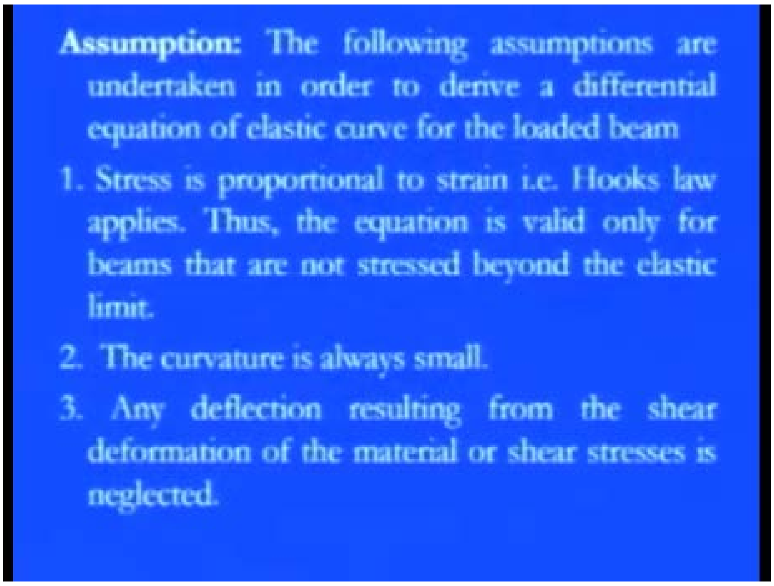

And for that first of all we need to put certain assumptions, and the following assumptions are undertaken in order to derive. The differential equation of the elastic curve for the loaded beam, because again one has to be very, very careful that, our all analysis is just based on the elastic deformation of the kind of beam.

So here, the first assumption is the stress is proportional to strain; that means, the Hooke's law applies and that is what I told you that whatever we are telling about the sigma by y is equals to M by I or equals to E by R for the bending theory or for shearing theory the tau is equals to Fa y bar divided by IZ.

All those theories with that concept like that it remains the plane remains plane or whatever this horizontal parts and all. These are only valid when we are saying that the stress whatever the stresses are coming the bending as well as the shear they are proportional to the kind of deformation or we can say the strain and in which the Hooke's law is valid. Thus the equation is valid only for the beams that are not stressed beyond the elastic limit.

Because, once it goes into the plastic region that just I told you that we have a permanent set of deflection; that means we have a deflected beam. So, when we are dealing with those kind of stresses which is already being there the deflected part; that means, more and more stress concentrations are there in that kind of beams.

Then, the theories are not at all valid or applicable to the deflected permanent deflected beam. So, this first assumption is very much valid and we have to be very careful when we applied the moment or shear forces, so that we the deformation or the deflection cannot go beyond this particular elastic region. The second which is again important part, the curvature is always small, because if more curvature is there, then definitely the kind of the analysis is not at all valid, because the fibers are going into the different region. And, we just want to be go we just want to the apply and we just want to go not go beyond the certain limit, so the fibers are well within the structured limit and we can apply the bending and shearing theory.

Then, third assumption is any deflection resulting from the shear deformation of a material or shear stresses is neglected. So, which is a very important part, whatever the deflections are coming due to the shear deformation or the kind of twisting or a couple, because, it disturb whole of the fibers of the beam.

So, it is it has to be neglected, because if it is coming then altogether it disturbed your all set up with the fibers, then whatever the bending is coming or the beam theories are there it is invalid in that cases. So now, it can be shown that the deflection, due to shear deformations are always usually small, but it when we are talking about the small deformation also it complicates your theory. So, that is what it is better to ignore that part. Now, come to the realistic picture that if we have a beam, which is under the deflection, and due to the deflection we have a deflected part. So, earlier when there is no bending and there is nothing is there, so we do not have any deflected part.

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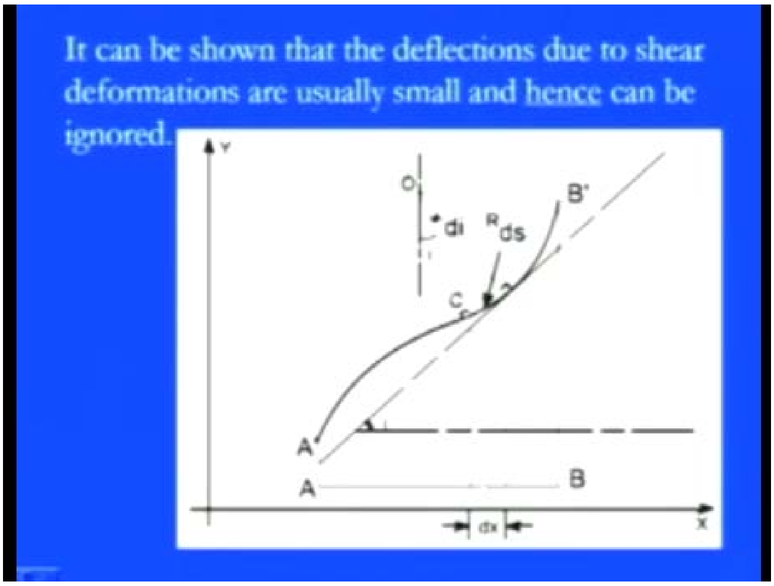

So, we have a straight beam that is let us say A and B and this the neutral axes are there for that. But, when we are saying that it is under deflection, so the A dash B dash is the deflected region and we have a small curvature part and if I am saying that these O is my let us say the origin is there from there the C just we are taking the surface area is ds and which has a radius is R. So, for that kind of regions and the angle is now i here just keep this thing the angle of curvature is i.

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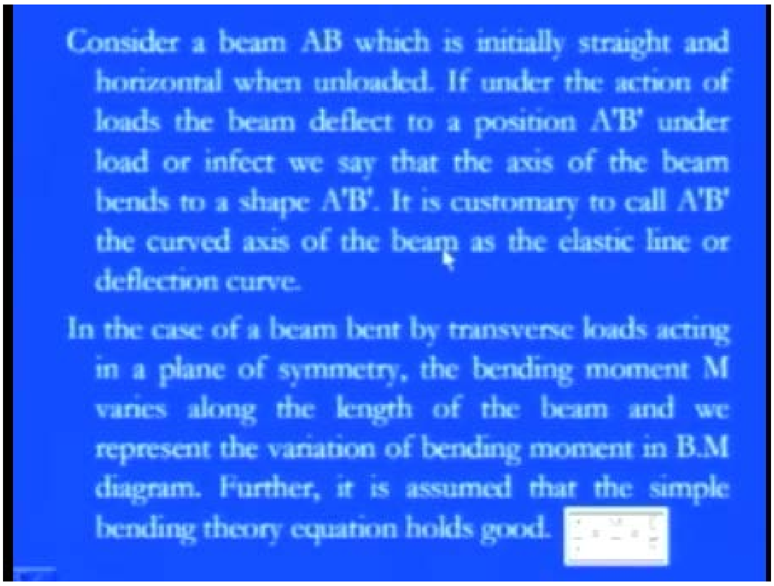

So, with these configuration, If I am saying that now that beam, which is initially the straight as I told you and it has the AB just horizontal part when there is no loading condition is there. But, if a it is under the action of any load that that particular beam deflects the position A dash B dash I just shown the curvature path under the load or we can say in just. In fact, we can say that the axis of the beam bends to a shape of A dash B dash, whatever the axes are there in between those thing now it is taken as any shape of elected part. It is customary to call A dash B dash the curved axis of the beam as an elastic line or we can say the deflection curve, then only the deflection criteria is a valid criteria.

In that case, just which I shown you a beam bent by transverse loading acting in a plane of symmetry the bending moment M varies along the length of a beam, whatever the this just along just these bending moment is varies along this particular axis or we can say the elastic line or the deflection a line along that. And, we represent the variation of bending moment in this particular the bending moment diagrams are there we can easily show that how the variation is there the with that deflection of a beam.

Further, it is assumed that the simple bending theory a equation holds the good relations within these deflection part because the curvature is very very small in that since. So, we can say that the sigma by y is equals to M by I is equals to E by R is also valid for these kind of cases.

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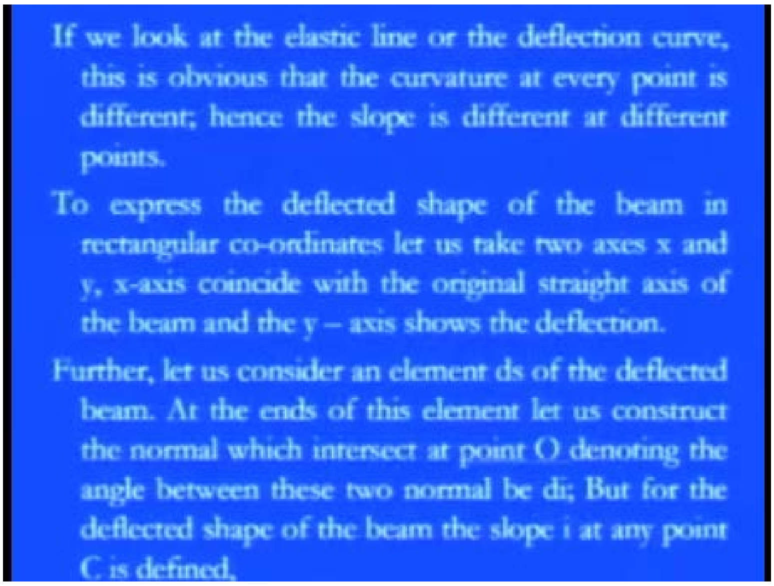

Now, if you look at the elastic line which is the deflection curve we can say or the deflection curve this is obvious that the curvature at every point is different, hence the slope is different at the different, different points. Obviously; the displacement of the points when it is under the action of any kind of loading, these deflections are altogether different.

So, we have to be very, very careful that actually how the deflections are coming at different, different points. So, that when you are meeting those points it gives you a clear picture of the curve that actually, how the elastic curves are there within those region. And, to express the deflected shape of a beam in a rectangular coordinates, because if we are saying that the rectangular cross section is there and now it is under the action of deflected one.

Let us, take the two different axes x and y we have now since it is in a rectangular section. So, we can easily take the two different axes x and y direction. So, x axis simply coincide with the original straight axis of the beam, that has to be there and y axis is just shows the deflection. So, when I am talking about the x y region this x is simply gives you a clear this axis of the beam. So, that we are just flowing with the real axis or the fibers are there in the horizontal set up, but y axis is clearly measure that how much deflection is there, how these points at deflected from the their origin point to the real point, So, the y axis will simply measure the kind of deflection.

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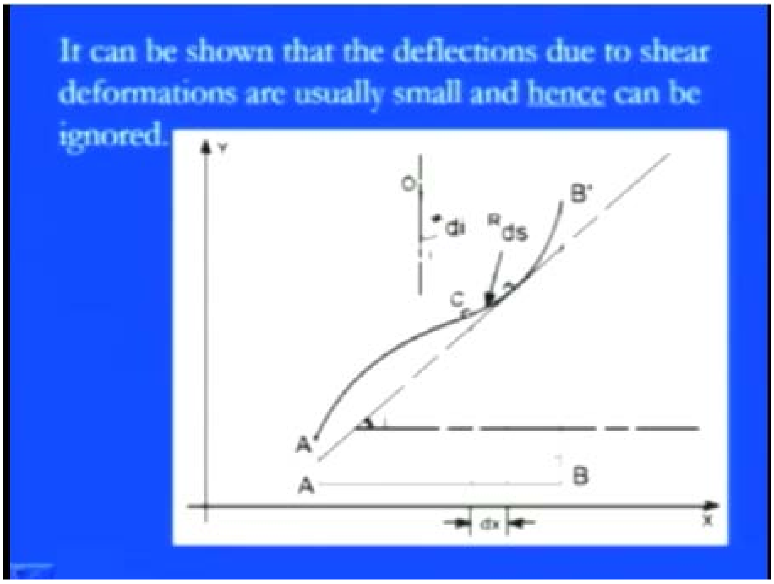

Further let us, consider an element ds, so that just the of the deflected beam. So, we can see in this diagram, here this is the kind of this ds is there which is the deflected part here, and this is the axis where as I told you as this simply flowing with those things.

And the y axis, if we are talking about this, then it is simply measuring that actually how much deflection is there in that. So, these x and y are different measures are there with the beam axis and the deflection part. And then these the radius of curvature or we can say this origin or the angle or the ds surfaces are clearly showing that actually how the relations are there.

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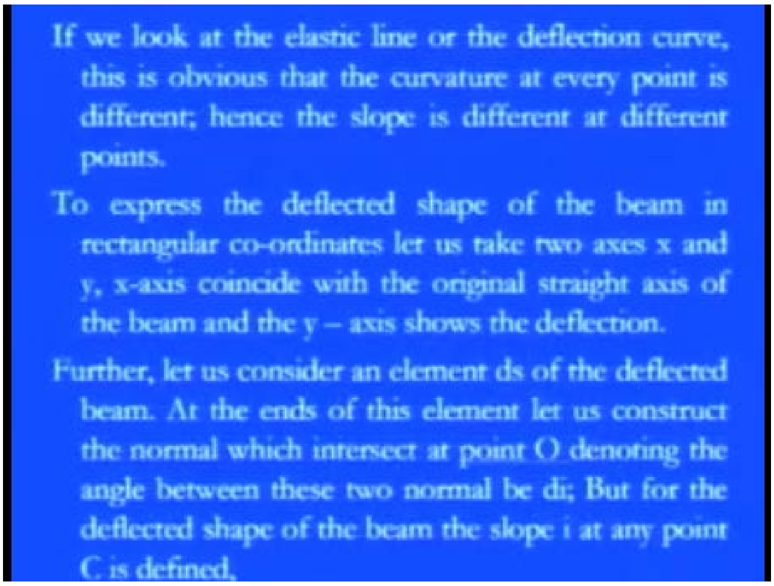

So for that, when we consider an element of ds of deflected part at these ends at the particular ends of the element. Let us, just construct in a normal, we just put the normal values there actually which is simply interacting at origin of these through which we have an angle between these to normal at di, because the main angle which we are measuring due to the deflection is i. But for the deflected shape of a beam the slope i at any point C is easily defined with the using of tangent.

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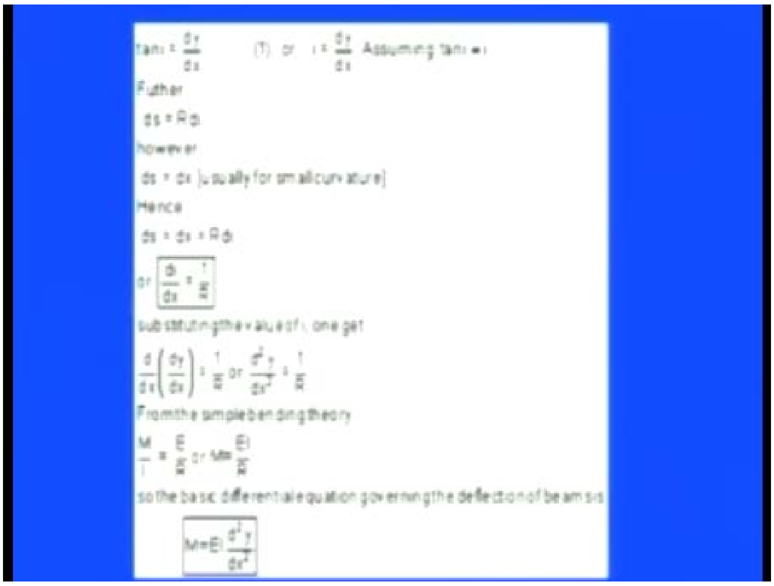

So, tan i is equals to dy by dx or we can say since it is a very very small curve path is there. So, tan i is equals to I, so we can say that the i is equals to dy by dx. So, now coming back to the curvature theory we know that when the radius, when the curvature path, so we can say that the ds, which is the small segment is there on the curvature path is equals to R into di. So, now by keeping those values we have i equals to dy by dx,we have ds equals to R di. So with those small curvature theory, we have ds is equals to dx.

So, by keeping those values in your mind, now we can simply say that dx ds the this is small segment ds is equals to dx or we can say that R di or we can say di by dx is equals to 1 by R. So, whatever the change is there in the angle within the x domain, because we are flowing with the beam will give you the 1 by R, the radius of the curvature which is a very well the valid theory is there as far as the curvatures are concern.

Now, if you substitute those things in the i we can get d by dx, now we are keeping those value dy by dx, because i is equals to dy by dx in the first segment here. So, if we are keeping those things d by dx into i is dy by dx which is equals to 1 by R or we can say that d 2 y by dx square is equals to 1 by R.

So now, we have a different relations now that 1 by R radius means the reciprocal of the radius is nothing but equals to d 2 y by dx square.The second derivative of the angle domain means or these things in the curvature path. So, now coming back to the main bending theory which is M by I is equals to E by R equals to sigma by y.

So now, what we have we have M is equals to E into I by R, so 1 by R can be easily replaced by d 2 y by dx square. So, now, we have the basic equation of the deflection is M is equals to EI into d 2 y by dx square, so coming back to this particular this equation what we have the bending moment is equals to EI which is the this rigidity. So, it is known as the rigidity E I into d 2 y by dx square.

Second derivative of the deflection in the y, because as I told you in the x axis we are flowing with the main axis of a beam but whatever the deflections are coming it is coming in terms of the y axis. So, the second derivative of the y will give you the exact that how much deflections are there what the exactly the variation is there in the x domain. So, moment is equal to E into I the flexural rigidity into d 2 y by dx square and this is the basic equation for the deflection.

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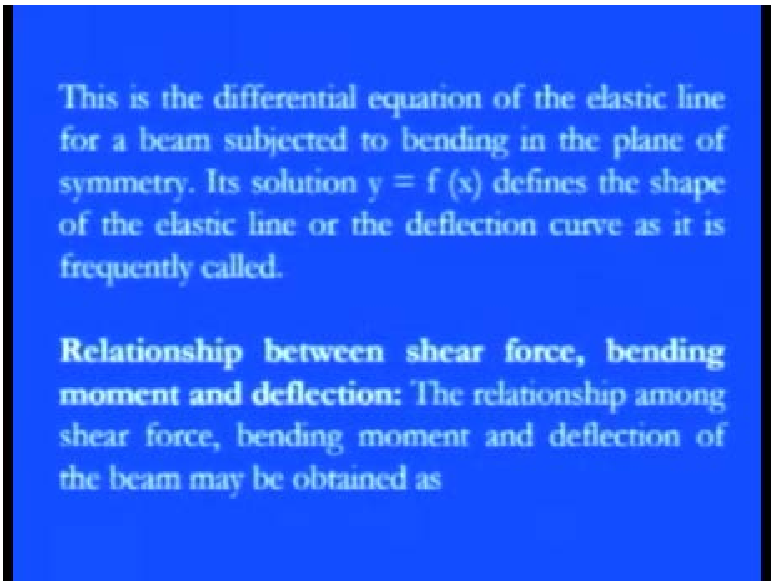

So now, what we are going to do, we are basically going to analyze these equations here, that how we can get the different position. So, this is the differential equation of the elastic line for a beams subjected to a bending of plane in a symmetrical way and it is the solution y equals to fx defines the shape of the elastic line or we can say the deflection curve as it is frequently called.

Now here, we would like to put the relations between the shear force bending moment and the deflection in our next lecture. So now, if you want to conclude this lecture our main focus was here, that if we have the different kind of sections then how we can first of all solve the numerically. So, we found that the numerically these can be easily solvable.

Once, the cross section of these beams that it is a rectangular cross section or i section or circular cross section and apart from that once that actually, what are the different potential regions where the maximum or minimum shearing or bending stresses are. So, we solve a numerical problem which was the in this lecture that we have a simply supported beam and if we want to find it out the principle stresses at certain location it can be easily calculated if the loading condition and the cross section of beam.

So, that is what sigma 1 comma sigma 2 which was sigma x plus sigma y by 2 plus minus square root of sigma x minus sigma y by 2 whole square plus 4 times of tau x square tau square xy by keeping this particular formula in your mind you can easily calculate the principle stresses at any section of a beam if it is under the loading condition.

Then, we discussed about that if we have the composite beam then how we can replace that composite by composite beam by any equivalent section which is very, very important. And then we found that in if you have a composite beam which carries two different kind of material and equivalent section is very, very important and it has to be follow the roles or the relations in between the stresses to thickness and the thickness to Young’s modulus.

Because, Young’s modulus difference is a key role here, and how the variation is there, in the Young’s modulus all always comes the picture when the thickness ratios are. So, this kind of discussion which we made and the last part was there the deflection, so what is the deflection is there what are the certain assumptions are there for the deflection theory, and then derived that the bending moment M is nothing but equals to E into I that is the flexural rigidity into d 2 y by dx square.

So, now once you have the relation, now we just want to put the relation in between the shear stresses bending moment and the deflection in the next lecture. And then also, we would like to once you put the relations then we just want to solve those relations. And then for a different the elements like if we have a cantilever if we have a simply supported beam with the UDL or point loaded, we can resolve those issues for the deflections with the using of shear stresses bending moment and that deflection criteria.

Thank you.