Strength of Materials
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Lecture – 3

Good morning, I am Dr. S. P. Harsha, faculty of mechanical and industrial department at Roorkee going to present today the lecture 3 of the strength of materials subject. So, in my previous lecture we had discussed about the two main components of the stresses like the normal stress component in which the force as well as the stress components are perpendicular to the area of concern; while the other thing is the other component was the shear stresses in which the forces as well as the stress setups which are there within those object are parallel to the matter of this area.

So, meaning is that you see you know like we discussed about the two major components, but sometimes it is not fruitful to just you know like disks only about these two stresses. Because as we have seen that there are the derived stresses like the bearing stresses, like the tensile, like the compress, stresses like the torsional stress or the thermal stresses. So, these are all the derived stresses we. So, if you want analyze any of the component or the object, we need to see that actually what other you know like the parts of the stresses are there, and how they are applying. Because the basic reason of the stress is inducing in an object or like what is the point of force of applications is.

So, how you see the force you know applying on those objects. So, today you see you know like we just want to see that actually you know like that what stresses are there. So, first thing is that the analysis of the stresses in which we just want to see that actually what is the general state of the stress at a particular point.

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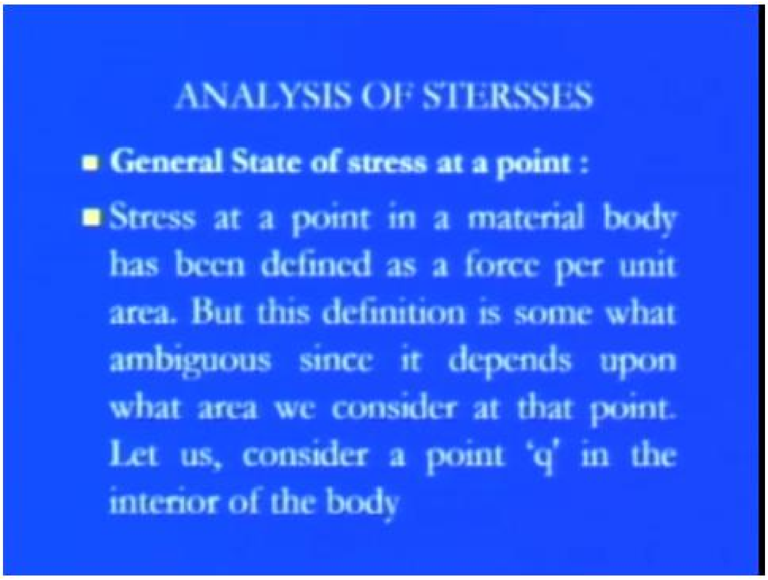

So, as you have seen the stress at a point in the material body has been defined as a force per unit area that is the internal resistances per unit area or we can say the internal intensity of the resistive forces. But this definition is somewhat ambiguous, because you see you know like we have seen that it depends upon what you know like we concern or we considered at a point. So, you see actually what the area is there, whether the effective area and the force application what the exact relation is there in between those things. So, now you see we just take a general case in which a regular body is there, and we have chosen one q point in the interior point of this body.

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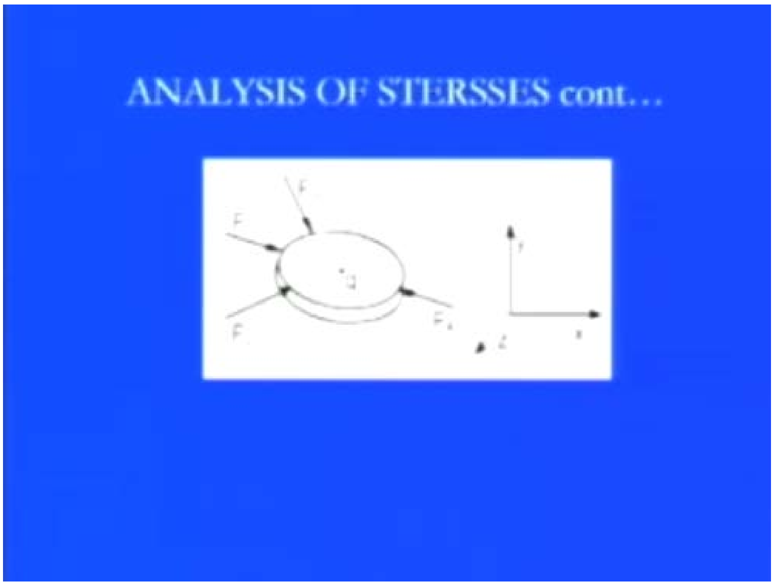

So, we have seen in this figure that actually this is my you know like the body is there; in the body somewhere not exactly at the centre but somewhere you see we have chosen a point q. This body is in equilibrium position under the variety of these forces you see. So, these force applications are there, but we assume that these forces are well set up because of the internal resistive forces. So, you see we have chosen all three axes, and we know that actually the forces are coming from the various you see you know like the aspects and at various varieties of the angles.

So, you see the F 1 F 2 F 3 F 4 they are coming you see under here we have chosen all the compressive forces, okay. So, now you see we just focus on these things that actually whatever the forces which are coming and what are the internal resistive forces which are inducing or which are resisting from this body because of the inherent property of the body, they are well set up and that is why this body is under equilibrium. Now you see to analyze the stresses, the internal intensity of the resistive forces to discuss about those things.

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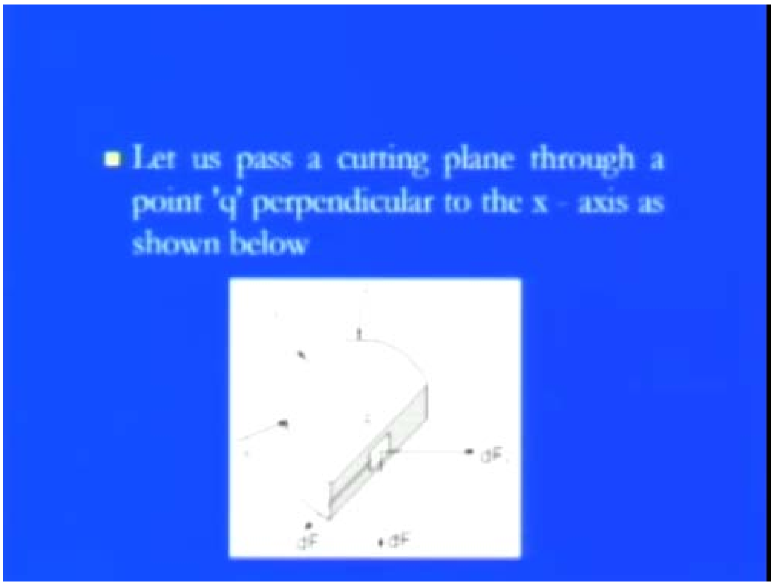

What we are doing here? Now we are passing a plane which is exactly perpendicular to the x axis. So, now you see here our matter of area concerned is the x or we can say now we are playing in the x domain. So, whatever the forces are coming our main nature of the field or the domain is x axis. So, now let us pass a cutting plane through a point q exactly; wherever it is the point q is there in this object perpendicular to the x axis as show in this. So, now you see you have seen in this figure that actually all the forces are being applied to the object.

And at this particular point now we have chosen a small segment and the area of the segment is delta a and whatever the forces are going because we have chosen three different directions. So, in those directions the forces are going like in x direction which is perpendicular to the x axis d F, but the other two forces because you see we have chosen the x axis. And this force is perpendicular to the x axis as we have seen in this figure like here you see this one.

But other two forces like d Fy d Fz they are not perpendicular to this concerned axis or the x axis; they are parallel to the x axis. And this is my d ax; if the x axis is there d ay; if the y axis is there and d az if the z axis is there. So, here you see you know like this is my matter of area concerned. This is my d Fx; this is my d Fy; this is my d Fz. So, if I see these things that, okay, these three forces are playing in the x domain only then what the stress components are.

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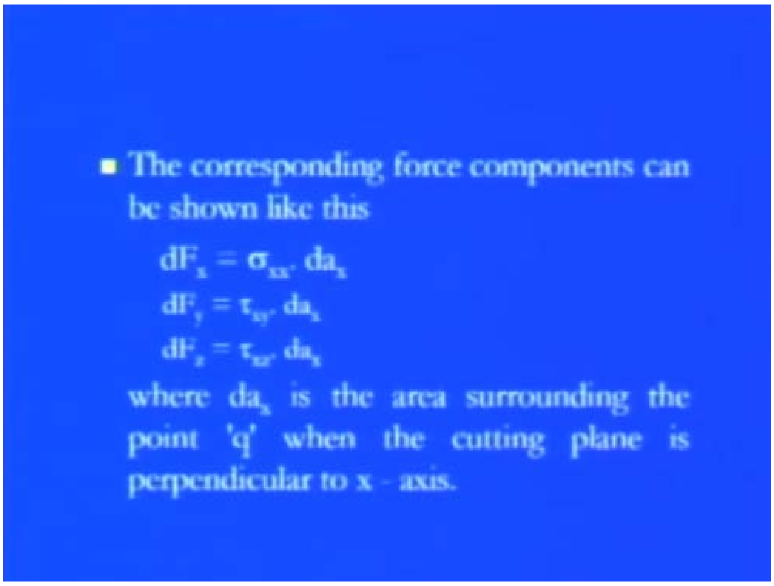

So, now you see in this the corresponding force components can be shown like that like if d Fx is there, then as we have discussed that actually wherever the force component is perpendicular to the area concerned, then we have the normal stress components. So, here we have sigma xx into d ax; that means d ax is the area surrounding to the point q when cutting plane is perpendicular to the x axis as we have discussed, okay. So, now you see here as far as the x component the force component in the x direction is concerned we have d Fx is sigma xx.

That means you see here in x direction we have normal stress because of the perpendicular axis, but the other two force components like d Fy and d Fz in these the forces are not perpendicular to the area concerned. So, here you see you know like the normal stress components are not being setup within the object or outside of the object; that means the surrounding part. So, here you see you know like the parallel stresses are there; that means the parallel forces are running just to the layers of the object. So, here we need to consider that the shear stresses are there, and that is why you see here we are denoting here the shear stress component as the tau xy.

Tau xy means here tau is the shear part. Now the subscript says that x into y; x is the domain as I told you now we have chosen the x domain, and y is the force direction because here we are concerned about the d Fy. So, force is going in the y direction, but the nature of the object is there in the x domain. So, here we can see that actually you know like this one. Here we have chosen this x which is the domain, and this force is going towards the y direction. So, as I told you that actually the shear stress is not the axially stress; shear stress is the plane stress.

So, this is my xy component in which we can say that the shear stress is there, but the area which I have chosen you now like the small area that is the d ax. Similarly, if you have chosen this d Fz in which also we have seen that actually this tau xz; x is the domain in which the force is there all the segments are there, but the force is going in direction of z. So, we have the shear component here. So, means that you see if we have chosen a small point just in the x domain, we found that there are three varieties of the stresses are there; one is the sigma xx tau xy and tau xz. So, these three components are there; these three forces are in the x domain.

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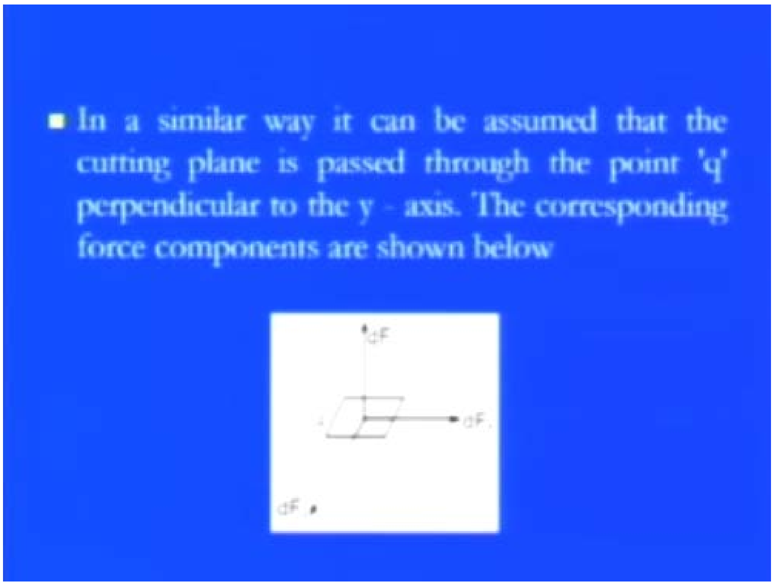

Similarly, you see you know like if we just go to the y axis; that means in the similar way it can be assumed the cutting plane pass through the point q perpendicular to the y axis; that means now we are playing in this y domain, okay. And now whatever the forces are there which is being setup irrespective of the internal or external forces, we just want to check that actually how these forces are acting in this plane. The corresponding force components as you seen here that at point q in this particular figure here this is matter of area concerned; this is the perpendicular force d Fy; this is the parallel force d Fx and d Fz. So, you can see that you see what the corresponding stresses are there as we have discussed.

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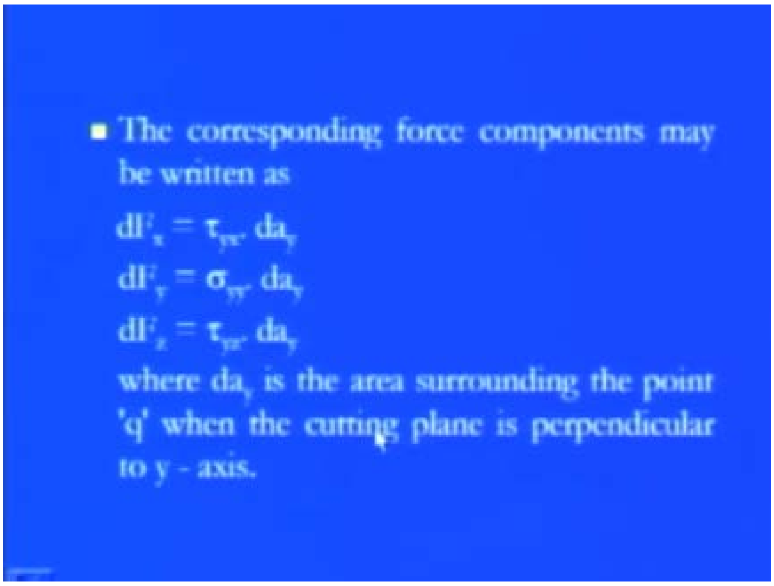

So, you see here first of all the d Fx; now in d Fx we know that we are now tau of y into x into d ay. D ay is you know the area of surrounding at point q where the cutting plan is perpendicular to y axis; that means you see here that now our domain is y while the force first of all if you are talking about d Fx; that means the direction of the force is in x direction, domain is y. So obviously, now the parallel stress components are there; that means there is no perpendicular things are there. Now you see the shear stresses are always dominating in this kind of things. So, d Fx is nothing but equals to tau times y into x into d ay.

Similarly, you see if you are going to d Fy which is nothing but you see you know like this is my y domain, and it is just perpendicular to the y d Fy. So obviously, we have the normal stress component. So, d Fy is nothing but equals to sigma yy into d ay, and similarly you see if you are talking about the d Fz which is you see you know like nothing but equal to tau times y into z into d ay. So, you see here we have the area in d ay direction; means the y direction as well as you see these other two components means that tau yx or tau yz. These are the shear stress components working in y domain, but going in the x as well as the y direction respectively, okay.

So, means you see here either in x domain or y domain, or similarly if you are you know like thinking about the z direction means it can be considered that the cutting plane pass through a point q perpendicular to the z axis. Okay, in the z axis you see now this is the area you know like which we are concerning; the point q is there in between that and d az is there because it is now perpendicular to the z axis. So, now you see the force is just going perpendicular to this axis. This is the d Fz while other two forces they are well setup either irrespective of the external or internal forces, they are well setup in the parallel forces.

So, you see if you resolve these forces in terms of the stresses then we would find that you see here that d Fx is nothing but equals to tau times zx into d az; d Fy is tau times d zy into d az, and tau Fz is sigma zz into d az. Means that now we have two shear stress component in x and y, because as I told you the tau zx because we are working in the z domain which is exactly like you see here the inclined plane towards this. So, this is my z plane, but in this you see this is my sigma z or we can say Fz.

So, I have sigma zz means the both, domain as well as the force direction in the both sides; that is what you see normal stress component is there in the z direction, while the shear stress component is there in the other two directions because we have the same domain, and we have the two different direction of the forces like in x and y. So, we have the shear stress component tau zx and tau zy, and you see d az because we are concerning about you know like the area which is perpendicular to the z axis. So, d az is the area surrounding the point q you know like because we are concerned about the point q when cutting plan is perpendicular to the z axis.

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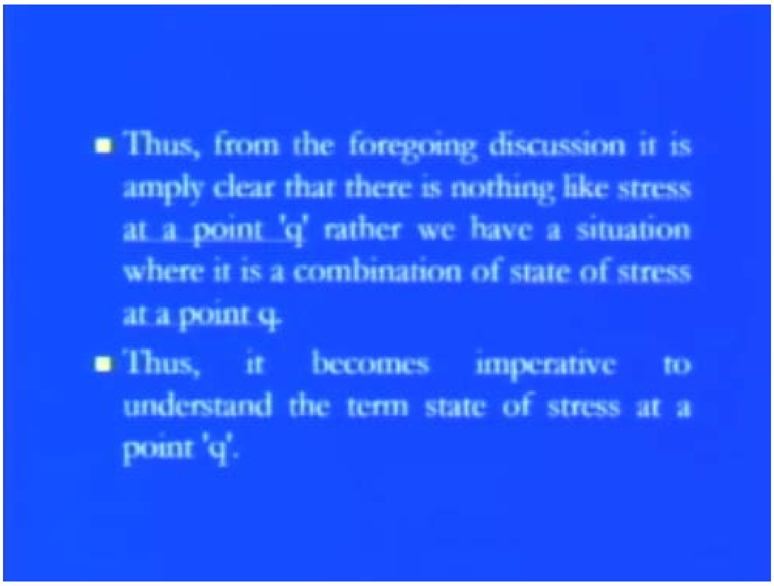

So, it is a clear meaning that you see from the foregoing discussion, it is amply you see you know clear that there is nothing like stress at a point q; means we cannot say that, okay, now like the force is there. Force is very you now like the phenomena in which we are just taking as I told you that actually what the directive part is is there. That means what is the point of application of these things are there, but it cannot be true for the stress. Because once we apply the force there is something you know like going on in the molecular weight.

So, once you apply something like this one. So, once you apply the force on any object this force is now okay, but what happens suddenly then in the object? We have to be very careful, because you see if you want to design something once the impact is there or even the regular forces are there. We have to be very careful that actually what does happen inside the object? Whether you see only the stresses are there or whether you see whatever the internal resistive forces are there, they can sustain these kind of impactive forces or the regular forces.

So, rather to discuss about a simple stress at a point q, we should discuss that actually what is the combination of the state of stresses; means what is the combination of the stresses or we can say the generalized state of stress at particular point. Means you see it is the combination of various stresses. Thus it becomes imperative to understand the term of state of stress at a point q.

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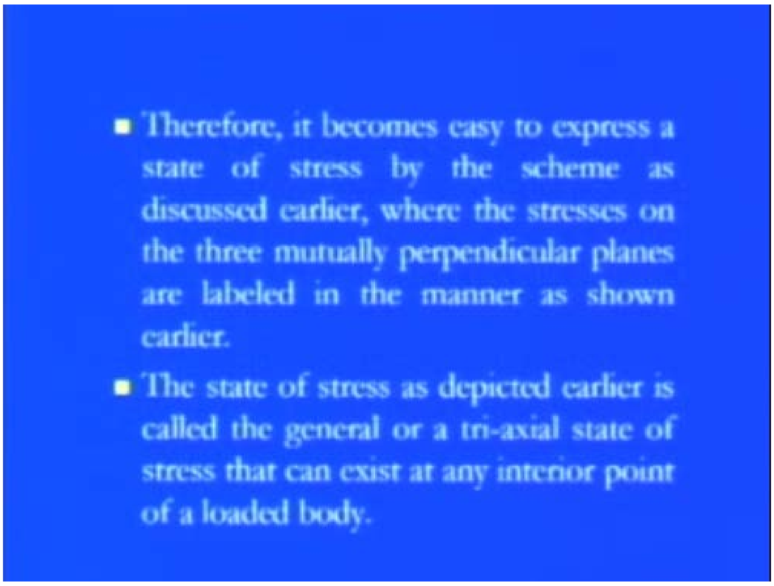

So, like you see here now if we go in that then; therefore, you see it becomes easy to express state of stress by scheme as discussed earlier you see, because we have seen that actually there are you know like the corresponding force directions are there d Fx d Fy d Fz in the x domain. Similarly, you know like in the y domain similarly we have the z domain of these forces three different forces d Fx d Fy d Fz. And we have found that in every domain the nature of the forces are being different because of the different directions of the stress components.

That is what you see you know like we can see that if you are choosing you know like the three dimensional part, then almost there are nine different components are there. That means you see you know like if we want to define a stress at a point, the single stress does not give you the clear understanding of the real phenomena. So, what we should do here? We need to check that how these generalized stresses are being formed within those things. So, the state of stress as depicted earlier is called general or triaxial state of the stress that can exist at any interior point of a loaded body.

So, that is the those you know like the generalized definitions of those things and it gives you, you know like the new phenomena of the stress that is stress we cannot say that stress is a scalar quantity or a vector quantity. Because in the vector only you see you know like just the similar directions are there or the scalar, there is only the magnitude is there. But in that you see it is now you see you know like it is going in the 3 into 3 matrix or we can say it is going in the different directions as well as the domains.

So, now we need to define the stress in a different manner. So, you see now you know like as there are two different types of the coordinate systems as we have discussed you know like in the first lecture that one is Cartesian coordinate in which we have chosen X Y Z, and other one is the polar coordinate in which we have chosen you know like we are generally using r theta and z.

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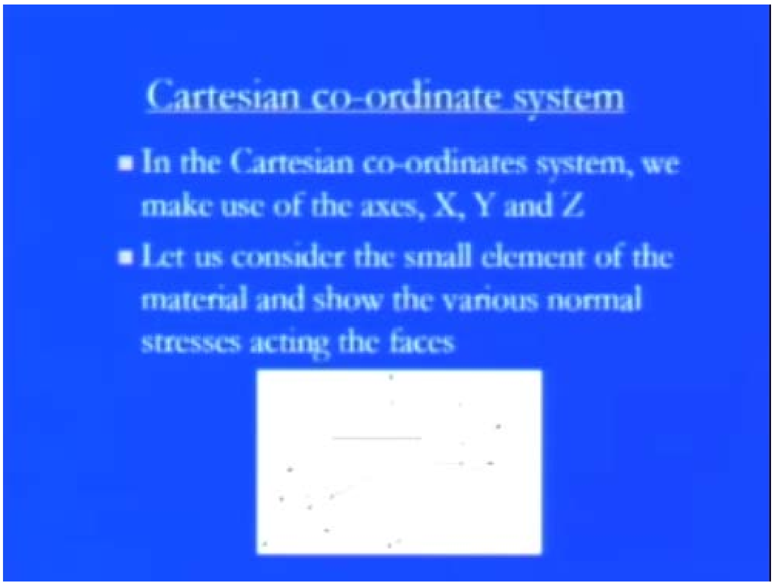

So, first if we discussed about the Cartesian coordinate system about the stresses because as we have seen that actually these stresses are being setup differently in these different directions like in you know like in X Y Z. So, now just consider these things that in the Cartesian coordinates again we are using three axis's X Y Z, and these three axes are mutually perpendicular to each axis and they are like this one. This is my X axis; this is you know like the Y axis, and this is Z axis which is perpendicular to the X Y plane, okay.

And now whatever the stresses are being setup which you can see even here like you see we have this one; the X is there, this is Y is there, and the Z is there which is perpendicular to this X Y plane, okay. So, now here we are also considering the unit element of the parallel pipe; that means you see you like why we have chosen this parallel pipe or we can say this unit cube, because in the cube we have the six phases and all the six phases are having the unit length, and it is symmetrical in all directions, because means that you see if you are talking about the three direction X Y Z. So, it has six phases.

So, three phases which are with the X YZ direction; in the frontal side they are having the unique feature just exactly like the rear part; means it is a symmetrical object having the unit width. And whatever the stress components are there, we can assume because if you have seen previously that we assume that whatever the area is there that is the uniform area. Because if irregular bodies are there, then what we need to do? We need to discretize the domain into different different domains and then integrate those things.

Because you see as we have discussed in the previous class that there is the stress concentration is there; means you see if we have a weakest region or we have some sort of you know like the cracks or some different kind of you know like the domain is there. That means the stresses are being more because of the force per unit area is there. So, area is always you know like an important part if you want to compute the stress. So, here also you see the basic reason to choose this unit cube or parallel pipe is simply because it is symmetrical in all three directions, or we can say all six phases you can see this one; this is 1 phase, 2 phase, 3 phase.

So, these three phases which we can see and the just opposite to these thing like this one or back of this or lower of this the other three phases; they are exactly similar to this diagonal; that means you see you know like this is a symmetrical object in which we can simply assume that how this stresses are being setup. Now as we have chosen three directions as you see X Y and Z. So, we can see that actually this sigma X, okay, which is a uniaxial stress because it is a normal stress component. So, this is sigma X, this is sigma X, okay.

And in Y direction this is sigma Y and sigma Y, and this is the sigma Z sigma Z which is perpendicular to the area of this X Y plane. So, these three you see you know like the small element which we have you know like chosen here of any material in these all three stresses, the normal stress components are well set up, and these are set up within these corresponding directions as it is shown in this diagram.

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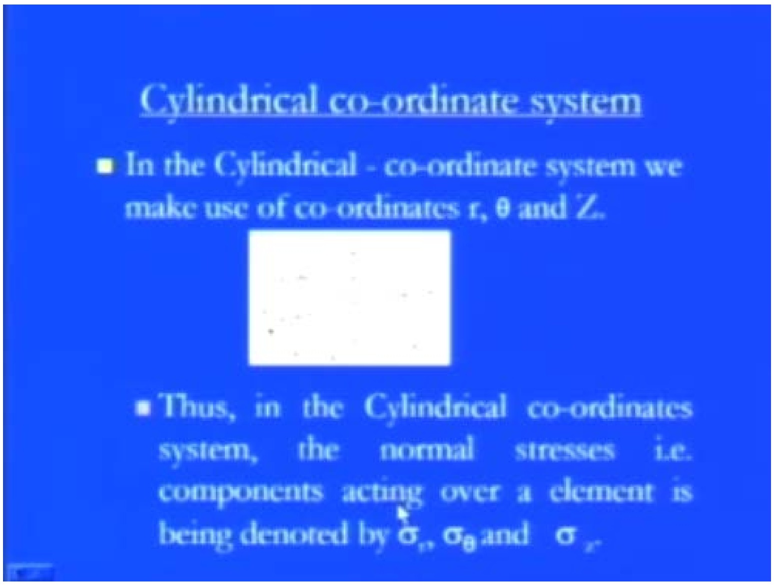

Now you see the other coordinate system as I was discussing about the cylindrical coordinate system. So, this is you see the different domain now which we have is r r theta and z. That means you see you know like we have just used here the Z coordinate, because the Z coordinate is there exactly in that, but we have the other two coordinate because of the cylindrical system which we are using here. So, we have the radius of that as well as this theta which is the angular displacement of this. So, here meaning is that if you see this diagram, then we have this Z axis, okay, which is exactly perpendicular to the other 2 axis.

That means we have this one; again if you go to this we have the Z axis, okay, and we have this r axis. So, this is Z, and this is r, and this the theta which is just revolving in this side, okay. So, now if you take a small segment of the cylindrical coordinate then you will find that we have as I told you the sigma r. So, this is sigma r which is a uniaxial stress in X direction. So, we have the normal stress component sigma r; we have another you see you know like the normal stress component that is the sigma theta and in the vertical direction as we have chosen the Z direction. So, we have sigma Z.

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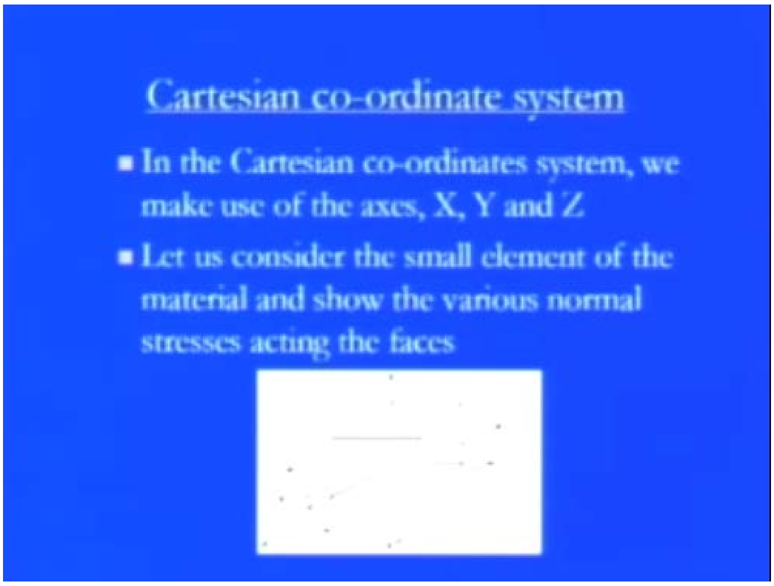

So, you see here either irrespective of you go to the previous slide, then we have the sigma X sigma Y and the sigma Y top and sigma Z. So, here the directions are just like this Cartesian way.

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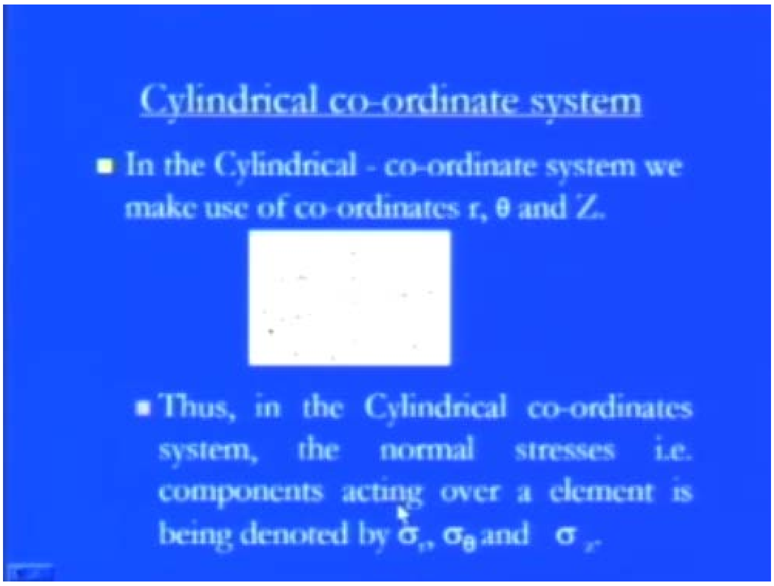

If you go this slide then you will find that we have r theta Z where the sigma r is there in this direction. So, we have to be very careful while we are choosing that which you know like the coordinate system we are using. So, if we are using the X Y Z then X Y Z, the Z is just perpendicular to this X Y plane; that means this one, this one. So, irrespective of this or this; this is my X Y plane, and this is just perpendicular to this direction or that direction.

While here you see I am just taking the vertical component of the Z, and r is the circumference radius of the cylindrical coordinate, and the theta is the angular location of these things. So, if you see these things then you will find that, okay, so these components and these other components how they are being setup within the cylindrical body. So, this is like that. So, this is sigma theta sigma Z and sigma r is there. Thus in the cylindrical coordinate system, the normal stress is because here we are only concerning about the normal stress component where the force is perpendicular to the concerned matter of area. It is the normal stress that is the component acting over element is being denoted to sigma r sigma theta and sigma Z.

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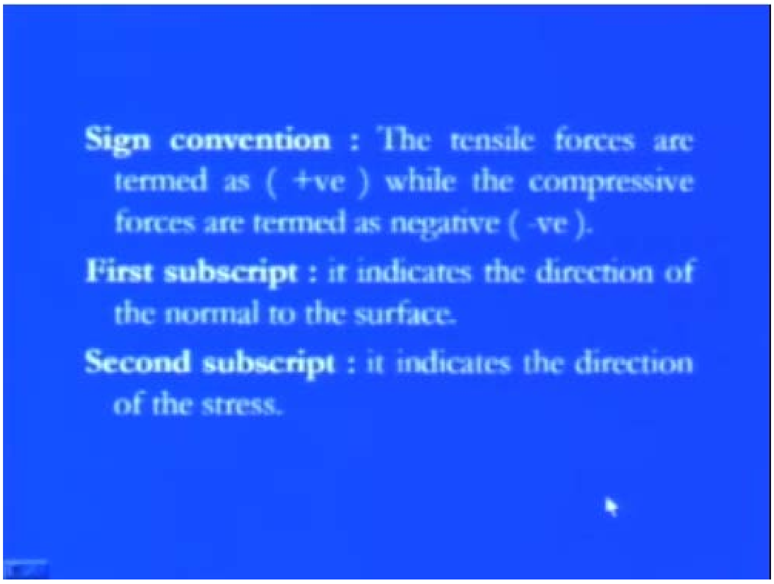

So, here you see you know like these two coordinate systems are well accepted in the universe, and that is why you see either we can go if the regular geometry is there; that means you see the square is there or any of you see the rectangular bar is there. It is pretty easy for us to analyze the stress components in Cartesian coordinates, but if we have you know like the surfaces like if aerofoil is there or any radial geometry is there this kind of thing. And if you want to analyze those things like you see you know like this turbine blade is there or aerofoil kind of you see the aerodynamic blades are there or any kind of you see this geometry always we need to choose the cylindrical part the polar coordinates. We can see this is polar coordinates also.

So, the polar coordinate system in which r theta Z is the coordinate system is. And then you see if you are you know like going for the sign convention of that as we discussed that actually that the stresses are being inducing due to the application of force. So, what is the point of application of the force is. If we have a regular or irregular geometry it does not matter you see, but if the force application is pulling; means you see the tensile forces are there always you see there is an extension is there, because whatever the forces are there the internal forces which are being setup within the object, they always tries to resist that pulling; that means they are well setup to oppose it to the pulling side.

So, we can use the sign convention for the tensile forces or the tensile stresses as the positive while in the compressive because it just tried to oppose it the tensile. So, that means you see we are now compressing the object. So obviously, whatever the internal forces are setup within the object, they are always try to repel that part; means you see we have whatever the resistive forces are there, they are just coming; the nature of these forces are just towards the outside of the object.

So, we need to choose because of the compression; compression is there the compressive forces or we can say the compression stresses as negative way. So, you see these are if we are talking about the normal stress component, then always we need to choose that the tensile stresses positive and the compression stresses as negative. But in that you see you know like if you just go back to this concept, we found that either if you are working in X domain or Y domain or in Z domain, the stress components were sigma; if it is in X domain then sigma XX tau XY tau XZ.

That means you see in these three stress components, there are two subscripts. One is you know like if we are talking about the sigma XX, then it is XX or tau XY or tau XZ. So, in that the first subscript indicates the direction of the normal to the surface. That you see what exactly the direction is what is the domain of the surface is how the cutting plane is cut. If it is cut in perpendicular to the X axis; that is what you see in the first case we have seen that all those first subscript sigma XX tau XY tau XZ is always X is there, because the cutting plane is perpendicular to the X axis, or similarly you see if you go the Y direction then you will find that the cutting plane was perpendicular to the Y axis.

Then that is why you see these three force component either this d Fx d Fy d Fz which you can compute as this tau YX sigma YY and tau YZ, okay. Means the first subscript is Y, because the cutting plane was perpendicular to the Y axis, or similarly you see if you go to the Z then also the similar terms are there for d Fx d Fy and d Fz like we have the tau ZX tau ZY while sigma ZZ, okay. So, means the first subscript just gives you that what exactly the direction of the normal to the surfaces; means you see what the domain is there or what is the area of the concern is.

But second subscript indicates the direction of the stress or I should say the direction of the force, then what is the point application of force is resistive force, okay. So, here you see you know like we can see that if domain as well as the directive stress or the directive force is in the same direction, then we have the normal stress component, because there is no parallel you know like the forces are there in the system; that means there is no sharing is there only irrespective of whether we are pulling or we are compression. Only these two uniaxial part is there, because it is the same axis.

The domain as well as the force in the same; means we are working in the same domain and the force is also there in the same domain. Means whatever the stresses are being setup in a sigma XX or sigma YY or sigma ZZ is on both in the same domain either the direction as well as this kind of this area is there, okay. But if we are talking about the other two components like either this shear stress component if we are working in the X domain like tau XY or we can say tau XZ. Then always we found that the force is always in the Y direction, or force is always there in the Z direction, but the domain is same.

So, you see here they have a real meaning or sometimes we can say that the normal stress, there is no need to define sigma XX sigma YY sigma ZZ. You can straightway go and simply you know like describe those things by sigma X sigma Y sigma Z, okay, because we know that the nature of force as well as the domain they have the same meaning, and they are working in the same area of the matter of area is concerned. So, like here we can simply you know like just based on our assessment.

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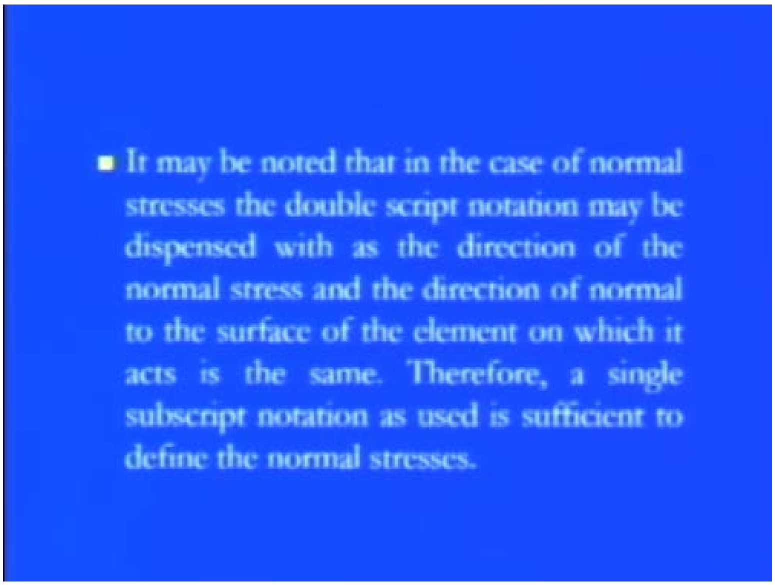

So, it may be noted that in the case normal stress as we have discussed just you see the double script notation may be you know like dispensed with the direction of normal stress, and the direction of normal to the surface of element on which it act is the same as I told you sigma XX sigma YY sigma ZZ. Therefore, and that is what you see we are using always. If we want to describe any stress in the normal components, always we are going for sigma X sigma Y sigma Z, while in other components always we need to choose that actually you know like how these other stresses are being set up and for that like for the shear stresses, because it is a plane stress you see; this stress is plane stress not the axially stress.

So, always we need to you know like the subscript just to show that what exactly what is the domain and what is the direction of the force is. And that is why you see all always the shear stress components are being you know like discussed in the two coordinate systems.

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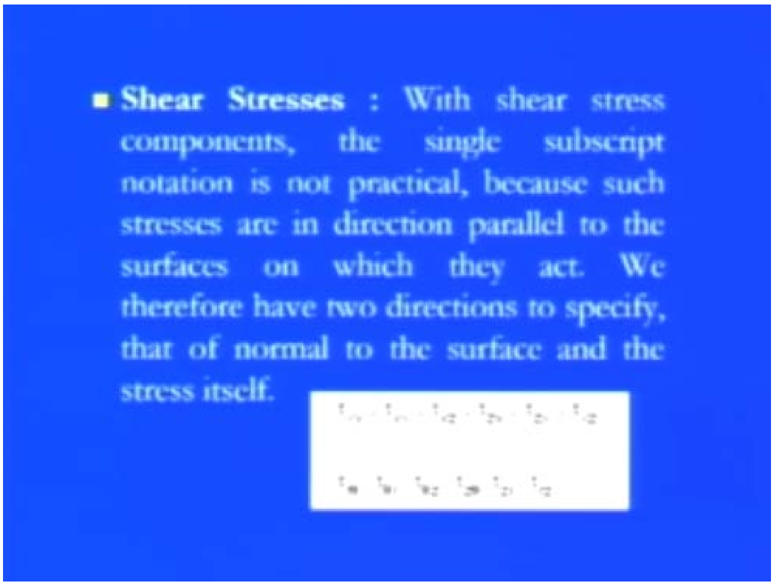

As you see here in this slide we can see with the shear stress component, the single subscript notation is not practical as you know I discussed, because such stresses are you know like in the direction of parallel to the surfaces on which they act. So, we therefore have two directions to specify that of the normal to the surface and the stress itself; means what is the domain and what is the direction of the force is. So, like you see here we can see that if you are working in the Cartesian coordinate, then we have like this shear stress component. If the X domain is there then this is sigma XY and sigma XZ.

So, this for the X domain, and if you are working with the Y domain then we have this tau YX and tau ZX and this tau YX and tau YZ. And if you are working in the Z domain then we have the two differential stress component that is the tau ZY and tau ZX. So, here you see that things are pretty simple because you see you know like there is now need to describe the two subscript for the normal stress component, but that is always essential to describe two subscripts for shear stress, because it will give you a clear-cut physical phenomenon about the shear stresses that how they are being well setup within the system irrespective of whether it is a Cartesian coordinate, or if you are talking about this polar coordinate or the cylindrical coordinate in which the r theta Z is there.

So, we can say that if you are working in this r domain; that means you see the cutting plane is just perpendicular to the R. Then we have this tau r theta or the tau r Z, or if you are working about the theta domain, then we have tau theta r or tau theta Z or if you are working or we can say if our domain is the Z which is just perpendicular to the normal surface of the Z domain. So, we can say tau Z theta and tau Z r. So, these are you see you know like the six different stress component which are there, and for which we need do describe in two different domains irrespective whether it is a Cartesian coordinate system or whether it is a cylindrical or we can say the polar coordinate system is.

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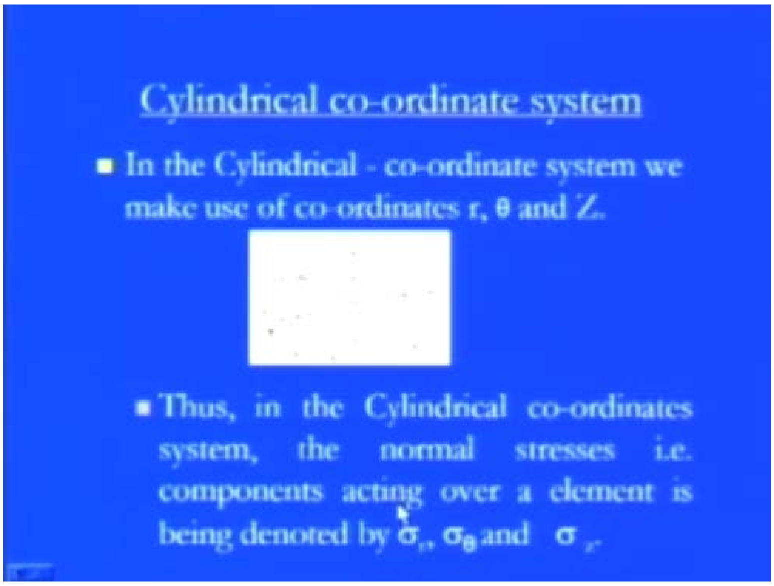

So, this is you see the normal stress in the previous case you see here this normal stress component is there or like here you see in the cylindrical coordinate or in this axis we can see that here the sign convention.

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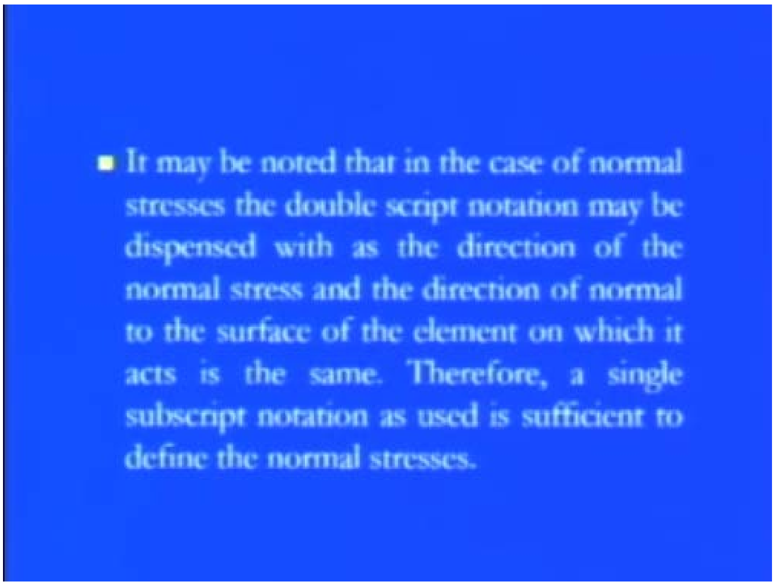

And we can see that actually either the normal stress component or the shear stress component always we need to go with the physical concept of this.

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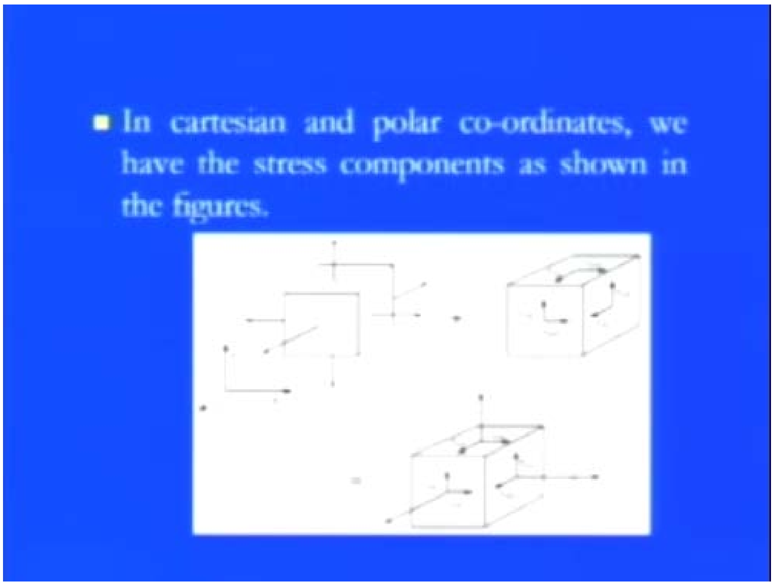

So, now you see come to the main point that in the Cartesian or the polar coordinates, if we want to combine the stress you know like we have the normal stress component, we have the shear stress component, how we can combine on a unit cube. Because you see you know like as we have discussed that the unit cube is a very symmetrical object. So, here you see just focus on this diagram which has all three mutually perpendicular axes;

the triaxial part is there in its all three you see the stress components are there in X as well as Y as well Z.

So, if you are working in the Cartesian coordinate system then you can see here, these are my three different normal stress component. See very carefully here we are only considering the tensile stresses, because they are all going towards this outward direction, okay. So, here this sigma X which is you see in the X direction, sigma Y which is you see in the vertical direction as you see and sigma Z which is exactly in the Z direction. So, this is my normal stress component, okay, which are well setup within this unit cube and if I add these things with the shear stress component.

So, you see here if you are working in this domain which is my z domain as you can see this is my Z domain. So, in that Z domain what the shear stress component are tau ZY because of Y direction is top of side. So, tau ZY because the Z is this is the area where you see the cutting plane is there the normal to the cutting plan. So, this is my tau ZY in the vertical and this is in horizontal way tau ZX. Similarly, you see if you are going towards this one where you see this X domain is there and sigma X is there. So, in the X direction you see we have two Shear stress component tau XZ and tau YZ.

So, here you can say that actually XZ because it is going in the X direction the stress is there, and it is going in the Y direction the stress is there, okay. So, this is showing the direction other subscript XZ and Z, but here we are always talking about this, okay. And if you are you know like going towards the other domain then we will find that you see here we have tau YZ as well as the tau YX, okay. So, as we have seen here that as well as the normal stress component or this shear stress component if you want to combine those things in these three domains XY and Z domain. So, we can straightway combine and the combination of these stresses because you see now we have six components from the shear stress; we have 3 components from these normal stress, okay.

So, we have now total the nine components. And if we want to define these nine components here you see this is the real diagram in which we can see that how these normal stresses as well as the shear stresses are being acting. And what is the combination of these stresses at a particular point in these three domains. So, you see here if you are talking about the Z domain then we have sigma ZZ tau ZY and tau ZX, and if you are talking about the X domain. Then we have sigma XX tau XZ and tau XZ and tau XY, or if we are taking about this Y domain in which you see we have the sigma YY tau YZ and tau YX; means that you see you know like these are the real nine components just to describe the state of stress in the Cartesian coordinate. Similarly, you see you know like the similar nine components we can also describe in the polar coordinates or we can say the cylindrical coordinates in r theta Z.

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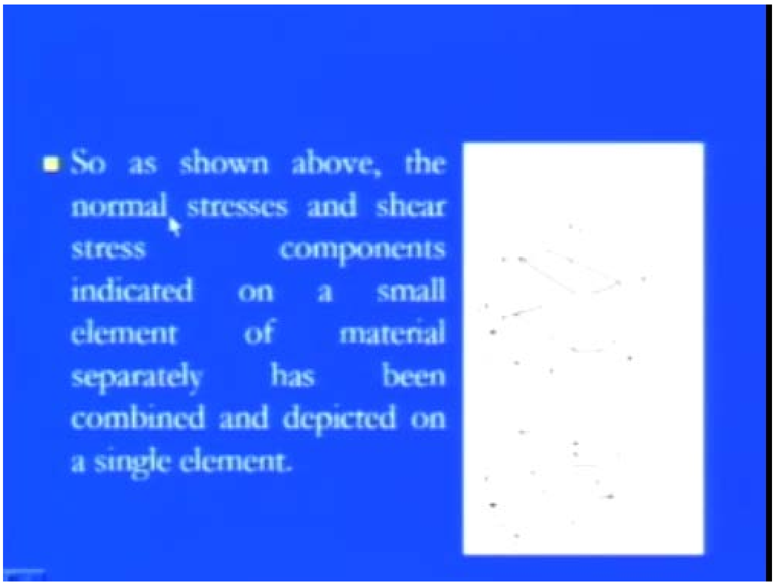

So, you see here we can see as shown above the normal stress and the shear stress components indicated on a small element, okay, in this material separately has been combined and depicted on a single element. So, here pretty simple; these are my normal stress component which are acting in r this sigma r, okay, or sigma theta or sigma Z. And if you go to the shear stress component then you have if you are working on this theta domain, then what we have tau theta Z and tau theta r. If you are working in r th domain then we have sigma r Z and sigma r theta, and if you are working in Z domain then we have tau Z theta and tau Z r. So, now means that we have nine different components either irrespective of the Cartesian coordinate or irrespective of this polar coordinate; we can simply describe the all nine components combinedly.

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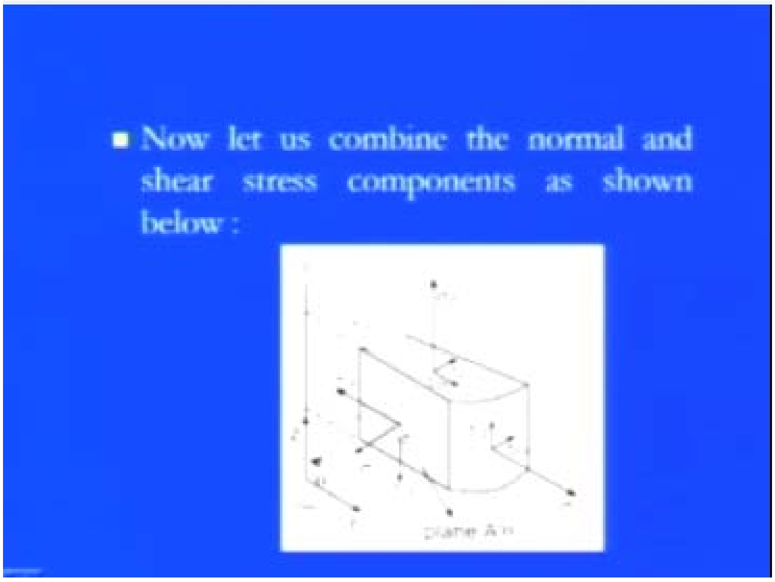

In this polar coordinates also or cylindrical coordinate like this; this is you know like as I told you the theta domain. So, in the theta domain we have sigma theta theta or also I told you that there is no need to go to the two subscripts. So, only one subscript is essential to describe the normal stress component. So, we have this sigma theta here sigma r here or sigma Z is here in theta r and Z domain for normal stress component, while for other two shear stress component if you are there in the theta domain, then we have tau theta r and tau theta Z. If we are in the r th domain then we have tau r Z and tau r theta; if you are in the Z domain then we have tau Z r and tau Z theta. So, here means all three directions in every direction or in every domain we have three different components. So, all and all we have total nine components. So, if we want to define the state of stress at least the minimum components which we need are the nine components as we have discussed in the Cartesian and the cylindrical coordinates.

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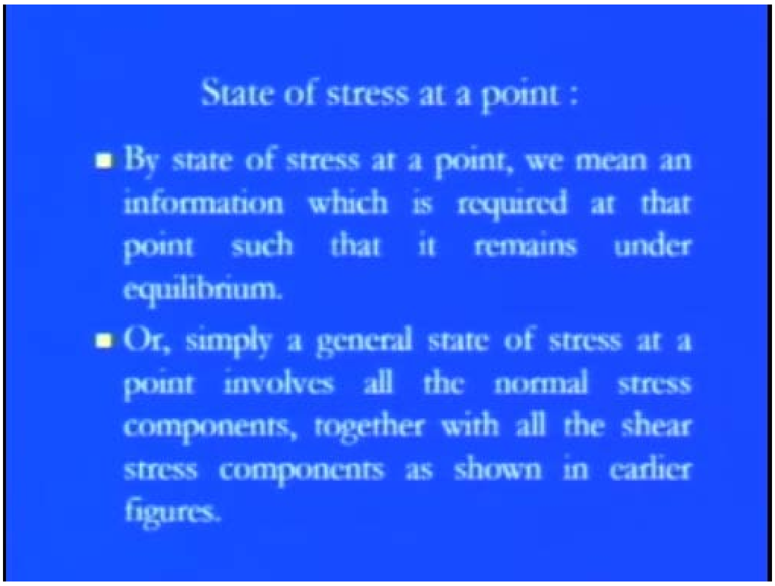

So, by the state of stress at a point we mean information which is required at that point such as it remains under equilibrium positions under the application of these forces, or simply a general state of stress at a point involves all normal stress components together with all the shear stress components as shown in the previous figures. That is what you see we have if we are talking about a general state of stress for any point or for any object; at least as I told you the nine components are there which are involving three normal stress components and six shear stress components.

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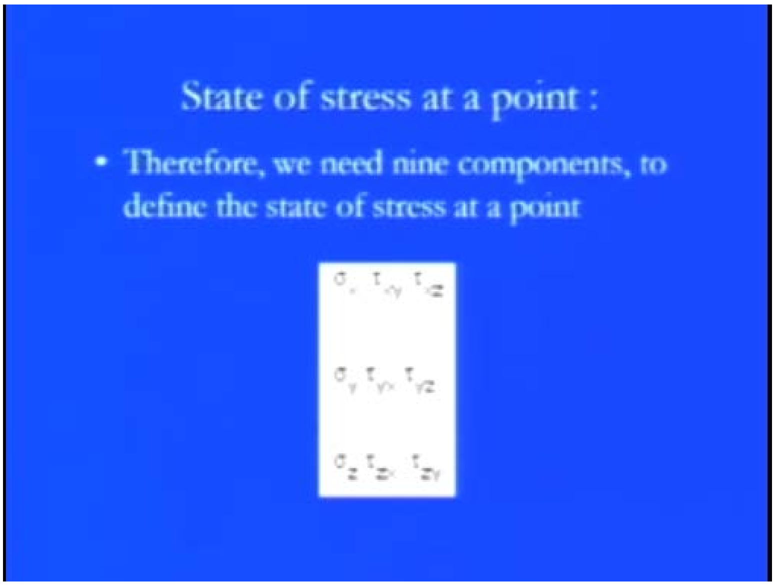

And then you see you know like this is a general phenomena that the state of the stress at point always needs nine minimum components to define the state of stress at a point like you see here the sigma X here you if are working in the X domains. So, this is for the X domain sigma X tau XY tau XZ, if you are working with the Y domain sigma Y tau YX tau YZ or if you are working in the Z domain then we have this sigma Z tau ZX tau ZY; means here if we are discussing about the stress component at a particular point always it needs nine components means you see all three different directions are there.

So, the stresses are always known as the tensile stress; that means you see it is neither scalar quantity nor vector quantity; it is a tensile quantity. So, that is what you see to describe up stress at a point we need at least 3 cross 3 matrix, or we can say the total nine elements for which we can say that how the stresses are being setup within the object, or how they are playing if this load application is there on an object. So, you see now it is a general phenomenon of the stress. Now you see because as we have discussed in the previous case we can see here if you see this figure, this figure is well symmetrical figure in the polar coordinate.

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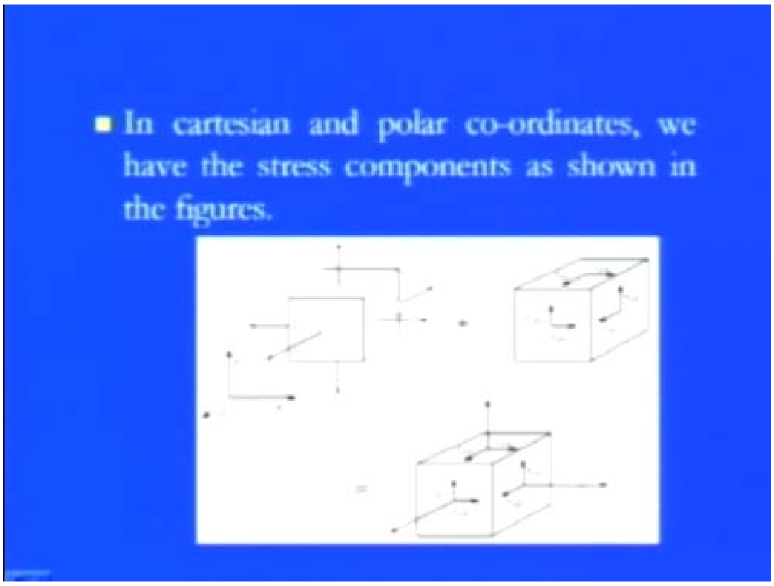

Of if you go to the Cartesian coordinate this figure or we can say this unit cube is well stabilized or we can say well equilibrium figure in which the symmetricity is there. So, if we are talking about these kind of things in which the symmetricity is there.

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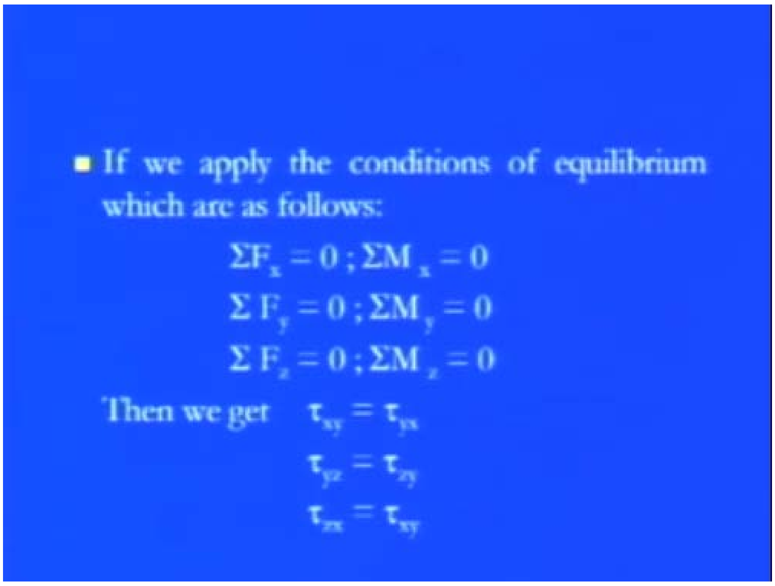

Then we do not have to need all those nine components to describe this stresses. So, what we can do here, we simply apply these symmetrical conditions to these equations, and we can reduce all nine components. Because you see at every point if we are analyzing or if we are calculating these nine components, then it pretty it is pretty you know like tough to check the stress every time. Because you see if the load application is changing how these nine components are changing, or you see you know like which one is dominating, which one is not dominating is pretty tough.

So, you see what we are doing here to make simplicity you know like what we are applying certain equilibrium conditions or we are choosing the elements, so that actually it has certain symmetricity towards either the X direction Y directions or Z directions in the Cartesian coordinate or r direction theta direction or Z direction in the cylindrical coordinate. So, here you see now see the first case if we apply the conditions of the equilibrium like if we have a point over which the summation of all you know like the forces in X direction is 0 or the summation of all forces in Y direction is 0 or summation of all forces in Z direction is zero; means under the force application there is you see no moment or there is no deformation is there within those object.

Since we apply the force, there are internal forces are there, and they are well established; that means you see our object is in equilibrium phase. Means that all the summation of forces internal forces if they are coming out of that or the external forces they are applying the summation of all the forces in X direction Y direction and in Y direction and the Z direction they are zero, or we can say if we take a point like q among these point means if we have pivoted this point q; you know like if we take the moment about these point in X direction like summation of these Mx or My or this Mz. That means if you are taking the moment about X Y or ZX, if you sum up those points if it is zero we can say that it is well under equilibrium conditions.

So, for that since you see here the two things are there; one, whatever the forces or the moments which are applying on the object they are well set up, all summation of the forces as well as the moments are zero, and we have the unit cube which has the symmetric geometry. So, if you apply those conditions then we can conclude that the shear forces which are applying in X direction tau XY or in Y direction tau YX; means you see now what we are doing here? We are simply changing the direction of force and we are simply changing the direction of force as well as the domain.

So, if we are talking about those thing then we can say that the tau XY is exactly equals to tau YX, because you see if you are talking about the frontal part and if you are talking about the real part they have the symmetricity and there is no change in the force; we can say whatever the parallel forces are there which are being set up in two different domains as well as the two different directions, they are equal. Because they are equal and opposite that is why you see we can say that the summation of the forces are equal. So, similarly you see if we apply the symmetricity to this object we can say tau XY equals to tau YX, tau YZ is equals to tau ZY and tau ZX equals to tau XZ.

So, you see here all these conditions are applicable when we have a symmetric geometry first, and when we have this summation of the forces, the reactive forces and the applied forces in X Y and Z are equal. Because that is why you see the Newton’s third law says that if reaction and actions are equal, then you can say, yeah, the body is in equilibrium position. Otherwise, there is always when you apply the force either the deformation is there, motion is there, whatever the things are there. So, we cannot apply these conditions here like that straightaway, okay. So, now you see if you know like conclude that part, then we will find that there are six components instead of nine components.

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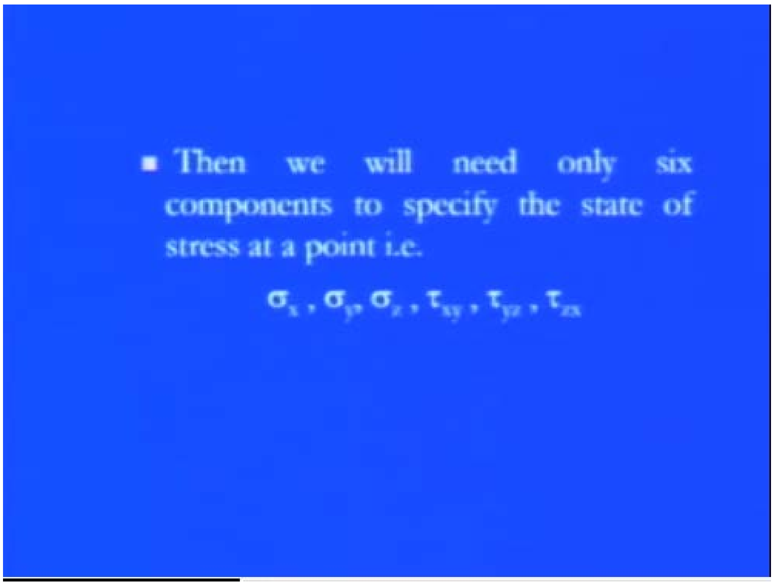

Only we need three normal stress component sigma XX sigma YY sigma ZZ or we can say sigma X sigma Y sigma Z in the normal stress component or we need only tau XY tau YZ and tau ZX for that. That means you see if we are talking about 2 dimensional way only you see sigma X sigma Y sigma Z three different directions stresses are there, and we have you see the three different components of the shear stress like these things. And they are well setup if we have the two previous conditions are applied. Means you see now you know like just six components are you know like well defined and we can say that actually the tensile stress is well defined if the object is symmetric by six different components, three normal stress three shear stress component. Now you see come to another point in the shear stress only that complimentary shear stress.

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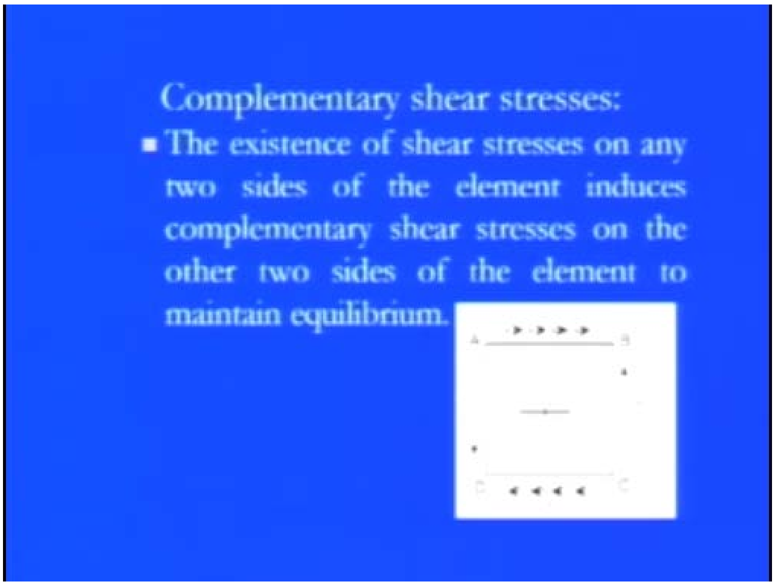

You know like as we have discussed that the shear stress is a plane stress. Plane stress means you see you know like it is always acting in either XY YZ or ZX plane as we have seen in the previous part you see here the shear stresses are there; so tau XY tau YZ and tau ZX. So, if you want analyze these things, what we need to define? The existing shear stress on any two sides of the element this that or whatever you see in all three sides of these things induce a complimentary shear stress on other two sides of element to maintain the equilibrium; like you see here if you have just chosen AB and CD in which you see the tau is acting.

So, on this we have this object and the shear stress is just going across this; they are parallel forces as I told you. So, they are just going in these directions. So, they will just tend to move this object in the clockwise direction, just it will move like that. So, we require an opposite component to compensate this or to resist this motion. So, whatever the stress component because as we have already discussed that it is well equilibrium position to you know like. So, that is what we need to define a new component.

So, whatever the resistive components which are coming due to this plane stress or we can the shear stress is known as the compliment stress. So, here you see you can see that in AB or in this CD whatever the tau dash is coming this is nothing but the complimentary shear stress.

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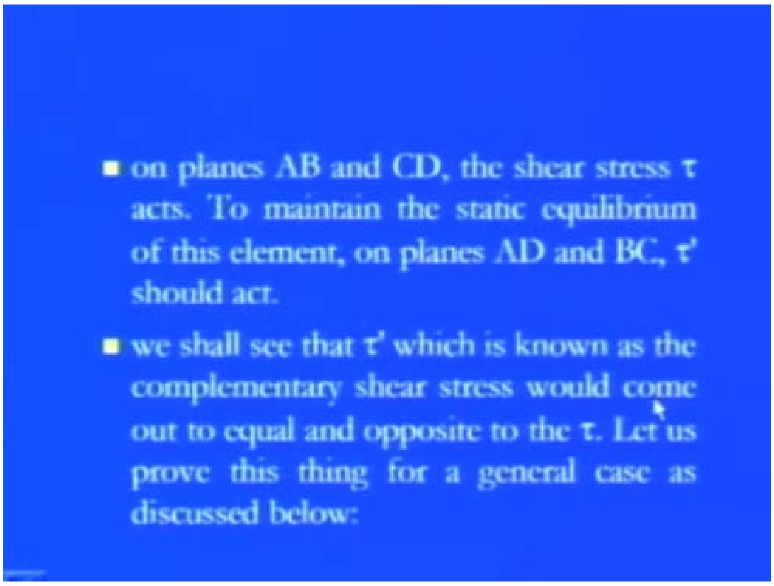

So, on plane AB or CD as we have seen in the previous figure, the shear stress tau was acting and due to that the object or the body tend to move towards the clockwise direction while to maintain the static equilibrium always we require it must be. We require the plane AD and BC must be acting towards opposite direction; that means in the anticlockwise direction. And to do that we need to you know like consider a shear stress that is known as the complimentary shear stress, and we are always denoting by tau dash. So, we shall see that the tau dash which is known as the complimentary shear stress as we discussed would come out equal and opposite to tau. Then only we can maintain the equilibrium, then only all those components whatever the conditions are there it is applicable to that.

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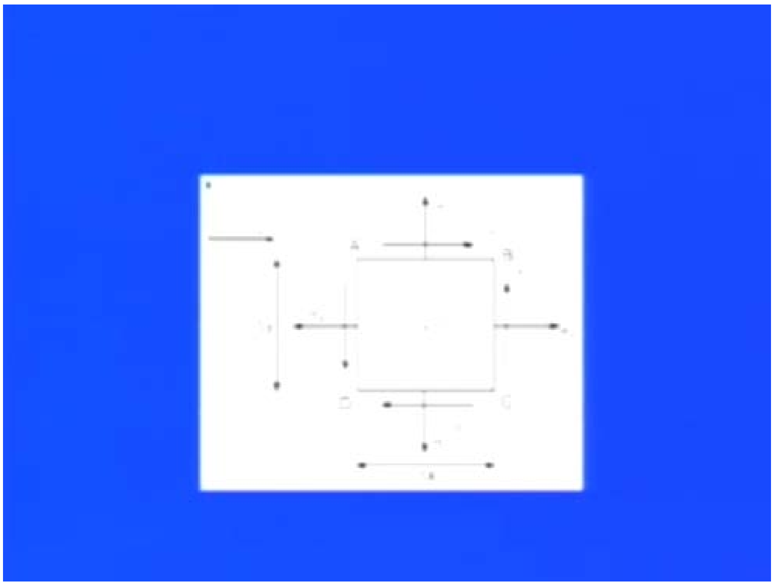

And let us prove that this thing for general case is as you see you know like this is there that if we are taking about previous case. So, what we need here only instead of nine components, six components are there; in the six components we have this you know like the sigma X sigma Y, this sigma X and sigma X in the X direction, while sigma Y and sigma Y is there, because now we are talking about a pin stress in which X and Y is there. So, whatever the stresses are being setup in XY plane, now this figure is showing here.

So, if you are talking about this, what we have? We have a unit cube which has the total width in X direction is delta X, total width in Y direction is delta Y. And if you are working in the X domain that is you see the X domain, what we have? We have two components; one is the sigma X which is the normal stress component; one, we have the tau XY you know here in the downward direction or tau XY which is the upward direction.

So, you see here since tau XY because the X is the domain, Y is the direction in upward as well as the lower direction. So, here these two components which are you know like on this side of this element in the X domain or this two elements these two elements the sigma X and tau XY are working on the other side of the element. Similarly, you see we can see that this component is well maintained because of the other two components are working in the Y direction like sigma Y is there in the vertical direction.

So, you know like both are going in the opposite direction, and they just want to maintain the equilibrium around the point A. And similarly you see we have tau YX and tau YX, because we are working in the Y domain. So, Y is there and X because they are going in the parallel to the X directions. So, these two shear stress components are there. So, you see if you are discussing about simple plane part where the X Y is there, how many stress components are there? Total we have two normal stress components and two shear stress component.

So, only four stress components which we need; no need to go for all nine you know like components, only four you know like the stress components are there; two of the normal stress sigma X and sigma Y and two tau XY and tau YX is sufficient to describe this stress component if we have the unit width sigma X and unit width this sigma Y in corresponding X and Y direction. And now you see if you are saying that it is maintained equilibrium, only possible if whatever the forces are there at around these structures and whatever the moments are there about this pivoting point O is 0.

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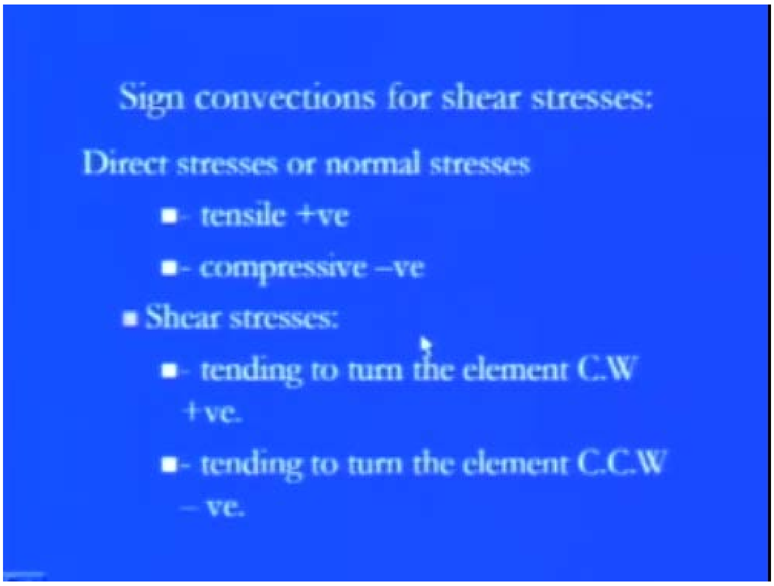

So, now you see you know like as we discussed first of all the sign conventions are there for the shear stresses, because you see for this normal stress component we already discussed about the sign convention just to remind you again that we have if the tensile forces are there with the pulling is there, we always consider the positive stress; if the compressive stress are there, we always consider the negative sign convection for the compressive force as well as the compressive stresses.

So, similarly you see you know like here the sign conventions for the shear stress the direct stress if we are talking about or the normal stress that is positive and negative for this or the Shear stress if it is tending to move towards the clockwise direction; means you see if object is moving towards the clockwise direction always see that if whatever the stress components are there which is always trying to you know like force towards the clockwise direction, consider the positive. And if you see the complimentary as we have you know like seen in the previous case, the complimentary shear stresses which just tries to oppose that part, and you know like tries to rotate the object in the anticlockwise direction. Consider the counterclockwise or we can say the negative direction is there for that.

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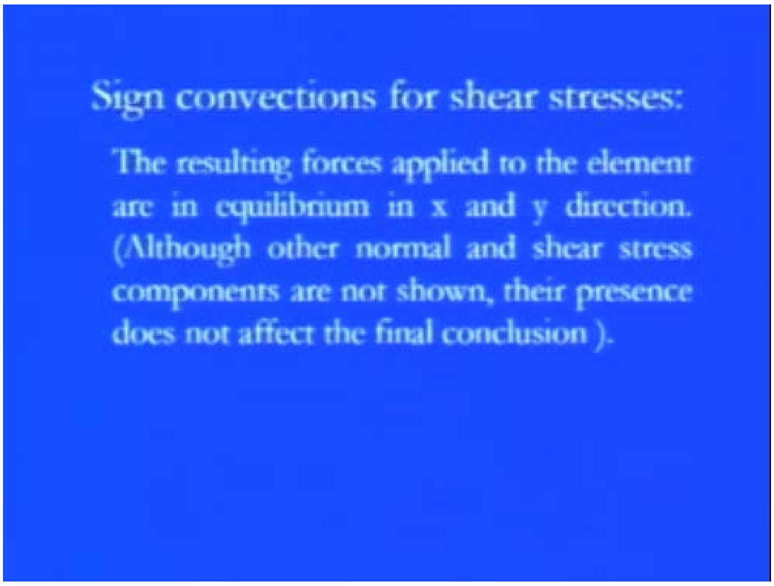

So, sign convections always gives you the resulting force applied on any element which is in equilibrium in X or Y direction. It just gives you that actually whether the shear stress is applying in the positive direction or not or complimentary shear stress is applying in the positive direction or not. So, all though you see you know like the normal and shear stress components are not you know like those shown in the present part, but you can simply conclude that actually if the resistive forces are not applying in another part then we cannot maintain the object in equilibrium manner. So, that is what you see you know like sign conventions is very very important to see those things.

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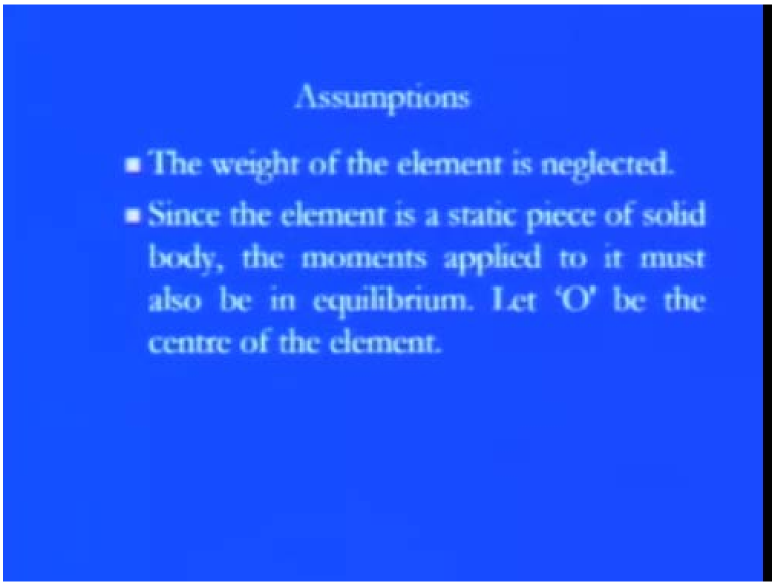

But to apply or to maintain the equilibrium position of those you see you know like the object, always certain assumptions are there. If those assumptions are applied then only we can say that, yes, this object is well maintained under the equilibrium conditions, and whatever the forces or the moments are applying they are well setup within the object. First, the weight of the element is neglected. We are not considering the weight here, because if we consider the weight, definitely, you see now due to the gravitational acceleration there is some more stresses are there.

And these stresses you see some times because you know like another force application is there and the weight is there these non-uniform stress distribution is there. And because of that you see if you want to define the stress by either the Cartesian or this cylindrical coordinate which is very, very tough and very, very complex, because the stresses are different at different points. Because of the displacement of each molecule, they are different and the resistive forces are different because of the self weight of the element. So, the first and the most foremost assumption is always neglect the weight of element.

So, that is what you see in further whatever we are going to discuss that this bending of beam or shear stress of any you know like the twisting part or torsion of shaft or any you know like either the deformation stress strain, always we neglect the weight for that of an object just to make easy of the analysis. Second, since the element is static piece because you see you know like under the application we always assume that there is any static part is there of solid body, the moment applied to you know like to on these things must be in equilibrium.

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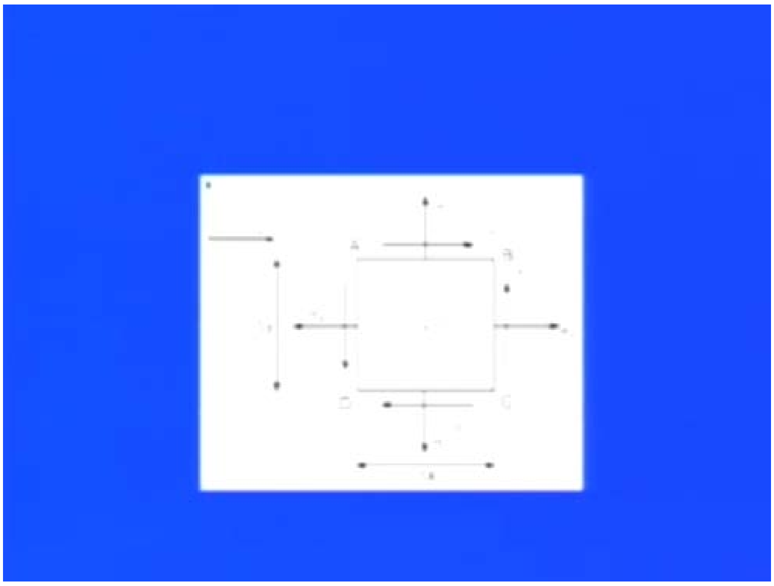

That means you see you know like as we discussed in the previous case here in this figure, whatever the moment is applying you know because these forces are there you see and if the shear stress components are working, there is a moment through this point. So, they are well set.

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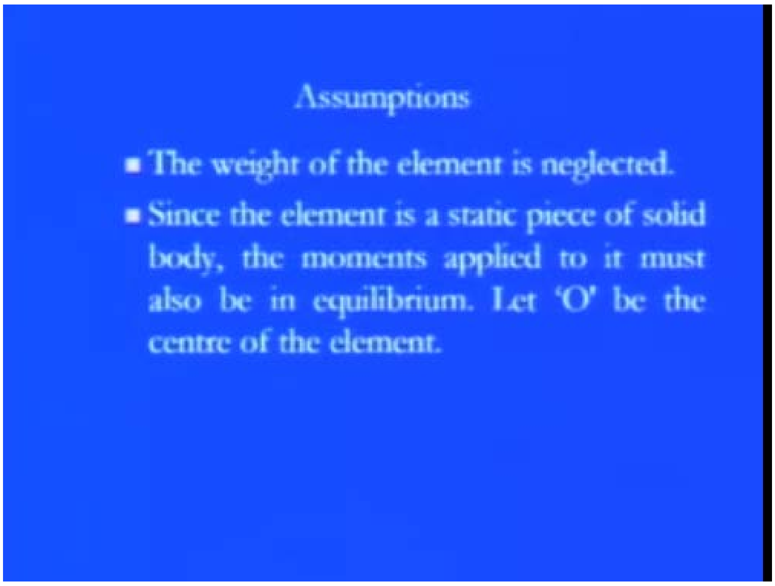

That means you see this object is in static way. So, we need to assume that whatever the moments are applying on this object because of the shear stress or normal stress component, they must be well setup, they must be equilibrium, or we can say the summation of all moments in different domain X Y Z, they must be 0.

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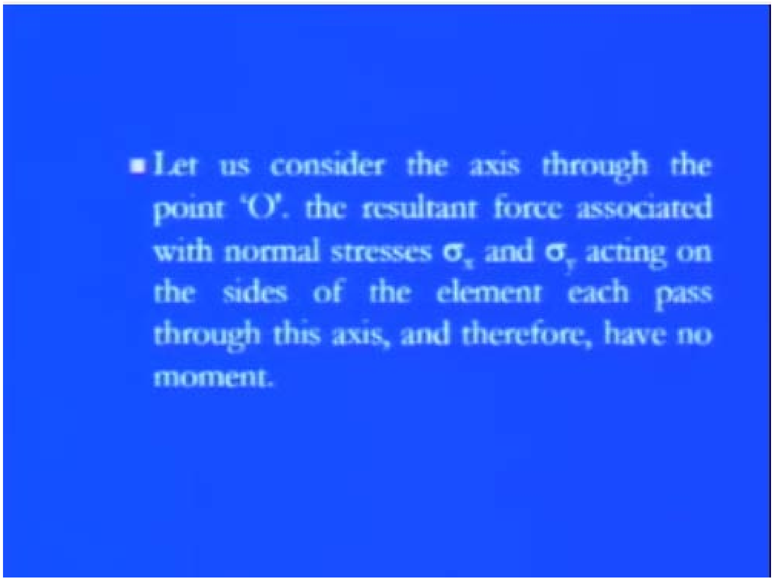

And you see you know like the third assumption is let us consider the axis through point O resultant force associated with the normal force acting on the sides of element passes through the axis; therefore, there is no moment. That is what you see I told you that whatever the moments will come in this kind of object where you see if you are considering the origin at centrally and this normal stress component in X axis Y axis or Z axis. If these axes are passing through origin then there is no moment because these axes are passing.

But as well as the shear stress component because they are the parallel to you know like these axis's or these domains, there is always you see that is what you see due to the moment they are turning to you know like rotate this component. But we need to assume that whatever you see the reactive as well as you see the active forces are there or these stresses are there, they must be well setup and the moments about O must be zero like that. So, now you see you know like we can see that now the forces on top and bottom surfaces produce a couple as I told you because of the shear forces,

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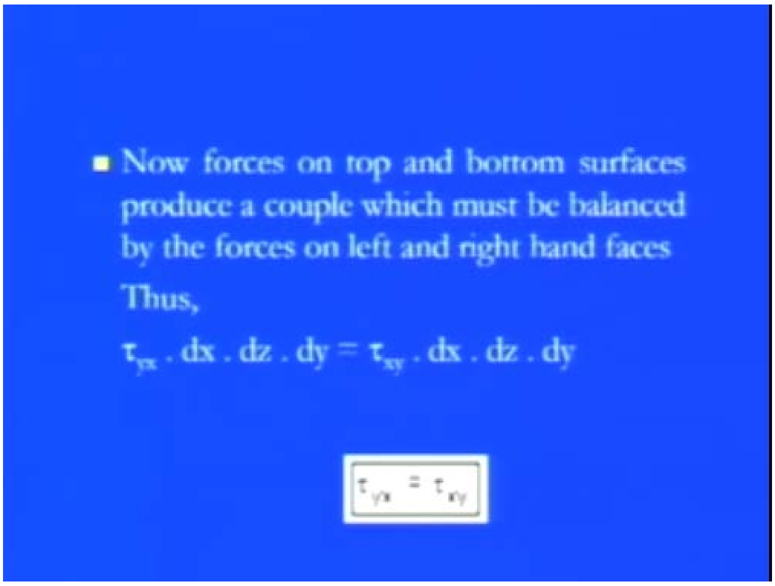

Which must be balanced by the forces like you see tau yx into the total domain dx dz and dy is equals to tau xy into dx dy and dz. So, if they are well balanced amongst these you see now if you go to back you see in this figure.

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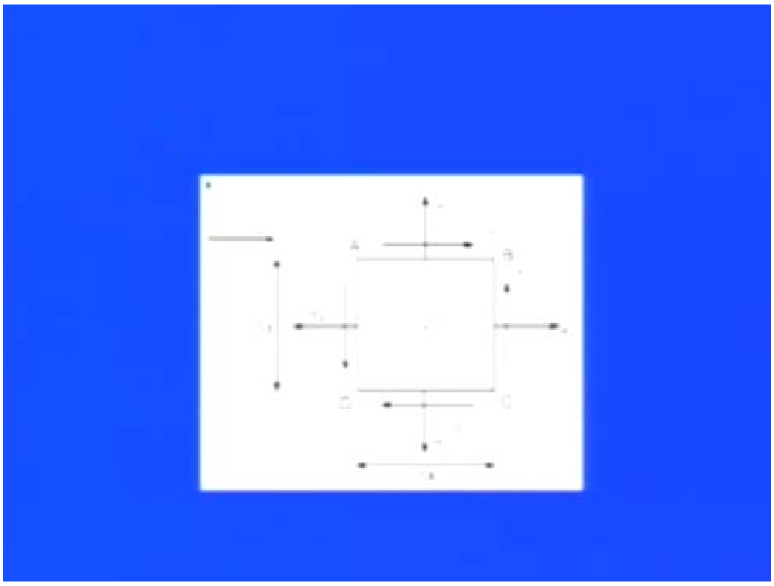

Then you will find that because of this tau xy; this tau you know like the tau yx into dx dy and dz on these particular surface or similarly you see this tau yx which is exactly you see know like in the bottom of that if it is well balanced, we can say that actually you know like it will always balance those things.

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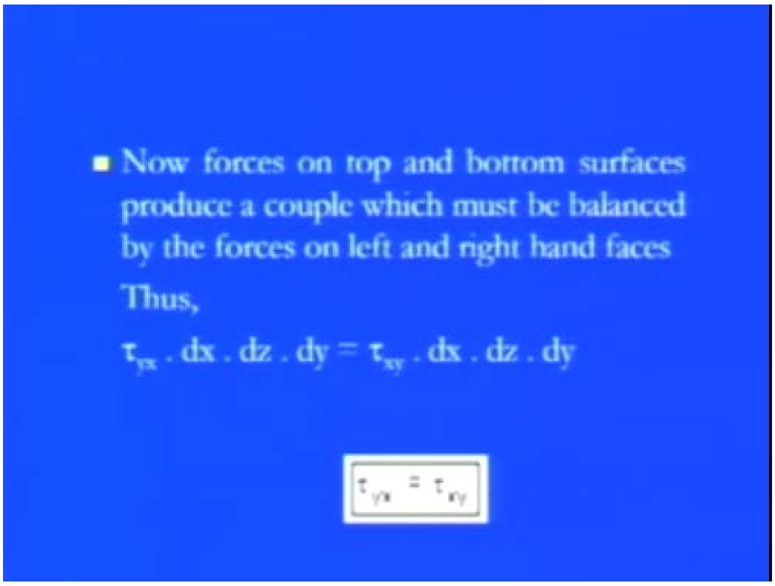

And we can say the symmetricity is there because of those things again the moments are zero, all the forces are zero. We can conclude that tau yx is exactly equals to tau xy, and this is one of the symmetric conditions that you see in the previous case we have discussed that if all the moments are 0, all the forces are zero. We can say that tau yx equals to tau xy, or we can prove by this way also, or similarly you see in other word the complimentary shear stresses you know like are equal in magnitude and the same form you know like they simply form. The relationship that can be obtained by the other two pairs of the shear stresses in the two different domains like tau zy is equals to tau yz or tau zx is equals to tau xz.

So, you see here irrespective of whether we are working in any of the domain, always the shear stress components are equal in the respective directions if we consider the symmetricity or if we consider that you see whatever the objects are there; you know like under these forces they are well maintained and the equilibrium. So, that is what you see you know like we discussed about all these forces that actually if we have a general state of stress at least we need the nine different components in which the three you know like the normal stress components are there and six the shear stress components are there, we want to describe the general state of stress.

But these you know like and even if we apply that the symmetric condition in which you see we need only six components to describe the state, three normal stress and three shear stress component. But this you know like whatever we discussed in this, the domain was very symmetric as we have seen you see only you know like at the outer part where the symmetricity is there where you see x domain or y domain or z domain these forces are well set up. But if we want to consider the stresses at the oblique plane; that means you see you know like if we cut the plane at theta angle. Now if we want to check that what the internal set up of the stresses are there then how you know like these normal stress as well as shear stress components will take place.

And you see you know what kind of analysis which we need to take you know which we need to consider, because at that point we cannot say that the moment is zero, because this is the internal part of an object, or we cannot say that all the summation of forces are zero. So, you know like what is exactly the relation are there and in between these two components of the stresses normal as well as shear stress component in xy plane or in triaxial form xyz plane which we are going to discuss in the next lecture.

Thank you, bye bye.