Strength of Materials
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Lecture – 29

Hi, this is Dr. S. P. Harsha from Mechanical and Industrial Department IIT Roorkee. I am going to deliver my Lecture 29 on the course of Strength of Materials and this course is developed under the National Program on Technological Enhanced Learning. Prior to start this course, which is basically on shearing stresses of the different cross sectional beams, like you see, if we have a rectangular cross section or if we have I section beam.

Then how we can find the maximum the shear stresses in those things and how we can say that, if we have the cross sectional beam of any kind, then shear stress distribution is of this kind and all like. So, this all information, which we are going to discuss in this lecture, but prior to start that you see would like to focus that what we have discussed. Because you see we are going to discuss, we are going to use all those theories, which we are discussed in the previous lecture.

In the previous lecture, you see we focused on two different issues of a beam; one was the pure bending, when pure bending is there, so for that you see we derived one equation that was this sigma by y, where the sigma is the bending stress. So, sigma by y is equals to m by I; and m is the applied bending moment and I was you see the mass moment of inertia was there or we can say this second moment of area, so like we can distribute according to what kind of sections are there.

So, sigma by y equals to m by i equals to e by r, and e is the Young's modulus of elasticity. So, whenever the pure bending is there, so in that we also assume that the plane will remain plane even before and after the bending. So that means you see there is no change in the plane section. So, that kind of discussion which we made and then we solve some of the numerical problem, that if we have a cantilever beam or if we have a simple supported beam. And if it is loaded with a single, you know like this uniformly distributed load or point loads are there, then how we can get the this all bending stresses for those things and you see how the distribution of this bending stresses are.

Because you see in the flanges in all those things, either we have this fibers of tough fibers or lower fibers they are experienced with the different kind of tensile and the compressive stresses, which are the flexible stresses. So, these discussion which we made in the previous lecture in the first part of that.

Then we found that if the shear stresses are there and due the shear stresses it causes warping action. So, whenever the warping is there in the beam, then the beam will not be exactly the plane of the beam, whatever you see under which the fibers are there and they are experiencing with those kind of stresses, they will not be remain exactly as the same of the plane as before and after of this stresses. So; that means, you see whatever the assumption which we made in the bending, whenever the pure bending is there here it is violating of some nature.

So, sometimes you see if you are saying that, if the uniform bending stresses or non-inform bending stresses are there are this kind of deformations are there due to the bending and the shearing. Then we cannot use those, the sigma by y equals to n by i equals to e by r, but due to the practical applications we found that their approximately equal means the warping effect has some impact on those things. But, this equations are somewhat valid for non-inform as well as the uniform bending distribution.

So, in that case you see we can use the sigma by y equals to m by i equals to e by r, then in the later section, you see we discuss that actually if we want to analyze the shear stresses for any section, then certain assumptions are there and those assumptions we applied for just to easy of our calculation. Because, you see if we are using all those the real theory, then it to complicate you see the basic bending.

Because, here in that section we the shearing forces are there at the same time the bending moments are there and you see it a complicates the system. So, we needs to if you want to analyze those things, we need to put certain assumptions to make it convenient for our easy of analysis. So, for that you see after applying all those assumptions in the shearing of the beam, which we discussed three assumptions.

Then you see we found that, the sigma whatever you see the tau which is the shearing stresses is equals to the f, which is shear force f into a into y divided by i into z. Where, f is the shear force and always you see shear force gives you the clear direction about the sign convention, that how the the beam will behave under the action of this shear forces is the area of whatever you see the area, which the concerned area or we can say the matter of the concern is there on that particular part and then y is the distance of the centroid from a neutral axis.

So, here you see this is the pretty typical thing is that when we are talking about y and y bar, y bar generally we are taking the distance between those 2 axis and y is the distance, you see generally we are talking about the where the shear stresses, which we want to calculate. Because, if we are saying that let us say above from neutral axis at you just calculate a 10 millimeters from neutral axis above or below or some other distance, it is pretty easy to calculate.

Once you know the y and, once you know the relation between the centroid as well as the neutral axis, then again it is pretty easy to stabilize those stresses within that object. Because, you see it is a kind of distribution and once you know you like here the centroid is occurs or here the neutral axis is there, then it is pretty easy to make the distribution of this kind of stresses. Because, you see ultimately distribution is very, very important for any kind of design for a beam under any kind of loading particular.

So, and then so f into a into y divided by we have i into z and i is the the second moment of inertia, which in the previous sections we discussed that actually it can be calculated by summation of a this y square into d a. So, this is the square term is there here into the multiplied by the area, so this is that is why it is known as the second area of moment or we can say that it is by integration also because, if we have a regular structure then we can also calculate this kind of thing by integration.

And into z, z was you see just the the width of a cross sectional area that actually what exactly the area, which the cross sectional part of the object we can say. So, width is giving by this z, so you see here it is influencing means the shear stresses which we are calculating it is always influencing by the shear forces and the this dimensional parameter of that, so this is which we discussed in the previous lecture.

Now, in this lecture we basically you know focused on first, that if we we have the shear stresses for this kind of thing and we are talking about the area. So, important thing is that actually what exactly the cross sectional area is because, in these shearing stresses we found that it not only depends on the shearing forces. But, also it depends on that what exactly the area is or dimensional parameter is...

So, first of all our main intention is just to focus that whatever the, area which we are concerning. So, that is why you see here in this lecture our main focus firstly is on that the what is the cross sectional area of a beam is like and that is why you see we are going to concern about two main areas, one is the rectangular cross section and second is the i section is there. And then you see will check it out that actually how the shear distribution is going on in that, so here it is you see first the shear stress distribution in typical cross sections.

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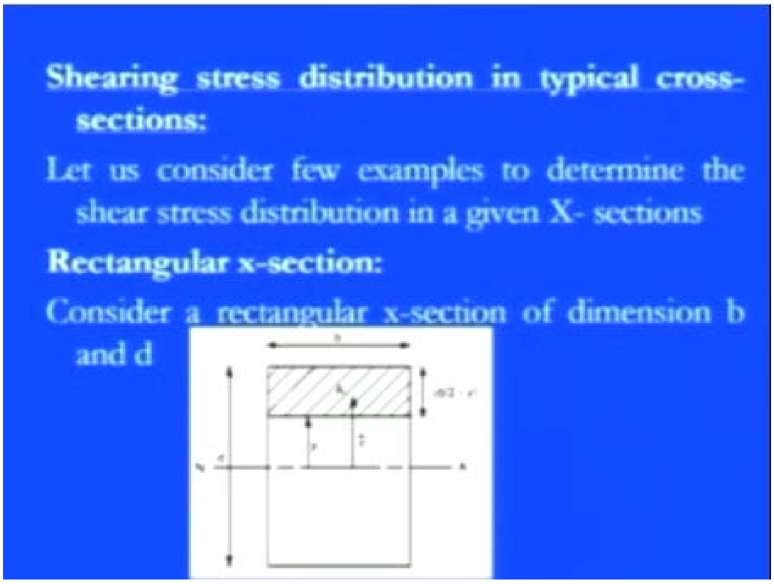

So, let us consider for few examples to determined the shear stresses distribution in a given cross section. So, first cross section as I told you that actually we are going to use that rectangular cross section, so considering this rectangular cross section, which has the height we can say is diameter or we can say this dimension d. And then you see we have a breadth that is the b is there and in the you can see that actually since it is a symmetrical structure, so in that we can easily calculate that where the neutral axis is existing.

So, at exactly at the half of means that d by 2 we have because, it is a symmetricity is there is no any kind of non-uniformities is there in the structure. So, we can easily put those things, so here at d by 2 we have the neutral axis, you can see on our screen that it is the dotted line is showing the neutral axis. And then now you see we just want to focus on a small element and then you see generally we are using that the integration of that.

So, that whatever the deformation or whatever the kind of shearing stress, which you know occurring in this kind of structure it can be simply simulated for entire structure, once you know for a small segment. So, for with using of that segment you see again we are using that if we have a segment or we can say the strip, which is just the distance of y.

So, you see here this from the neutral axis, we have the distance of the strip is y and you see this centroid of the strip is exerting at somewhere and the distance for between centroid of this strip with the or with this rectangular section to the neutral axis is y bar. So, here you see we can clearly visualize the difference between the y and y bar and as I told you see, this is the real difference and in the shearing stresses we are using those thing particular.

And then as I told you this is just the total is d, so it is d by 2 here there, so this is d by 2 and this is the total is this y. So, this distance will be clearly the d by 2 minus y, so we have the height of this strip is d 2 minus y. And there is a point a where the centroid is located, so now just keep these information in your mind you would like to calculate the shearing stress for that...

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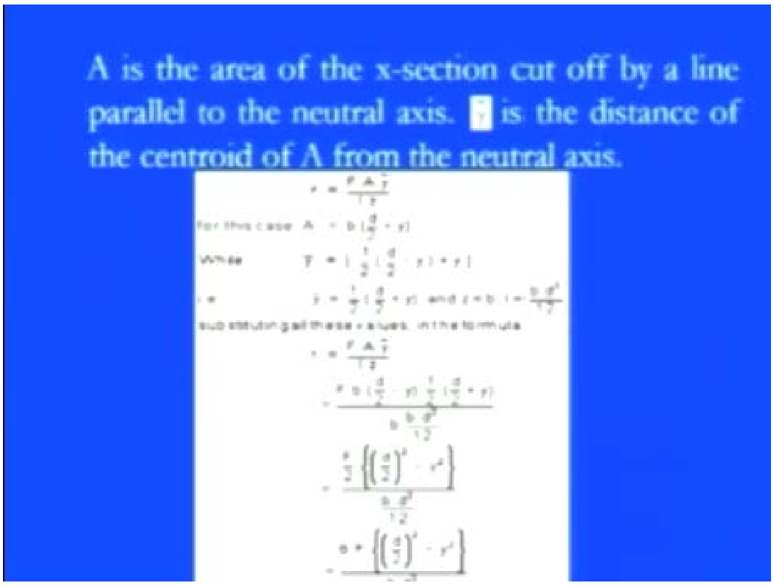

So, A is the area of cross section generally, which we are using cutting by the line parallel to this neutral axis that is the y bar and y bar is real the distance as I told you between the centroid of the this cross sectional to the neutral axis. And then you see with the using of that information, we have the shear stresses is nothing but, equals to f into a into y bar divided by i into z as we discussed. And now since we are talking about that is strip in which the breadth is b and the height was d by 2 minus y.

So, you can calculate the area because, it is a simple rectangular cross section is there, so it is nothing but, equals to b into d by 2 minus y. So, now we have the cross sectional area of this we have the shear stresses, in this shear stresses now what we need, we need y bar. So, again you can calculate the y bar because, y bar is exactly just existing in the in the midway of the strip, so now half of because, you see we have the distances from the neutral axis is this y bar, so y bar we can calculated just by midpoint of those things.

So, first of all we have the strip this height that is d by 2 minus y and the total from that to that we have the y's. If you remember those things we have the total height of these things is y plus d by 2 minus y's, so if divided by 2 then we have the exact location, where the centroid is locating. So, y bar is nothing but equals to d by 2 minus y 1 segment plus y divided by 2 or if manipulate those things, then we have the y bar, which is the distanced of the centroid to neutral axis is equals to half of d by 2 plus y.

So, now you see we have area we have y bar, so in this shear stresses what we need, we need first now i. Since, it is a rectangular cross section, so i can be easily calculated and the neutral axis passing from the diameter, just keep this thing now I just in this d basically it is a height know diameters it is height d. So, now we have the i is b d cube by 12 and then one more parameter is left that is the this width, so width is again you see it is given as p, so z is equals to p.

So, now keeping those parameters in our mind, now we just want put those things in our first formula that is shearing stresses is equal to f into a into y bar divided by i into z. So, now if you keep those things, then what we have in place f that is again shear forces are there which is pretty common. And then we have the area, area we calculated b into d by 2 minus y, so we just simply put those things and then we have the y bar, which we have calculated the half of d by 2 plus y divided by i which is b d cube by 12 and we have the b that is z.

So, now you see by manipulating those things what we have, we have after calculation, we have f by 2 that is the shear stresses is equals to f by 2 inside the bracket d by 2 whole square minus y square divided by b d cube by 12 and again if you keep those things on top of side, then we have 6 into f d by 2 whole square minus y square divided by b d cube.

So, if you look at those the parameter, then you will find that you see the shear stresses for this kind of rectangular section is absolutely depends on, that how these the dimensions are varying. That means, you see here that y you know that actually that y is always a varying distance that how much depth you want to use and what is the distance is there from the neutral axis to that strip. So, that is the y square and d by 2 is always you see that where how much height, which you are using for our cross sectional area, the cross sectional section.

And then we have the b d cube these are also the dimensional parameter of those things. so here if you look at this picture we will find that we have a parabolic kind of distribution because, it is a square term is there. So, shear stress is also whenever you see the kind of this, the distribution is there in the rectangular structure, then you will find that we have a kind of this parabolic distribution is there.

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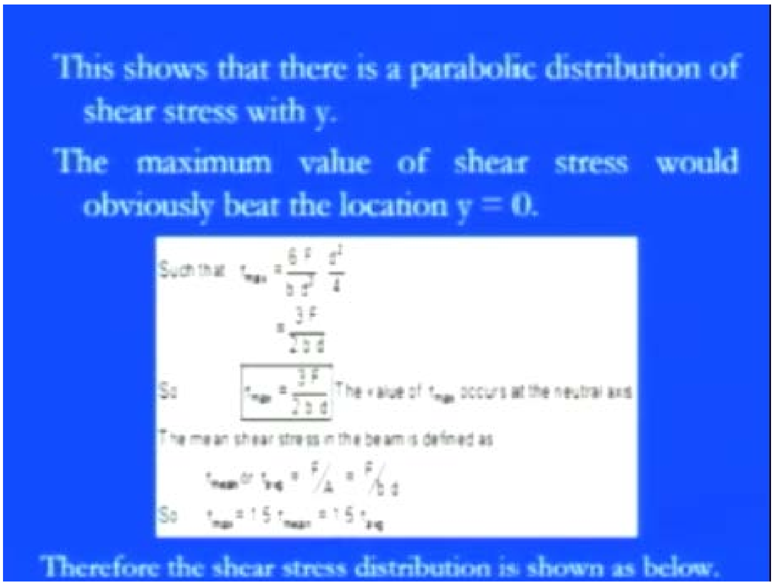

So, with that parabolic distribution of shear stresses with y, now our main intension is to find it out that now with this parabolic distribution, where the minimum and where the maximum shearing stresses are occurring. So, the maximum value of shear stresses; obviously, beat at to the location y equals to the 0; that means, you see now if you are keeping y equals to 0; that means, exactly it is somewhere, if you visualize this you know y equals to 0, then we will find that there is no distance from neutral axis that is strip you are simply deleting.

So, now by putting those values what we have, we have the tau maximum, which is the shear stress maximum is 6 f by b d cube this was the constant parameter and we had d by 2 whole square minus y square. So, y is gone, so we have d square by 4 or we can say by manipulating we have 3 f divided by 2 b d, so tau maximum is equals to 3 f divided by 2 b d and the value of tau maximum occurs at the neutral axis because, we are already illuminating this y, so there is no gap between the neutral axis and the height of the strip, so now this tau maximum is occurring at the neutral axis.

So, if you just go back then you will find that just remember those things, then we were discussing about the bending stresses and when a beam is there and it is under the application of the bending moment. So, due to the bending moment just I am refreshing those things due to the bending moment, the upper area of the beam is you know experiencing this tensile stresses and the lower area of the lower portion of the beam is experiencing the compressive stresses.

But, there are some locus of the points where there is no stress component and due to that there is no strain is there. So, but here you see the maximum when actually this and the same beam, when it is experienced by the shear forces, the maximum shear forces are at the neutral axis only. So, see the comparison, we have f equals to d m by d x; that means, you see here whenever if I am saying that the shear stresses are maximum, what about the bending moment or bending stresses you see, so that kind of relation is always very, very important for that thing.

So, now come to that point we have the shear stresses, which is equals to 3 f divided by 2 b d. So, in that also the maximum shear stresses, which are occurring at the neutral axis is absolutely based on that how much shear force, which you are applying and then you see the two main dimensional parameter that if we have more b or the height, then definitely the corresponding changes are there in the shear stresses.

Now, the main shear stresses in the beam can also be defined as you see the tau mean or we can say tau average, generally you see we are using the tau average as which is equals to f by a or f by b d. So, now it is once this 3 by 2 times tau mean; that means, you see what we have, it is a mean structure means it is a simple f by b d tau maximum is a we have 3 by 2 f by b d. So, the relation is the maximum the shear stresses, which are occurring at the neutral axis is 1.5 times or 3 by 2 times tau mean or we can say the tau average.

So, this is exactly the this relation between this maximum shear stresses and the average shear stresses, which are occurring in the beam the rectangular cross section of beam. But, the important feature is that, if you want to calculate the shear stresses maximum or first the normal shear stresses for rectangular cross section of beam, then first of all we need to find it out that what is the shear force is there and what are the dimensional parameter b and d.

And then you see if we are calculating a normal shear stress, then only we need to know that actually what is the height or breadth and how much distance you see of the this strip is there from the neutral axis. That means what is the value of y, but if you want to calculate the maximum one because, it is exactly at the neutral axis, so it is only we need to know that 3 by 2 f divided by b d. So, you see these are the simple formula which can one easily calculate the maximum shear stresses, which are existing at the neutral axis of the beam. So, now you see just keep those things in your mind, we just we would like to describe those things by graphical way. So, we again we are focusing on the same rectangular structure, which we used in a previous action.

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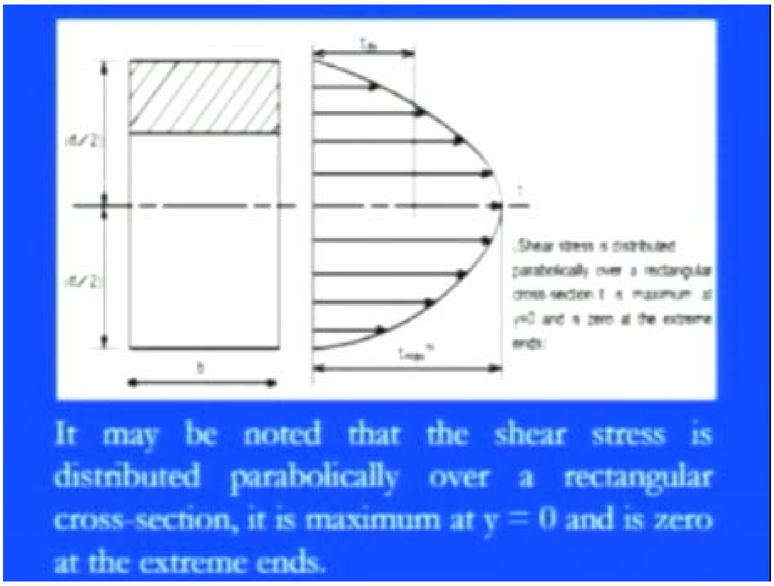

So, we have you see this is a rectangular cross section and we have this particular strip, you can see this is our matter of concern on which we have focus is there d by 2 d by 2 is height is there. So, total is the d height, now at this neutral axis as I told you the maximum shear stresses are occurring, so the value of that is 3 by 2 f divided by b d, so you see here one can easily calculate this maximum. And since the distribution, if you just remember the formula the tau which is 6 f divided by b d cube into d by 2 whole square minus y square.

That means you see the square terms are there, we have the distribution of shear stresses in parabolic way. So, you see starting from because, at these two extreme ends it is a 0 shear stress and it has a maximum value of the neutral axis starting from the 0 value, now the distribution is parabolic one. So, now we can easily visualize those things, so this is the maximum value of shear stress, these are some average value of the shear stresses. So, we can easily get those thing because, very simple average value is f by b d, so it can be easily calculate.

So, it may be noted that the shear stresses distribution is parabolic, as we discussed our rectangular cross section and it is the maximum at y equals to 0; that means, at the neutral axis and the shear stresses are 0 at where the you see, this y is maximum or we can say at the extreme corner, so this is a kind of shear stress distribution is there. Now, so this all discussion which we made it is just for a rectangular cross section, which has a uniform structure, there is no and it has a symmetricity all across the neutral axis, so it is pretty easy to find all those issues.

But, now you see if we focus on the I section, which has a different segment altogether from the rectangular section, in that you see we have the two flanges and we have the web is there in between those things. So, two flanges even this structure is also symmetry, if you just cut the this neutral axis right from the middle portion, then you will find that the one flange and half of the wave and half of the wave and one flange is there all across this neutral axis, so we can say that it is pretty symmetry.

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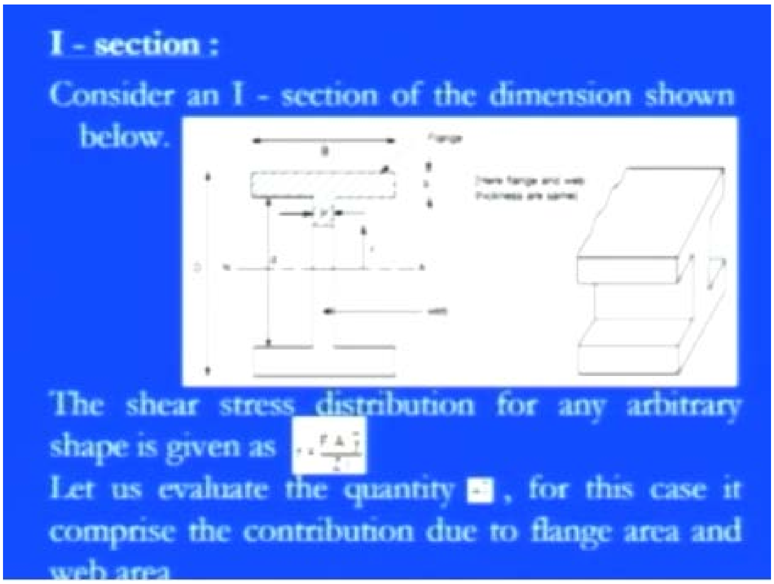

So, we can see on your screen that we have the real feasible I section, which we are using generally for almost all kind of constructional way that if we have this I section there on our top of the roof or for balancing the roof. Generally this I section is usable because, it has a less mass moment of inertia because, what we are doing here in comparison with this rectangular section, the mass is very, very less you see we are simply cutting those the side mass from the web ways.

So, we have the less mass and when it is the inertia is there, inertia is very, very less as compare to that one, but it has certain unique properties, which we are going to discuss in the next sections. So, right now you see we have the I section, so in that I section you see again the same height is the d and the breadth is b, so in that this breadth is there of the flange. So, for that even this the small height of the flange is the b, so the total I section height is d, but for these two flanges which have both of the flanges have the common kind of uniform structure.

So, for those things we can define a single parameter, which is valid for both of the side, so we have you see the breadth is b for both section, this height is there for the flanges, the flange height is small b and then you see you have the web. So, for web also you see what we have this breadth is small b and then this height we can simply go with this small d. So, there is a difference now if you are going for the overall picture, then we have the capital B and capital D are the dimensional parameter.

But, if we are going for the different sections of this I section, like for flanges then we have you see the two main parameter, the small b and the capital B. And if we are going for the web portion, then we have the dimensional parameter is small b and small d, so just keep these information in your mind. Now, you see our main focus is we are again taking a small section of the this I section, so for that you see here, we have this small section of I which has the distance you see from the neutral axis is y.

So, now here again our main focus is on this particular section, which is the this ((Refer Time: 21:47)) section is there; that means, cutting portion which we are showing here. So, now if you focus on these things our main intension is to find out that how the shear stress distribution is there all along that, first we want to calculate the shear stress for this small element and then we integrate those portion for the entire structure because, it is a uniform structure altogether.

So, the shear stress distribution for any arbitrary shape is given as usual general formula is f into a into y by divided by z into i. So, now see here focus on this our main because, it is a rectangular this it is I section is there, so altogether you see our main focus is on the dimensional parameter. So, now focus on the quantity a y because for this case it comprises the contribution due to the flange area and web area. So, you see here it now our main focus is to combine the two different area altogether and then see the impact of these things.

Because, in the previous case we had a uniform structure that uniform means actually a simple structure, which has there no kind of sections are there. That means, there is no two different parts are there just there was a one part rectangular part, but here you see we have in I section two parts are there as I told you, one is the flange, one is the web. So, now we need to integrate those things to calculate these stresses for that you see first of all our focus is on the flange area.

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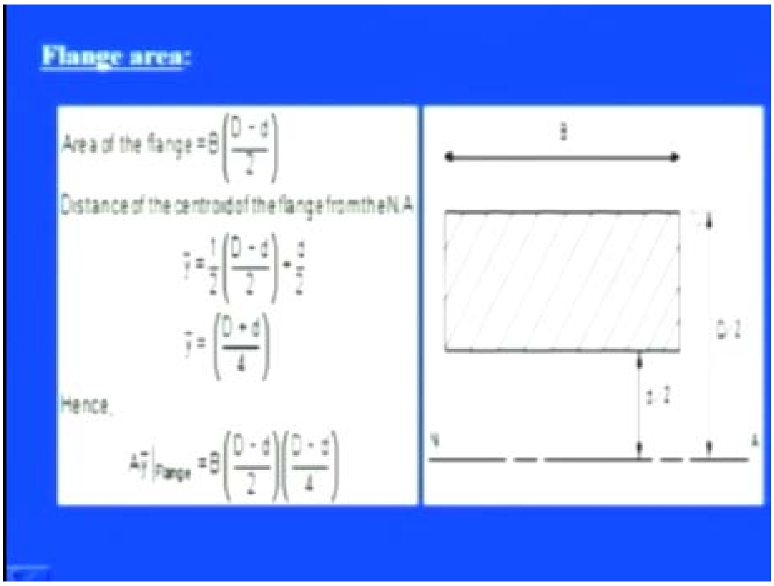

So, now if we are taking the flange; that means, the upper part will find that we have already describe that the dimensional parameter are these capital B and you see the the total height the top height is D by 2 from the neutral axis and the this 1 is small d by 2 from the web portion. So, now you see here, if you want to focus on the height of this one, which we have assumed that, that is the small b is there, even from that also we can calculate that this is capital D by 2 minus small d by 2 will give you the height and the width is B.

So, area of length can be easily calculated by B into D minus d by 2, so this is the area, now you see the distance from the centroid of these just we want to calculate the y bar that wave the centroid is occurring. Because, now you see in the previous case, it was pretty simple that now it is exactly at those things, but here you see we have a web and we have the flange.

So, with the consideration of these two different kind of sections, we can again calculate the distance of the centroid from the neutral axis, which is y bar equals to half of now this area of the flange, which we calculated D minus d by 2 plus d by 2.Because, you see here somewhere it is exactly exerting on this, you can see this diagram it is here. So, half of this D minus d by 2, so this height is D minus d by 2 half of this plus this distance d by 2. So, if you want to calculate the distance from this neutral axis to any of the centroid first d by 2 plus you see here half of the distance.

So, if you are manipulating those things, we have y bar is equals to D plus capital D plus small d by 4. And then you see we can also just put those things there, then we can calculate the area a y bar flange because, that this is the main the dimensional parameter is there for calculating the shear stresses for different sections. So, as far as the flange is concerned, we have a y bar for flange is equals to B into D minus d by 2 that is the area into D minus d by 4 that is the y bar. So, it is you see this parameter can be easily calculated with the using of these dimensional parameter. Now, the second section is the web area, so once we have the you know flange area multiplied by 2 you have the total the a y bar for the overall both flanges, now you see focused on the intermediate section that is the web.

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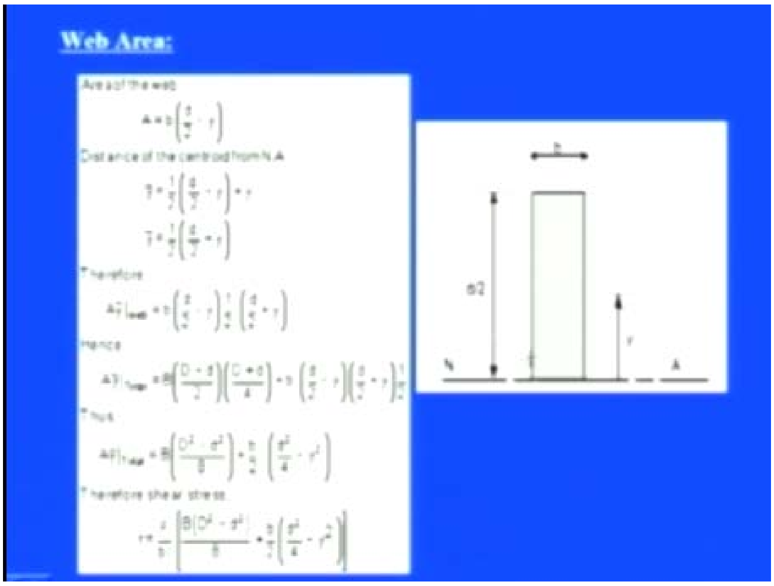

So, while by considering web you can see on your screen that the diagram of web we have you see the total height is small d. So, if you cut the neutral axis then we have the d by 2 we have already assume that the width of this small b and now you see here if already we put that this flange and web portion, which is our matter of concern is having the height of y. So, with the using of this information now we can simply find it out that what will be the area of that.

So, now, this particular this height is d by 2 minus y, so this is height this width b, so area is b into d by 2 minus y. So, with the consideration of this area, now our focus is where what is the distance of the centroid from these things, so again you see we are using the same information that half of this portion particular. So, half of this height is half into d by 2 minus y plus whatever the distance is there of this y.

Because, you see this distance is there from the neutral axis, so this y plus half of this portion will give you the distance of the centroid from the neutral axis. So, half of d by 2 minus y plus y or we can say by manipulating these things we have y bar equals to half of d by 2 plus y. Now, you have area you have y bar, you can calculate the a into y bar for this particular web portion, so it is pretty easy to calculate those things. So, b into d by 2 minus y that is the area and then we have half of d by 2 plus y is the y bar, so now if we keep those things together then we have this area a y bar for this web portion.

Now, you see here if you want to calculate for the entire region, means you see 1 web 2 flanges, so for those things a y bar total is nothing but, equals to b that is now for flanges first. So, b into d minus d by 2 d plus d by 2 plus this the top portion, which we calculated for the web b small b into d by 2 plus y and you see into the half of the portion. So, just manipulating those things we have the total a y bar for web plus flange is equals to b into d square minus d square, if you multiple those things here.

So, D square minus d square by 8 plus b by 2 here just calculate those b by 2 we have d square by 4 plus minus y square because, this plus and this is minus. So, a plus b into a minus b a square minus b square, so now you see here we have the total this a y bar for both of the sections for 1 web 2 flanges. So, with consideration of those things, now we are keeping these things this values in the main formula we have tau, tau which is the shear stress is equals to f by b into i we are keeping those things b into i because, z is b i is i, so now, you see we are not manipulating this i as for these things.

Till now you see we are keeping i as i b is z, so it is always coming like that, so this f divided by b into i know this is b into d square minus d square by 8 plus b by 2 d square minus d square by 4 minus y square. So, now you see here, we have the shear stresses in that way; that means, you see it is again if you look at those picture, then you will find that it is again going in term of the parabolic path because, you see the square term are there of d square by 4 minus y square. So, again it always reflect the kind of distribution of the shear stresses all along with the i section in terms of the parabolic way, so look at the maximum or minimum value of the shear stresses.

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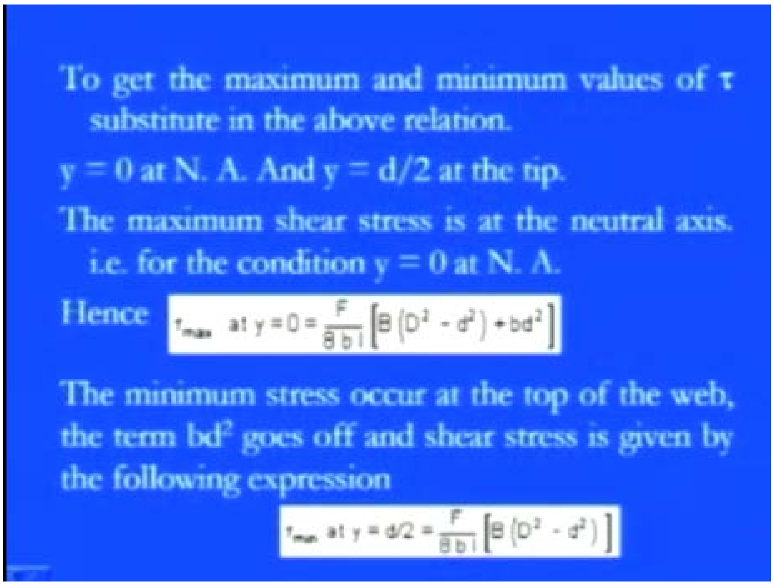

Again you see we need to keep those the different, boundary values constraints and we can get those values. So, at y equals to 0; that means, we taking at neutral axis, the y equals to d by 2 at you see now we have the 2 main condition, when we are talking about the y equals to 0, we have an always at the neutral axis and we know that the maximum shear stresses are always occurring at the neutral axis.

So, if you keep y equals to 0 we have the tau maximum that is f by 8 same you see this b into i b d square minus d square plus b d square because, the y square has already gone in this particular first expression. So, now you see we have the maximum shear stresses and the other thing, you see if you are just going for the tip that what will be the shear stresses at tip, because of flanges are there at the tip, so we need to keep the y equals to small d by 2.

So, you see here the minimum stresses they are occurs at the top of the web, means you see just near to the flanges area and the term b d square goes off and the shear stress is given as tau minimum, which is exactly at the y equals to d by 2 means the top of this web portion is equals to f by 8 b into i b times of the capital B that is the width of the flange total into b d square minus d square.

That means you see now, you can pretty easily calculate that, where the maximum shear stresses are there and where the minimum shear stresses are there. Meaning is very simple these stresses are all together varying with the web only, you can see here at the neutral axis, which is passing from the web is having the maximum shear stresses that is equal to f by 8 b d f by 8 b into i you know b times of d square minus d square plus b d square.

So, this gives you a maximum value which is occurring at the neutral axis, so you have the maximum. And then the parabolic distribution is there and now you see we have the minimum value of the shear stresses exactly at top and bottom of the web, means d by 2 d by 2 from both of the neutral axis. Because, we are assuming that it is a symmetrical section by considering those things now or focus is that now how the distribution is...

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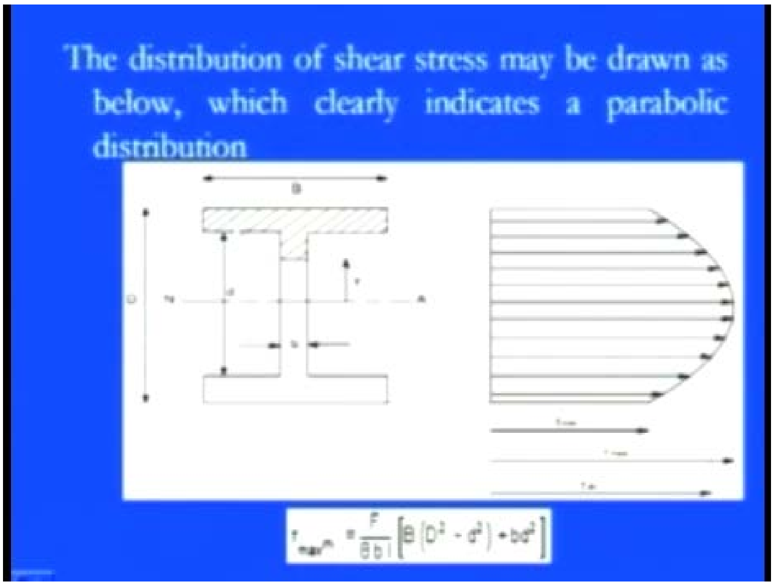

So, now you look at those things that we have the same structure this i section this portion was which we covered and then we integrated that part, this is the y distance you go this things. So, now you see here at these two portion, you see we have certain value that is the tau minimum, so these are the tau minimum and these thing as I discussed and then we have at these neutral axis we have the maximum shear stresses all together.

And then you see, since this tau average which we already discuss that the tau maximum is 1.5 times tau average. So, we can also calculate the tau average here, so look at this picture we have a clear feeling that, if we want to design any beam which has a cross section of i. Then you see our main focus is to strengthen these the web portion because, you see here the maximum, this shearing stresses are occurring and it is starting from this portions that from the minimum section at the flanges part, we have you see this web which is experiencing the maximum shearing stresses.

And this maximum shearing stresses the values are pretty simple f by 8 b into i into this b and i adjust these are the focused area or the focus parameters, within the web portion and then we have see the region is coming within between the flanges and this web. So, we have the capital B that is the width of the flange into d square minus d square plus b times of d square. So, you see here it we have clear feeling about the shear stresses that how the variation of this stresses are there in that.

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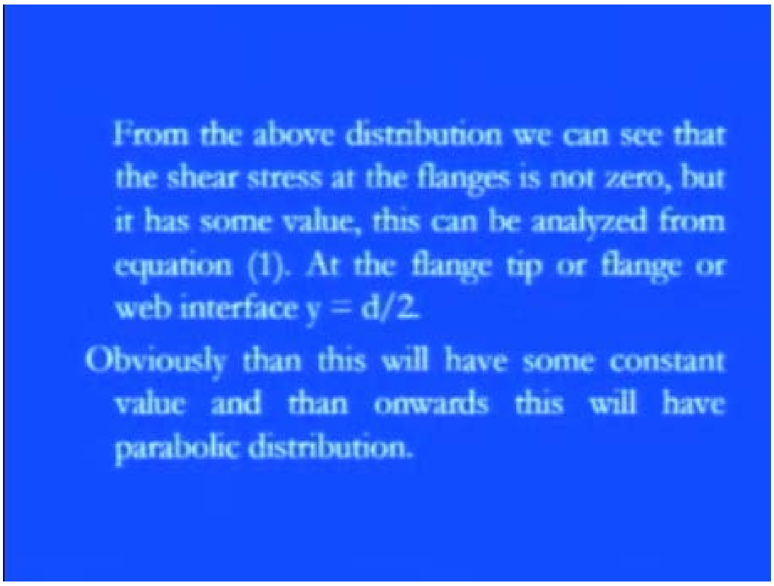

So, from the above distribution we can see that the shear stresses at the flanges is not 0 they have the minimum values and we discuss all ready. But, it has some value this can be analyzed from the equation 1 and at the flange tip or we can say the whatever the flanges are there or web surfaces are there at y equals to d by 2 we have certain the value of this tau minimum, which we calculated. And for the design purpose also we need to be very careful that actually though the we have the maximum shear stresses all along with the neutral axis.

But, these flanges also having some effect or some impact of these shear stresses, so; obviously, then this will have some the constant value and then onwards this will have a parabolic distribution from that. So, with that consideration you see again we have to be very careful that actually how this distribution is there, because you see if you look at those formula for tau minimum also, we have the square term. That means, you see some sort of the distribution is there in the flanges also, all together you see we have a clear parabolic distribution in the web, but the distribution is there. So, in practice it is usually found that most of the shearing stresses is usually about 95 percent is carried by the web only.

So, just 5 percent is there by both of the flanges, so when we are designing those things one has to be very careful about the web portion because, it pears the maximum shear stresses and it is maximum at the neutral axis. Hence the shear stress in the flange is negligible; however, if we have concrete analysis, means if you are going for; that means, if you just want to write the expression for the shear stresses for flanges and web separately, we will have this type of variations, means some variation you can see this thing we have some variation in the flanges.

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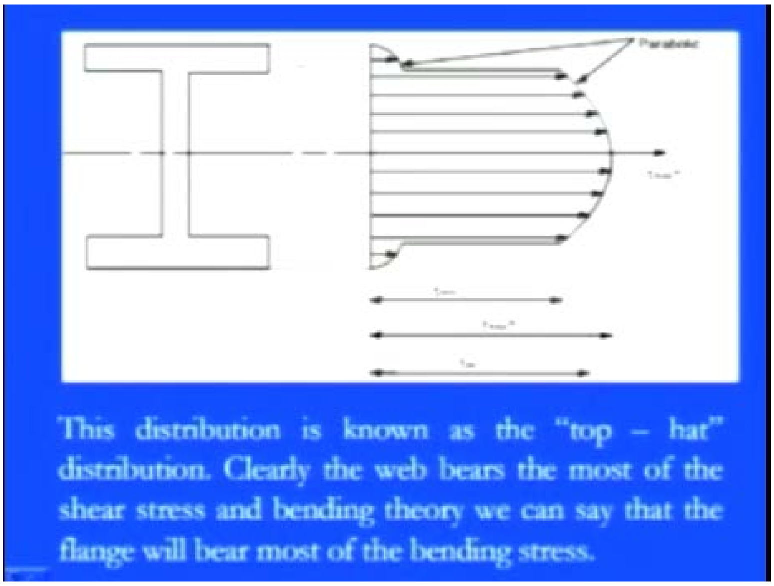

But, the real variation with the maximum value of the shear stresses in the web portion only, so ((Refer Time: 34:11)) you see here as I told you that, we have some sort of distribution in the flanges. So, the distribution in the flanges of this nature in the both of side, then we have you see this is starting from these, since they have the square term as I told you, so that since the square terms are coming in the tau minimum or tau maximum means the minimum and the maximum value of the shear stress; obviously, the parabolic distributions are there.

So, even in the flanges though it carries only the 5 percent of the shearing stresses always the distribution is the this parabolic one. So, this is parabolic here the and then you see we have the minimum tau minimum this is 1 as I told you and it describe the parabolic one with the maximum value at the neutral axis and then we can calculate the tau average also once you know the tau maximum. So, this distribution is known as this is pretty the common phenomena the top hat because, you see hat is on the top of that and the bottom side you see even it follows similar kind of a structure.

So, this distribution is known as the top hat distribution for shearing stresses, clearly the web bears the most of the shear stresses; obviously, and the with the bending theory we can say that the flange will be your most of the bending stresses. So, this is a pretty important phenomena, just keep this thing in our mind that you see whenever we are talking about the bending stresses, along with the neutral axis the bending stresses are not having the impact on the neutral axis.

So, if you want design for bending, like the bending stresses are more you see and if you want to design any beam for bending portion, like you see for safe in the bending always go for this i section. Because, you see here the loose portion is the y which carries the minimum bending shearing, most of the bending is coming on the flanges part. So, that is way you see just make the flanges strong and go for this and as I told you see it has a less mass as computed this rectangular cross section.

But, if you want to design the beam for the shearing stresses here, the weakest portion is the web and in that the maximum shearing stresses are coming the 95 percent of shear stresses, shearing stresses are coming on the web, so do not go for the i section where ever the maximum twisting or the shearing or the couple or twisting movement are coming on the beam, otherwise you see the failure will be occur in that particular stresses.

So, this is as for the practical applications are concerned we are using correspondingly to the i section for the beam design for bending and you see the rectangular section for shearing part. So, now you see here, so these two sections which we discussed about first was the rectangular cross section and second was the i section.

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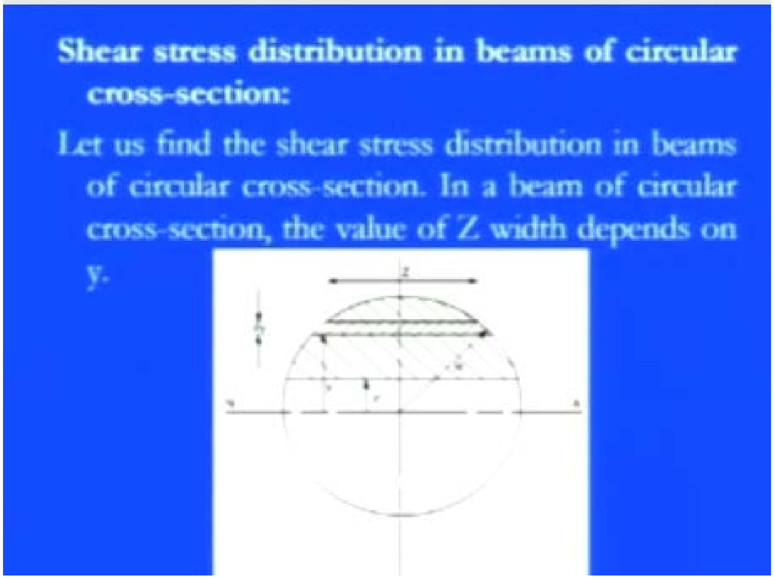

Now, third cross section which we have the shear stress distribution in the beam for circular cross section. So, and the circular section again we need to check it out that how the neutral axis is occurring and once you find out the neutral axis then what the distances are there between the centroid and neutral axis and how the distribution is there, that is our prime focus. So, let us find the shear stress distribution in the beam, which has the circular cross section.

So, in a beam of the circular cross section again the value of the Z will absolutely depends on the variation. Because, again this you can see on the diagram that this is the Z, Z is there because, you see here we are concerning about this particular strip, so for this strip we have this particular Z, which has the distance of y from this neutral axis. So, here you see now this is the circular cross section, which has the radius of capital R, now you see for that we are taking a strip which has height or we can say the unit width is there d y and for that you see it has a distance from the neutral axis is y.

But, you see the area which we are concerning R the particular segment, which we are concerning which has the distance of y 1. So, it has a variation from y 1 to i y 1 is minimum to r maximum, so you see here this is pretty clear the dimensional values are there in that, that what we are concerning. So, we are concerning about a area which or we can say the portion, which has the minimum value of the distance of y 1 from the neutral axis at the maximum is capital R, which is the radius of the extreme end.

And then you see, if you are concerning about this particular segment, let us say this segment which we are taking at the one part of that. Because, it has a symmetricity on the other side, then we can find out this is the Z by 2 and this distance is y, so this vertical distance we have the y in a horizontal distance, we have the Z and this is you see this is the radius is there from any end this is our neutral axis is there for that, so this is the main circular section of the matter on which we just want to focus.

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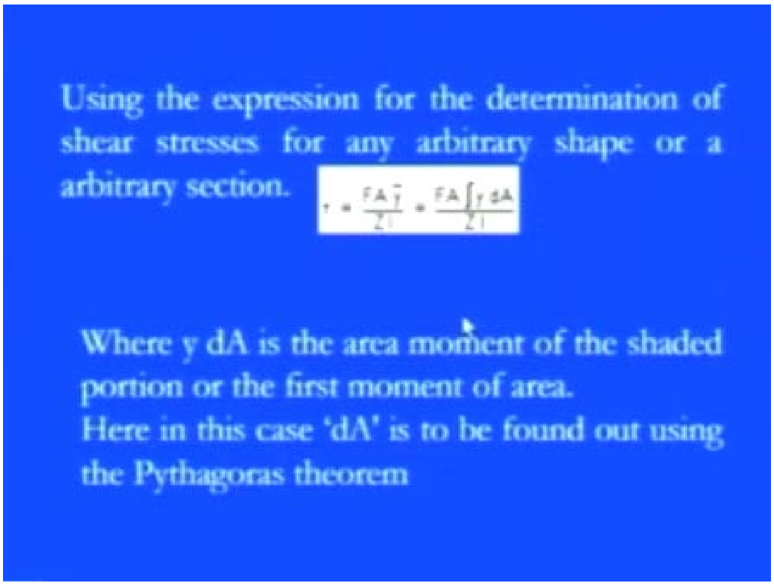

So, using the expression for the determination of shear stresses for any arbitrary shape as we discussed or any section this tau is equals to a into a into y bar by divided by i into z or we can say that you see here, since we are talking about circular section, which has a symmetricity in all along with the radius. So, for that you see now you we would like to go for the first area moment, means you see we are not going for the second moment of area here the first moment of area.

So, here just by describing those things what we, have we have f into a integration of now y bar is simple replacing by integration of y into d a as usual divided by i z, where y into d a is a area movement of shaded portion or we can say that as usually the first movement of area. Hence, in this case you see the d a, which is to be very we can say the main thing for the sided area of portion of area.

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So, for that you see we are going to use the Pythagoras theorem as I told you that we have a height of this y and we have this width is z by 2 because z is the total portion half of the z 2. So, while applying the Pythagoras theorem for that particular portion, we have z by 2 square plus y square is equals to r square or we can say that this z by 2 whole square is equals to r square minus y square or we can calculated the z by 2 or z is equals to 2 times of a square root of r square minus y square.

So, now you see we have because, this is the pretty typical phenomena here that which portion you are covering according to the value of the z is coming as I told you. So, here we are taking that particular strip, which has the width is d y and it has a distance of y from that. So, with that consideration you see we have the value of z, which is equals to 2 times of square root of r square minus y square, now you see we can calculate the d a area we because, you see it is simple the circular section is there.

So, it is equals to z times of d y because, z is the total height and the strip width is d a is there, so z into d y will give you the area d a. So, now if you are keeping those things then we have 2 times of square root of this i square minus y square into d y. So, now you see you have the area d a, now we can again calculate the mass movement of inertia i at this neutral axis for the circular section is pretty clear pi into r to the 4 divided by 4. So, with that consideration now going into the back that what was the main formula was tau equals to f in to a into y bar divided by z into i.

So, keeping those values there in that what we have, we have f as usual you see now for the area because, you see as I told you in that particular figure our minimum limit is y 1 that 1 and the maximum limit is r. So, for entire region of these we have the integration of y 1 to r into now this area, so it is 2 into square root of r square minus y square into d y into this y is there. So, with that you see we have 2 y the square root of r square minus y square into d y divided by we have this z into this i.

So, z we have already calculated this is 2 times of square root of r square minus y square into i which is pi into r to the power 4 divided by 4. So, keeping those values here we have this expression now for shear stresses, where you see r is the radius of a circle and you see now as I told you the limiting conditions are y 1 2 r. So, we are keeping those things, now by manipulating those things what we have, we have 4 times of f divided by pi into r to the power 4 into square root of r square minus y square.

Now, integrating you see again y 1 2 r, this y times of square root of r square minus y square into d y. So, now you see our expression is pretty clear now that, now we want to define for these things for this particular region we have the kind of shear stresses of this nature. But, you see this integration just keeping those things that, now keep the y 1 here and put the r 1 value, we have the final value of tau is equals to 4 times of f r 1 square minus y 1 square divided by 3 times of pi r 1 to r to power 4.

That means, you see, now if you look at this particular picture what we have, we have this shear stresses, which is depending on the shear forces f. But, you see whatever the variations are there in the dimensional parameter, it is again in terms of the square term r square minus y 1 square. So, means you see if you want to again go for the distribution of the shear stresses, again you see it is a kind of parabolic distribution along with that. Now, we would like to see that actually where the maximum and the minimum shear stresses are occurring.

So, that if you want distribute those things, we have the clear picture that these are the location, which are bearing the maximum shearing stresses of the circular section and these are the portion, where the minimum shear stress are occurring. So, for that you see again the similar kind of this theories are to be applied for calculating the maximum shear stresses or minimum shear stresses, put you see for maximum value of the shear stresses put y 1 equals to 0, as you seen in the previous case we put the y equals to 0.

So, by keeping y 1 equals to 0 we have the shear stress is maximum and which is equals to 4 f times of 4 into f divided by 3 pi into r 2 power square. So, now you see here what we have, we have 4 by 3 into f divided by pi r square, so if you want to calculate the maximum shear stresses what we need to do, only we need to know that what is the value of shear force and what is the means what is the radius are there of that., so that you see the distribution is coming accordingly.

So, by keeping those things again our main intension is where this will occur and this will occur because, you see we are keeping y 1 is equals to 0. So, if you go for feasible there is no distance between the minimum the portion to the neutral axis; that means, the maximum shear stresses are occurring at the neutral axis only and the value of these shear stresses, which are coming at the neutral axis, which has a maximum the value and it has a parabolic distribution.

So, the maximum top of this parabola is coming exactly at the neutral axis and the value of this is 4 times of f divided by 3 pi into r square. So, now you see this is one position portion, which we gives you the maximum value, now if you want to calculate the minimum value for which you see the diameter is y 1 is equals to plus minus r, the plus 1 is top and the minimum is minus r. So, if you are keeping those things we have the just if you keep those values that y 1 equals to r, we have tau equal to 0.

That means, you see here now clear description is there, about even if you want to calculate the tau minimum also it is tau 0 at those extreme corners. Now, you see distribution comes, then you will find that we have the maximum distribution all along with the neutral axis, the minimum values of the shear stresses are there exactly on the top of and the bottom corner. So, now it pretty clear distribution is there parabolic distribution coming right from the minus r to plus r 0 and then it is going to the maximum at the neutral axis.

Now, you see if want to calculate the average value of the shear stresses, so for that you see, again the average value tau average or tau mean is nothing but, equals to f by f by a are we can say that since it is a cross sectional area is the circular once. So, f by pi r square, so now we see if you want to compare then comparison in the previous case when the i section was there, it was 1.5 times of tau maximum. But, here you see the tau maximum is nothing but, equals to 4 by 3 tau average.

That means, you see if you are keeping those things by 4 by 3; that means, by 1.33 times multiply the average value you can get the maximum value.

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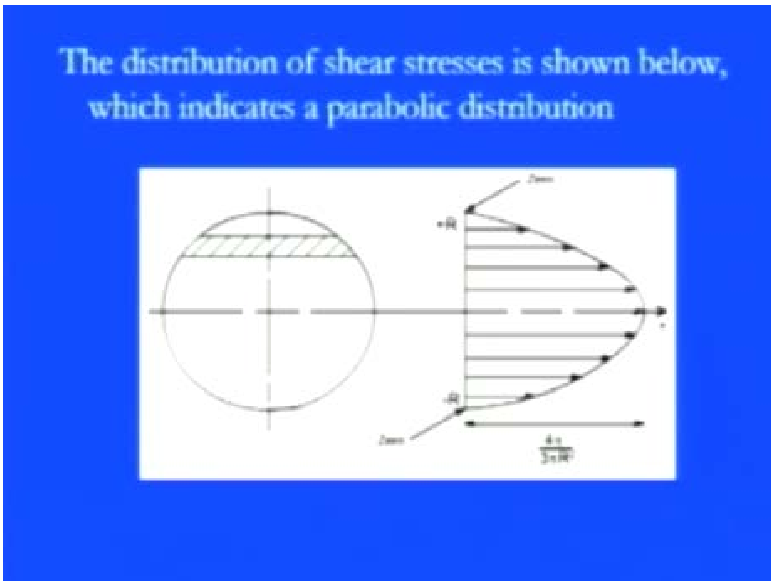

So, you see now these all numerical values come into the real the graphical way and the clear description is like that, we have a circular sections of this nature at as I told you at plus r and minus r we have the 0 shear stresses. But, the maximum shear stresses are coming all the way to the neutral axis, so this is the neutral axis and this plus minus r and you see it is a parabolic distribution was there, which has the value of 4 f divided by 3 pi r square. So, with that you see you can calculate these the parabolic distribution of the shear stresses, which has the maximum value of or we can say the top of these things, we have this 4 f divided by 3 pi r square, so this is you see the shear stress distribution.

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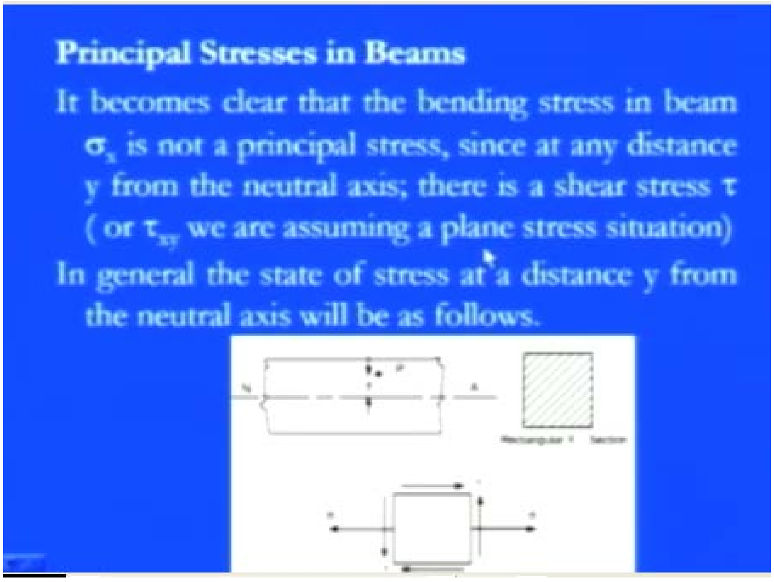

So, once you know about the shear stress distribution for any cross sectionals area, you can design the beam accordingly. Because, the different, different portions are bearing different cross sections, different the value of shear stresses and corresponding factor of safety as end the corresponding applications are coming all the way, like how we can design those things after that you see, we have the principal stresses.

Because, you see in this beam as we got to know that the two main types of stresses are there, one is the bending stresses and beam, which is you know just all along to the longitudinal way and we have another part is the tau that is the shear stresses. So, when you have the normal stress component due to the bending, when we have the shear stress component due to the shear part, we can easily go for the principle stresses and in the beam. So, it becomes clear that the bending stresses and beam sigma x is not principal stress, obvious it is normal stress or flexural stress component is there.

Since, at any distance y from the neutral axis and there is a shear stress tau or tau x y we are using for the plane stress component in x and y direction. So, in general the state of stress, which we discussed in the initial lectures, which has a distance y form the neutral axis can be easily calculated. And you can see in this particular diagram, we have a simple structure of this as usual and you see this is our neutral axes, which is passing from this and if you are assuming that we have a cross section of beam is rectangular one.

So, this is the rectangular cross section and you see it has a distance, where you see we want to calculate the a stress component is the distance of y. And this p value is we can say where the centroid or you see the kind of this fibers are just passing on those things and we want to calculate the bending as well as the shear stresses on that. So, with the consideration of those things, you see again we simple describe those stress component on simple segment with the as I told you the longitudinal 1 is there at the bending. So, these are the flexural stresses on all along with this particular the longitudinal 1 and we have the shearing stresses this tau. So, this is a perfect combination of the stresses of a beam which has the rectangular cross section and carries both the normal as well as the shear stresses component.

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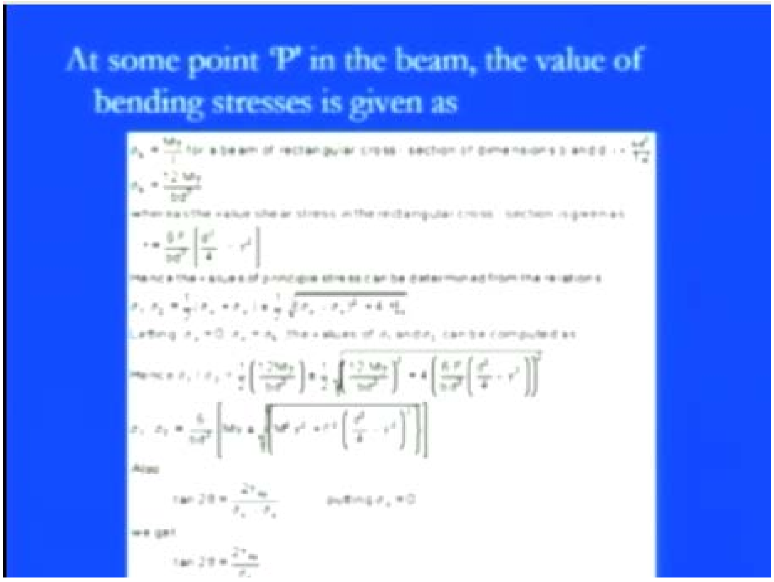

And now you see when both components are there, we can easily go for the principle stresses. And for that you see again first of all as I told you the normal stress component is coming due to the bending stresses and with the bending stress formula tau by y is equals to m by y, we can calculate tau b which bending stress is equal to m into y divided by y. And you see since the rectangular cross section is there, so i is nothing but, equals to b d cube by 12.

So, we have you see the sigma b is equals to 12 into M into y divided by b d cube or with that particular, you see we can again easily found the flexible stresses are all along with this particular x axis. So, we have you see the normal stress component here with the value of 12 times of m into y divided by b d cube, but you see we have the shear stress component all the way from the complimentary as well as the normal shear stress component. And you see the value of the shear stress component is nothing but, equals to 6 times of f divided by b d cube and d square by 4 minus y square.

Because, you see it has a parabolic distribution and has a maximum value you see all along that particular neutral axis. So, we can calculate both the thing all together see, so we have the value of normal stress component, we have the value of shear stress component, now you see know that if we have both the component all together. So, we can calculate the principal stresses sigma 1 or sigma 2, you see the maximum or minimum you see is equals to sigma x plus sigma y by 2 plus minus half of the square root of sigma x minus sigma y by 2 plus 4 times of tau x y square.

So, now you see we have the standard one, but here you see as you can see in the previous diagram only we have the longitudinal stresses in the x direction only. So, there is no y component, so sigma y will; obviously, be 0 because only we have the bending or longitudinal 1 in this x x component. So, sigma x is there which is equals to sigma b which is equals to 12s times of m into y divided by b d cube and we have the tau 1 which you see in the previous section, we shown you that it is 6 f divided by b d square into you know b square by 4 minus y square.

So, from that you see we can simple replace those figures and we can get tau 1 oblique tau 2 the minimum or maximum is equals to half of, because half of sigma x plus sigma y is there, so sigma y 0, so in terms of sigma x, we can simple put this 12 times of m y divided by b d cube plus minus half of the square root of this sigma x square that is 12 m y divided by b d cube whole square plus 4 times of you see this tau is there that is 6 f divide by b d cube d square by 4 minus y square.

So, now by manipulating those things what we have, we have tau 1 or tau 2 basically is equal to it is divided by b d cube. So, you see it is coming out and then you seen in that you see have m y plus minus square root of because, m is the bending movement it is due to that the bending stresses are coming. So, m into y plus minus square root of you see the m square y square plus f square times d square or 4 minus y square whole square.

So, you see here we have the clear picture that in the combined you see, if you want to calculate the bending this principal stresses due to the bending as well as the shear it has a clear picture that both bending movement as well as shear force is clearly influenced your principle stresses. And now you see with that, if you want to calculate the plane that where you see these stresses are coming all the way. Then you see the it is pretty straight formula is there the tan 2 theta is equal to 2 times of tau x y divided by sigma x minus sigma y since sigma y 0. So, you can put tan 2 theta is equal to 2 times of tau x y divided by sigma a sigma x you can say.

So, now you see here in this tau x y is again, the same value 6 f divided by b d cube into d square by 4 minus y square and tau b, which is the bending stress is 12 times of m y divided by b d cube. So, by keeping those things you can get the location of the principal plane, there where these stresses are there and you have both the values sigma 1 and sigma 2, so that you see we can easily get those principal stress for this kind of...

So, now you see we would like to conclude this chapter, that we discussed the important phenomena about the shearing stresses, that it is very sensitive to which cross section, which we are using like if you are using the rectangular cross section or if you are using I section or if you are using circular section, then how the distribution is there. Generally you found that the distribution is circular and it has the maximum value at the neutral axis only.

But, as far as the this the applications are concerned again we are thinking about that, if you want to use any beam, then what kind of applications are then how the bending as well as the shear stresses are occurring. And in the later section we discussed about the principal stresses, that if a beam is subjected by both bending as well as shear stresses, then how we can get value of this principle stress and at what location they are located.

So, this all the way we discussed about the bending as well as the shearing stresses on a beam, if beam is loaded with the point load or UDL like that. So, now in the next section our main intension first use those the formula for numerical derivations that actually will practices those numerical problems. And then go for the further clarification about the deflections that actually if the beam is there under this kind of load then how the deflection will take place.

Thank you.