Strength of Materials
Prof. Dr. Suraj Prakash Harsha
Mechanical and Industrial Engineering Department
Indian Institute of Technology, Roorkee

Lecture - 28

Hi, this is Dr. S. P. Harsha from Mechanical and Industrial Department, IIT Roorkee. I am going to deliver my lecture 28 of the course of The Strength of Materials, and this course is developed under the National Program on Technological Enhanced Learning, NPTEL. Prior to start this lecture 28, actually I would like to refresh you know like the concept of previous lecture in which we discussed mainly about if we have a beam and like if it is under the consideration of bending moment, then what will happen, you see you know like means when the bending moment is applied, then what exactly the relations were there that part we discussed. So, I would like to refresh those things because you see in that lecture, we are going to discuss mainly on the numerical problems based on the previous formula which we derived, and also you see we would like to discuss what the shear stresses in this lecture.

So, you see in the previous lecture if you just remind, then you will find that we had a beam is there, and when this under you know like the consideration of or under the application of the bending moment M, only pure bending moment, there is no other forces are there. Then you see you know like simple bending equations which you derived that sigma by y equals to M by I equals to E by R in which sigma was the bending stress which induce due to the application of this bending moment. So, sigma by y - y is the distance you see on which we just want to find it out the bending stresses, and that fiber is having a distance from the neutral axis y. So, this y is the distance from the neutral axis of that particular fiber.

So, sigma by y equals to M by I. M was you see you know like the bending moment which is the applied bending moment, and you know like due to that the bending stresses are coming sigma, and then M by I was there. So, I was you see you know like this second moment of area was there which we defined the new term which was nothing but equals to summation of y square dA. So, there were you see the two main theories which we described to calculate this I. That was you say the second moment of area because it has you know like the summation of y square dA. The y square term is there that is what you see we defined that as the second moment of area.

So, you see by integrating, if you have a regular structure, then you know like this tapered bar or the soft or that kind of thing, then it is pretty easy to find it out by integration for y square dA or if we have different kind of selections, then you see summation of y square y square dA will give you this second moment of area. Then, you see the third term is sigma by y equals to M by I equals to this E by R. So, E is the Young's modulus of elasticity and R is the curvature radius. So, you see this is a simple bending like the moment equation is there, and if you see this equation, then you will find that only the pure bending is there.

There is no shearing part is there, there is no other stress components. Only these stress components due to the bending is there in this particular equation and you see due to that we can also find it out the maximum bending stresses. The sigma maximum which is nothing but equals to you see you know like we discussed that this M by I into you like the y maximum distance. So, M and I are you know the applied part. So, this is maximum bending. This bending moment is applied and I is the second moment of area is there. So, it can be easily calculated with the using of the dimensional one and the applied one, but the key feature is that actually what is the location is there at which this maximum bending stresses are coming. That part we discussed on that, and also we discussed that actually M is equal to Z times the sigma maximum.

So, Z was the section modulus and it can be easily calculated based on you see M by I because you see the section modulus absolutely depends on that. What is the exact structure is there of that kind in which we want to find it out the maximum bending moment with corresponding to the maximum bending stresses. So, that part you see we discussed in the first section of those things and then, you see you know like other section which came up with that if we have a bending moment in that or if we have a bending you know like these, this second moment of area is there on the neutral axis, then also we can calculate these second moment of area at any section, or any axes using perpendicular theory and using parallel theory.

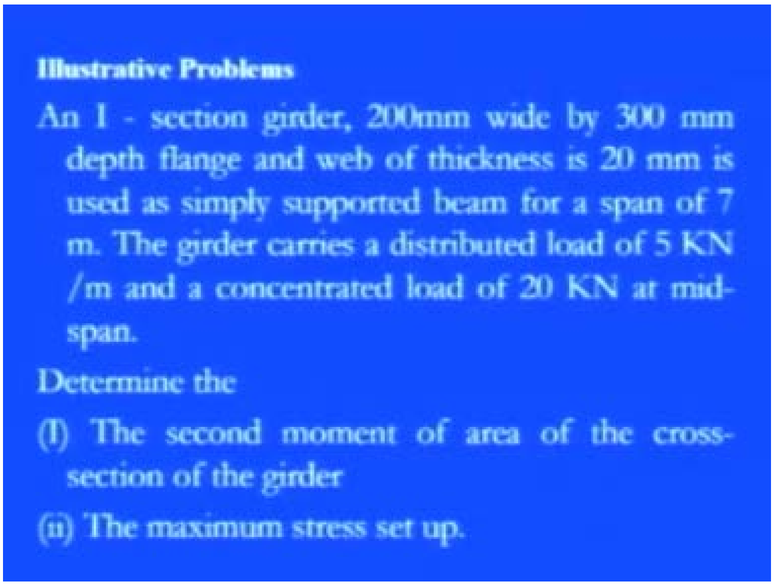
So, in perpendicular theory we described that we are using the perpendicular axis then like a polar moment of section. That means, you see if you are taking area and you see these two axes on which we want to find it out the section area IX and IY, we can simply you know like make the relations. The polar section of this inertia with this because the polar section of inertia, or we can say this is J, J is perpendicular to the area concerned and this IX and IY are the two corresponding axes to this moment of inertia. So, with that you see you know like if we want to relate those things using this perpendicular axes, we calculate that Z is equal to IX plus IY. So, we can calculate that one, and the another relation which we describe using one theorem that was the parallel axis theorem.

So, that was you see you know like it is pretty easy that if you want to find out at any axis, let us say Z axis. So, IZ is equal to IX plus integration of this you know this Y into dA, or we can say that this we described that H distance was there. So, H square into A. So, meaning is pretty simple that we can calculate you know like this second moment of area or the moment of inertia for any section. Once this moment of inertia for you know like the axis which is passing from the centroid or we can say the neutral axis, so these theorems which we discussed and also, we find it out that actually if we have a circular section or if we have a rectangular section, then also we can calculate those things. So, for rectangular section which has this height is D and breadth is B.

So, for that you see the calculated INA. That means, you see the moment of inertia at this neutral axis is nothing but equals to cube by 12, or also you see if we have you know like the uniform diameter from 0 to d, then we have you see is equal to bd cube by 3. So, these kinds of discussion you know like a derivation which we described. So, in this lecture you see we are going to use again the similar kind of formula for that, and then the last section of the previous lecture which we described that if we have you know like big blocks are there in which the bricks are to be settled, and there is you know like the small gaps are there in between those things for setting up any pillars for that kind of structure you know like we can calculate this INA for the bigger portion and the shaded portion.

So, I is the total you know like INA is nothing but equals to I for total rectangle section minus I is the shaded portion. So, while comparing those things, we can calculate bd cube by 12 minus two times because you see the two different bricks were there if you remember two times of bd cube by 12 or we can say bd cube by 6. So, you see here these kinds of discussions which we made, but in those discussions you see there was no stress component due to the shear. So, in this lecture you see first we will discuss about you know like what will happen when you know like all these shear stress component is there. That is a second part, but first part is focused on you know like whatever we discussed in the previous lecture, the formulae for the bending stresses and the bending moment we are going to use you know like for the numerical problem.

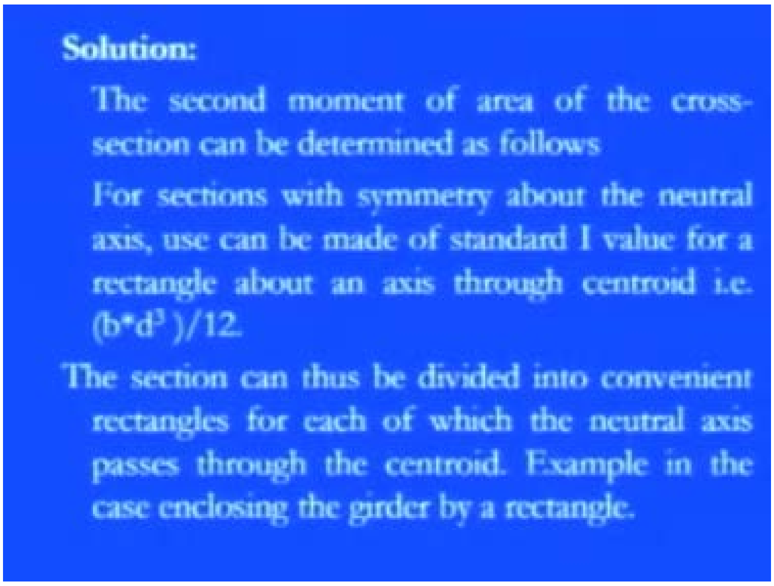
(Refer Slide Time: 07:51)



So, here you see the first numerical problem is that we have I section girder. So, we have you see the girder for you know like the basis of any building, you see you will find that on the top of roof you see the girder is there which has the dimension of 200 millimeter wide by 300 millimeter depth flange. So, one is the flange which has this diameter and the web in between. The web, it has the thickness is 20 millimeters is used to simply support beam for a span of 7 meters. So, you see here for flanges, both of the flanges you see the dimensions are 200 millimeter wide by 300 millimeter depth, and there you see the web thickness is 20 millimeter is given for the total span of beam is 7 meters.

The girder carries a distributed load of 5 kilo Newton. So, you see per meter here the load distribution is given on that particular girder which has the intensity of 5 kilo Newton per meter, and the concentrated load. That means, you see we have a combined load here. So, one is the UDL in which 5 kilo Newton per meter is there, and distributed load is there and other one is the concentrated load or we can say the point load that is 20 kilo Newton at the mid span. So, this is the configuration of the problem. Now, our main intension is to find out the second moment of area of the cross-section of the girder because I cross-section is there. So, for that we need to find out and also, we want to find out the maximum shear stress, maximum stresses which are being set up due to this load application, the combined UDL as well as the concentrated load.

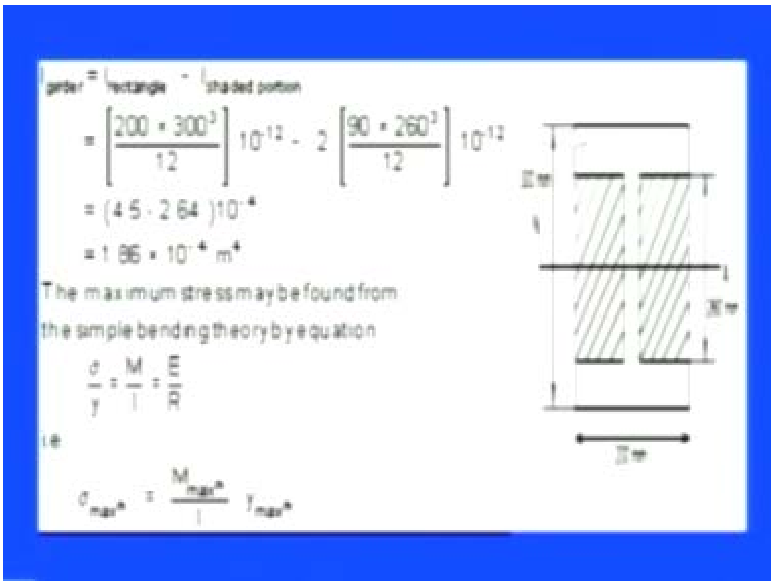
(Refer Slide Time: 09:09)



So, for that you see here the second moment of area of the cross-section can be easily described when you see we know the total regular structures. So, for that we have taken the section with the symmetricity about the neutral axis can be easily made if we have I section. So, for that you see you know like since it is I, so centroid is absolutely passing from the rectangular section of the mid-portion of this I which has the total this I is nothing but equals to bd cube by 12 which we have described for you know like the rectangular section, and here you see you know like it is this middle portion of the web, I should say is rectangular portion.

So, we can simply get those things bd cube by 12, and the section can be divided into convenient rectangles because you see these are all the rectangles for each of which the neutral axis passes through the centroid because it is well balanced structure. So, all the neutral axes just passing from the centroid of those I section. For example, you know like in the case enclosing by girder is just by rectangular. So, you see what are the description there of this kind of problem, it is absolutely based on I section. In I section, you see this neutral axis is just passing from this particular centroid and you see you know like it has symmetricity all along, both of the side. So, we can say that whatever the things we will be analyzing for one section, it has symmetricity on the other section also. So, for that you can see this particular diagram. So, in that what is that you see here?

(Refer Slide Time: 10:30)

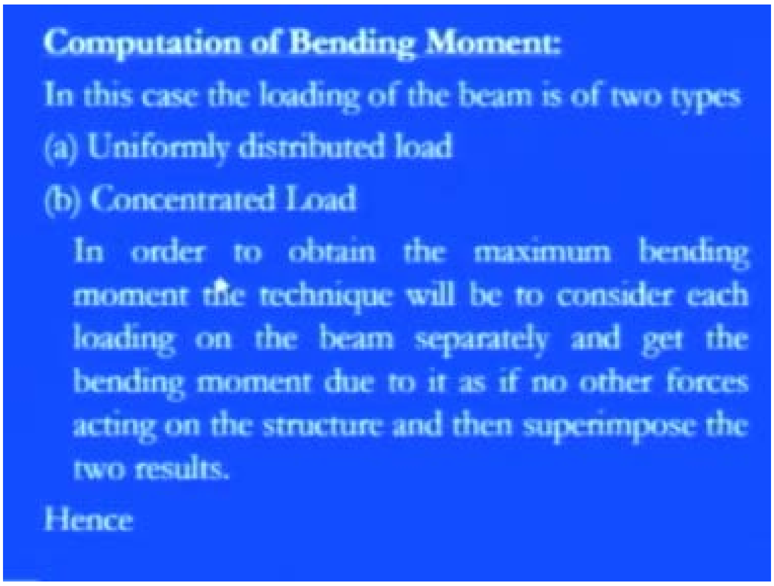


This is I section you see and this is the web part, and this is I section. So, you know like just go with my cursor, you will find I structure here. So, this is I structure here you see these two, this is one flange and this is another flange and this is the web part. So, you see here you know like as it is being discussed here that this is 300 millimeter of this, and this is 200 millimeter and 20 millimeter, this web is there. So, this distance is 90 and 90. So, total is 180 and this is 20. So, this is 200. So, just keep this thing in your mind that this is I section and the neutral axis is just passing from this. As I told you, it is symmetricity on both the side.

So, for that we can simply calculate I girder which is nothing but equals to I rectangular. The total rectangular part you know like minus I shaded area. If you remove this shaded area, then you have I section. So, for that you see you know like if you are talking about the total rectangular area of this, then we have 200 into 300. So, 200 into 300 cube divided bd cube by 12, so b into d cube by 12. So, this is there into 10 to the power minus 12 because it is in the millimeter. So, we just want to calculate in the meter side. So, we have this 1 minus two times because these two shaded areas are there. So, two times into 90 is this as I told you because this is 20. So, 90-90 and 12, the total is 200.

So, that is what 90 into 260 as it is given you see you know like this is 260 for this description. So, 260 cubes of bd cube divided by 12 into 10 to the power minus cube. So, now if you analyze those things by that, then you have 1.68 into 10 to the power minus 4 meter 4. So, now you see here since I section is well symmetrical section all along this two flanges and one web. So, it is pretty easy to calculate you see with the use of this. First you calculate for entire rectangle and then, you know like reduce the shaded portion out of that. So, you will be handicapped with this kind of structure, and the maximum stress may be found from you know like the simple bending theory of equation. As you see we discussed in the previous case that sigma by Y is equal to M by I is equal to E by R, or we can say that if you want to find out the maximum bending stresses, then it is nothing but equal to the maximum bending moment M by I into Y maximum.

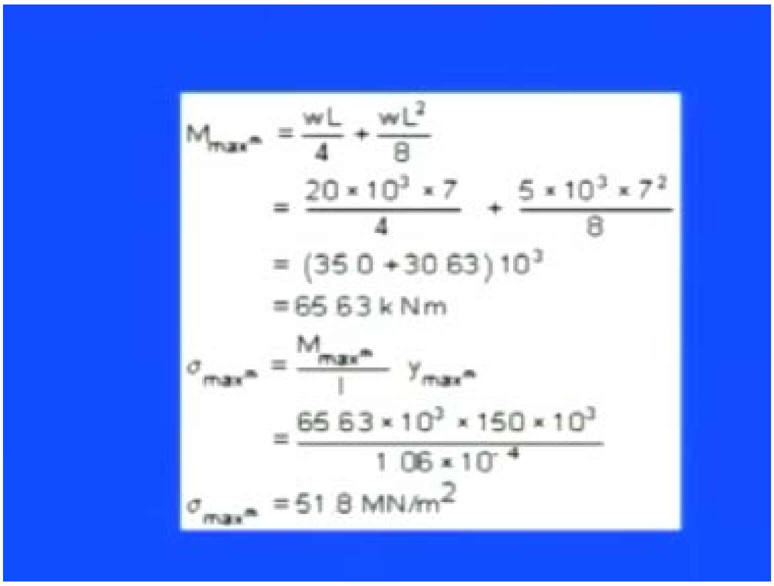
(Refer Slide Time: 12:43)



So, for that you see you know like it is pretty easy to put those values and get the final thing because you see here there are two different kind of loadings. So, first of all our main focus is to find out this bending moment. So, for that you see here the two main loadings. As I told you, one is the UDL which has you know like the intensity of 5 kilo Newton per meter, and then we have a concentrated load you know like some magnitude. So, in order to obtain the maximum bending moment, the technique will be you know like considered each loading on a beam separately and get the bending. You know like we need to just go with the separate kind of loading and get the bending moment due to these loadings as if no other forces acting on the structure, and then you like superimpose of these two results because you see first the UDL is there and then, the concentrated load is there.

So, we need to calculate different segments and then, combine it because our main focus is that there is no shearing occurs on that. Only this bending stresses are there due to this bending moment. So, that is why you see we need to compute these bending moments or bending stresses very carefully along with this particular kind of load.

(Refer Slide Time: 13:51)



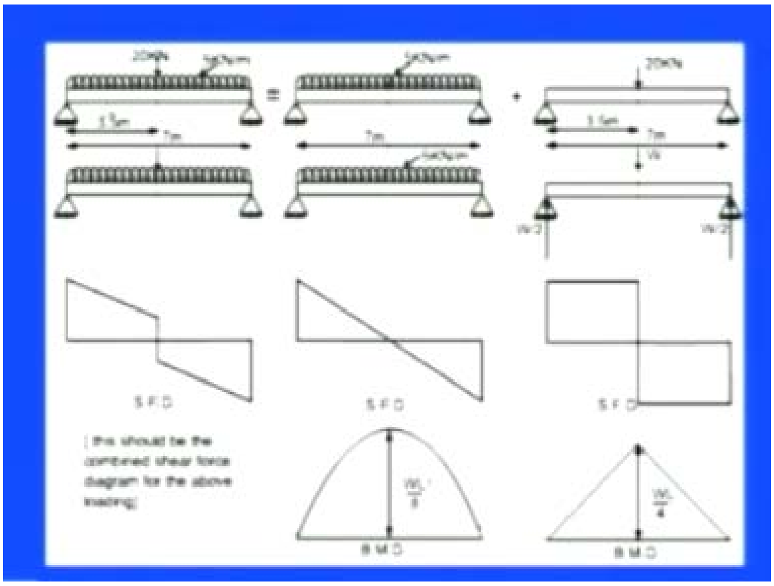
That is why you see you know like the maximum bending moment is nothing but equals to wL by 4 plus wL square by 8. WL by 4 is coming due to this point load, or we can say the concentrated load plus you see whatever the bending moment is coming due to the UDL. So, this distributed load wL square by 8 is there. So, if you are keeping those things, then we have this w which is 20 kilo Newton of 20 into 10 to the power this cube. So, that is in you know like the Newton, sorry in kilo Newton. Then, we have L that is 7 meter divided by 4 plus w which has the 5 kilo Newton per meter. So, 5 into this 10 to the power cube into 7 square because the total length of the beam is 7. So, 7 square divided by 8.

So, if you compute, then we have the maximum bending moment is 65.63 kilo Newton meter. Once we have these things, then we can pretty easily calculate what will be the maximum bending stresses and for that the sigma maximum is nothing but equals to M maximum divided by I into y maximum or if you are keeping those value 65.63 into 10 to the power cube. So, this is you see the maximum bending which we calculated on the above side into you know like we want to find out at this 150. You know like this millimeter from that 150 meter from that side.

So, you see here we are keeping those things, and you see we just calculated 1.06 into 10 to power minus 4. We are keeping for I this value for this I section which we calculated. So, if you are putting all those values, then we have y maximum is 51.8 mega Newton per meter square. So, you see this is the exact processor to calculate first of all what is I because you see the second area, second moment of area is very important for any kind of structure that what exactly the cross-section is. Here, we have just chosen the cross-section of the beam is I section. So, that is why you see first of all we need to apply that part and then, you see you know like what will be the maximum bending moment is there. So, it comes only due to that what kind of loading is there. So, once you know the loading or combination of loading, then again segregate those parts and calculate that combined effect of those things.

So, we have you see the maximum bending moment here. Once you have the bending moment, once you have you see the second moment of area, then it is pretty easy to calculate the maximum bending stresses by using the simple bending equation theory. So, you see here these values are again you see like all these things, it can be easily predictable for these kinds of examples.

(Refer Slide Time: 16:22)



So, you see here we have an example that if we have you know like simply supported beam on which the UDL is there, so one is UDL which has the intensity of 5 kilo Newton meter as we discussed in the previous case, and you see this at the middle mid of this pan, what we have. We have 20 kilo Newton of the point load, and you see you know like this distance because the total distance is 7 meters. So, at the central point, we have the 3.5 meters. So, with that now first of all our main intention is to combine those loading because you see here we have the UDL as well as we have the point load.

So, with the combination of those things as I told you, see the maximum bending moment is coming by wL by 4 or wL square by 8. So, with that you see here we simply combine those loads here you know like with the UDL plus these things and then, you see reaction forces are coming out of those things, and then once we have the reaction forces, we can go for shear force diagram. And the bending moment diagram also as well because here if you go for these reactions here, all these you know like the joint points, this reaction point is just going on the top of side. This is you see are just going on the bottom of side. So, we have you see this positive direction here, and if you go for this right hand direction, this reaction is going on the top of side. This is going on the bottom of side.

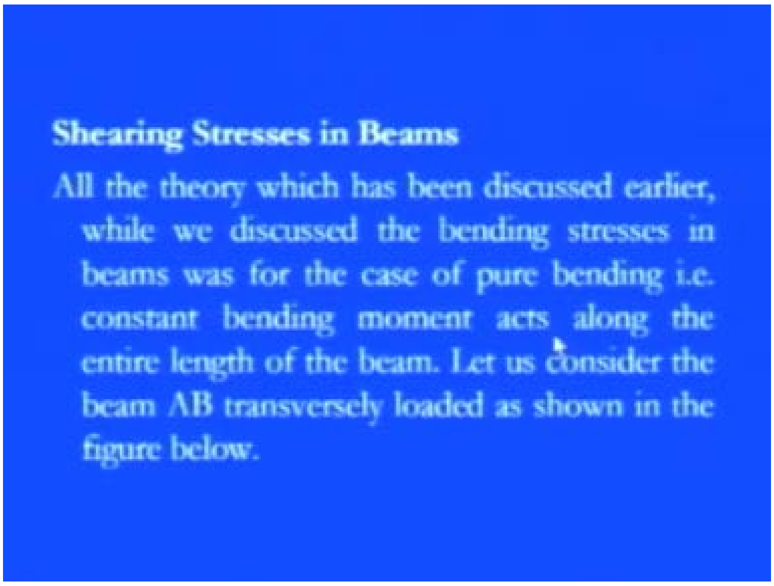
So, you see here we have the magnitude directions, and you see in between there is a point load which will give you abrupt change in shear force. So, just with that concept you see we can simply draw shear force diagram. So, this is you see at this particular point, we have the maximum shear force due to the reaction, and then it is just going to reduce here up to this point and then, there is 20 kilo Newton you know like the force is there which is on the downward direction. So, there is you know like the change is there in shear force and then, there is a negative direction because of the sign convention. So, we can calculate those things pretty easily. Now, you see you know like because as I told you this is pretty simple to calculate. We can again segregate into two parts. As we discussed in the previous case, you see here that first just go with the UDL.

So, this is you see the first case with the UDL only. So, in this 5 kilo Newton per meter is there and the 7 meter is the total length for that. This CFD is there as we discussed in the previous cases you see and this is the bending moment diagram. Then, you see here this is one part another part is that that this beam is influenced by also this twenty kilo Newton. So, this is a simple simply supported beam point load is there at the mid of this span. So, for that you see this w is coming and then, you know like since it is segregating two exact equal parts. So, we have one positive, one negative part in shear force diagram. So, this is the positive and the negative part and since, there is no UDL, so simple triangle is coming because of the point load.

So, now you see if we combine those things, then you see it is very important thing that once you combine both of the things, then we are ending up with those things. So, you see here this is the combination of maximum these things are there, then it is decreasing and you see up to that point, it is decreasing. So, you see up to this it is bisecting here, it is also bisecting here. So, this you see the changing area is there. So, it is very interesting you know like if you want to calculate those things, then you know like just on the top of that you can see that this is the real problem. Our problem equating to one problem by this UDL plus one problem by point load.

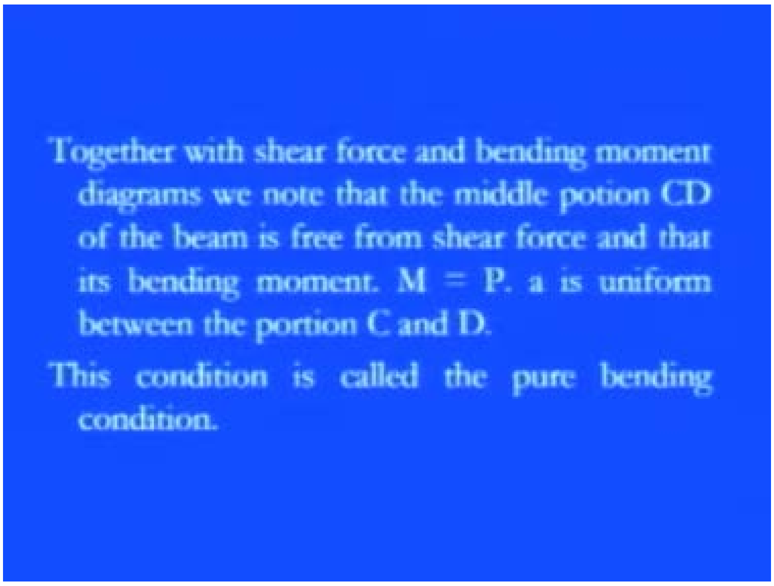
So, you see here we can simply analyze those things or in the bending moment if you go for the previous slide. Then, you will find that the maximum bending moment was wL by 4 due to this point load, 20 kilo Newton plus wL square by 8 due to this UDL of 5 kilo Newton per meter, and we can calculate those things. So, you see here we have shear force diagram for both things and we have the bending moment diagram for both the cases.

(Refer Slide Time: 20:05)



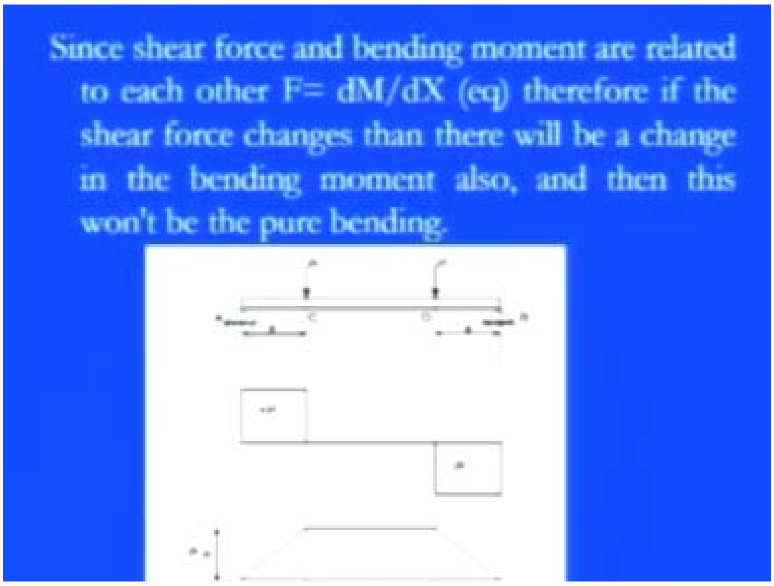
So, you see here in these cases, we only focused on the bending moment as well as the bending stresses which are coming due to you know like this applied moments at this particular beam in irrespective whether it is cantilever beam or simply supported beam. Now, you see here main the main key feature of this particular lecture is to focus on shearing stresses in the beam, and all of the theory, which has been discussed earlier. As I told you just on the bending stresses in the beam you know like for any kind of pure bending is there. That means, constant bending moment is just acting along the entire length of the beam. Now, you see here we would like to focus that we have a beam you know like just let us say AB beam is there which is transversely loaded, and then you see you know like with that we have kind of forces which is just creating the shearing.

(Refer Slide Time: 20:51)



So, together with the shearing force and the bending moment, we can simply note that the middle portion CD of the beam is free from shear force, and that you see the bending moment always is loaded into whatever the distance is there at which the load is acting P into a is always uniform between these two particular sections. That is why you see we are always saying that whenever the pure bending is there, it always gives you the bending moment, which is applied to both the sections.

(Refer Slide Time: 21:22)



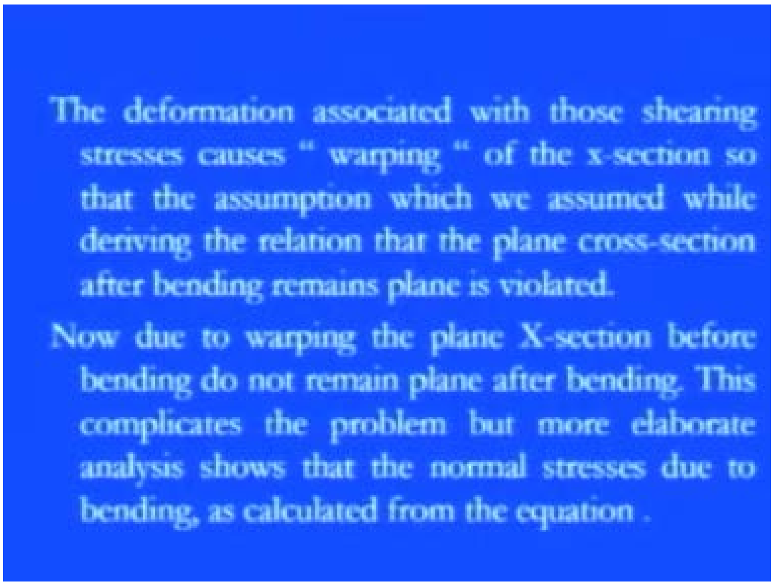
So, you see here since we know that this point loads are coming along with this particular section in which I just told you that you see here the constant shear forces are there for which there is no bending. The dM by dX is 0, since there is no shearing. So, for that the bending moment is constant. So, you see here when we are talking about those things, this is pretty simple diagram on which the two point loads are there. Both are just going downward direction. The reactions are coming from this which is on upward direction.

So, if you are going for the bending moment, then you will find that this and this direction you see on the left hand force, so we have the positive shear force. So, this is the positive shear force and this is the negative shear force, but there is no load application in between. So, there is no shearing or there is no resultants shear forces existing in between this action. So, when we have you know like more shearing force, so F is equal to dM by dX. If you are keeping F equals to 0, dM by dX is equal to 0. That means M is constant. When M is constant, then that means only bending moment is applied on a beam and this is the main condition of pure bending. When you see only you know like the bending moment is applied like the total stresses which are inducing in this particular beam is just the bending stresses due to this M.

So, you can visualize this thing in this bending moment diagram. So, this bending moment is P into a. As we discussed M is equal to P into a, which has constant value of this particular section and these are there due to the shear forces. So, this is you see one of the example in which we can say that there is a condition in which the pure bending is there, but it is also computing with certain kinds of this shearing stresses. So, our main focus is just to focus on that actually. If we have this kind of loading, then how we can conclude you know like the shearing altogether with the bending stresses. So, conclusion is hence one of the conclusion which is coming from the pure bending theory was shear force at each cross-section is 0 because you see you know like all the pure bending is coming, no shearing is there. And the normal stresses are due to you know like this bending are there only which are producing on the top of the fiber, we have the tensile stresses on the bottom of the fiber, we have the compressive stresses.

So, this flexural stresses or we can say this normal stresses are always coming, but there is no shear stresses are there. In that case you see the non-uniform bending of a beam, where the bending moment varies from one cross-section to another, there is a shearing force on each of the cross-section and shearing stresses are also inducing in the material. That means, you see whenever a non-uniform you know like the bending is there if it is a uniform bending, there is no point to go with any kind of shearing any kind of twisting or any kind of couple is there. But whenever non-uniform bending is there, always it gives for different cross-section at different kind of shearing forces are there, and corresponding shearing stresses are there in the material.

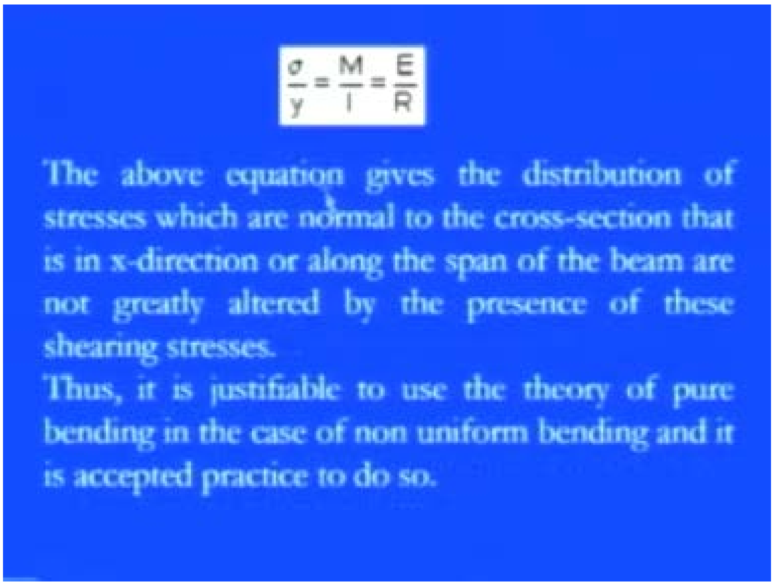
(Refer Slide Time: 24:19)



The deformation associated with those shearing stresses causes warping because you see like whatever deformations are coming, they always cause the warping of the cross-section, so that the assumption which we assume while deriving the relation for this plane cross-section after bending remains plane will be violated because you see after once it is warping, once it is deviating from the original position, then we cannot say that you see once you remove the load, it will be in the main form of all those fibers. It would not be possible.

So, for that you see this assumption is violated. That means, this condition, whatever the condition sigma by sigma y equals to M by I equals to E by R is absolutely violated for this kind of relations. Now, due to the warping, the plane cross-section before bending does not remain. The plane after the bending as we told you and this come you know like this complicates the problem, but more we can say elaborated analysis shows that normal stress due to the bending as calculated by a simple equation like sigma by y is equal to M by I equal to E by R.

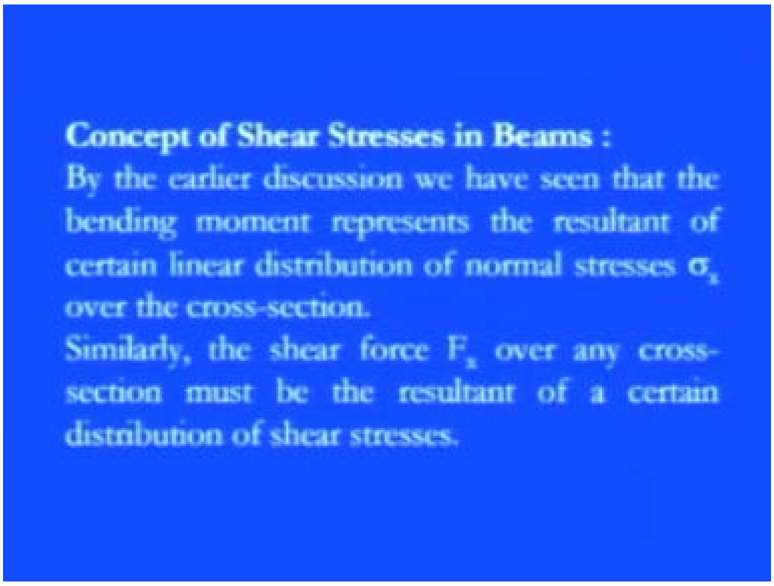
(Refer Slide Time: 25:18)



This equation you know like gives the distribution of the stresses which are normal to the cross-section. That is in the cross-section or we can say along the span of a beam are not greatly alerted because by the presence of these shearing stresses. Thus, you see you know it is just to use the theory of pure bending in case of non-uniform bending and it is accepted in practices also to do these things. That means you see sometimes when we know that non-uniform bending stresses are also there in those cross-sections, then also we can apply these normal stress concept that these normal stresses are there along with the particular fibers. On top of the fibers we have the tensile stresses, and on the bottom of the fibers we have this kind of compressive stresses and we have a neutral axis.

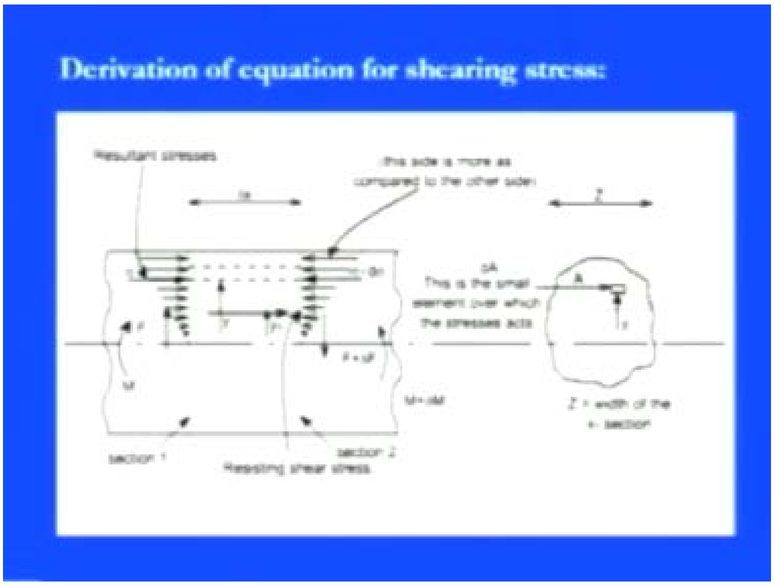
So, we can apply all these perpendicular axis theorems or this parallel axis theorem for those things and also, we can apply this equation sigma by y equals to M by I equals to E by R, so that this is justified case. Therefore, this kind of theory in which you see the non-uniform bending is there, and we can say that whenever pure bending is there, it is just due to this particular M concept. Now, you see here in that you see we just focused on the uniform, on non-uniform bending or now, our focus is that shear stresses are there in the beam.

(Refer Slide Time: 26:44)



So, by the earlier discussion which we had, we found that all in the bending moment represent the resultant of certain linear distribution of the normal stresses sigma x or the cross-section that you see. The upper fibers are there in the tensile stresses which is one form of the normal stress, and lower fibers are there of the compressive stresses which is also one of the form of the normal stresses. Similarly, the shear forces Fx over any cross-section must be resultant of a certain distribution of shear stresses because you see whenever shearing is there, whenever the kind of twisting is there, always induces some sort of the shearing stresses, and these stresses are always coming along with this particular bending stresses and our main focus is on that only.

(Refer Slide Time: 27:28)



So, you can see here this particular diagram as we discussed, we have simple structure of a particular kind of strain being in which this cross-section is like that, and on that particular we have this particular section influenced by shearing force and on top of that this cross-section. After this you know like the delta x segment, we have the cross-section or at this particular shearing forces are F plus delta F, and then you see we have the bending moment which in the sagging form. So, M and this is at this particular side after this delta x, we have M plus dM.

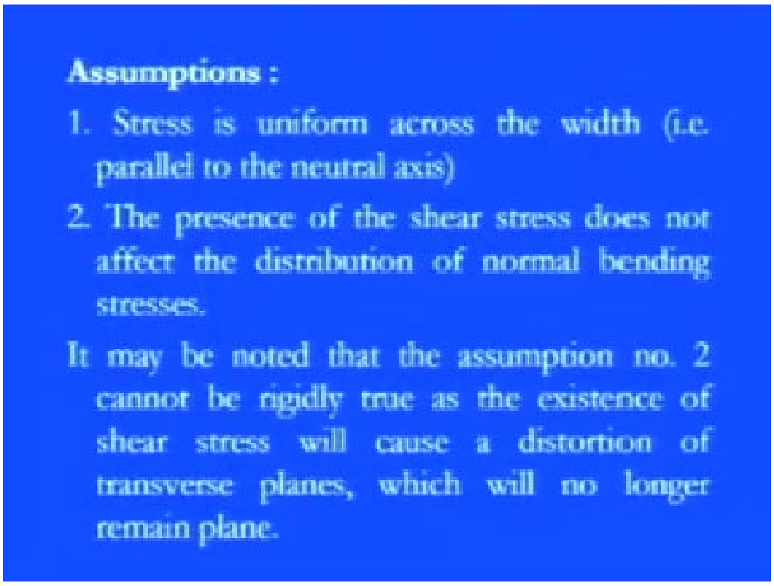
So, with that our main focus is that you know like that will happen along with the neutral axis and on above. So, this is the neutral axis which you see you know like our main plane of the matter of this mean plan is concerning, or which our main focus is coming on this particular way. So, that is what will happen, and how these fibers will react due to this F, and this F plus delta F along with this way or we can say in sagging form that M on this particular way and M plus dM will act.

So, if you see those things, then you will find that the resultant stresses are because you see here not only the bending stresses are there, but along with this we have the shearing stresses. So, that means, you see whatever these things are coming, these are coming with the resultant part. So, we have the resultant stresses of these particular things. So, resultant stresses are there, and you can see here they are coming in the triangular form at this particular point and at this particular point, the triangular form. So, we can say that the resultant stresses the sigma at this point, and here it is sigma plus delta sigma along with this point. And this side you know like just to be compared with the other side in the sense that this small element is in the equilibrium position under the action of the shearing force as well as the bending moment.

So, we need to concern that you see when it is coming along with this x and y axis, so what are the stress components along the x axis, what are the stress components along the y axis, and how this twisting will take place? Due to this twisting and shearing forces, what kind of interactions are there? So, now if you focus on this element, let us say the y distance is there from this neutral axis, and we have you see the AB element. So, now you see of this fiber our main focus is that now if you are taking a small element dA, you can see this particular part. So, we have a small element on which the stresses are acting.

So, when the stresses are acting or just the clear cut you know like the description is there, it has the area dA is there which has a distance from you know like the neutral axis y, and the section modulus is Z for this kind of thing. So, now with this particular width of the cross-section and width is dA area of this small element, and with these stress components from the left hand side, the sigma from right hand side, the sigma plus delta sigma, and under all these forces, the shearing forces and the sagging moments, these objectives in the equilibrium portion. So, now our main analysis will come that what will be the resultant bending stresses are there and what will be the resultant shearing stresses are there to calculate those you see and to put the relation in between those things, certain assumptions are always needed because to avoid the complication in the system.

(Refer Slide Time: 30:34)

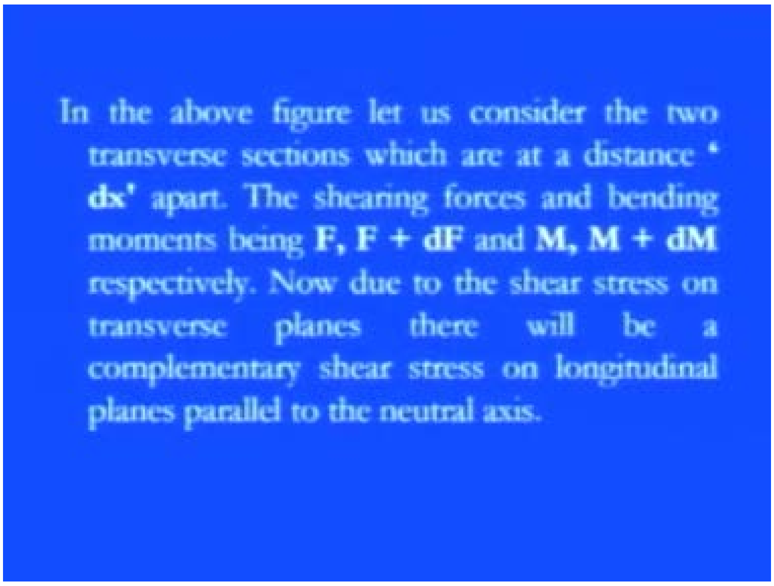


So, for that you see first of all the first assumption is that stress is uniform all across the width of those thing. That means, you see you know like it should be parallel to the neutral axis. If it is not parallel, then you see you know like the fibers are playing in the different way and then, you see whatever the stresses are coming on the fibers due to these combinations of the shear stresses and the bending moment all together, it is different. So, that is why you see first of all the first assumption is very important that whatever the stresses are occurring or inducing in this particular beam, it has to be uniform all across the width. That means you see they have to be you know like parallel, just parallel to that neutral axis. This is one.

The second is the presence of the shear stress does not affect the distribution of the normal stresses. That means, you see you know like since we have shear stress presence, and along with that there is also this bending moment and due to that the bending stresses are there, but whatever the presence of shear stresses are there, they will not affect in any way to the distribution of the normal bending stresses. So, normal bending stresses are always coming as per own concept. That is what we can again apply the same theory, the sigma by y which equals to M by I equals to E by R in this equation also, and it may be noted that the assumption number two cannot be rigidly true. Sometimes you see as an existence of shear stresses will cause distortion of the transverse plane, and that is what you see you know like which will not be exactly there with the same plane. That means, you see once we apply, once we remove the load, it will not be exactly in the same plane before and after bending.

So, sometimes this assumption two will not be valid for certain cases, but in general cases, we can you know like agreed upon those things that these are valid for certain things. And in the above you see you know like all those figure which we described in the previous one that we have simple segment, and we cut one portion or which one portion is affected. One side is affected by sigma and one side is affected by sigma plus delta sigma.

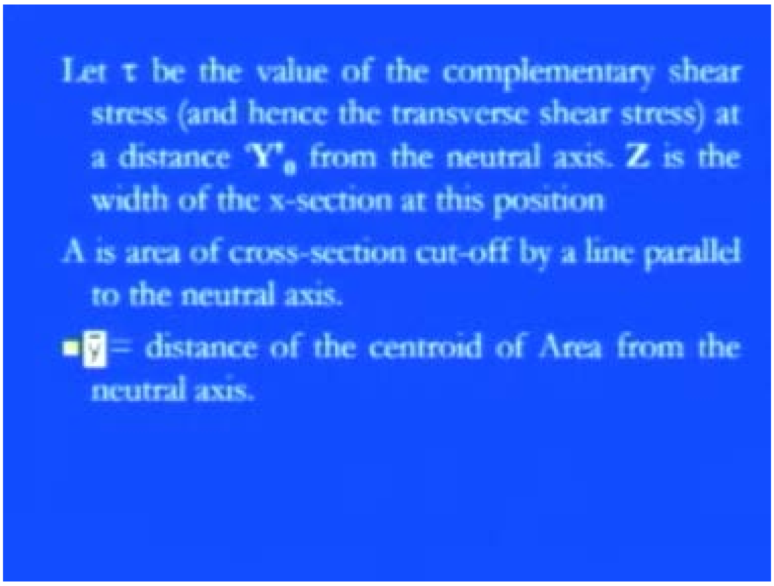
(Refer Slide Time: 32:44)



So, for those you see you know like for the two main transverse sections are there which we concern which has a distance of delta x apart from one point to another point and the shearing forces and the bending moment as we discussed, they are at one point. It is F and in this direction and another point, we have F plus dF in this particular direction. Then, you see we have the bending moment also. At one point, it is M which is going in this direction and M plus dM which is going in this direction. So, we you know like both of the sides are being influenced by both shear forces as well as the bending moment in that.

Now, due to the shear stresses on the transverse plane, there will be complementary shear stresses are always be there on the longitudinal planes parallel to the neutral axis just to balance the equation. Otherwise, you see what will happen if you know like because of this shear forces. And because of these things, the bending moment, the sagging bending moment, we cannot you know like gear to exact equilibrium position because if for any analyses, this is a must condition that the equilibrium position must be coming within those things, and then only we can apply all those equations. So, for that you see there is complimentary shear stresses are there just to balance these shear stresses acting. So, to counter the shear stresses or we can say the complimentary shear stresses are always there on the longitudinal planes, they are which are just parallel to the neutral axis and they will form always you see just to balance these equations.

(Refer Slide Time: 34:09)



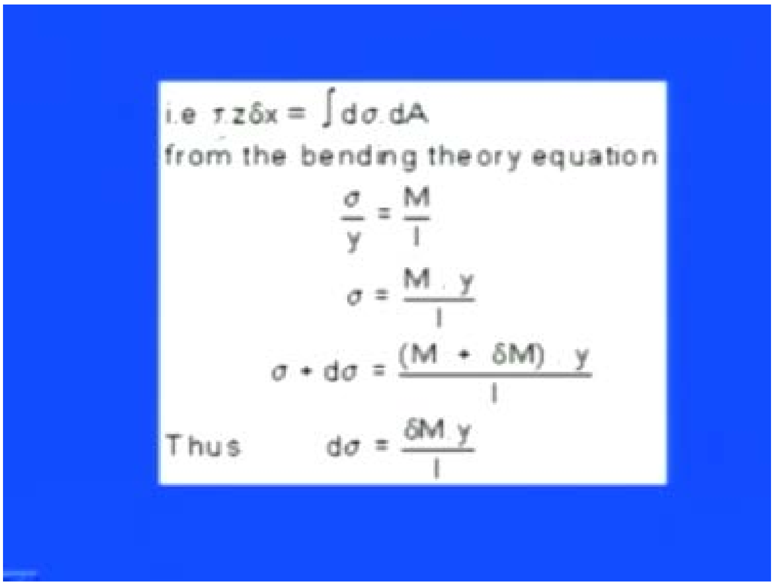
If I am saying that tau is the value of the complimentary shear stresses which are coming you know like in the transverse shear stresses at any distance of Y 0 from the neutral axis Z, then from the neutral axis, then we can say that it is one more component which has to be defined which is Z which is nothing but equals to the width of the cross-section at any position. So, now you see we have two-dimensional parameters. One is where the tau is acting. That means, you see where the complimentary shear stresses are acting and that is now at the distance of Y 0, and another one is from the neutral axis and another one is the width of the cross. This width is there of any cross-section which we are defining by the Z like it is absolutely based on what kind of cross-section which we are using. So, the Z value is varying.

A is the area as I told you like in the cross-section cut off by the line parallel to the neutral axis, and Y bar is the distance of the centroid of the area from the neutral axis. So, we found that these are some of the geometrical parameters which can be analyzed for any kind of numerical problems while we are dealing with those things. So, here you know like this tau, which we found that the complimentary shear stresses are there which are always inducing in that kind of structure, whenever even the shear forces are there or we can say that these bending moments are there for that kind of thing. So, here you know like with those components as we discussed for this left hand side and right hand side for this small element, again we found that sigma and sigma plus delta sigma which is you know like the resultant stresses are there due to shear force.

Bending moment are just normal stresses on the element on the area dA at the two transverse sections as we have shown in the previous diagram, and then you see if we want to find out the difference of the longitudinal forces. On one side we have the force sigma into dA; on another side we have the force of sigma plus delta sigma into dA. So, if you want to find out the differences of the longitudinal forces which is always equal to delta sigma into delta dA for the small element, and you see here we just want to find out this quantity is d sigma into dA is the quantity which is sum up over the entire area. This area A is in equilibrium with the transverse shear stress tau on a longitudinal plane of area which has Z into dx. That means you see you know like our main focus is how these stresses are being setup, and one side you see we have you know like this because our total thickness which we have assumed is delta x.

So, at one side we have this sigma, at one side we have sigma plus delta sigma and then, you see what will be the total you know like the resultant longitudinal forces are there which is being setup due to the this shear forces as well as the bending moment. So, that that is why you see it is coming like that and then, after focusing on those things what we have is this tau z into delta x which is the resultant part is there the longitudinal forces.

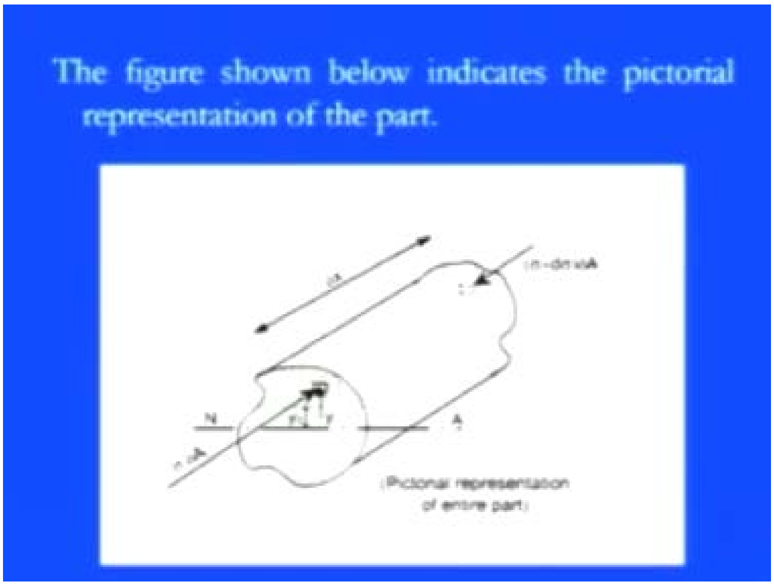
(Refer Slide Time: 37:04)



So, tau which is coming due to the complimentary shear stresses is the width, the Z is the width there and dx which is the total thickness which we are assuming. So, this tau which is stress into this area will give you the force which equals to the integration of d delta into dA because you see these are the longitudinal transverse forces are coming due to the resultant forces, difference is there. So, if you integrate for entire region, then you will be ending up with the integration of d delta into dA. So, from that particular bending theory, again we can simply apply the sigma by y which is equal to M by I. So, from that we have the bending stresses sigma which is nothing but equals to M into y divided by I or we can say that since this is on the first section.

On the second section, you see we have sigma plus d sigma. This is the total stresses on the other side and due to that we have the bending which is coming on the sagging way M plus delta M into y divided by I. So, from those above two equations, we can conclude that d sigma is nothing but equals to delta M into y divided by I. So, here you see we have shear stresses tau into Z into delta x equals to integration of d sigma into dy dA, and we have you see the total another stress formula. This d sigma which is nothing but equals to delta M into y divided by I. So, if you compare both of the equations, then you will find that these stresses are the resultant stresses coming due to the shear part of the shear forces, that is tau is there in one figure and due to the bending moment, delta M is there. That means you see you know like whatever normal and shear stresses are coming, they are the resultant part due to these two portions.

(Refer Slide Time: 38:44)

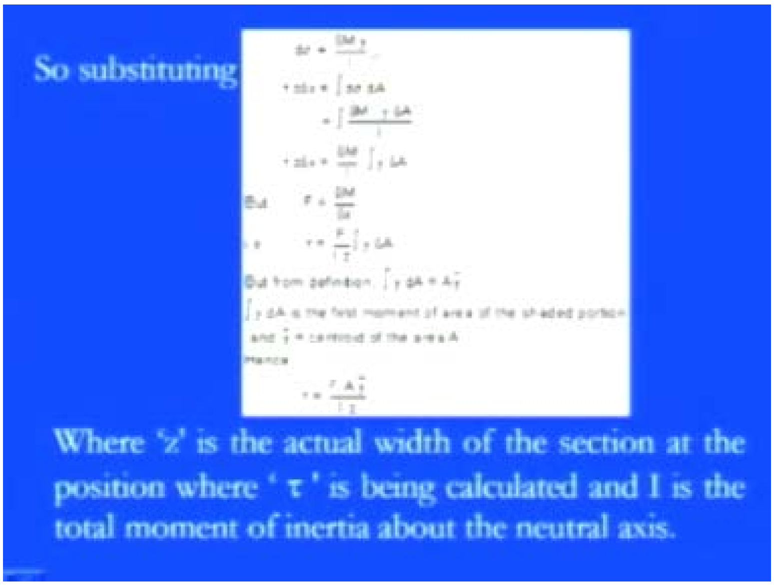


So, you can again visualize those equations from this particular figure, and this figure says that we have a small rectangular section like this cross-section, and the total length is delta x. And for that you see this neutral axis passing from this and this is our main strip of you know like which our main focus is that how the stresses will play. So, out of one side of that, we have sigma into dA and on another side, we have sigma plus del sigma into del A.

So, you see here how they will balance. So, exactly you see we discussed this particular section that this is there and then, we have the distances from you know like the neutral axis of this particular area that is the y distance is there, and this y 0 is the initial distance, or we can say y bar is there that we define for which you see this Ix axis is there is nothing but equals to integration of y square dA. So, for that you see you know like along with those neutral axis and along with this particular axis, since both are coming altogether you know like in the same way, and there is a combined effect of shear stresses as well as you know like the shear forces as well as the bending moment.

So, we can simply visualize that actually when these fibers are being influenced by both of the way, then the resultant is coming from this side sigma into dA. That is the force is there and on this side, sigma plus delta sigma into dA from both the side. So, the resultant since we are saying that this and this small element is in equilibrium position along with this particular you know like whatever the forces are coming. So, they will you know like cancel each other, and that is why you see complimentary shear stresses also come in tau which are there at all across the parallel to you know like this cross-section, or we can say that the parallel to the fibers, and that is what you see we know like we assume in the previous section.

(Refer Slide Time: 40:39)



So, here if you substitute those things which we got d sigma is delta M into y by I, and from the second equation by you know like the basic bending, this equation and the second part you see the first thing was there due to this tau into z into delta x which is equal to you know like integration of this d sigma into dA. So, now if I replace this d sigma, then what we have? We have this tau into z into dx equals to integration of delta M into y divided by A into delta A. So, now you see here you know like if you consider this particular equation, then you will find that we have both of the components, we have shear stress component, we have the bending component and due to that you see you know like the result is coming.

So, if you analyze those things, then dM by I simply taken out, and we have you see this tau into z into dx equals to delta M by I integration of y into dA, or you see you know like just go to the main relation between shear force and the bending moment. We have shear force F equals to dM by dx. So, with that you see you know like again we can simply put those things here. So, by keeping those values here, this F is equal to dM by dx. We are ending up with tau which is equal to F. Shear force is there divided by I into z because z will come, this delta M and delta x will be giving you the F. The F will come here I into z integration of y into dA, but from the definition of integration of y into dA as we discussed in the previous section, which is equal to A times of y bar.

Again you see with this y integration of y into dA is nothing but the first moment of area because you see in the previous section, we discussed about the second moment of inertia that is the integration of y square into dA, and the second moment of inertia will give you the real bending feeling. Here you see you know like we have the first moment of area that is the integration of y into dA of the particular shaded portion which is there, and y dash is as I just told you in the previous diagram that it is nothing but just the location of the centroid from the neutral axis.

So, you see how much distance is there like the centroid from this neutral axis. If they are common, then you see you know like this particular first moment of area, it is just you know like coincide with each other. We can calculate correspondingly, but here you see whatever the distance is there of the y bar which has you know like the main distance from the centroid to neutral axis always coming up in the co-operation of this first moment of area. So, with that you see you know like we can simply put this A into y bar here. So, we are ending up with shear stress formula. Shear stress is nothing but equals to shear force F into the total area A into y bar which has the distance of the centroid from the neutral axis divided by I into z, where z is the actual width of the section at which the position you know like at any position is there, and tau is you know like this complimentary shear stress are there which can be easily calculated, and I is the total moment of inertia above the neutral axis.

So, now you see if you focus on this formula, then the shear stresses mainly you know like is coming due to the shear forces that what are the shear forces are there in what direction this shear forces are there, and you see what is the affected area under which these you know like the shear forces are acting and then, you see again the key feature is that what is the exact distance is there of the centroid from the neutral axis. Then, you know like this total moment of inertia I is key feature because what kind of the cross-section which we are taking, correspondingly you see you know like the shear stresses are coming, and then another part is you know like what will be the total width is there of the cross-section.

So, the dimensional parameter of cross-section is very important to visualize the shear stresses in those things. So, now I would like to conclude this chapter. So, you know like in the first section of this chapter, we discussed about when a beam is under the combined load, then how we can you know like the bending moment as well as you know like the bending stresses. So, in that you see we found that if you know UDL is there and the point load is there, then how we can segregate both the issues and then, we need to calculate you know like the different sections. The maximum bending moment like you see you know like in case of point load, we found that wL by 4 is there in case of you know like this UDL wL square by 8 is there, and then we can calculate the maximum like the shearing stresses that is you know like this M by M maximum divided by I into this Y maximum.

So, you know like this kind of situation, it is pretty easy to analyze and then, in the middle portion you see our main focus was there that whenever a beam is under the action of you know like shear force as well as bending moment, then the main assumption of this bending like once you remove the load or once you remove this bending moment, then the situation is exactly same as the previous. That means, you see you know like the fibers are having the same kind of plane of the section as before and after the bending. So, whenever this shear forces are acting, then this situation will not remain exactly as the previous case.

So, that is what you see this theory is sometimes uniform and non-uniform bending. In non uniform bending, sometimes it is not valid, but you know like it depends on what kind of structure which we are using. Based on that you see this sigma by y is equal to you know like M by I equals to E by R. It can be valid for uniform and the non-uniform bending part. Then, in the final part you see we discussed about the shearing stresses of a beam. In this you know like some of the assumptions which we made that whenever you see the forces are applying on this particular you know like the beam, then how we can you know like put the cross-section of this particular beam and how we can analyze for the shearing forces.

So, these two assumptions sometimes you know like it is valid sometimes depends on what kind of structures or cross-sections which we are using. These assumptions are valid and then, you know like in the last section of this, we found that the shearing stresses are always like they are the resultant stresses coming due to shear force as well as the bending moment, and you know like if you want to calculate the shearing stresses, then it has you know like the total component of shear force F into area into y bar which is the distance of the centroid from the neutral axis divided by I which is like this total moment of inertia into the total whatever the actual width is there on the cross-section.

So, in that shearing force, the shearing stresses if you want to calculate, it is highly sensitive to first of all that what is the cross-section of your like the objectives. That means, you see whatever the beam which we are taking, what exactly the cross-section is and then, the second thing which is very important that how you are you know like configurating those things means actually if you are talking about I section or if you are talking about the rectangular section, then what is our affected area on which our main study is. So, based on that you see you know like we can simply calculate the shearing stresses. We can calculate the bending stresses. This is sigma which is nothing but equals to sigma by y equals to M by I or equals to E by R.

So, based on either that equation or this equation tau which is F a y bar divided by I z, we can simply calculate the bending stresses and the shearing stresses for a beam which is under the influence of shear force and the bending moment. So, you see here this in this section, we simply you know like go with the simple assumptions and we simply put certain shearing stress equation, and the bending moment equations for bending stress equations.

In the next chapter, our main focus is that if you have different section as I discussed like if you have a circular cross action, if you have this I section or if I have a rectangular cross-section, then how can one can be calculated, you see this shearing stresses and the bending stresses for that, and which you see if I am just taking this I section or any section, then which of the area is having you know like the maximum shear stress or the bending stresses. Then, how this interaction is there you know like this bending stress and shearing stresses, and what is the real relation between those stresses. These are our main matter of concern and that part you see we are going to discuss about different kind of cross-section of a beam.

Thank you.