Strength of Materials
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Lecture – 27

Hi, this is Dr. S. P. Harsha from Mechanical and Industrial Engineering Department, IIT Roorkee. I am going to deliver my lecture 27 on the course of Strength of Materials which is developed under the National Programs on Technological Enhanced Learning, NPTEL. Prior to start this lecture, I would like to refresh you know like the previous concept which we discussed in the previous lecture. We discussed you see you know like that if a beam is under the combined loads like you see if we have the point load and the U dl plus, if we have you see you know like that we have the U dl plus the triangular element, then how we can analyze those things.

So, in the previous lecture, you know like that at which point the maximum bending moment is there, at which point the minimum bending moment is there, and what is the relation between shear force and the bending moment. That kind of discussion we made in the previous lecture, and we found that there is a point where there is a change in the bending moment diagram from positive to negative. That means, you see when there is a change from the sagging to hogging of the bending, the bending moment diagram, then this point is known as the point of contra flexure. That means, you see you know like we have the drastic change is there from the positive sign of bending moment to the negative sign of bending moment, so that you see we define and even actually we found that at those points, where the drastic changes are there in shear force diagram, we found the maximum bending moment.

So, this kind of relations even we derived in the previous lectures that when F is equal to dM by dX. Then what exactly you see whatever the changes are there in the bending moment with respect to the domain x, it gives you shear force and if we are keeping shear force 0, then the bending moment will remain constant. M will be constant because dM by dX is equal to 0. So, that discussion you see we made in the first part of our lecture in the previous class, and then you see we derive a simple bending theory that with certain assumptions that you see like when the beam has to be in straight form and the plane should be symmetrical towards the longitudinal axis, and also you see the plane should remain plane before and after the bending. And whatever the material which we are using for the bending, the beam material, it should be homogeneous isotropic material.

So, this kind of assumptions we made and then you see we concluded that actually you know like that if we have the straight beam and if we have the curved beam, there are certain portions which are affected by the stresses. That means you see when there is a straight beam, no bending moment is there. Then you see all the fibers of that particular beam is you know like just existed in the symmetrical way, but when you see you know like a bending moment is applied at the two extreme corners. Then the upper fiber is always you know like experienced by the tensile forces, and the tensile stresses are occurring on those fibers while the lower fibers or we can say the bottom fibers because it is just like the curved part.

So, the bottom fibers you see the experienced by these compressive forces, or we can say we have the compressive stresses, but within those tensile and compressive, there is intermediate layer on which there is no impact of this shear stresses, and if we will focus those points you know like if we just find those trajectory of those particular points, these trajectory or these locus is known as the neutral surfaces. We can say actually whatever the axis which is forming on these particular surfaces is known as the neutral axis. So, neutral axis is that axis you see which you can say that whatever the impact is there on the bending moment, it has no impact on that particular thing because there is no shear, there is no deformation and there is no strain component all along this particular line.

That is what you see we define the neutral axis and since, it is a curvature part, you see we define that actually if you want to define the radius, then always starting from the locus, this radius of curvature of the locus to this curvature path. So, whatever the distance will come that distance is known as the R radius of curvature. So, you see now our reference point is the neutral axis, and now you see whatever the things will be going to happen with respect of the bending moment and all those things, our main focus will be there on the neutral axis to define the neutral axis and then correspondingly you see you know like whatever the changes are there, we need to define. So, that kind of discussion we made in the previous lecture.

Now, you see in this lecture, we are going to again analyze those things that actually what will happen you see if we have a different cross section of the beam irrespective of you see if we have the rectangular cross section or I section or whatever, then how we can find out the neutral axis for those sections. First, we will also go for the numerical exercises and then you see the key feature is that actually what will be the bending stresses in that because still now you see we discussed only about the bending moment that what will happen to the bending moment, what will be the relation between the bending moments and shear force diagrams.

So, all those bla bla, these bla bla discussions which we made only with that shear force and bending moment diagram, still we did not discuss about what will happen when the shear forces are existing within the beam element, and then due to that what will happen with the bending stresses, or how they induce the bending stresses within those components, and then how these neutral axis will react when the bending stresses are there, and what are the potential area within the beam when it is under subjected by various forces through which we can say that, yeah this is the potential area which the maximum bending stresses are there. This is the minimum bending stresses are there.

So, if you want to design the beam irrespective whether cantilever beam or simply supported beam, you can simply you know like focus on those potential areas and use the factor safely higher recording to the safety reasons. So, this kind of you know like discussions which we are going to discuss in this particular first lecture. So, first of all as we discuss the bending stresses in the beams or the derivation of the elastic flexure formulas because you see first of all we need to see that how the bending stresses will form and then how it will be distributed all along the entire span of this beam.

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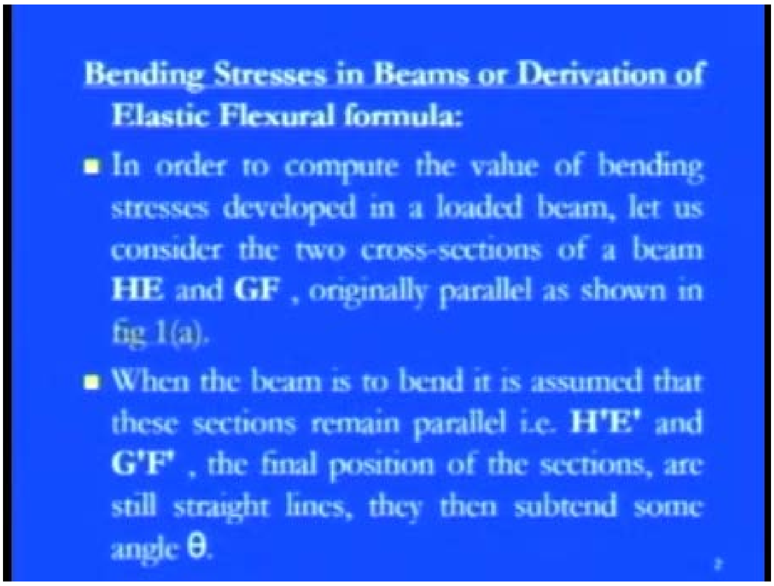

So, in order to compare the value of bending stresses developed in a loaded beam, first of all we need to consider the two cross-sections of the beam like HE and GF originally parallel because there is no bending moment and then they are absolutely parallel to each other.

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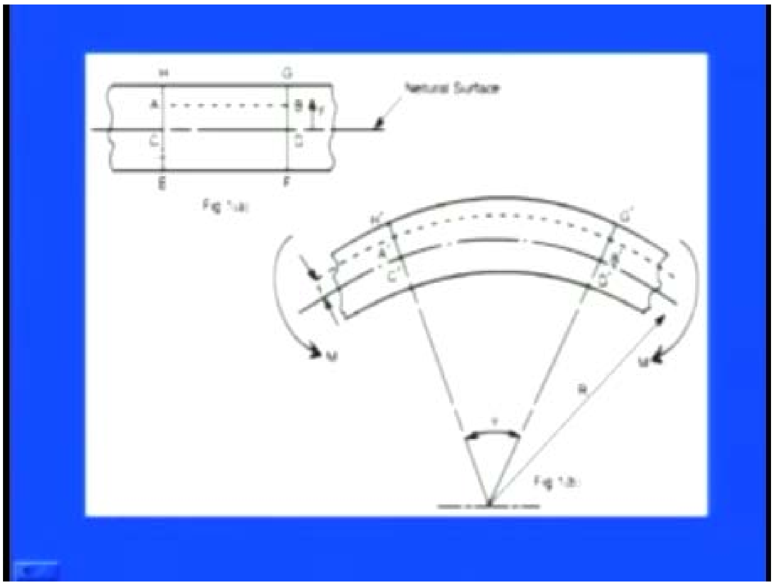

You can see the first diagram here on the screen that these two you know like we are taking these two elements in which these two lines are very, they are simply parallel to this EH and GF. And you see you know like these as we discussed already in the previous chapter that the middle section which is passing from you know like entirely just you know like this neutral surfaces. This is nothing, but our neutral axis and we just want to focus that actually if we take element, let us say from distance y from the neutral axis, this AB, then what will happen is means how the fibers will behave when it is under subjected by bending moment. That is our prime focus.

So, for that again you see what we are doing here since it is plane structure in which this first figure 1 A plane structure in which there is no bending moment, there is no shear forcing is applied. A straight beam is there. Well it means equilibrium manner and you see for that we have simple structure. There is no deformation you can see on the diagram, but when a beam is to bend, that means if your bending moment is applied, it is assumed that this section remain parallel. That means, you see H dash, E dash and G dash, F dash, they have to be there in the parallel because you know like we assume that even the plane should remain plane even before and after the bending.

So, with that particular assumptions, we can assume this kind of theory and the final position of the sections are still together you know like the straight lines are there corresponding the previous lines. And they also you know like subtended by the angle theta, and you can see this particular diagram figure 1B as we discussed previously also that when you apply the bending moment at the extreme corners, it will bend. And then you see the extreme top corner will be extended due to the tensile forces, and the tensile stresses occurring on this particular surface, but you see this particular surface which is under the compression, we have the compression stresses on these particular surface, but we have the neutral axis line, this A dash, this B dash, this C dash and D dash. There is no strain component on that. That means you see C dash, D dash is equal to CD. Just keep this thing in your mind because you see there is no deformation occurring along this particular line. This is the neutral axis is there.

So, C dash, D dash which we are denoting in the figure 1B is exactly similar to this figure, this point CD in figure 1A, but there is a change in A dash, B dash to AB because you see these certain deformations are there, the extension is there. So, we have the tensile deformation and this is due to you see you know like we have a kind of you know like some extension is there, and this extension we can easily form because we know that this distance is R, and you see this distance because the neutral axis distance as we defined that we can always you know like suppose to define the radius from this locus point to the neutral axis.

So, this distance is R and also, we define in the previous section that actually this AB distance, this AB section has a distance from CD as y. So, this 1 is y. So, the total distance if you want to find out for the curvature part is nothing, but equals to R plus y and then this theta is there. So, we can define with the radius and R definition for that, and then you see you know like we have one more thing that we have this H dash and G dash which is also in the extensive part. So, we have the maximum deformation in tensile way on these particular points, and this is the minimum deformation at these points you know like or we can say the 0 deformation at C to D point, and you see like again some compressiveness is there of this E dash, F dash points in this particular figure.

So, these figures you simply justify that actually whenever there is a kind of bending moment, then how you know like the fibers are playing in this particular way, and how the tensile and the bending this compressive stresses are coming due to the bending only. So, that is why you see we are saying that this normal stresses which are coming due to the bending action is always the flexural stresses.

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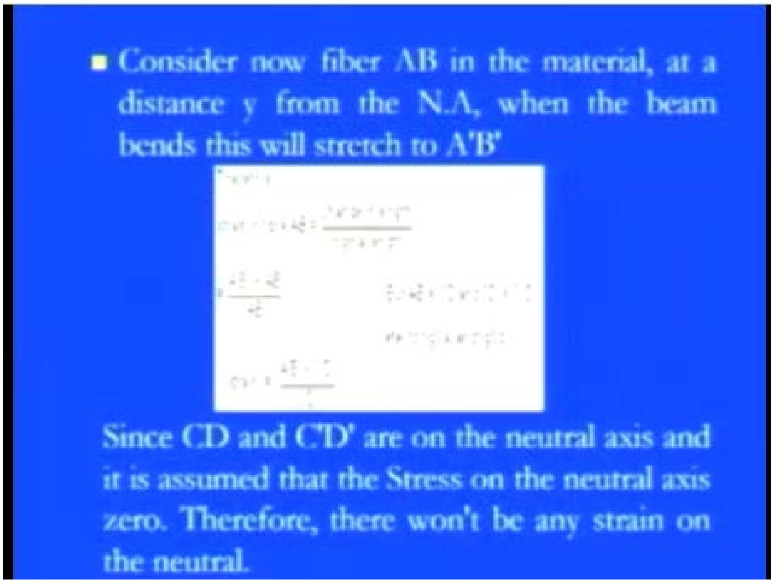

Now, you see consider the fiber AB of the material, you see as I told you in the intermediate section, if you are just focused on the fibers of that particular path of the AB line at the particular distance y from the neutral axis, then the bend will be stress to A dash B dash. That means, you see now due to the bending action, there is a deformation in this beam right from AB to A dash B dash. So, now our main intension is that actually how the deformation can be measured, and how we can you know relate this strain to the stress component with the using off because you see the important feature here is whatever the deformation is there, due to these bending moment is always the elastic deformation. And that is what you see we can straight way apply the Hooke's law for that in which the stress is proportional to strain, and since here the bending moment is applied.

So, we have the bending stresses which is correspondingly you know like proportional to the strain component. So, here with that particular theory, again what we are doing here is the strain in the fiber AB which is our main focus is on fiber AB only that actually how the strain will be coming and what kind of deformations are there along that particular path. So, strain in the component or we can say the fiber of AB which is nothing, but equals to change in length divided by original length, and the final distance or final length of this fiber AB is A dash B dash earlier distance, or we can say earlier length of the fiber AB was AB. So, A dash B dash minus AB divided by A AB.

So, that means, you see whatever the original path is there means how much deformation is there divided by how much original length is there. This will give you the total component means how much deformation is there, and this deformation generally we are measuring in terms of the strain. So, strain definition is like that, but as far as the neutral axis is concerned, we already assume that AB is equal to CD because you see you know like there is a straight part. When there is no bending moment is there, the straight regions are there. So, AB is just cutting exactly as a CD.

So, there is no difference in that same distances are there. So, because there is no bending moment, there is no deformation in that particular you know like either the neutral axis and above. So, we have AB equals to CD, but CD equals to C dash D dash because the main thing is that CD and C dash D dash are the points, which are always lying on the neutral axis, and as you know like we know that there is no strain or there is no deformation is coming on this particular neutral axis. So, we can straight way you know like conclude that whatever the distance is there before bending moment is applied and after bending moment is applied on the neutral axis. They have the same distance without any deformation or without any strain.

So, we can say that CD is equal to C dash D dash and if we apply those things here, what we have? We have the strain component is nothing, but equals to A dash B dash which is the previous thing and AB is equal to CD and CD is equal to C dash D dash. So, we have A dash B dash minus C dash D dash divided by C dash D dash. So, now you see here our main conclusion for the strain component is just focused on what will happen when the strain component is there. That means, you see A dash B dash is nothing, but the total deformed path along the line A, B. C dash D dash is the new location for the points on the neutral axis, and you see you know like when both are comparing, then we can get the strain component.

So, since CD and C dash D dash as I told you like they are on the neutral axis, and it is assumed that the stress on the neutral axis is always you see line 0 and since, there is no stress component is there on that particular part. So, there is no strain, no deformation is there on that particular part. Therefore, they would not have, they would not be having any strain component on the neutral axis and there is no deformation there. So, we cannot measure anything. So, whatever the points are there before bending moment and after bending moment, they have the equal distances on the neutral axis.

So, for that now our main intentions to find out with the use of the curvature theory that what will be the distance of A dash B dash and C dash D dash, so that we can straightway get the strain and once you have the strain, then you can easily go with the stress component with using of the modulus of elasticity. Because whatever the deformation which we are assuming for this particular kind of analysis is always the elastic deformation, and all those Hooke’s law is valid for that, and you see all those elastic modulus of Young’s modulus of elasticity or whatever the components which are valid for the elastic deformation, they are also valid for this kind of analysis. So, come to the main point that the strain is nothing, but equals to A dash B dash minus C dash D dash divided by C dash D dash. So, if you focus on the A dash B dash in the previous diagram, then we will find that the total distance for A dash B dash was R plus y.

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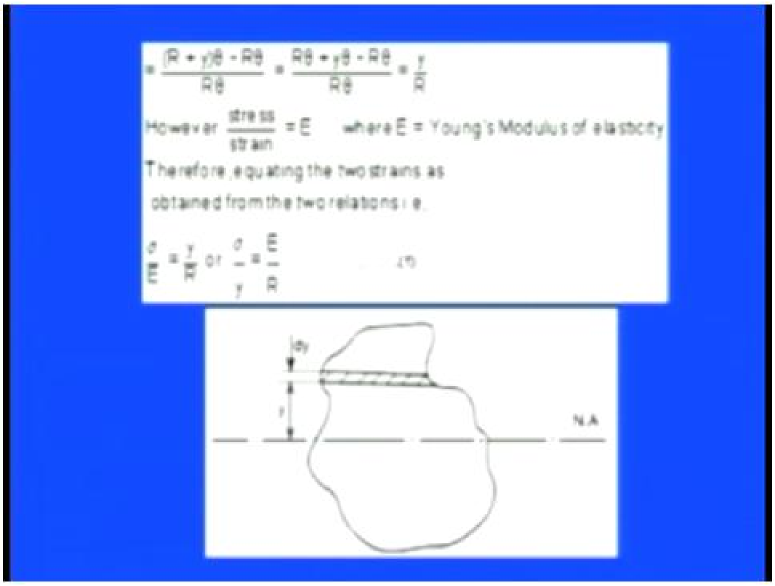

Now, you see this is the curvature path was there. So, if you multiply this arc into this particular theta, then you have the total distance. So, R plus y into theta, that is A dash B dash minus C dash D dash, straightway you see it is on the neutral axis where the distance is R. So, R theta will give you the total distance. So, R plus y into theta will be A dash B dash minus R theta is the C dash D dash. So, if we put these things in this particular formula, the strain formula, then we have R plus y into theta minus R theta divided by R theta and if we generalize those things, then we have the strain equals to y by R, where y is the distance of the fiber from the neutral axis.

So, you see here always it is above you know like neutral axis, and you see we just want to observe the effect of this bending moment on those fibers from the neutral axis, they have the distance of y. So, now you see as I told you that it is the elastic deformation where the Hooke's law is valid. So, we can again use the stress is proportional strain or you see stress by strain will give you Young's modulus of elasticity E, and then you see if we are keeping those values there, then what we have is this stress. The sigma is always denoting the stress. So, sigma by E will give you the strain and you see in the previous formula, we just derived that the strain is nothing, but equals to y by R.

So, we have now real formula which is the basic formula for any bending stresses which are occurring in any beam kind of you see irrespective whether it is a cantilever beam or simply supported beam. The bending stress is sigma divided by E which is equal to y by R, or we can say that this bending stress sigma is nothing, but equals to y by R into E, or we can say that actually E which is Young's modulus of elasticity R which is the radius of curvature into if you multiply y as, so as y is you know like just increasing, definitely you have more bending stresses. Obviously we have because you see if you compare the realistic way this bending of beam, then we will find that the upper surfaces because as you are moving from upper direction from the neutral axis, we have more and more stresses, and this will you know like conclude from this formula also that sigma is equal to E by R into y.

So, as you increase y, again you have more and more bending stresses in that particular beam, and whatever the fibers are there on the extreme corner, they are experienced with the maximum bending stresses in that particular way, and we will describe many more things in the further lecture. So, you see here now again if you focus on just distance at y, but if we cut the section of dy at this particular y, so you see we have just you know like unsymmetrical section and this you see for that we have a neutral axis for which is passing from this particular way, and we have a segment for which actually our main studies focused that we have this particular small segment, and it has you know like the total width is dy which has a distance y from the neutral axis. So, now you see what will happen when the bending moment is applied on that particular section, and how you see the area will affect of that.

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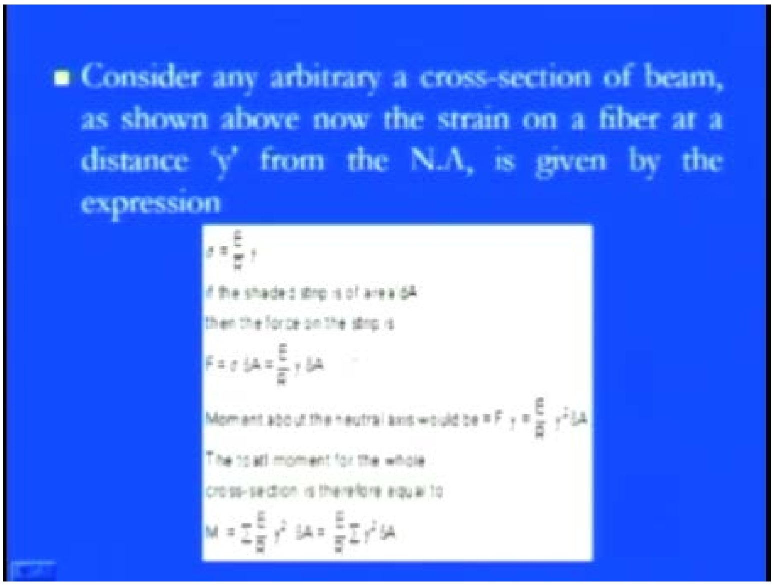

So, considering this particular arbitrary cross section of beam, as you know like shown in that main you see now what will be the strain on the fiber at distance y. That means, you see you know like what will happen when the bending moment will apply, how these fibers will play under the action of this bending moment? Then you see you know like again we would like to start from the same formula that when it is under the action of bending moment, then we have the bending stresses sigma which is equal to E by R into y, and you see if the shaded area which I just shown you and that dA. Now, if our main focus is on that, what will happen? You know like when you apply the bending moment, then you know how fibers will play their key role in set up of the bending stresses in this particular strip dy.

So, for that you see again our main focus is that actually what will be the force on that particular strip. So, the force is nothing, but equals to whatever the stresses are there into the area and area we consider dA. So, sigma into dA will give you the force, or we can say sigma since already we have described that when it is under the action of bending moment. The bending stresses are there. So, sigma is the bending stress which is equal to E by R into y into delta A. So, now you see here we have the force. Once you have the force, then you can easily get the moment for those things.

So, for moment about the neutral axis, this will give you force into the distance because you see what we are doing here is, we are simply taking strip which has distance y from the neutral axis. So, we can easily compute the moment you know like along with this particular neutral axis because we have a distance. So, when we apply the force and the distance is there, this will give you the moment. So, F into y will be the moment or you see once you have the force, you can easily replace that force E which equals to E by R y into del A into you see this distance is y. So, what we have? We have the moment which is equal to E by R y square del A.

So, now you see if I just want to compute this moment for whole of the section because till now you see we discussed about a small strip which has you know like the area elemental area dA which has the total you know like the width of the strip is dy. So, for that you see this moment is there which is equal to E by R y square into del A, but now it is sum up all those small element which has a distance you see certain distance from the neutral axis, and if we sum up with the summation, these thing we have the total moment which is there you know like moment applied on that particular section, and due to that we have the bending stresses. So, for that this total bending moment over the cross section is summation of E by A y square dA or since, thus total summation is always focused on y square into dA.

So, E and R are the constant one. As we discussed E is the modulus of elasticity which is a property of the material which you are using. So, Young's modulus of elasticity will come and the curvature is always focus that actually how what the dimensional parameter which we are using for that. So, if you just take it out once, then we have the summation of y square dA, E is the total you know like will give you when you sum up those things, the small element, it will give you a real feeling about the moment.

So, once you do that, then you have the total moment for that section and you see in this there are two components. One component E by R which is as I told you is subjected to like what kind of material and the dimension which you are using, but another component in that is the summation of y square dA. This is a specific property and this property you know like of a material is known as the second moment of area. That means, you see you know like it is a very different thing because it has the square term, y square and it has area term. So, you see here we are saying at different way. You see first area, it is second moment of area of the cross section and generally, we are denoting I. So, I is equal to summation of y square dA.

So, now if you are keeping those things, then we have the total moment y M is nothing, but equal to E by R. That was the first component. Second component was the summation of y square dA which we replaced by I. So, we have new equation of bending for you know like any kind of cross section which is equal to M by I equal to E by R, and you know like E by R, we already concluded that sigma by y is there in the first equation.

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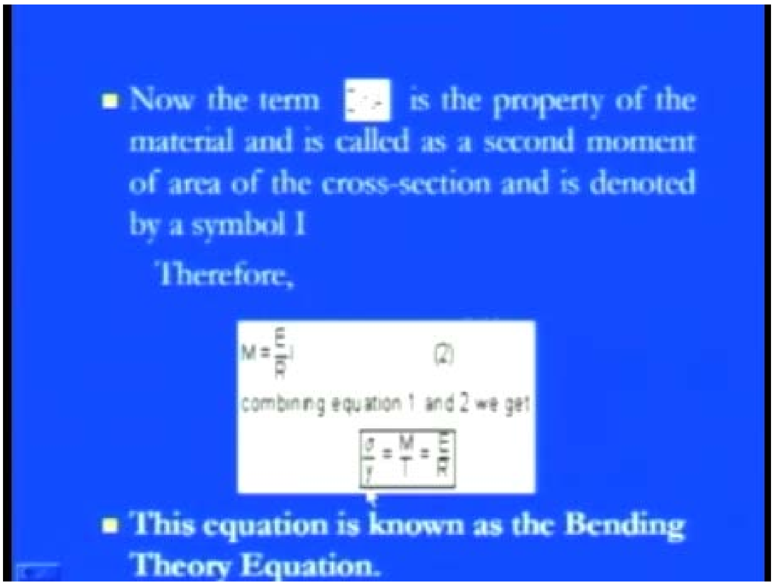

So, the total you know like the bending equation for this generalized bending equation, I should say nothing but equals to Sigma by y which is equal to M by I which is equal to E by R in which sigma is the bending stresses which is coming due to the bending moment, and this bending moment is M you know like, which is always applied at the extreme end or you see because due to the couple also, it can be formed y is the distance which you know like the fiber which we just want to find out this bending stress, and the distance of that fiber from the neutral axis is y. So, this y is there, I is IE. I just told you that it is the second moment of area which is nothing but equals to summation of y square dA.

So, one can easily calculate those things and E is the Young's modulus of elasticity. It is specifically you know like the property of a material which you are using for that elastic property is there and R is the radius of curvature. So, with all this six points or all the six properties, we can easily describe generalized theory of the bending. Sigma by y is equal to M by I which is equal to E by R, and it is a very good you know like equation for solving many of the numerical problems, and this equation is known as the bending theory equation, generalized bending theory. So, whatever the structure which we are using, if it is under the action of bending moment, then this equation is valid if we are considering the deformation up to the elastic one.

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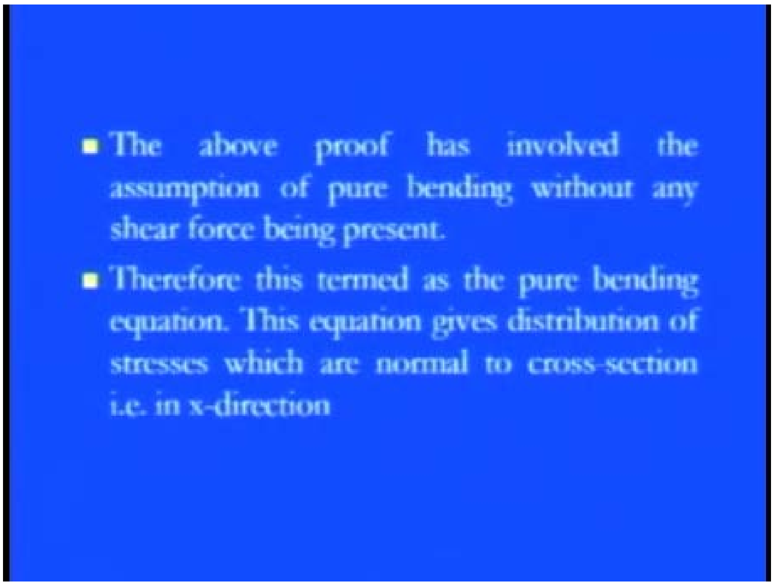

So, the above proof you know like has involved the assumption. That pure bending without any shear force being present because you see like we have sigma, it is the bending stress M which is the bending moment and you see other sections are the dimensional parameters like y I E and R and E is nothing, but the property of material. So, all those components if you conclude, then you will find that there is no other stress component is presenting. Only we have bending moment and the bending stresses, and that is why you see we can say that when the pure bending is there, there is no shear force occurring within that.

Therefore, you see this term as the pure bending equation as we discussed, and this equation gives the description you know like about the stresses which are normal to the cross section. That means in the x direction. So, that is why you see all these normal stresses are there as we discussed that whenever bending moment is there, they have the tensile and the compressive stresses. They are the normal stresses or we can say that they are the flexural stresses. So, this theory which is known as the pure bending theory has only the component related to the bending moment, bending stresses. There is no shear component is occurring in that. We have the normal stress component along with the bending stresses. So, now you see in that we found that there are some special components within this particular structure. So, we would like to first focus on those components.

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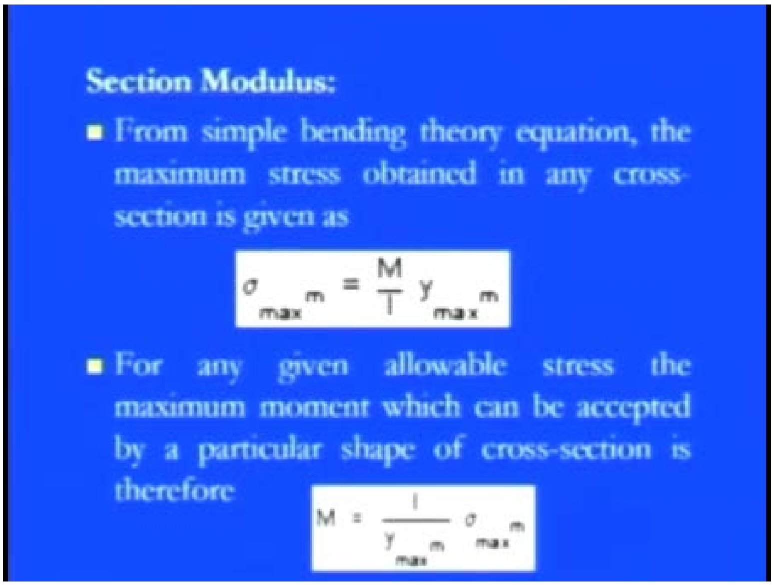

First is the section modulus. In that you see from the simple bending theory, whatever we discussed in the previous equation sigma by y is equal to M by I is equal to E by R, from that equation you see our main intention is that when there is you know like the bending moment is applied on this particular beam, then which area is influenced by the maximum bending stresses or the minimum bending stresses. So, for that you see here the simple cases just give you the maximum stresses which can be obtained at any cross section sigma maximum. That is the bending stress maximum is equal to M by I, whatever the bending moment which you are applying I is nothing, but the summation of y square dA.

So, these are the constant one, but the key feature is this distance. So, y maximum means you see you know like where this maximum bending stresses are occurring means which fiber is experiencing this maximum bending stresses which can be easily calculated. Once you know the maximum value or once you know the location, you can get the maximum value. So, sigma maximum is basically depending on y maximum. That is the distance from the neutral axis. So, sigma maximum is equal to M by I y maximum.

Second equation is that for any given allowable stresses means you see we have these allowable stresses for the bending of beam, and for that you see we just want to find out the maximum moment which is acceptable to any particular shape of the cross-section we can easily get because now our beam can be you know like absorb this much amount of bending stresses. So, we can get once we know that you know like this is the allowable stresses for this kind of beam.

So, for that we can easily get the maximum bending moment and the formula for maximum bending moment is nothing, but equals to I by y maximum into sigma maximum. That means, you see I which is you know like having all the summation of the cross-sectional like summation of y square dA, but you see what will be the distance of the fiber at which this maximum bending moment is coming. So, first of all, this y maximum which will influence this M, and second is how much bending stresses are occurring on those fibers, the value of that. So, that will also you know like depending parameter is there for calculating the maximum bending moment.

So, you see M is nothing, but equals to I by y maximum into sigma maximum. These formulas are very important to calculate the remaining parameter because you see in the question they can ask you that now calculate the maximum bending stresses. So, for that first of all you need to obtain that what will be y maximum is there. Once you know the y maximum because other parameters are very constant M by y M by I. So, you see first of all you need to locate the location y maximum and then you can calculate the sigma maximum. That means the maximum bending stresses. Similarly, you see you can also calculate the maximum bending moment with that formula as I shown you I by y maximum into sigma maximum.

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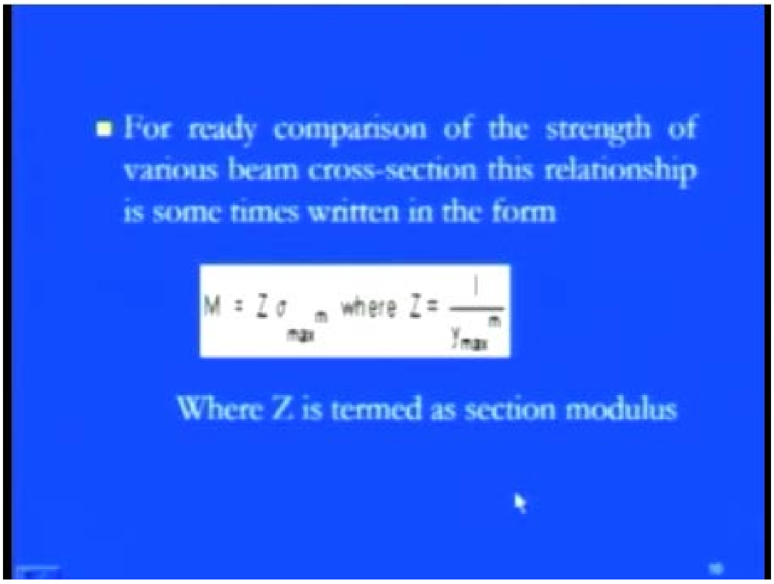

In the last one whatever you see you know like these compressions are coming in that particular curved beam of the strength, this particular beam of any cross section we can simply give with some relation and that relation is coming with the section modulus Z. As we defined earlier also that the section modulus is always you know computing the geometrical parameters like you see you know like if we know that, yeah this is the location at which the fiber is there which is experiencing the maximum bending stresses.

So, once you calculate that one and once you know that what will be I is there, you can simply compute this section modulus, and then you see whatever the bending moment is coming which is always maximum focused on the section modulus into whatever the maximum stresses are coming. So, if you conclude all that part, then you will be having M equals to Z into sigma maximum, where Z is the section modulus which is equal to I by y maximum. I is nothing, but you know like the second moment of area in which we can calculate the summation of y square dA. Y maximum is the distance from the neutral axis. So, we can calculate all those things with the corresponding way.

So, you see here we have three different ways to calculate, either the maximum bending stress or maximum bending moment or also, you see with the using of section modulus, we can calculate you see how we can relate the maximum bending moment and the maximum bending stresses. Those issues are discussed here because of you know like once you have the basic theory of beam, you know that actually because of the bending moment, we have the bending stresses at different locations, but to get the maximum value or the minimum value. We should first identify the location that which fiber is experiencing because in that particular if you know the concept of the bending, then you know that we have tensile stresses, we have the compressive stresses, but we have some of the section, which does not exhibit any kind of stresses.

So, for within those limits, you see our main intension is to get that which fiber is experiencing maximum bending stresses, and what will happen you see once you locate that part, how we can relate you know like with the maximum stresses or bending moment or bending stresses with the section modulus Z. So, all these you know like relations, one has to remember and these relations are there in front of you.

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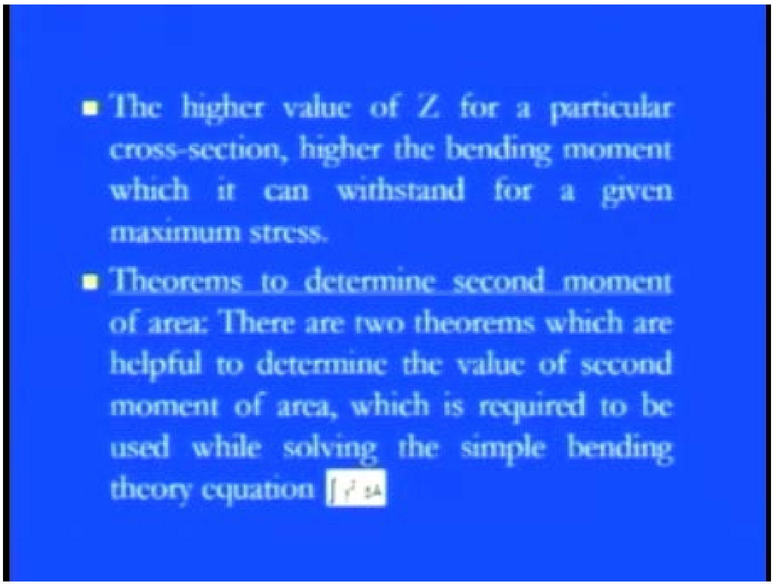

So, the higher value of Z, now you see we have M is equal to Z into sigma maximum. So, M is straight way depending on Z section modulus. So, higher the value of Z for a particular cross section, higher the bending moment is there because it is directly proportional to that which it can withstand for given maximum stresses. That means you see if you want to find out that what will be the bearing capacity of the beam is.

First of all, our main intention is to find out that how much you know like the maximum value will come for the section modulus, and once you know the section modulus, maximum value you can say that we can go up to this much maximum. The maximum value of bending moment and the corresponding change is there in the bending stresses. So, that is why you see Z which is the section modulus is a very important parameter to design you know like this particular beam for loading conditions. And now you see we would like to put one theorem to determine the second moment of area because you see in previous formula, you will find that I is a very important parameter.

So, our main intention is that how to find out those things because practically you see if you are going for individual sections and then sum up is not an easy thing because you see we have simply linearly varying this uniform structure or any kind of you know like this kind of structure is there in which we can simply put the integration from 0 to some value. Then, we can get here. We have a different uniform section in which the different segments are there. So, it is not easy to get all the small segment area, this y square dA and then sum up those things.

So, here you see like we need to go with certain theorems. So, here there are two different methods. One method which I just discussed that you know like there are two theorems which are helpful to determine the value of the second moment of area, and which is required to be you know like used while solving simple bending theory of the equation, the integration of y square dA. So, first is summation of y square dA, and second is integration of y square dA which is helpful in which condition this is the prime you know like the matter of concern.

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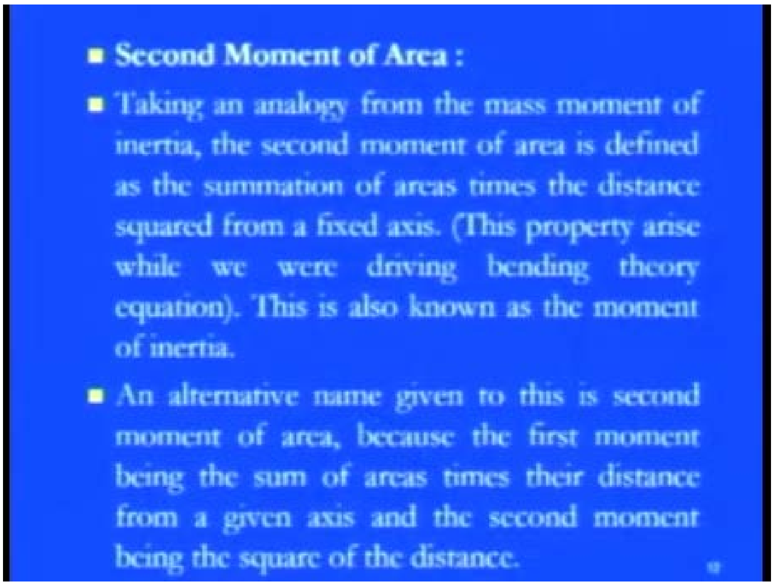

So, first of all we first focus on the second moment of area in which taking an analogy from the mass moment of inertia as we discussed in the previous case, the second moment of area is also defined as the summation of area times distance squared from the fixed axis because you see here what we have is y square into dA. So, first we need to calculate if we are taking any small segment.

So, what will be the area of that segment? Once you know the segment area, then how much distance is there of that particular segment from the neutral axis square term. So, y square into dA will give you this property arises while we were deriving bending theory equation and this is also known as the moment of inertia because you see here we are using the same concept of the mass moment of inertia in which we are taking individual sections and sum up those things. So, here also this moment of inertia which we are calculating, the same concept, alternate name is given to the second moment of area because the first moment of area is different than second moment of area. It is the sum of the area times there distances from the given axis.

So, if we are you know like if we are summating the summation y into dA, then that is your first this moment of area, but we are saying that the second moment of area because here we are multiplying this first moment of area into distance, or we can say that we are doing the summation of y square into dA. That is why we can say it is the second moment of this area or we can say that is a moment of inertia. Both have the same meaning. Only our focus is that actually what we are doing, how we are dealing this problem and what are the influencing parameters which are coming within that particular you know like things. So, that is what you see we need to cover those things.

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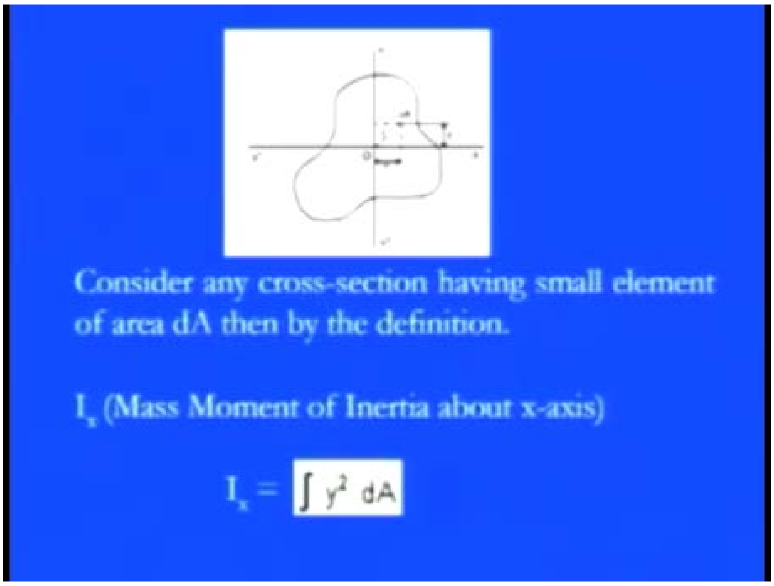

So, for that to calculate the second moment of area, again we are coming with the same structure that we have. Any structure in which you know like just the O is the origin which is coming from both of the axis x, x to x dash y to y dash. Then, you see we have a small segment which we are taking for which there is a small element, dA is the area is there and for that we have the distance R. So, R can be you know like the polar coordinate is there which can be again related with x to y Cartesian coordinate.

So, it has a distance x and y. So, you say that R is nothing, but equals to x plus y. Sorry, square root of this x square plus y square and then you see you know like once you focus on this small element of area, then once you know like we just have the bending moment and then there is a fiber which is influencing by the bending moment. So, for this particular element area, how these fibers will you know like deviating from their original shape. Under the action of this bending moment, it is a prime matter of concern. So, considering any cross-section having the small elemental area dA, we need to define what will be the mass moment of inertia is there, because what we are doing here is the same as we discussed in the previous slide that because we are summing up all those things, so it can also be known as the second moment of area, and is also be known as the mass moment of inertia So, Ix equals to integration of y square dA. So, in that again there is a term y square is coming because it is a second moment of inertia and dA is the total area which is our focused area in that.

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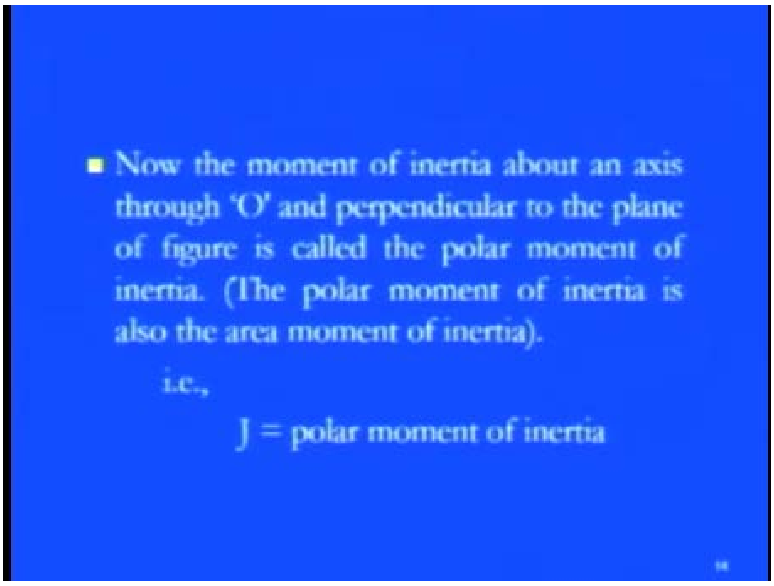

So, if we focus on this, the second moment of area, we would also always see that it is a kind of integration because we have a regular structure. Now, the moment of inertia about the axis which is you know like passing through the O, as you know like I just told you that x to x dash, it is passing or y to y dash, it is passing from O and perpendicular to the plane which you know like of the figure like this one perpendicular to that is also called the polar moment of inertia, or polar moment of inertia is also you know like known as the area moment of inertia. So, this is the new term which is you know like the concern of area is just perpendicular to this plane of figure.

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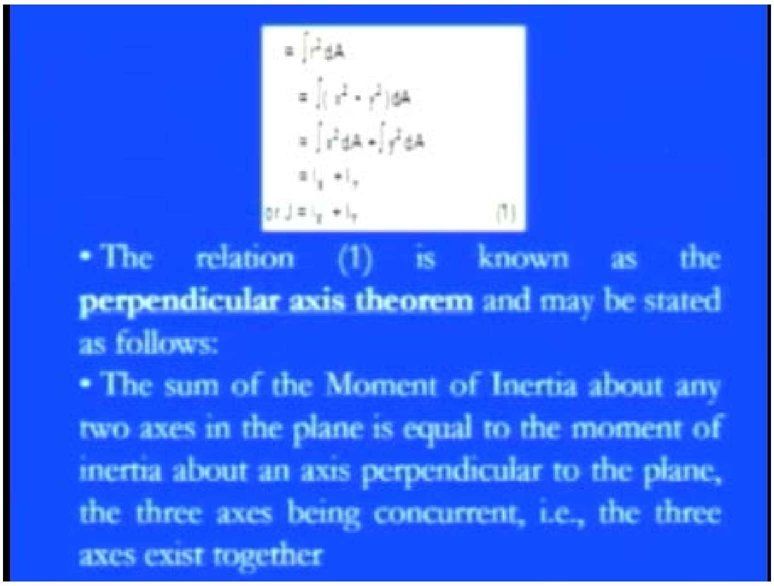

So, for that matter you see always whatever the second moment of area is coming, the new term is there for that which is the J which is also known as the polar moment of inertia, and it can be you know like calculated because it is just focused on this. We have the R of the distance for that which I have shown you this R distance from O to our elemental area. So, this integration of R square dA will be equal to J or since, R is nothing, but was you see x and y was there.

So, R square is nothing, but equals to x square plus y square into dA, or you see if we segregate those matters integration of x square dA plus integration of y square dA will give you the new terms of the second area moment. So, polar moment of area is nothing, but equals to this Ix plus Iy or we can say J J is equal to Ix plus Iy. So, you see all three, one is along the x axis, along the y axis and the third one is just perpendicular to the plane of figure. All three you know like this second area of the second moment of area is coming together, and this you know like relation is coming you know like due to that is known as the perpendicular axis theorem, and it is just stated like that some of the moment of inertia about two axis is x or y.

Whatever you see you know like which we are concerning here is the plane is equal to the moment of inertia about an axis perpendicular to plane. That is the polar moment of inertia that is J, but condition is that all three axis must being you know concurrent. That means, they must exist together in the particular plane, so that we can say that Z is equal to Ix plus Iy. So, this theorem is only valid when they are you know like concurrent or we can say they are just existing together. So, in that we can say that Ix plus Iy will give you J value that is just perpendicular to these two planes.

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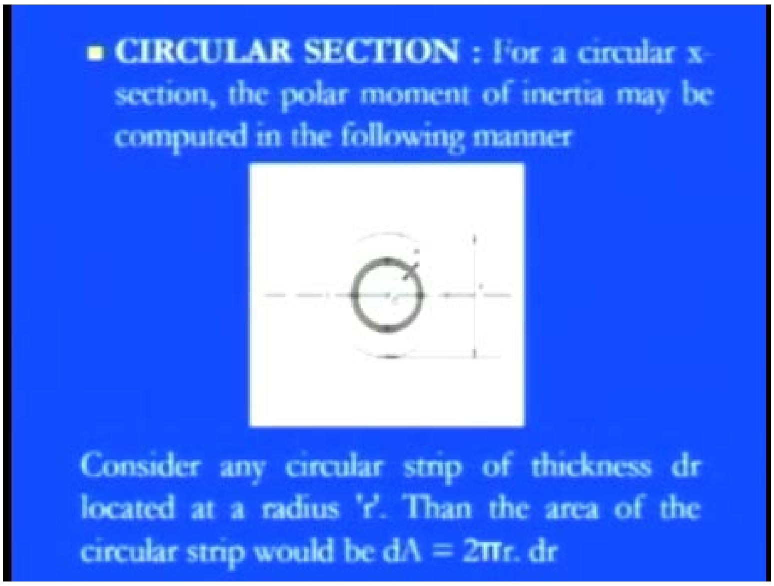

So, you see here these two relations are there with the polar moment of area for that kind of section, but now you see here if we are focused on the circular section, so you can see the figure for a circular section. The polar moment of inertia can be easily you know like computed in just similar manner that again we need to just concern on the circular portion of that. So, what we have? We have you know like a circular portion having the diameter d, and you see our main focus is with the small strip which has a thickness dr and which is located at the distance of r from the O.

So, with that you see you know like we can simply calculate the circular strip area dA which is nothing, but equals to 2 into pi into r into this thickness dr. So, 2 pi r will give you the total length, whatever is there into dr will be the area. So, dA is equal to 2 pi r dr for the circular section and with that now we know that the polar moment of inertia is nothing, but equals to integration of r square dr.

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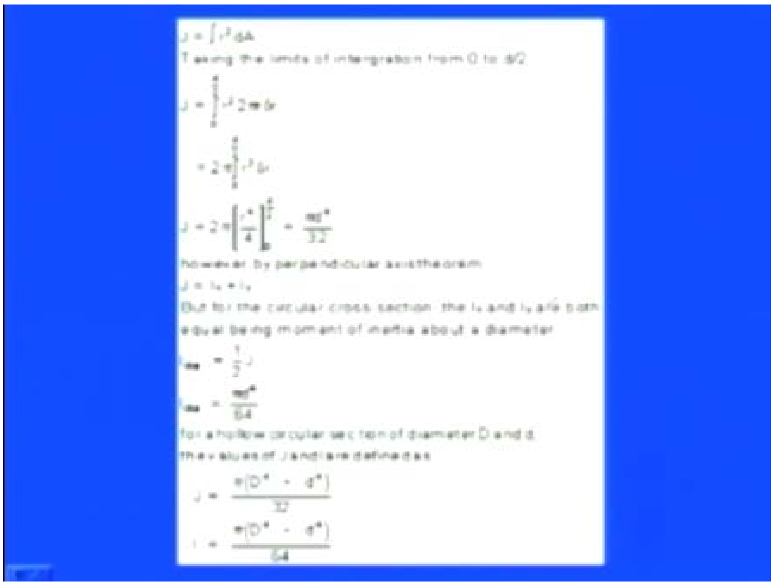

Now, you have these you know like the total symmetrical sections. For that we can simply integrate those things within those definite integrals 0 to d by 2 or 0 to r, I should say. So, 0 to d by 2 r square into 2 pi r dr because of the small segment, our main focus is there in which the thickness of dr, small dr. So, this one you see that is what dA can be easily replaced by as we discussed in the previous section. 2 pi r dr r square is there and since, we are defining from 0 2 r, so 0 2 d by 2 pi is the constant one. So, it can be simply taken out from that. So, we have integration 0 2 d by 2 r square dr, or we can say that 2 pi r you know like since it is a cubic term is there from these multiplications, so we have r to the power 4 by 4 0 to d by 2.

So, we have the polar moment of inertia for circular section is pi d 4 by 32 and it is just for you know like since it is perpendicular axis theorem, it is coming. So, we can say that J is nothing, but equals to Ix plus Iy because we are simply concerning about the rth value, but in the previous section, we found that when you know like all three axes are existing together and you see they have the relation Ix plus Iy is equals to J. So, by applying those things, what we have? Since, it is a symmetrical section of both thing, Ix and Ir always must be equal to the moment of inertia about this particular diameter because we are simply concerning about a symmetrical geometry. So, for that we can say that whatever Ix and Ir Iy is coming, they have the equal value.

So, you know like I diameter because we are considering about the diameter. So, I diameter equals to J by 2 or we can say that the real I is equal to pi d 4 by 64, and this for you know like this is for the solid circulars section in which we have the radius r. We can say the diameter is d. So, for that we can simply calculate the value. The second moment of area that is I is equal to pi d 4 by 64, but if we have a hollow section in which there are two diameters, the capital diameter, the outer diameter and the small d is the inner diameter. So, if we have this kind of thing, then we can calculate this J that is the polar moment of inertia is nothing, but equals to pi d 4 D to the power capital D to the power 4 minus small d to the power 4 divided by 32, Or we can say I which is second moment of area is nothing, but equals to pi D to the power minus D 2, the power 4 small d to the power 4 divided by 64.

So, you see here these two formulas are very important for solid as well as the circular shaft because once you know the value of J and I, you can easily calculate the different property described for the beam. Because you have the circular cross-section of the beam and you see you know like all those property, the bending stresses, the bending moment, they are absolutely focused on what the values of I and J are. So, for that you see we need to calculate those things and for calculation of I and J, the parallel axis theorem was very important to the perpendicular axis.

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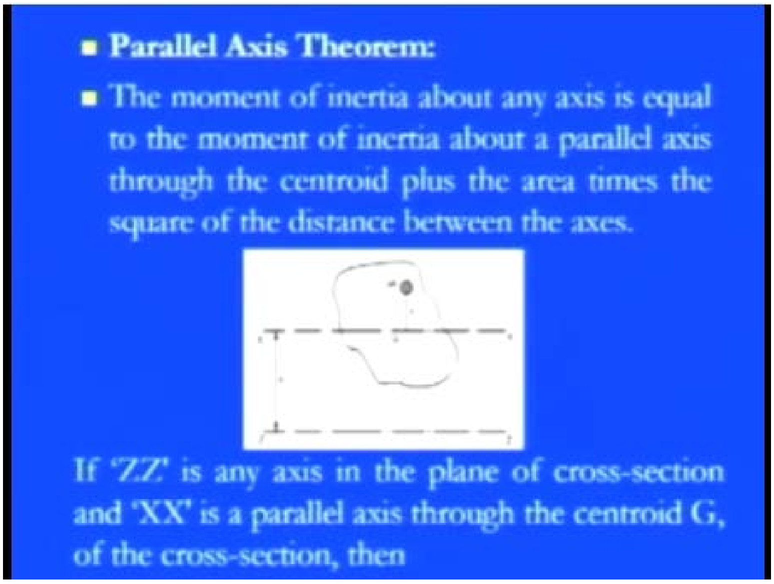

Now, you see we have the next theorem that is the parallel axis theorem So, in that the moment of inertia about any axis is equal to the moment inertia about a parallel axis through you know like which is passing through the centroid plus the area times square. You see what the integration of area into distance whatever is there between the axes. So, you see this kind of you know like theorems are always useful if we want to find out the moment of inertia at a particular section of the total object because you see here we know the distance. Once you know the distance, multiply the distance by the area and you have one component. Then, you see you know like the moment of inertia which is for the axis which is passing through from the centroid or we can say the neutral axis.

So, now you have I plus you know like integration of y into d. So, this will give you the new moment of inertia about the particular axis. So, if in this particular diagram which you can see here, what we have? We have a particular diagram in which G is you know like the center of gravity through which we just want to find out. If we are taking a small element, this one which has a diameter dA and which has area dA and the distance is y is there from this axis. So, now our main focus is to find out you know like that at the ZZ dash with this particular section, what will be the moment of inertia because we know this XX dash this moment of inertia because it is passing from the centroid. So, it can be easily computed once you know these things, you know the distance you can get the moment of inertia by this particular formula.

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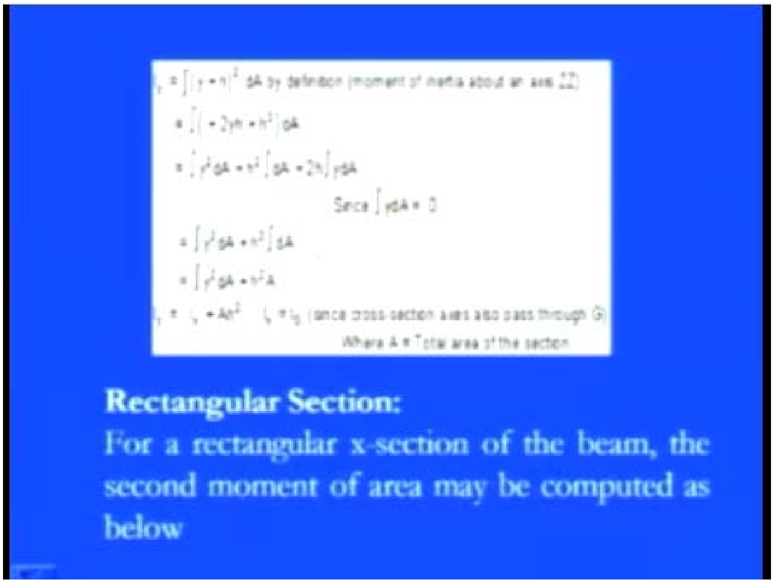

So, here you see we have Ix is equal to integration of y plus h, this square term into del A. So, del A is the area because our main focus is that one. So, with that here we are defining the moment of inertia about the axis ZZ dash because we know that XX dash, this moment of inertia can be easily calculated because it is passing through the centroid. Now, what will be the distance there? The distance we are assuming is h. So, with that y plus h into whole square, this y plus h whole square into dA will give you the Iz. So, once you have the Iz, again you can describe those things y square plus h square plus 2 y h into dA and since, we know that y into dA because you see y is nothing, but you know like it is passing from this centroid.

So, y into dA integration will give you 0 because it is always you see there is no deformation within that particular element because it is passing from the centroid. So, with that particular assumptions, now we have Iz equals to integration y square dA plus h square integration dA. So, integration dA is nothing, but you see the total area which will give you, so we have A and here you see integration y dA is always we are saying that since it is passing from this square, it is a second area moment for the neutral axis, or we can say the centroid. So, it is Ix plus A times h square.

So, you see here we have with the using of this parallel axis theorem, we can calculate this moment of inertia for any section which has a parallel relation from this neutral axis or we can say centroidal axis. So, for that Iz is nothing, but equals to Ix plus area which is the total area into h square and Ix is nothing, but equals to Iz as I told you which is passing from the centroid, and A is the total area of the selection. So, this was therefore circular section. Now, if we have a rectangular section for this one you know like if you want to calculate, so for that rectangular cross section of beam, the second moment of area can be easily calculated for this.

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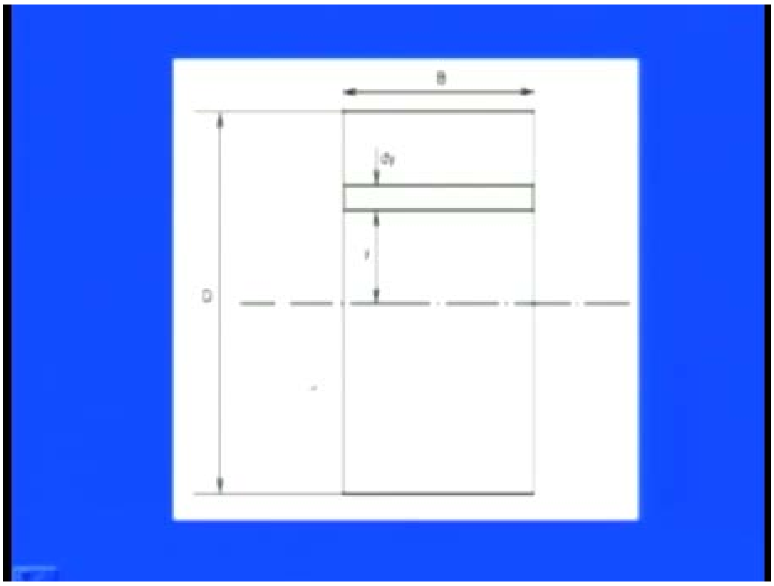

So, for that the diagram is pretty simple. We have cross-section here. You see the rectangular cross-section. This is the total distance is D in the vertical way and B is the width is there for this, and the neutral axis is passing from this or we can say the centroidal axis is passing from this. It is showing by the dotted line and this we are taking strip on which our main focus is there which has total width is dy in this particular direction and the total distance of this strip from neutral axis is y as we are using generally.

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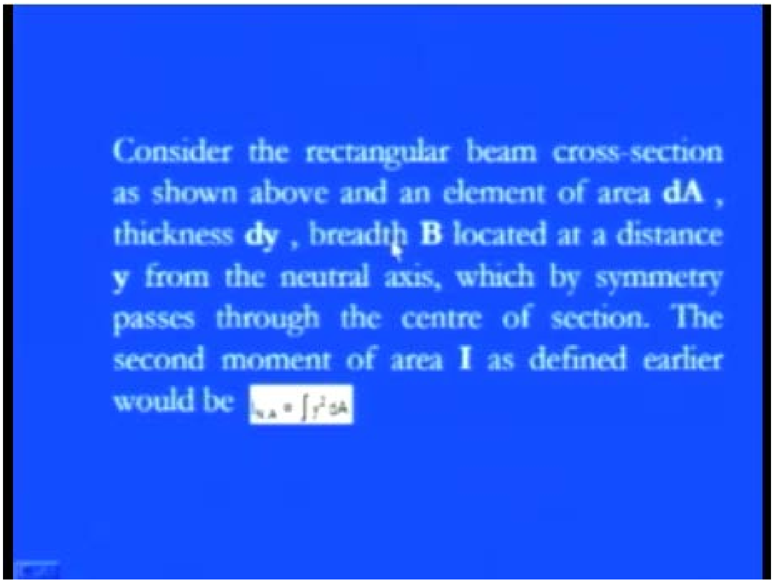

So, with this configuration, now our main focus is to calculate that what will be the second moment of inertia is there for that. So, considering this rectangular cross-section which has the area dA, and the thickness is dy as we have taken and this breadth is B which is located at distance of y from the neutral axis, for that the second moment of area is I. Generally, it can be easily defined that this integration of y square d.

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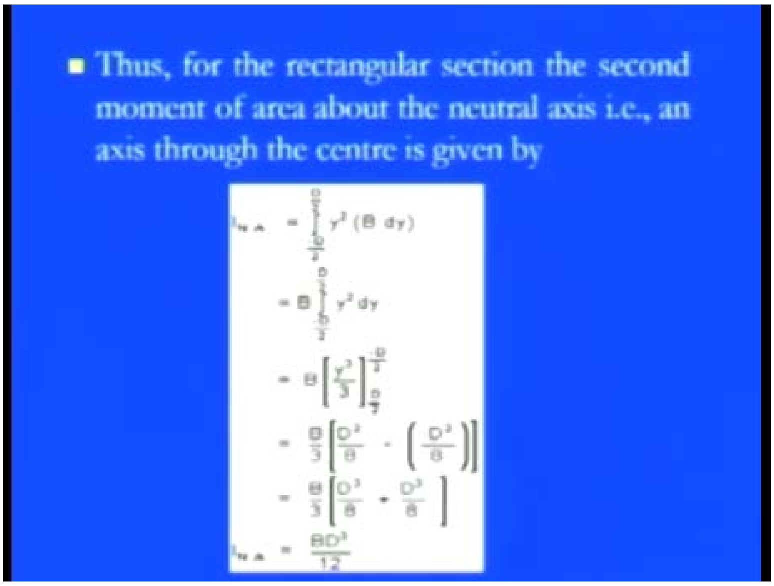

So, with that you see you know like the moment of inertia at the neutral axis from minus d by 2 d 2 because you see the neutral axis is this centroidal part. So, it is coming from minus d by 2 to d by 2 like y square into B into dy because this is the area, the width into the thickness of the strip. So, B into dy is the area y square you see over because it is a second moment of inertia is there. So, with that minus D by 2 to D by 2 or we can say that since the B is the constant one, there we are not changing any breadth of this particular rectangular one. So, B is taken out. So, minus D by 2 to D by 2 y square dy or we can say that y cube y 3 and when we are keeping those things, we are ending up with B by 3 D cube by 8 plus D cube by 8, or we can say that when we are calculating those things, we have D cube by 4 and outside is 3.

So, we have BD cube by 12. So, here what we have? We have the second moment of area about the neutral axis is BD cube by 12 when you know like the neutral axis is just passing through the centroidal part and distance is this D. And this is the width and if you are changing those things because if you are taking the neutral axis which is passing from B and D is seemed parallel. This is a very special case. In this case what we assume is, we are taking you know like the dimension that the diameter is this, and the neutral axis is just cutting the diameter in this way that B width is parallel. For that you see the second moment of area about this neutral axis is BD cube by 12, but in the other case, if I am taking the same rectangular cross-section and the neutral axis passing from the B which is parallel to D, so in that case we have I neutral axis is DB cube by 12. So, you see here we need to be very conscious about those things that what we are taking and how it will come. So, you see again you know like what we discuss that actually if it is passing from those things.

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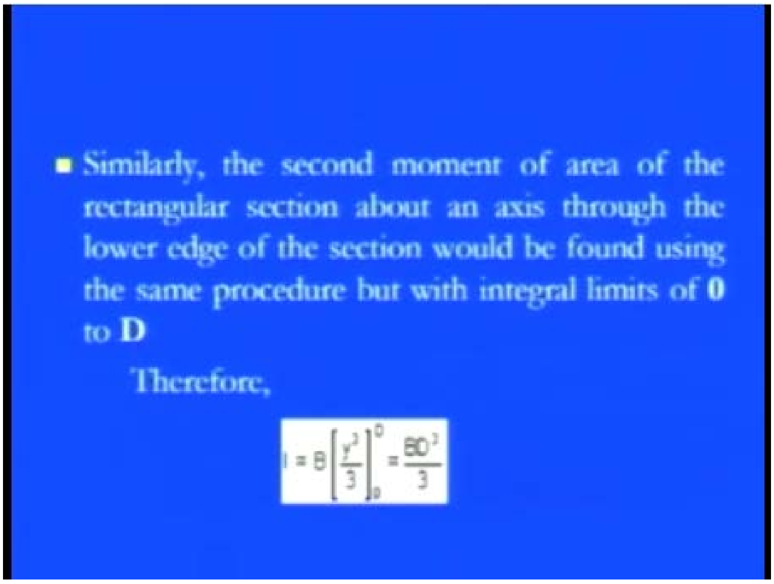

The second moment of area for this particular rectangular section which has the axis passing from the lower edge of the section, it can be easily found with use of the same procedure which we are used from you know like 0 to D, or we can say minus D by 2 to D in the previous case. So, for that we have I which is equal to B y cube by 3, 0 to D because we are concerning about the whole diameter from 0 to D 1. So, we have BD cube by D 3 because that kind of cross-section is symmetrical along 0 to diameter.

These standard formulas prove very convenient you know like in determination of this second moment of area about the neutral axis I NA to build up the section which can conveniently divide into rectangular because you see in the real structure, what we are doing here is, we have a different cross-section. So, we can easily divide into the rectangular shape and then we can find out this second moment of area along the neutral axis, and once you know these things, then you can apply the parallel and the perpendicular axis theorem to get the other, this second moment of area at the particular section.

For instance, you know like if we just want to find out the moment of inertia about the I section, then we can use the above relation very easily because I section is having a first two rectangular section and then there is a middle rectangular section. So, it is you see two horizontal and one is middle. So, how the compatibility is there is based on that we can easily get those values of I for those kind of sections.

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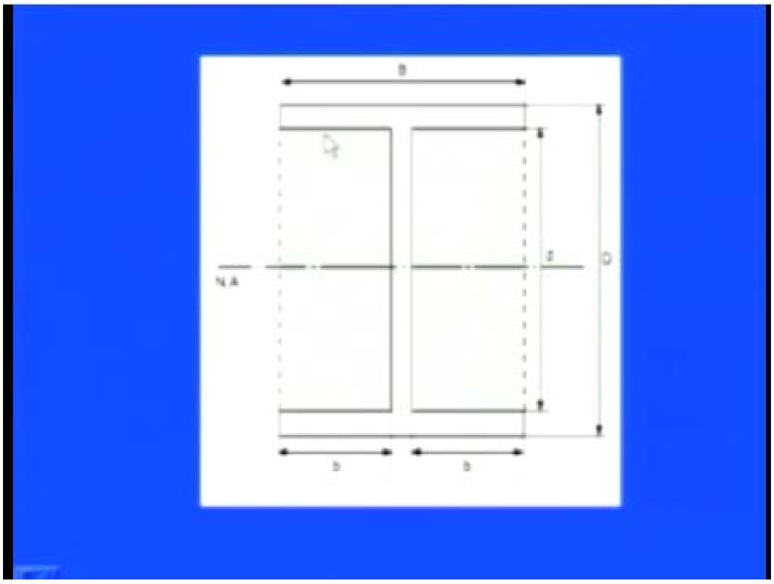

So, now here you see we have a different structure all together if we have the neutral axis which is passing from the D. There are two different sections, which have same breadth, small B and the total B is this one. So, as I told you see you know like if you want to find out for really structure that what will be I NA is there. So, for that kind of thing you know like it is just giving you the small kind of gap.

So, the total, the brick we can say if we are using the brick or any kind of the rectangular structure, the breadth is B. So, for that and the total breadth is capital B, the small diameter is d and the capital diameter is D for the whole section for that we can calculate.

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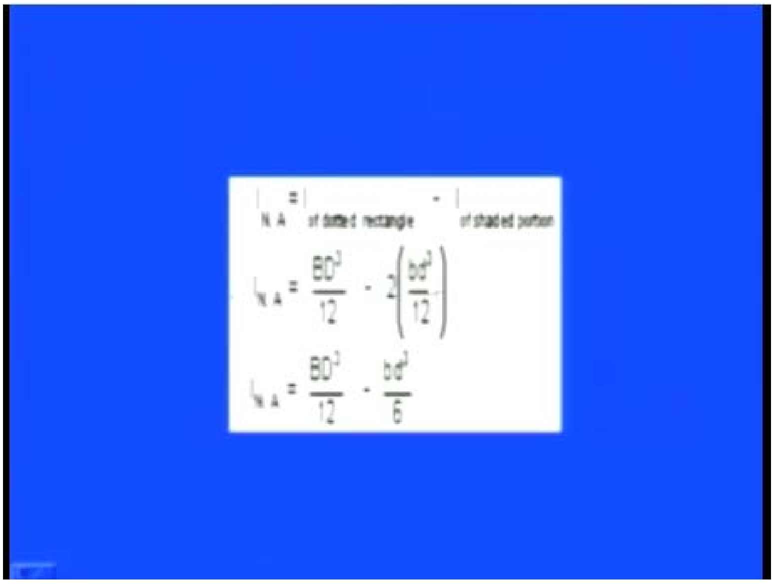

You see I NA is nothing, but equals to I of the total dotted rectangular minus I of the standard area. For this you see there are two different standard areas, and for dotted one this capital B and the capital D are the two main parameters. So, BD cube by 12 minus two times of BD cube by 12 for a small section standard area, or we can calculate the overall result. From these two different kinds of section which is coming in this moment of second moment of inertia for this I NA is equal to BD cube by 12 minus BD cube by 6. So, this is you see the resultant like the mass moment of inertia for this rectangular cross-section.

So, in this you know like lecture we discussed mainly about you know like what will happen when beam is there under various kind of forces, then what exactly the relations are there when we are only talking about the bending you know like only there is no other forces, only bending moment is there. Then, you see what will be the relation like sigma by y is equal to M by I is equal to E by r. So, this is the pure bending you know like the relation is there. The equation is there through which we can easily calculate the different parameters once we know the bending stresses of moment etcetera.

In second section, we discussed about the section modulus and also we discussed about the second moment of area. That means, you see integration of y square dA. So, in these sections, we discussed about the basic concept of the bending stresses, but still we did not discuss anything about when you see the bending moment or any bending stresses are there along with that always there is shear stress.

So, in the next section, our main focus is first is to solve some of the numerical problems based on the geometrical parameters, and all those you know like the attempted values of these bending moment and bending stresses. So, we just want to again use those you know like formula just to refresh our concept.

So, in the next lecture our main focus is to solve first the numerical problem and then the second section is like dealing with the shearing stresses that when you see we have the bending moment with the shearing forces and then how these shearing stresses are coming all together with this particular bending stresses. Then, what exactly the relations are there, and how the interaction is there in between these bending and the shearing stresses, and what exactly the relations are there in those two stresses. That is our main focus. So, in the next lecture, we are going to discuss all these things.

Thank you.