Strength of Materials
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Lecture - 26

Hi, this is Dr. S. P. Harsha from mechanical and industrial engineering department, IIT Roorkee. I am going to deliver my lecture twenty sixth on the course of the strength of material; and this course is developed under the national programme on technological enhanced learning. Prior to start this lecture, I would like to brief something about the previous lecture, because see in this lecture, we are going to discuss same extension of the previous lecture. So, in the previous lecture, we discussed about when a beam is there; and we discussed that, actually how we can categorize a beam. So, when a beam is there and if it is under the eccentric load or if it is under the coupled – couple is there or any twisting movement is there; then how we can get the different bending moment as well as the different shearing forces at the different locations. And how can get also the maximum-minimum – those components which are occurring along the length of the beam. So, actually when a couple is there, then there is a kind of twisting moments are there. And we need to fix up one end and then actually we need to calculate irrespective of whether it is a cantilever beam or a simply supported beam.

And then, we discussed that, actually if we have an eccentric loading; that means if we have a load like the lever is there – lever is connected on a particular surface of beam, and the load is applied; then not only the vertical compressive loads are there towards the downward direction, but also along with the force, we have a couple. So, we need to check it out that, actually when this kind of loading is there in which there is a load is applied on the extreme corner of this kind of lever. Then, we need to check it out that, the same amount of the force can be also applied on that particular point – the point of application is exactly same, where the connection is there of the lever as well as this beam. But also, along with that, we have the kind of couple also. And then, corresponding, the calculations are there for the reactions – support reactions if we have a simply supported beam or if we have a cantilever; then, actually the other, where the rigid fixed end is there; actually, we need to calculate the reaction forces. So, this kind of discussion, which we discussed.

And then, we found that, actually if we have… because till now, we discussed that, actually if we have this – all single loads are there. But, if there is a combination; combination means if we have the concentrated load plus we have the UDL – means uniformly distributed load; if both are acting combinedly, then we need to check it out that, actually which of the portion is affected or it is under the same load. That means if we are saying that, the combination of load is there like UDL is there; and if UDL is expanding all across the beam; and then, the point loads are there; then, we have to be very careful that, actually which section is coming under the UDL and which section is coming under the action of both loads.

And then what is the interaction is there of the both loads; corresponding, the resultant will come accordingly, So, one has to be very careful while checking the impact of the load, so that whatever the portion, which is coming under these kind of loads, we have to be calculated. And then, accordingly, we can easily design the kind of beam. So, this was an important discussion, which we discussed that, actually if we have the combination of uniformly distributed load, where the intensity of loading is there, the rate of loading is there; and the second – if we have concentrated load and both are acting with the same time on a beam; then, how we can analyze those things; what will be the resultant. So, that kind of discussion, which we made and this is very important.

And, the second thing, which we discussed that, actually if we have a triangular load; that means if the load intensity is not uniform as we discussed that, the concentrated load is there or the rate of loading like q, which is uniform. So, that was the different cases. But, if we have a triangular load; that means at one end, it is 0; at another end, it has a maximum and it is simply like uniformly varying all across the section; then, how we can calculate and what will be the centroid is there through which this load is applying on a beam. So, we found that actually. We need to check it out that, two-third of the length – total length will be the location, where this center the mass is acting or we can say this is like the center of point is there, where the load application is there. And then, corresponding, we need to check it out that, what will be these shearing forces as well as the bending moment is. So, this kind of discussion, which we made.

And, not only that, but also we have… If we have this kind of loading; that means, you see at both ends, we have a zero loading. But, at the extreme center point, we have the maximum loading. If this kind of triangular loading is there, then again the intensity of load is different at different locations. So, we have to be very careful that, how these loadings are to be distributed all across the beam of length and how we can consider these shearing forces, because we already discussed that, actually what will be these sign conventions are there for the this shearing force as well as the bending moment. So, for calculating whatever the kind of loading is there for calculating shearing force and bending moment, we have to be very careful that, what will be sign convention is and how they are interacting to each other; corresponding analysis will come.

So, this kind of discussion, which we made in our previous lecture; and also, we solved some of the numerical problems in which if pure loading is there; that means only the concentrated and this UDL is there. Or, if there is an interaction is there; then how we can proceed; how we can solve this shear force as well as the bending moment component, different-different locations And as well also, we can resolve those issues that, actually which of the portion is influenced by maximum shear force or maximum bending moment; and which of the portion, which will influence by minimum those portions; and what will be the interaction is there of these two kind of shear force as well as bending moment diagram. So, this kind of discussion, which we made in our previous lecture.

And, in this lecture, again we would like to discuss the same issues that, actually if this kind of loading is there; and then, how we can interact those forces and how we can analyze those forces. Along with that, actually we are going to discuss that, actually if we have a simple beam and if we have a curved beam, then what will be the difference is there in the analysis; what are the factors are there, which are influencing in the analysis of this kind of curved beam; what will be the assumptions, which we need to make to simplify those analysis for a curved beam. So, this kind of discussion, which we are going to discuss in our lecture.

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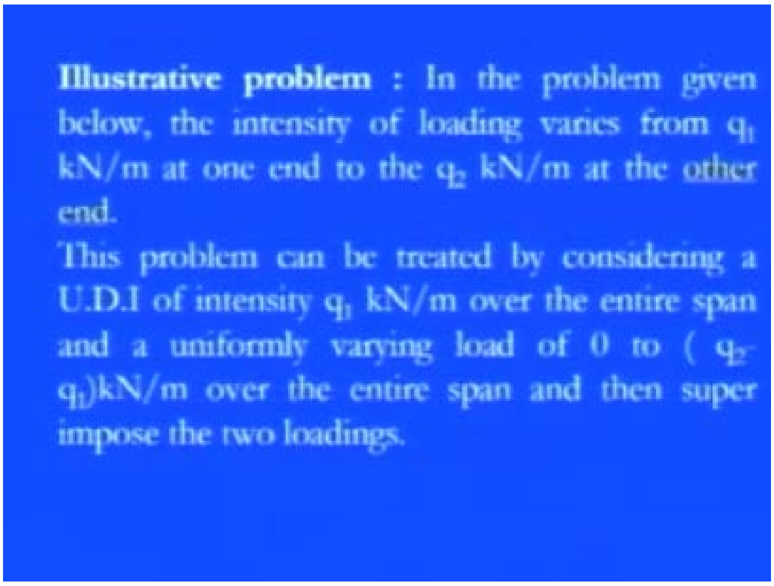

So, before these things, first the problem is that, we have illustrative problem that, in this particular problem, we have the intensity of loading, which is varying from q 1 kilo newton per meter at one end to q 2 kilo newton per meter.

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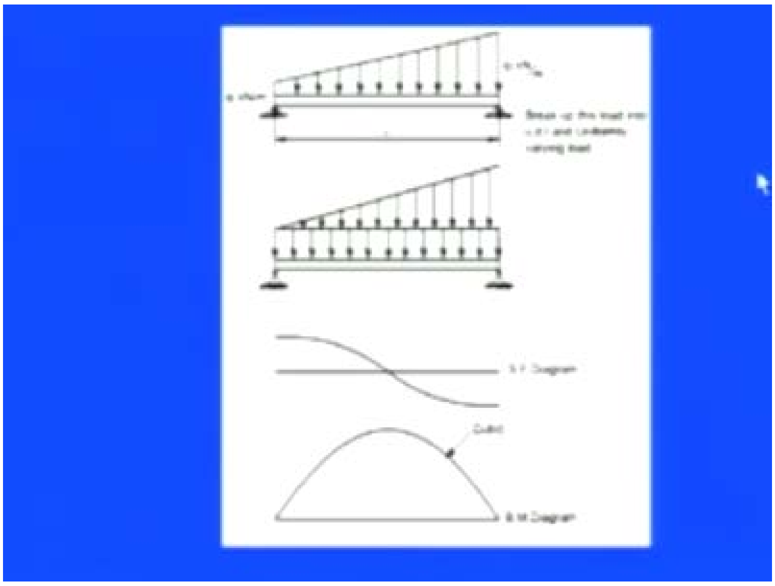

So, if you see this particular diagram, you will find that, we have triangular kind of loading. At this, again it is not 0. We have some sort loading in which it has a magnitude of q 1 kilo newton per meter and this is the q 2 kilo newton per meter. And this total length of beam is l. So, we need to consider that, actually which of the portion is influenced because it is a uniformly varying load. So, again we have to be very careful that, actually which portion is influenced by maximum loading, because this is – the intensity is there. So, we need to calculate the total load. So, if I am saying that, these different loadings are there; then, how we can go?

So, here this is one kind of analysis that, we can first… What we can do here; we can simply put q 1, because this is the q 1. So, again we need to cut this portion. So, if you cut the portion, then you have this is the q 1. So, this is the q 1 loading, which has a uniform distributed load all along this particular length of the beam. So, this is q 1, which spread it out along this particular beam. And then, this is the remaining load. If you focused on this particular part, then you have this is the q 2 minus q 1, because this is the whole q 2. And if you deduct this q 1, then the remaining one is q 2 minus q 1 and the different intensities are. So, this problem can be treated by considering a UDL as I told you of the intensity of q 1. If you just deduct that part over the entire span of the beam and a uniformly varying load right from 0 at extreme left end to the extreme right end; that is, q 2 minus q 1 kilo newton per meter.

So, now, if you want to analyze those kind of problem, again you can simplify in the regular structured way that, one is the pure rectangular part in which the UDL is there of the intensity of q 1; and another is a pure rectangular starting from 0 at one end; and another end is some magnitude. So, here in this case, we have q 1, which is a UDL. So, analyzed by that way, and then additionally. So, first, q 1 will come independently; and the second part – it is this UDL plus this triangular element, which has 0 at one end and q 2 minus q 1 intensity at the other end. So, by considering those things, we can simply analyze and we can draw the shear force as well as the bending moment diagram.

So, here by combining those things here, if you focus on this part, only we have a q 1 load, which this rate of loading q 1 intensity is there in kilo Newton per meter we need to consider here. But, as well as this is concerned, we have q 2 at one end; otherwise, it seems like we have first q 1 for this thing, and then we have a triangular part – q 2 minus q 3. So, for that, we need to calculate for q 2 minus q 1; there is the two-third of location – means two-third l; we have the centre of the loading. And corresponding, the elliptical part will come. So, that is what – if you consider the combined effect of those things, then we have… because here this is the reaction. So, it is going in this direction. And because of this rate of loading, it is coming in this direction. So, we have this shearing force. And for this shearing force, we have a positive direction. So, for that we have a positive direction here. But, at this point, because there is an interaction is there in between UDL as well as this uniformly varying load. So, we have the kind of this confliction is there, or we can say the convection is there. So, for that, there is a change in the direction.

And, at the end, when there is an integrated load or we can say either the q 2 or this q 1 for this one and this is for q 2 minus q 1; for this combined load, we have a negative, because here this is the reaction. So, it is going in this direction. And these shearing force – they are coming in this direction. So, all and all, this sign convention is there. So, for that reason, we have a negative shear. So, it will come in the negative region. So, starting from this and since it is a UDL and varying this uniformly varying load, for both of the things, we have a parabolic curve. As I told you, this square term is there in terms of the distance with the load. So, for that, we have a parabolic path. So, in the shear force diagram, starting from this with the reaction of this point in the positive, we have this shape. And at this particular point, we have a reaction in the negative shear force. So, here we have this. So, this is a shear force diagram for that.

And, if we want to calculate the bending moment diagram, again at both of the junctions or we can say both of the extreme ends at these two, in this particular diagram, both ends are like pin joint. So, at this point of time, there is no bending moment at both of the points – both of the extreme corners. And we have a combined part of the UDL as well as this uniformly varying load. So, for that, we have a parabolic curve is there in the bending moment diagram and which is always maximum wherever the shear force is always changing its sign from positive to negative. So, by considering that concept, even… because we discussed that, actually this shear force F is nothing but equals to dM by dx. So, wherever the change in the domain is there, the bending moment – always it gives you a shear force changing; that means, the 0.

So, here if I am saying that, it is 0; at this particular point, there is a maximum bending moment at this particular point. So, this is even conceptual. You can say analytical way or by this mathematical way, if we calculate for this q 1 UDL and this q 2 minus q 1 in a triangular way, you can easily get this particular shape of the bending moment. And starting from and ending end, they are 0 and it is maximum at the middle point and they are the cubic one because of there is a combined load of this UDL as well as the triangular elements. So, here it is a cubic term is there, which varies along with the length of the load application. So, this is kind of analysis, which we made; only it is a different kind of analysis, because it is a combination of UDL as well as a triangular element.

Now, as we found that, there is a change of the sign is there in shear force as well as the bending moment; but, in previous diagram, it could not happen in bending moment. But, in general case, you can find that, there is a change of the sign in both of the diagram. And if this is happened, then there is a point is coming. And that point, where this change of sign is there is known as the point of contra flexure. So, point of contra flexure is always the point, where there is a change of sign. And we are going to discuss this kind of concept right now. So, here you see in front of you, we have a kind of simply supported beam at the two extreme corners: A and D, where the reaction forces are there – RA and RD. And the total length of beam is 10 meter. And then, there are two points. The point forces are acting of the magnitude of 7 kilo newton and 6 kilo newton at two different locations. And the locations are well-defined within this particular beam.

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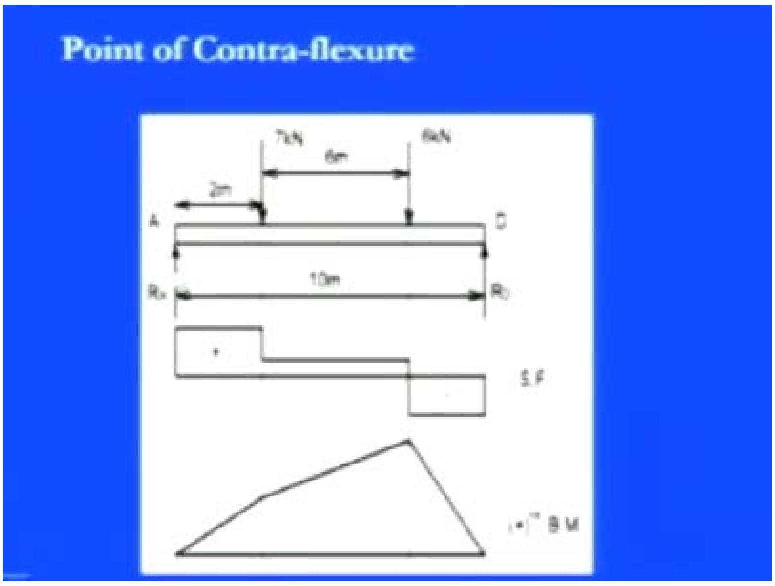

So, you see here starting from those first, we simply by equating the forces and the equating of the moment at particular points, we can easily calculate the reactions at these two points with the force summation as well as the moment summation. And then, once you have both of the reactions, then it is pretty easy to calculate the sign convention for shearing force, because at the RA point, it is going in the top-ward direction from the extreme left. And for this particular force, it is going in the downward direction. So, this kind of sign convention is there and it is always positive. So, for this reason, we have the positive shear force.

And, the similar kind of thing – it is a kind of a hogging part is there as well as the bending moment is concerned. So, for hogging, always we have a positive. And for sagging, we have a negative bending moment. So, for that, again for this particular thing, it is tending to move this particular beam in this direction. And here also it is in this direction. So, we have a hogging shape of a beam due to these kind of application of these forces. So, for that, we have a positive bending moment diagram.

So, by calculating those magnitudes, we can easily draw the shear force diagram, because it is starting from this. So, it has a total magnitude of RA; then, it is going up to… There is no force in between these two. So, up to that, there is nothing; but, then you see there is a kind of 7 kilo newton force is going in the downward direction. So, this is a reduction, is there of 7 kilo newton. And then, again there is no force is there in between these two forces. So, a consistent approach is there right from this point to that point. And then, we have again abrupt change is there because of this 6 kilo newton load application. And then, it is going… Again the sign convention is there because of the load magnitude. And then, we have this, because here we have this reaction force in this direction and the load application is this direction. So, we have the negative shear force direction. So, we have this negative shear force – this. So, this is pretty simple.

And, as we discussed almost many times that, if these sign conventions are there and this kind of load applications are there irrespective of whether it is a point load or this UDL, we can simply calculate this shear force directions that, actually how it will react and how it will come out of this shear force diagram. And then, corresponding, as I told you, like the shear force is nothing but equals to dM by dx. So, with that domain… because the moment is changing in the x domain with respect to this whatever x or yz is there in all three directions. So, for that reason, we can say whatever the shear force – the change of the direction is there, always it gives you the maximum bending moment. So, here even it is changing the direction; we have the maximum bending moment at that particular thing. But, the key feature is that, here the is no abrupt changes are there right from positive to negative in the bending moment.

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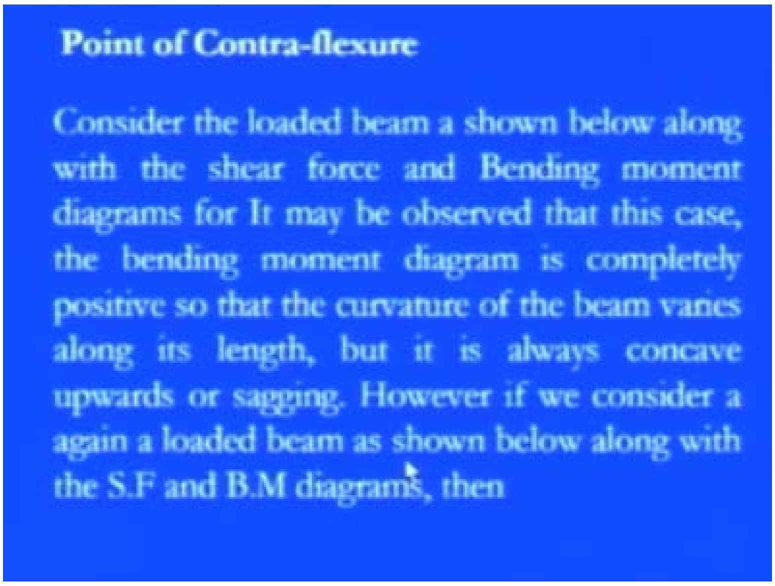

So, here consider these – the loaded beam as shown in this particular diagram along with the shear force and bending moment diagram. We have to concern that, actually if there is a kind of abrupt changes, then we need to define that, here this change in the maximum to minimum shear forces are there. And corresponding, the changes are there in the bending moment from negative to positive. So, in these conditions, we need to define different points, because whenever we want to design these kind of things, how the fibers are playing… because if you are saying that, the bending moment is changing from negative to positive; that means there is a drastic changes are there of the fibers of a beam. So, how these changes are taken place and how the setups are there of these shear force as well as the bending moment – that is the key feature.

And, for that, first of all, we need to observe there that, actually the bending moment is… If it is completely positive, then the curvature of the beam varies along with the particular beam, but it is always concave upwards or we can say it is a kind of sagging position. However, if we consider again a loaded beam, just I am going to show you that one with the different kind of loading. Then, you will find that, the shear force and bending moment is entirely different.

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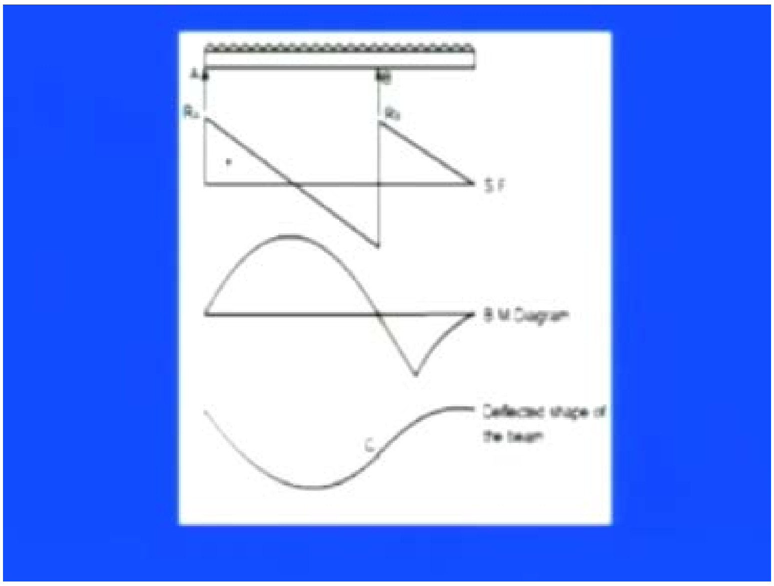

Like here we have this UDL is there. Instead of point load now, we have a uniformly distributed load all across the span of the beam. And for that, again what we need to do here? Here instead of putting the simply supported beam of the extreme corners here, we simply put at this particular corner – the left end extreme corner. And somewhere in this right end portion, not exactly at the extreme corner, but some displaced portion is there at this particular… So, now, for this, again the UDL is there. So, we need to calculate what is that, because it gives you intensity of the load. So, for that, first of all, we need to multiply the total length of the beam and then we have a total load; that actually how much load is applied on that. And corresponding the changes are coming in, this shear force as well as the bending moment diagram.

So, for that, again since here it is going for this left end corner, the reactions are going on a top side, and the UDL, which is putting the load on the downward direction. So, we have again the positive shear force in the starting direction. So, starting from this, we have RA. And then, it is changing right from here to here. So, this portion is one portion where the reaction is coming on the top-ward direction. So, for that, this is a change, is there. And since it is a UDL; so we have a straight line all across that point. So, this is the sign changing is there from positive to negative. And this is one point, where the kind of reaction is there. So, again, there is a pushing. And this one – it will come, because of the reaction forces coming at that particular point. And due to that, again since there is no support is there at this particular point, but there is a load application due to this rate of loading the q 1. So, again we have direct line and it is going up to the 0.

With this shear force, we found that, there is the change of… You can see here the change of shear force from positive to negative are there at the two different locations. At this one location, there is a clear change is there from this to this. So, here this – one location; and another location is because of the reaction force, which is supporting part is there. So, for that, we have this kind of shear forces. So, at these two points, you will find that, the shear force is changing from one direction to another direction. And if we look at the bending moment, then again we found that, there is a different phenomenon is there. And due to that, actually the bending moment diagram is altogether different as compared to the previously obtained bending moment diagram.

And, here it is at these two corners – again these two supported reactions, always we have a 0 bending moment. So, here at these particular two points, we have 0 bending moment. And since there is a change of sign here; so obviously… And due to the UDL, we have this parabolic path is there. So, this is parabolic path and it has a maximum value wherever the change of sign of this shear force is. But, again you will find that, here there is also the abrupt change is there in the shear force; then, corresponding changes are also coming in the bending moment and will find that, it is… Since it is going in the sagging way in the another corner, because there is no… This if you see at the right end corner, then there is no support is there in the extreme corner, that is, a free end. So, at the free end, since there is no support; so it is going in the sagging part. Since it is a sagging part; so we need to consider the negative bending moment diagram. And it has the value in corresponding to that. So, if we extend up to this point and it is a maximum point, where we can say that, actually this beam will be deflect in a sagging part; and then, it is a convective part is there, which is going up to the extreme end.

With that, we have the bending moment is going right from positive to negative direction. And wherever the sign is changing, we can say that, there is a kind of different point is there. And that point where the change of the sign is there in the bending moment diagram is known as the point of contra-flexure. And here again you can simply visualize that part in the deflected manner that, how this deflected shape of the beam is. Due to this load, here it will deflect in a very structured manner, so that we have a kind of this deflection right from this point. And since it is a UDL, because here and here we have a support; so it will kind of… – the support will be provided by these two reaction forces. So, we have enough strength at these two points. But, in that, in the middle region, because of the UDL, it will try to deflect in this manner.

But, again since it is supporting and then again it is kind of at the extreme corner since there is no reaction support or there is no simply supported part is there – the pin joint; so again it will try to deflect. So, again it will go up to a certain region because of the support. And then, it will try to deflect. So, this is well-structured phenomenon as far as the deflection of the beam is concerned and we can get because of these UDL loading. But, the key feature is that, this bending moment diagram is always coming from positive to negative direction and there is a change of the sign is there. And this point – the C point is known as the contra-flexure point.

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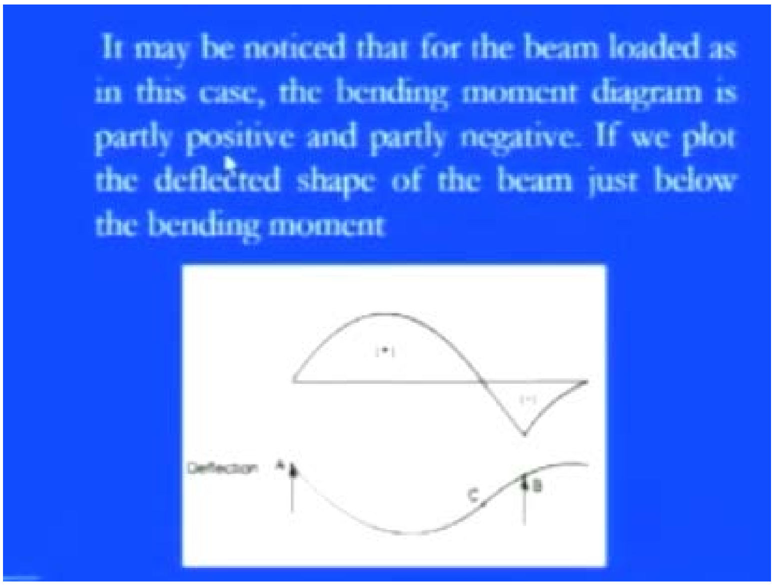

And, it may be noticed for the beam loaded as the previous case that, the UDL and the two supports are there – not exactly at the last corner; the bending moment diagram is partly positive and partly negative as we shown this. And we plotted the deflection shape also – the beam just below this particular thing. So, again you can see this; actually we are simply plotted that. And in this particular region, we have a positive bending moment and the negative bending moment. This is the kind of deflection of the beam is coming. And we can say this is the deflection shape of the beam is there under the UDL loading.

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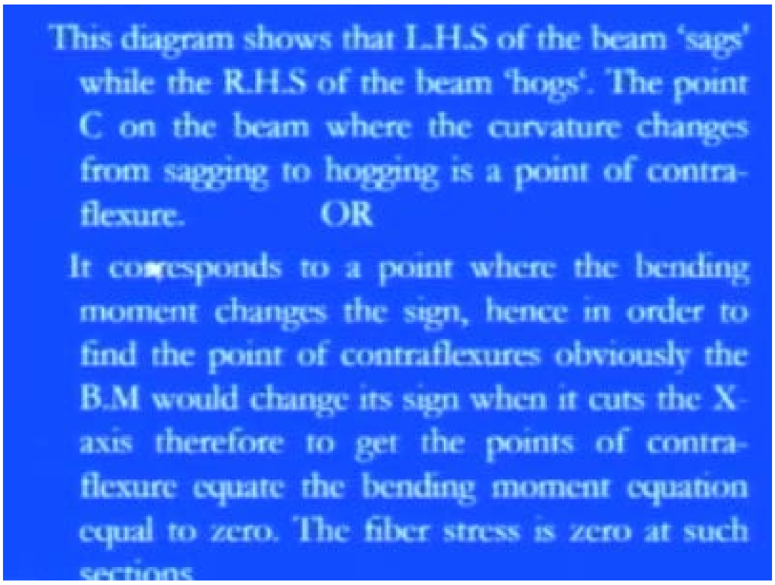

So, the diagram shows that, actually the left-hand side of a beam sags, because it is the kind of sagging; while right-hand side of the beam is hogs. And that is why it is a positive side of that and the point C. Since it is a sagging to hogging portion is coming in the same kind of loading; so the point at which this is happened – that means sagging to the change from sagging to hogging – this point C on the beam, where curvature changes from sagging to hogging is known as the point of contra-flexure. As I told you, it corresponds to a point, where the bending moment diagram changes the sign. This is also the same meaning; the sagging to hogging are positive to negative; or, when it is changing the sign.

Now in order to find the point of contra-flexure, always the bending moment would be changes its sign when it cuts this cross-sectional axis of the beam; that means whatever the cross-section part is there, how it is changing; and since it is changing only in the x direction. So, whatever the changes are coming along the main line, it is always coming in the positive to negative direction. So, here the point of contra-flexure – if we want to found it out, first of all, our focus is that, actually whether we need to check it out that, whether there is a change in the sign convention in this bending moment diagram or not. If it is changing, then what is the point? And we need to find it out the location for that. Again we need to check it out that, actually how the relation is there in between the load as well as the bending moment. And then, how the shear forces are playing their role in between; and what interaction is there in between the shear forces and the bending moment.

And, the key feature is that, in this… which is very important – the fiber stresses is always 0 at this section, because it is in… If you are talking about the positive bending moment, then the fibers are well-settled in the sagging portion. And if I am saying that, in the abrupt changes are there for the fibers of the same beam, then you can imagine that, actually how these fibers are being setup-ed within the beam under these actions; because at one point of time, they are in the positive direction; and one point of time, they are in the negative direction; and they are coming exactly along the change of this particular moment diagram. So, it is a bending and it is changing its direction.

So, we have to be very careful to get the point of contra-flexure, which always equate the bending moment equations to 0, because we need to find it out the C point. And for that, actually we need to calculate that actually the bending moment if it is 0; that means the fibers – this particular fiber stresses due… Or, we can say the bending stresses on these fibers – they have to be settled at 0; then all we can get these points. So, this is a very good phenomenon, which one has to be very careful to consider those points for a proper design of the beam.

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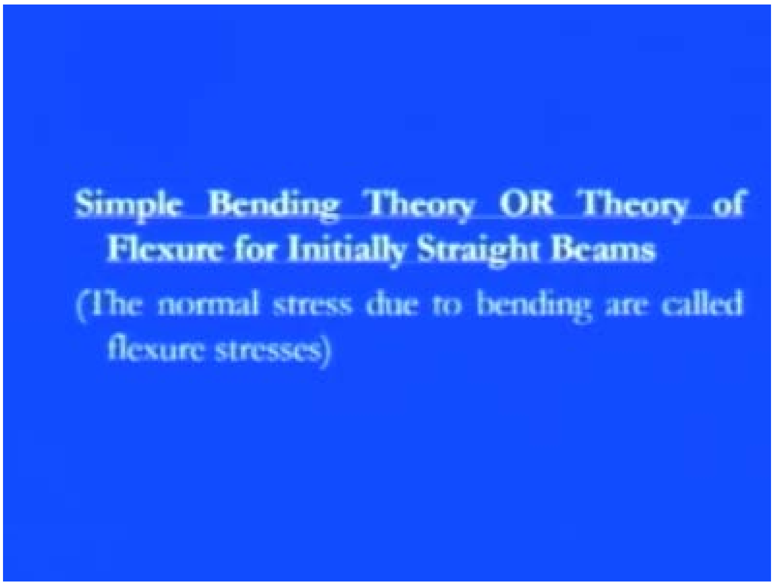

Now, you see here our main focus is that, actually what will be the bending theory or theory of flexure for initially straight beam; that means our focus is on the normal stresses due to the bending always that, actually whether it is a compressive stress, whether it is the direct – we can say the tensile stresses. So, how these stresses are coming on the beam? Because if beam is under the load, then always some portion is under the action of this compressive; and some portions are under the action of tensile. So, how this… What combination is there and how we can say that, with the interaction of these two stresses, we can analyze the forces? So, these stresses as you see, it is known as the flexural stresses; these are normal stresses. So, how we can analyze those things.

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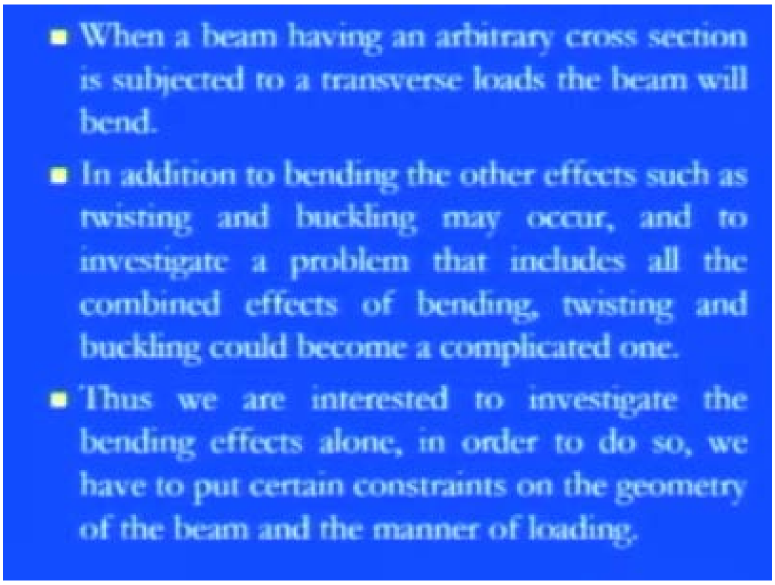

Now, we are going for the analysis of a simple bending of the beam theory. And when a beam is having arbitrary cross-section, it is always subjected to a transverse load of the beam, which will just act on this beam and it will always bend the beam. So, if we are saying that, the beam is bending under these actions, then how these fibers are well-settled within this particular beam, because under the action of these forces, we always assume that, the beam is well-established; that means the kind of deformation is there or the kind of displacement are there of these fibers. But, they are – even apart from these things, we are saying that, the beam is stable or the equilibrium forces are there; that means all these displacement of the fibers or the disturbances of the fibers are well-settled under the action of these forces.

In addition to bending, the other effects such as the twisting or buckling will also be occurred. So, only we cannot say that, only the bending is there. But, along with that; since there is the couple is there or there is a kind of this eccentric loadings are there; and because of that, there is a good chance to be happen of twisting as well as the buckling in this particular beam. And to investigate a problem that includes all the combined effects of bending, twisting or buckling, it could become a complicated one, because all these effects are coming together; and then, our focus is there, actually that how these fibers are playing under the action of these forces. And then, it is very complicated to be seen that, actually which one is dominating and which one is not; and then, corresponding to what is the factor of safety is there for this kind of loading. So, it is very complicated.

Thus, we are interested to investigate only the bending effects along. Because when these kind of load applications are there, always bending is a key feature to design with. So, that is what of our focus is that, actually how this bending will occur. And due to this, actually how these interactions are there of the normal stresses like the tensile as well as the compressive one. In order to do so we have to put certain constraints on the geometry of the beam and the manner of the loading, so that only the bending will come and we can only analyze the bending or whatever the fibers are there of the beam; there we only exerted, we only experienced the bending portion. So, for that, again since as I told you, there are certain constraints. And for that, we need to put certain assumptions to do so.

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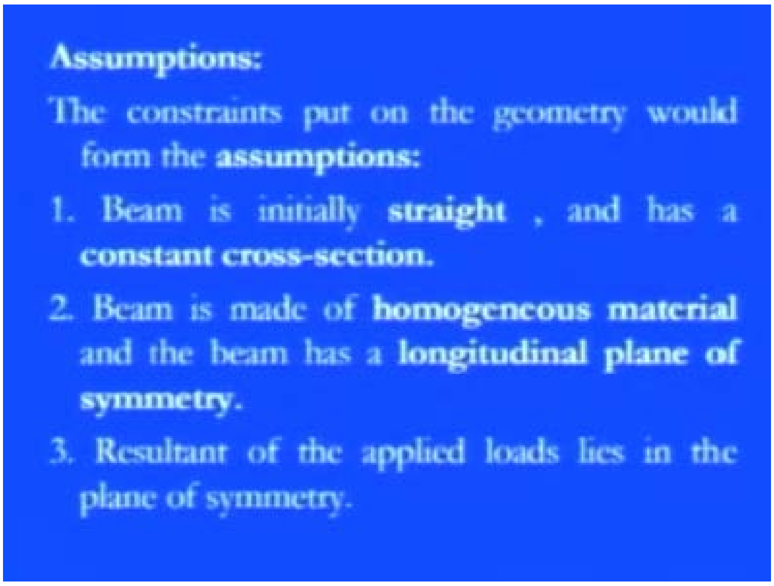

So, for that, the constraint put on the geometry would be certain assumptions as I told you. And the beam… The first assumption is the beam is initially straight and has a constant cross-section; because if it is a uniformly varying cross-section and any kind of couple is there, then there is a fairly good chance to happen as a buckling as well as the twisting part; which is very complicated. And the other thing is that, they sometimes… Or, we can say in some of the portions, they will be even more dominant way as compared to the bending. So, we need to assume that, the beam is initially very straight beam under no load actually. It has to be very straight all… And the cross-section must be uniform all along that particular path of the beam.

The second is beam is to be made of a homogeneous isentropic or isotropic material; that means whatever the stress components are coming, they have to be equal in all the directions. And then, the beam has a longitudinal plane of the symmetricity; that means if we are talking… because the main part of the beam is the longitudinal plane. So, it has to be a symmetrical one; it does not have to be unsymmetrical, because if we are applying the load, then definitely the non-kind of… If it is not symmetry, then whatever the displacements are there of the fiber within the beam – they are not infirmly displaced; and we will get the stress concentration at the different factors. And whatever the analysis will come, it will mislead the results. And whatever the results, which we will get – it will not be truly…

So, first, the important thing is also for this beam theory is beam is to be made with the homogeneous isotropic material, so that actually whatever the load application is there, it has to be followed the generalized Hooke's law. Or, whatever the stress components are coming, we can apply all those theories, which we applied for the Hooke's law.

And, the second beam has to have the longitudinal plane of this plane in the symmetrical way. And then, the resultant of the applied load lies in the plane of symmetricity. Whatever the resultants are coming with… they will simply lie along this particular plane of symmetry in the longitudinal way, so that whatever the fibers are there, we can say that, under the action of these loads now, it is well-settled; we have the equilibrium… this beam is there. So, whatever the analysis will come; whatever the even the small segments are there; they are under the equilibrium portions. So, we can simply apply this total summation of force is 0; total moments – the summation of the moments are equals to 0; and then, all the analysis will come, because we are doing all the same exercises for this beam theory also. So, we have to apply these kind of assumptions.

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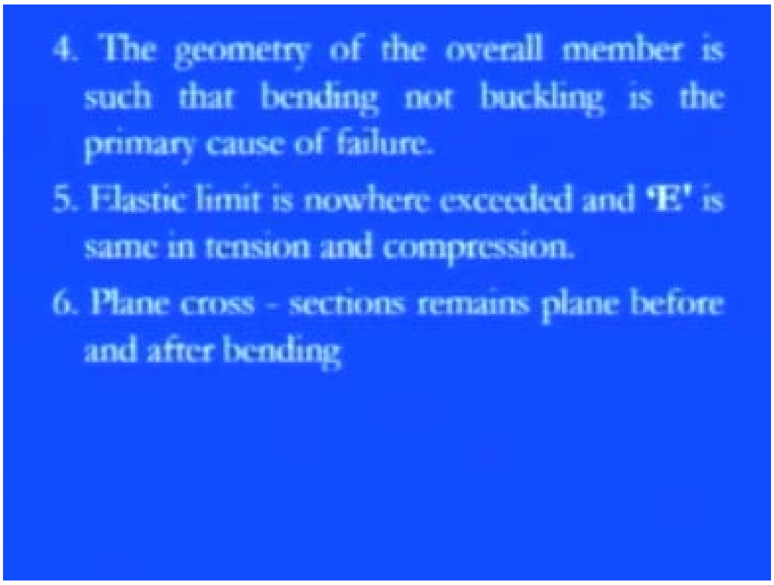

And then, certain assumptions are the geometry of the overall member is such that, just only the bending will come; nor buckling or the twisting will be there – the primary cause of failure; that means if you are saying that, we are taking a constant geometry and the load application is there, then only we have to be focused on the bending; neither buckling nor twisting; because if these two other phenomena are coming, then we need to consider other stresses and other stresses when… And we need to check it out that, what the interactions are there in between these different stress components, which is very complicated.

And then, the fifth one – as I told you, the elastic limit is nowhere exceeded and E is the same in tension and compression; that means whatever the laws, which we are applying; they have to follow the Hooke's law; and whatever the load applications are there, they will be followed the elastic deformation only; elastic limit is the key limit for that. And we apply the load up to only the elastic limit is followed, because once it is going with the plastic limit, then we have a permanent set of deformation within the beam. And whatever the theory, which we are applying; whatever the generalized Hooke's law or the Young's modulus of elasticity, bulk modulus of elasticity, Poisson ratio – whatever the constants are there; they are highly invalid and we cannot apply all those theories, which we applied for even calculating the shear force and bending moment also.

And, the last one is the plane – this plane cross-section remains plane before and after bending. It should… If there is a change in the plane cross-section, then again the different kinds of the forces as well as the stresses will come. And then, again we need to analyze those parts along with the bending stresses, which is even… Whatever the bending theory is coming; this will be untrue in those conditions.

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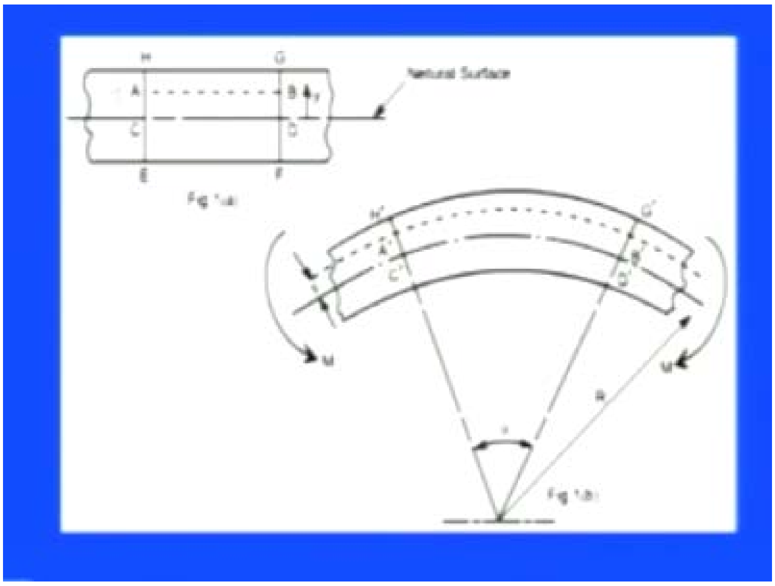

Now, here with those 6 assumptions, now, we have a perfect beam. And this beam – the first diagram on top of one figure 1 a, is a straight beam. And if it is a straight beam and it is following the same assumptions as we are saying; then, in the middle of the portion, there is one dotted line is there. And this dotted line is known as this natural surface; that means it is always along the neutral axis. And whatever the surface is there along this neutral axis is known as the neutral surfaces. And for the analysis, always we assume that, if you cut any portion here, we have this A to B and C to D; that means if this portion is there; since the CD portion is always along the neutral surfaces; if I am saying that, along any fiber, if I am cutting this AB portion; so for that, if I simply along…

If we apply the load and if it is in the bending portion, then the bending portion of this particular beam will be shown in this particular diagram figure 1 b. So, since it is a curvature part; so again we need to first of all focus the locus of this curvature. So, this is the locus of the curvature. And since this portion A to C and B to D – since it is in the curvature form; so they are now the A dash C dash and B dash D dash. And they are always focused on the locus of the curvature. So, this is the locus of the curvature. And whatever the deflection of the curvature will come, it can be easily measured by the theta. So, this is the theta, which is the angle of the curvature.

And then, corresponding whatever the changes are there; again the important thing is that, under the action of this bending moment, you can see this diagram – this M and M on the both of the portion. Only the change will come in the fiber action; but, the neutral axis is the position; this neutral axis – this axis position or we can say whatever the portion, which we concern y; now, it has this particular y bar. So, here either this A dash to C dash or B dash to D dash, there is a curvature path is there; their positions are fixed; only it is changing their deflection portion, because of this bending moment. And the focus of this deflection is always along the locus of this particular point. So, this is the locus of the curvature.

And, we can simply visualize that, actually under the action of this bending moment, how these fibers will play; and how they can simply shown their justification that, since this dotted portions are there, there is no change in the length as far as this A dash C dash is concerned or A to C is concerned. So, we can easily get those values by even applying the theory of the curvature.

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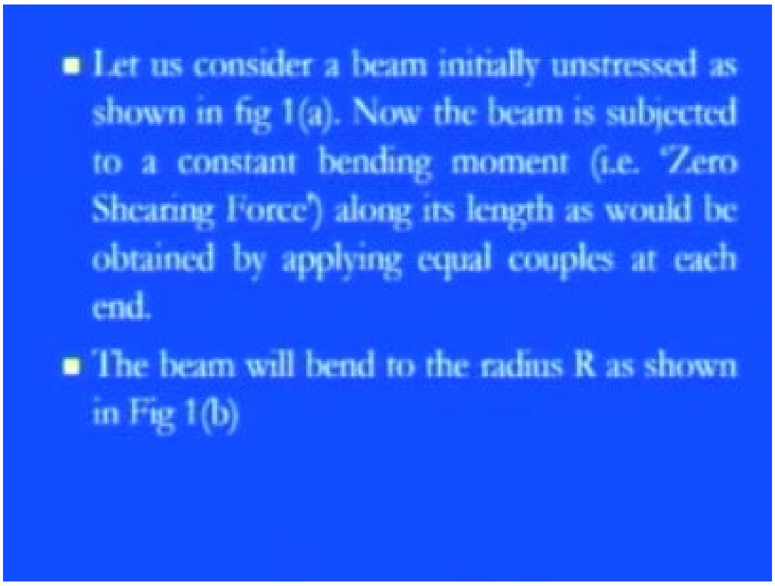

So, the beam was initially unstressed as you see in the figure 1 a, which we shown. And here when we apply the load – the constant applied bending moment, there is… And since it is a constant bending moment. So, it was zero shear force. And then, there is the fibers are under the action of the bending and we have the curved beam is there. And for that, this radius of curvature is coming, which we have shown as the R.

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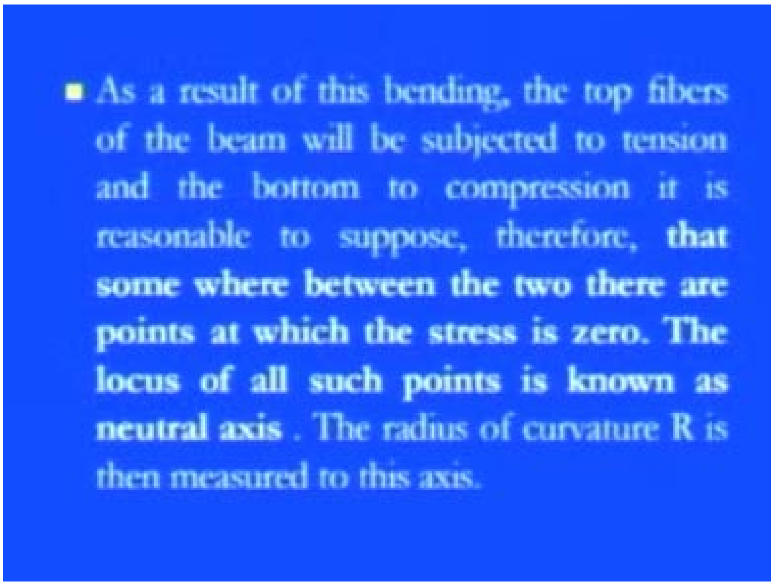

So, as a result of this bending – because of the bending moment, the top fibers of the beam will be subjected by the tensile load, because it is now under the action of the tension and it is expended. But, under this particular axis – the dotted axis, which I have shown you; the load portion or we can say the bottom portion is to be subjected by the compression. And it is reasonable to suppose that. Therefore, we have the two different actions. On top of that, we have the tensile part; the tensile stresses are there. And bottom of the portion, we have the compression. So, we have the compressive stresses are there. So, somewhere in between, there are two points at which the stress is 0, because it is changing from tensile to compression; that means it is… – the point of contra-flexure we can say as far as bending moment is concerned.

But, here the change is there from the tensile to compression. But, there is a kind of the points are there at which there is no stresses are there irrespective of whether it is a tensile or shear. And the locus of all those points, where the stresses are 0 are known as the neutral axis. That is why the meaning of the neutral axis is there is no stresses are occurring irrespective of the tensile or compressive along the neutral axis. And that is why it is all the locus of the points, where there is no stresses are there. The radius of the curvature R is always then measured from this particular axis, because this is the neutral axis through which there is no stresses are there irrespective of whatever the amount of this bending moment is. So, always right from the locus, where the curvature is there to this particular axis – neutral axis, it is to be measured that, this is the distance, which gives you the radius of curvature. And that is the R – capital R is the distance.

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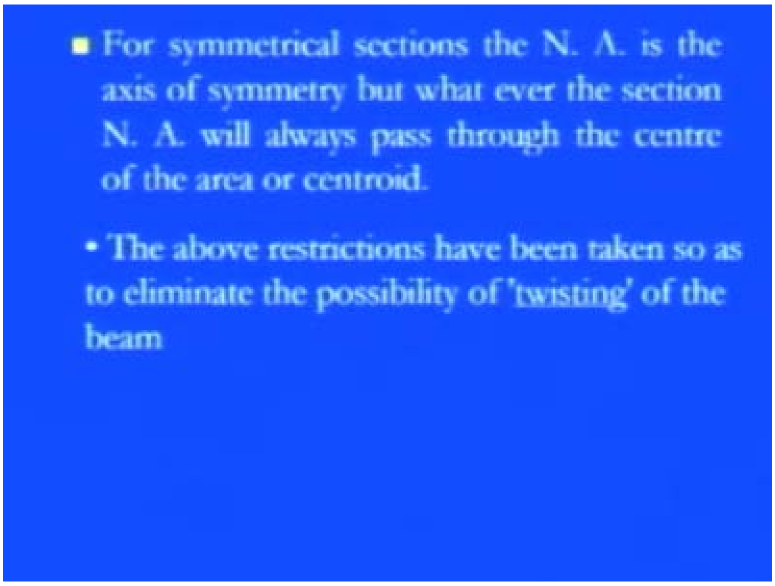

For symmetrical section, the neutral axis is always the axis of symmetricity, because it is a symmetrical section as I shown you in the figure 1 a; that it is a uniform structured; and the neutral axis is well-passed from the neutral surfaces. This is a natural path. And it is under the well-established phenomenon. So, there is no change as such in the neutral axis part. But, wherever the section, where the neutral axis is; it is a not the symmetrical part is there; it will always pass through the centre of the area of the centered. Or, we can say it is the centroid part is there, which the neutral axis is belonging; that means when we are saying that, the neutral axis is there; somewhere it is existing; centroid is always falling on this neutral axis. Or, we can say that, the centre of area is always along… Or, we can say the centre of area is always passing from this particular axis. So, neutral axis is always – we can say the key feature for analysis of any kind of cross section for the beam irrespective of whether it is a symmetry.

And, if it unsymmetrical, then actually we need to calculate that, actually what the symmetricity is there in this particular… means we need to cut the different segments and then we need to see that, what is the overall impact is there on the symmetricity. So, the above restriction have been taken so as to eliminate the possibility of twisting of the beam. And that is why all those analysis in the particular beam – this bending of beam, is focused along the neutral axis, so that whenever we are saying that, the moment is there, then what will happen?

What the beam is behaving along the neutral axis is to be focused. And that is why we are always watching that, the normal stresses are there. And that is why these normal stresses are known as the flexural stresses; that means there is a point of flexure, where there is no stresses are there; that means there is no bending moment is there. Under the action of bending moment, there is no stresses are occurring on those things. And that is why these axes are known as the neutral axes. And only when we are focusing on the neutral axes, the bending is only coming; there is no twisting is there in that particular picture or for the analysis.

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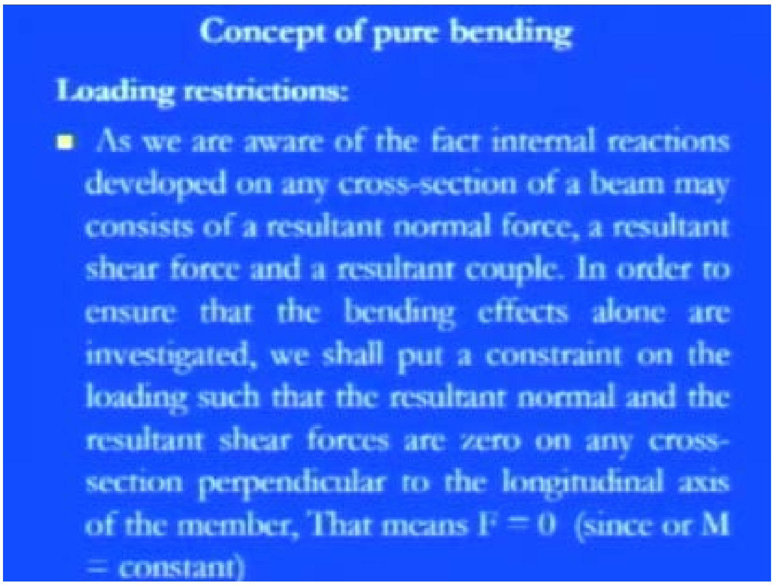

So, now you see the concept of the pure bending as we discussed So, the loading restriction is very important because our focus is only for the bending, because otherwise, if the different eccentricity is there, then definitely… Or, if different locations are there, then the twisting or buckling may happen. So, now, the focus is that, actually what will be the loading restrictions are for the pure bending. So, as we are aware of the fact that, the internal reactions developed on any cross-section of a beam may consist of a resultant normal force and a resultant shear force and a resultant couple; that means if I am saying that, a beam is under the action of any kind of loading, then what will happen? First of all, there is a resultant normal force. And based on that, we are calculating the shearing force as we discussed.

And then, since there is… we are applying the load at different locations. And definitely corresponding, the bending moments are there. Or, we can say the couples are there; due to that, the bending moment will occur. So, our main focus is that, what will be the resultant force; due to that, what will be the shearing force; and due to that, what will be the resultant couples, so that it will conclude or it will end up with the bending moment. So, in order to ensure that, the bending effects – just alone; we are investigating; no other twisting or buckling is there. We shall put a constraint on the loading such that, the resultant normal and the resultant shear forces are 0 at any cross-section perpendicular to the longitudinal axis of the member; otherwise, what will happen if the shear forces are not be 0. Then, it will end up with the twisting kind of that. And due to that, the torsional stresses are coming into the picture. So, along with the bending stresses, we need to consider the torsional stresses. And then, what will be the interaction is there in between the torsional as well as the bending stresses, one has to carry out those calculations.

So, in order to avoid that situation, always our important thing is that, one has to put the resultant shear stress 0 just to avoid the torsional stresses. This means the F is equals to 0; the shear force is the resultant shear forces is equals to 0, and M is the constant one, because F is nothing but the dM by dx. So, if we are keeping F is equals to 0, dM by dx is 0; that means M is constant. So, here this is whatever the bending moment is coming, which is the constant, it will only end up with the bending stresses or we can say the bending moment. So, our focus is only on the bending. So, here this is the clear cut situation for the pure bending that, if whatever the resultant normal and the resultant shear forces are coming; and if you are… because as I told you, whenever the beam is under the load condition…

Again I am repeating those things, so that actually there would not be any confusion in that situation. If a beam is under the load situation, what we have? We are ending up with the normal resultant force, normal resultant shear force, and the normal resultant couple. But, if we are keeping this normal force – resultant normal force and resultant of shear force is equals to 0; that means the total force component is 0; that means there is no torsional part is coming; then, that means F is equals to 0. And we already discussed that, F is nothing but equals to dM by dx. So, dM by dx is 0; that means M is constant. So, whatever the load is coming, it is coming out of only in terms of the constant bending moment. So, here whatever the deformation or whatever the changes are there in the fibers of this beam, it is just coming due to the constant bending moment or we can say the bending only. So, this is the concept of the pure bending.

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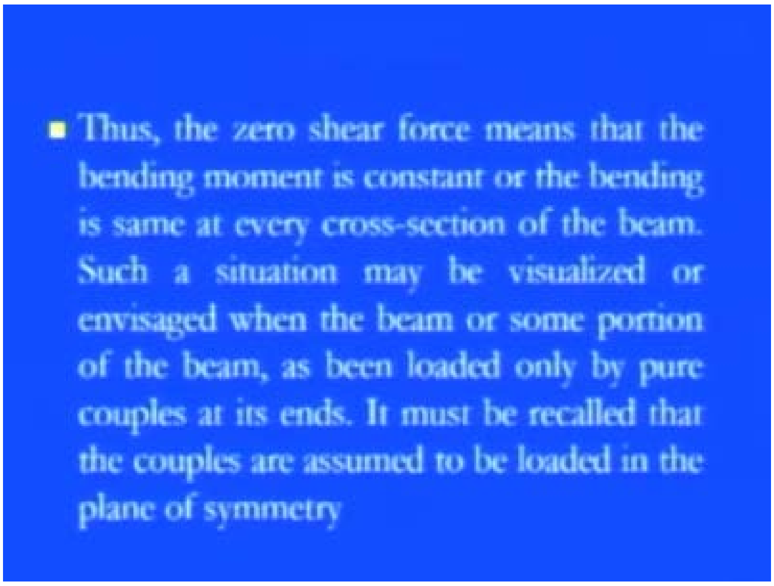

Thus, the zero shear force means that, the bending moment is constant as I told you. Or, bending is same at every cross-section of the beam. So, whatever the extension is there on the top of the fibers of the beam or the compression is there on the bottom of the fibers of the beam, they are only exerting the pure bending moment. And that is why it is known as the pure…

The situation is known as the conditions for pure bending. Such a situation may visualize or envisaged when the beam or some portion of a beam or we can say this kind of beam is only loaded when the bending is occurring on that particular part. Irrespective of whatever the couples are coming, they have the constant magnitude; and they will end up with the simple bending moment. There is no other shearing as well as the normal forces are coming into the picture; or, there is no interaction is there in between these other components; only the bending is there. And due to the bending, as we discussed that, the normal stresses are coming or the flexural stresses are coming on top of the fibers or the bottom of the fibers of a beam. And it must be…

We recall that, the couples are assumed to be localized in the plane of symmetry only, so that whatever the couples they are coming, they have to be well-settled with this particular…; because if they are acting at a different plane, then again there is good chance of the torsion. Or, we can say some kind of buckling may occur. So, whatever the couples or the bending moment if… Or, we can say these kind of moments are coming in the particular beam – they have to be considered within the same plane of geometry. Or, we can say the symmetric plane is there; otherwise, if it is not symmetricity is there, then the kind of eccentricity may come or we can say it will end up with the kind of twisting moment or we can say the shearing moment kind of that. So, we can visualize that situation in these diagrams.

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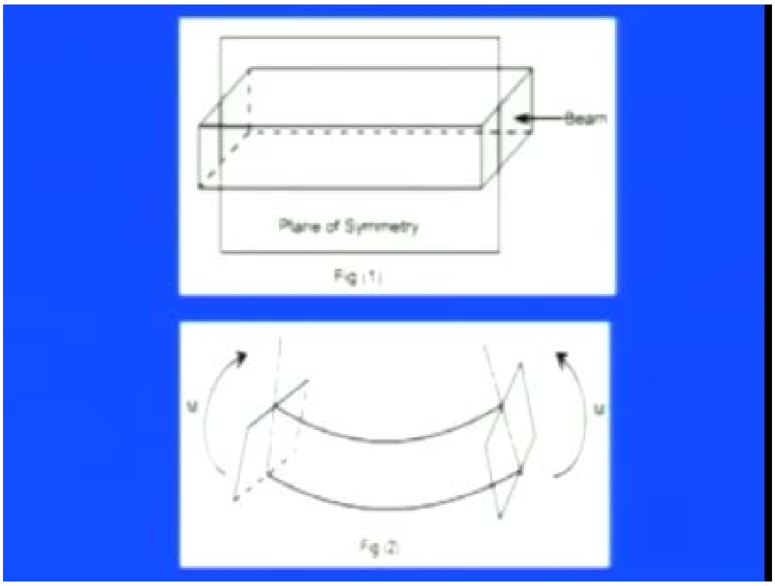

We have a kind of beam and this is our plane of symmetry. So, whatever the couples are coming, they have to be coming within those particular plane. So, we can see in the figure 2 that, we have the beam. And this – if we are saying that, the beam is under the curved part or we can say the constant bending moment is there, this is a kind of under the action of pure bending. But, these M – they have to have within this particular plane of symmetry only. If it is not, then it will end up with the different twisting part and kind of twisting or we can say a kind of buckling may happen in the beam. So, for justifying those conditions, this is a plane of symmetry. And for that, these bending moments are there within that particular plane of symmetry to carry out only pure bending in this particular beam. So, this figure 2 is simply showing the same kind of concept for a pure bending in the beam situation.

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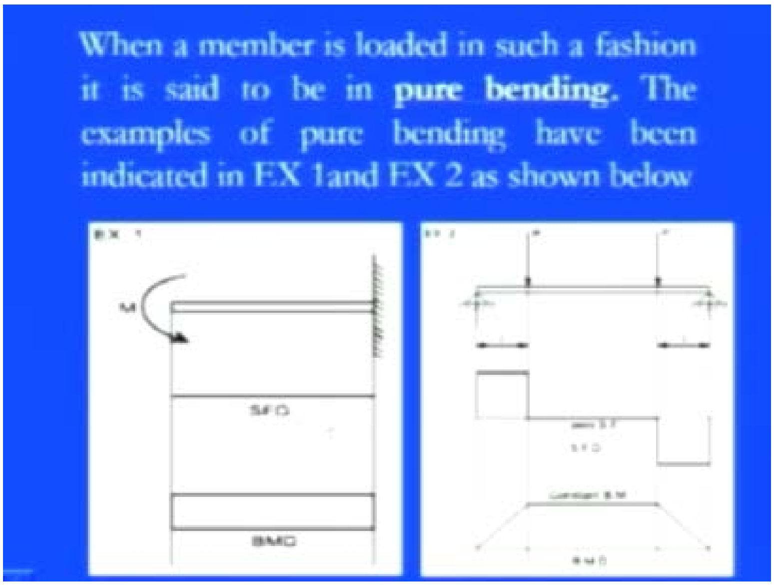

When a member is loaded in such a fashion that, it is said to be pure bending as we discussed that, only irrespective of whether whatever this bending moment is coming, they have to be there in the plane of symmetry. Or, whatever the resultant force or this normal or shear forces – if we are keeping 0; in those conditions, if you are saying that, if the member is loaded in this situation, we have a situation of the pure bending. The example for the pure bending, we can simply visualize in these two examples. Example 1 is simply a cantilever beam; and one end is fixed and one end is free.

And in this particular end, again whatever the moment is coming, this moment will be along the plane of symmetry. So, due to that, we have the shear force, which is pretty… Since it is only moment is there at the free end, this shear force is there on that and we have a bending moment. So, you can see a simple bending moment is there; there is no other shearing force is there. So, shear force diagram – you can see a straight line. But, if the bending moment – since it is a constant moment is applied at a free end; so whatever the bending moment will come at this particular end, it is constant throughout the cross-section of the beam. So, we have a straight rectangular portion is there in that. So, this is a situation, where the pure bending is occurring in the cantilever beam.

In the next situation, in the example 2, we have a simply supported beam. There are two reactions at the two extreme corners. And the two points – at these particular points, we have the point loads are coming from the top of the portion. So, here for that, since there is a shearing is occurring; so for that, this is the positive region of shearing; this is the negative region of shearing. And it is changing from positive to negative for this. This is a constant region. So, for that, here wherever the 0 shear force is there; that means there is… When positive to negative, there is no shearing has occurred. For this portion, we have a pure bending situation. So, we can say that, whatever the bending moment, whatever the constant is there; this situation, where only the bending is occurring; there is no shear force is there. But, for this situation, we have the positive shear force right from 0 to maximum and there is a constant bending moment.

So, no shearing has occurred for this situation. So, this is the idealistic situation for pure bending. And then, here again there is due to this negative shear force, this is going to be zero. So, there is a direct impact is there of the shear force F on the changing of beams in the bending moment F is equals to dM by dx. And you can visualize those changes in these two different examples, which are there for pure bending situation or where you see the interaction is there with the shear force to bending moment. So, you see these two examples.

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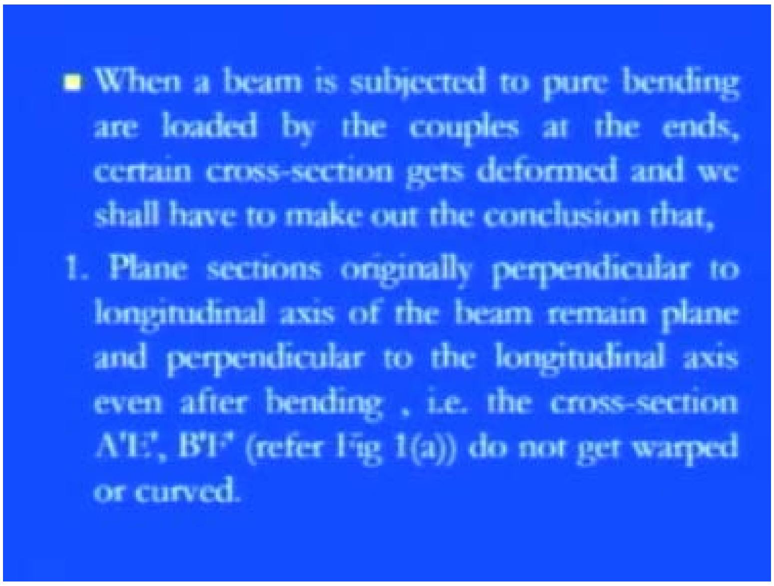

And then, when a beam is subjected to a pure bending, which are loaded by the couples at that extreme end; if it is the simply supported or we can say this cantilever, certain cross-sections get deformed simply, because it is quite visualized that, when you apply the load, it is to be deformed certain sections; the fibers are to be deformed. And we shall have to make certain conclusions out of that that, actually what will happen under those actions. So, these conclusions are like that. First, the plane section – originally perpendicular to the longitudinal action always; whatever the plane sections are there, the longitudinal plane sections – they have to be perpendicular to the longitudinal axis of the beam, remains plane after the bending; because once you remove the load and if they are not exactly perpendicular to the this plane after even the bending, then these theories – whatever the formula, which we are solving – they are not at all applicable. And whatever this elastic limit will not be valid for this region. That is the cross section.

Whatever AE or BF in the first figure as I have shown; they do not have to be wrapped out or curved – means they do not have to remain in the curved once you remove the bending moment. They have to come there in original portion. Or, we can say whatever the deformation is coming – the bending deformation, it has to come to its original shape after this removal of this particular bending moment. So, it has to be there in the elastic deformation under the action of these loads. So, this is the first conclusion. And it is very much valid for any kind of bending action.

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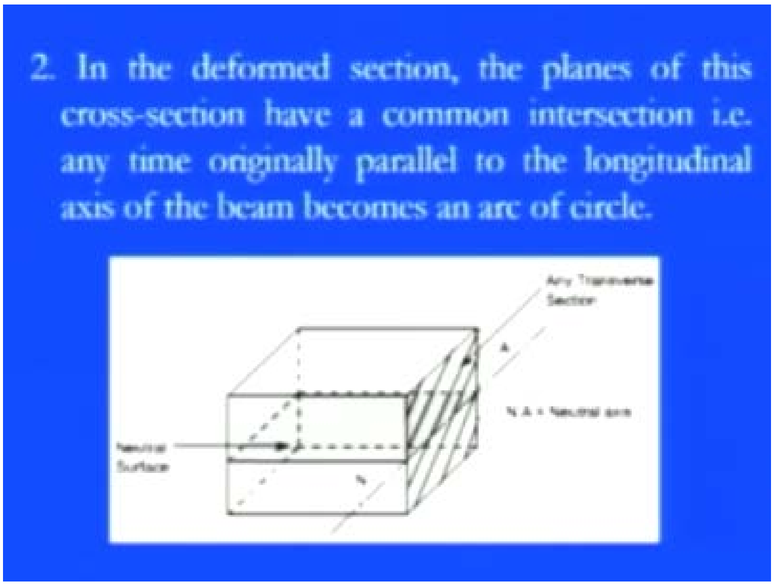

Second is in the deformed section, the planes of this particular cross-section – whatever you covered, have a common intersection; that means at any time, originally if we are thinking about the original plane, which is parallel to the longitudinal axis, it always becomes the arc of the circle, because we are always assuming that, under the action of the beam, it is under the curvature path. So, if it is a path of the particular curvature, then whatever the theories of the curvature, which we are applying for analyzing the beam theory; it has to be followed at any instant of the point; that means it is not that, when you apply the load, these theories are valid; when you remove the load, it is not valid. It has to be removed, so that we can say that, at any time, whatever the cross-section of the beam is there, they have to be parallel to the longitudinal axis of beam. And it becomes to be the arc of the circle; that means if you see this particular figure; you can simply say that, what we have.

We have… This is the straight beam and this is the neutral axis. So, along this path, if you apply the load, this upper portion will be under the action of this transfers action; and we have these tensile stresses. And for this – the load portion, if the bending moment is applying, it is under the action of these compressive loads are there; and the fibers are well close to each other. So, in both of the sections, because it is the arc of the circle in the second section and the first section as we told that, actually that, whatever the load applications are there or moment applications are there – they have to be limiting by the elastic deformation. So, these are a real good – this point. And whatever we are going to discuss about those things, this is very much valid.

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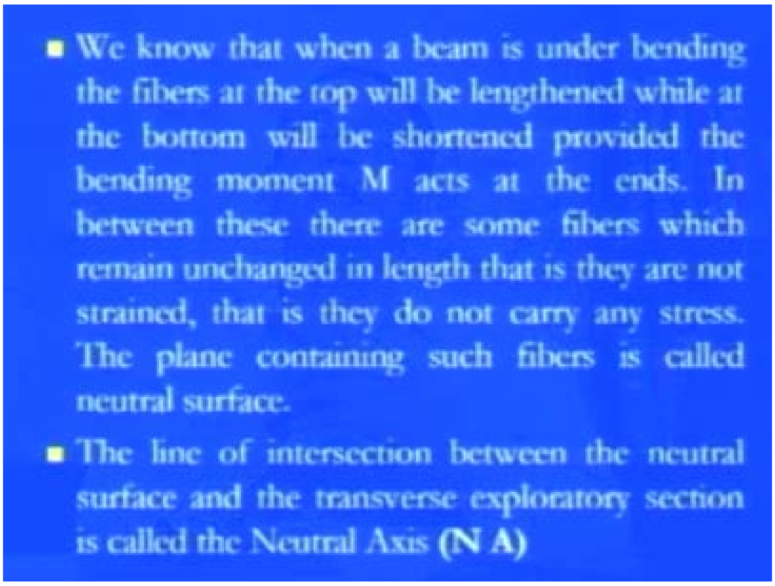

So, now, just we want to conclude those things prior to finish this lecture that, we know that, when a beam is under bending, the fibers on the top will be lengthened. As we discussed, while at the bottom, it will be shortened; that means, on the top of that, we have a tensile stresses or tensile forces. On the bottom portion, we have the compressive stresses or the compressive forces. And they are just providing the bending moment, which acts at the particular end. And in between these, there are some fibers which remains unchanged in the length; that means there is no effect of these bending moments are there. And the plane – where there is no impact of this bending moment, these planes are known as the natural surfaces.

And, the line of intersection between the natural surfaces and the transverse exploratory section, which I have just shown in the previous diagram is known as the neutral axis. And always, in the particular bending action, we have both kind of the flexural stresses are there on the top and bottom of the neutral axis. And that is why all the analysis is simply focused on the neutral axis. So, we have seen in this lecture that, the bending only… If you are thinking about pure bending, then it is simply focused on that, actually what is the neutral axis and where it is to be placed. And along the neutral axis that, actually what will happen on the top of the fiber and the load of the fiber.

And also, we discussed in this lecture that, actually if we have different kind of loadings on a beam irrespective of whether it is a UDL or triangular load or if we combined of those things, then actually how we can analyze a shear force and the bending moment diagram as well. So, in the next lecture, our main focus is that, with the basis of the neutral axis, how we can calculate the bending stresses and what will be the relation in between the bending stresses and the shearing stresses; because till now, we did not discuss anything about the stress component; only we discussed about the shear force and the bending moment diagram.

So, now here again… But, prior to calculate the shear stress and bending stresses, our main focus is that, for any kind of structured or unstructured structure, first to locate the neutral axis to analyze the forces along the neutral axis, and then how these stresses are forming and how we can visualize those stresses for this kind of beam under the action of any load. That is our prime focus. So, in the next lecture, we are going to discuss all those issues about the beam and the neutral axis.

Thank you.