Strength of Materials
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Lecture - 25

Hi, this is Dr. S. P. Harsha from mechanical and industrial engineering department, IIT Roorkee. I am going to deliver my lecture 25 on the course of the strength of materials. And this course is developed under the scheme of national programs on technological enhanced learning. Prior to start this lecture, I would like to refresh the concept on this BMD and SFD; that means the this bending moment diagram and the shear force diagram on a beam, because in this lecture also we are going to discuss again; that actually if there are the interaction of the different loads are there, then how we can draw these two diagrams – the shear force as well as the bending moment diagrams.

So, see in the previous lecture, we discussed the four different cases of that, and as we told that as we discussed that if a beam is there and there are vertical forces acting on towards the downward direction, then always it depends on what kind of the supports are there; the reaction forces are dominating. That means if we have a cantilever beam, then definitely one end is free and one end is the rigidly fixed at a particular point. Then at this particular junction where these two – the beam as well as this support are joining, the maximum bending is there at that particular point and we discussed that point. So, in that, we found that, in the first case, which we discussed that, if we have a cantilever beam and a point load is there at the extreme end of this cantilever, then shear force is nothing but… because on this particular cantilever, the applied force is going downward direction and the reactions are going on the vertical upward.

And due to the sign notations, which we took in the previous cases also, the shear force is in negative way. So, F is equals to minus W, and we got the rectangular portion with the minus w with the shear force. And then we found that, if we have the similar kind of arrangement. And if we want to find it out the bending moment, then it was 0 at that particular point and it is simply going in this linear relation F equals to minus w into x or F equals to minus w into l, if we are taking the entire length of a beam. And we found that, it has a linear relationship. So, we have a triangular form of this bending moment diagram. And at the maximum, as I told you that, wherever the junction is there, the maximum bending moment was minus w into l. So, this was the first case. And then we changed the support.

So, if we have a simply supported beam; that means, this beam is supported at both ends by a pin joint. And if we apply the w load at the exactly midpoint of the beam, and then we found that, the reactions were wl by 2 and wl by 2. And then corresponding those shear force diagrams were there. So, if we were taking the left portion of this kind of beam, then we had a rectangular form, which is w in a positive way. And if we are taking the right-hand side, which is the negative portion, because of the sign convention of the shear force and exactly the… But, the amount is exactly same as we have taken in the first one. So, here what we have in that kind of thing, we have minus w by 2 on this one in shear force diagram going up to the middle point portion. Then, it is simply abrupt change is there, because of the applied load and it goes up to a minus w by 2. So, plus w by 2 to minus w by 2, there is a straight reduction of w. And then at the end, we have the same minus w by 2. So, these two triangles were showing that, actually how the shear force is changing from positive to negative.

And then corresponding this bending moment diagram was there that, actually starting from at both reactions, it was 0 and it was maximum at the middle point. So, it is just like wl by 2. So, it is… Just in a… The exact perfectly, the triangle is there – 0, and then just peak is there at the middle point. So, these two cases, which we discussed in the previous lectures when the point loads are there; and then we just simply change the type of loading; that means if we had this uniformly distributed load all across this length the beam of length, then we found that, it is not… As far as the shear force is concerned… because now the load is not point load, it is distributed over across the particular length or the entire length of a beam, then the total load is w into l.

So, now, we found that, the either if we have a simply supported beam or a cantilever, then altogether it is a different kind of shear force and bending moment diagram is. And we observed that, as far as the shear force is concerned, there would not be an issue, because it is changing from the middle portion and then the symmetric is there – means positive to negative change is there in both the cases. But, this bending moment diagram is concerned, it is absolutely now, the relation is not linear as we have observed in the previous two cases when the point load was there.

Now, this time we have absolutely the parabolic relation is there, because the square terms is coming and we can say that, it is nothing but the x square equals to 4 a y or whatever. This kind of a parabola will come in that picture. So, both of the… Either we have a cantilever beam and UDL is there or if we have a simply supported beam and UDL is there on that particular; we had… Even if we have simply supported beam, then the shear force diagram is just starting from wl and then it is going to minus WL. So, what we have – we have a point, where there is an abrupt change is there. And at that point, we observed that, the maximum bending moment is occurring.

Or, if you draw the bending moment, then starting from this to 0 point and it is meeting like at this point. So, wherever the shear force is changing from positive to negative, at that junction, we have the maximum bending point. And we can simply observe that, if mathematically that, if we have the shear force F, which is equals to dM by dx, we can simply keep either F equals to 0 or F equals to constant. There is the corresponding changes are there in this bending moment. Or, we can say… We already discussed about that, if we have this load W, which is equals to dF by dx, which is also equals to d 2 M by dx square. So, this relation is perfectly okay for all those four cases, which we discussed.

And then in this lecture, we are going to discuss that, apart from these point load or UDL on this cantilever beam or simply supported beam; now, if we put the couple or if we change not only the couple, but if we have UDL and the combination with the point load. Then, what will happen? What kind of interaction is there in between these kind of loadings plus what is the resultant effect is there on shear force as well as the bending moment. And then we would like to draw those diagrams, so that actually we can easily visualize that, this kind of interactions are there and we can get this kind of shear force as well as the bending moment diagram. So, in this lecture, we are starting from the couple itself.

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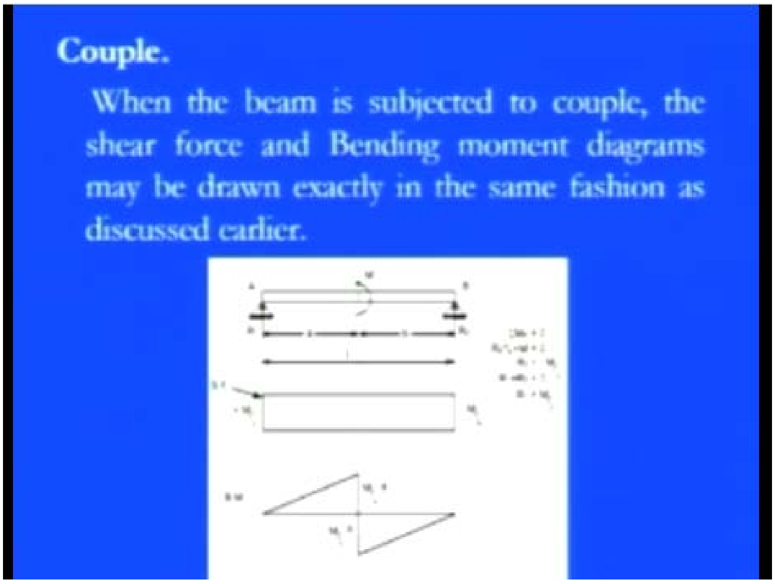

So, here when a beam is subjected to a couple, the shear force and the bending moment diagrams may be drawn exactly in the same fashion as we discussed for the four cases, where the point loads were there or UDL was there in a simply supported beam and a cantilever. So, here we have a simply supported beam; and at these two these two extreme corners, we have the pin joints. And at the middle of the portion, we have a couple. So, couple is acting. So, due to the couple, always there is a kind of shearing is occur… all across this entire length of beam and also there is a bending moment is there, because when you apply the couple, a kind of a twisting is there from both of the ends. And since we have the supports, how the reaction will react on these kind of couple?

So, we just want to balance those things, because whatever the analysis, which we are going to discuss or we discussed in the previous cases also, it is just for the equilibrium position of the beam under the application of load. So, come to the point; you can see on the figure that, we have this AB length of the beam – a simply supported beam is there. So, at these two extreme corners, we have the supports – pin joints are there. And at the middle of the portion, we have a bending moment, which has a magnitude of M. So, here what we have? We have a distance from left end is A when this point of application of this bending moment is. And from right end, we have a distance of B.

Now, we would like to see that, how these moments will be reacted at these reaction points. So, we found that, just assume that, we have the summation of bending moment at this particular point A is 0. So, after applying that particular condition, if you just… When we know that, actually this is the junction point; and whatever the moment is coming from, it is coming from middle distance A or a distance A plus B means that, entire length. So, how these moments will act on these particular points. So, according to that, we can starting from… Then, if we have a reaction at this particular point A, it is R 1; and at his particular point B is R 2. So, we can straightaway start from that R 2 into l; l is nothing but as we discussed A plus B. So, R into l plus… And this… Because of that, since we are applying this moment; so the reaction will be vertically upward. So, even this R 2 will also act in a vertical direction. And since… If you are saying that, this is moment; so entire…

When I am saying that this is the junction and it is starting from that; so the total moments will be from this reaction is R 2 into l plus this moment M, which is equals to 0. So, we can calculate this R 2 is equals to minus M by L. And now, since… If we are saying that, the total; there is no force applications there, only the moment is there. So, the total force R 1 plus R 2 is to be 0. So, you can simply keep this R 2, which is minus M by L. So, we have R 1, which is equals to minus R 2. So, we have R 1 is also M by L. So, now, what we have? We have reaction forces at this point A and point B. So, at this particular point A, we have the reaction – this plus M by l; and we have the reaction at point B, which is R 2, is minus M by L. So, corresponding to that, we can simply find it out that, what will be the shear force is there in that cases.

So, if we want to draw the shear force diagram, we can easily get those things by these diagrams; that we have the centre line or which this shearing is occur. And then at this particular point, at one point of time, what we have? We have the reaction is going in upward direction. And due to moment, it is just going towards that direction; that means we have a positive one. So, plus M by l is there. And in other direction, it is going in upward direction. And due to that, it is going in a vertical direction. Since there is no force relation is there straight, at one point, we have a moment; and these two forces are just going in upward direction. So, you can say that, the shear force is exactly equal to this. At this particular point, we have plus M by l; at this point, we have minus M by l and it has a rectangular shape altogether.

But, there is an addition of a moment at these two junctions, because at these two reactions, always we need to assume or we have to calculate… If you calculate, then definitely you will get that, wherever the pin joints are there, there is no moment is there – all those things, because we always take the moment from that. Even if you see… We just take an example of a door; that always wherever the junction is there of a door, where the wall is, attachment is there, if you apply the moment near to this particular junction or if you just go towards outward direction, you will find that. Whatever the force into the distance… As you increase the distance, the moment will increase So, similarly… But, if you apply the load wherever the junction is there, there is no moment will take place even in that door or any of the cases. So, here also we can apply the similar concept to get the bending moment. So, we found that, at these two junctions, where the pin joints are there, always these are the reference points. So, there no bending moment is there at these particular two points. So, starting from these two points, we have the 0, 0.

Now, as we move further, we just to focus on that, whether it is going in a sagging or whether it is going in a hogging way; so if you just analyze those things for left-hand portion, then will find that, this will be as you apply the moment. So, it is acting in this way. And due to that, it will also try to tend in this way. So, what we have? We have a positive; it is simply a sagging motion is there at this particular left-hand portion .So, we have a positive bending moment, but it is 0 at this particular portion and it is maximum at middle point. And this maximum bending moment is M by M, because it is a distance of A. So, M by L, which is force into distance is l; So, M by l into A. So, this is the maximum bending… A moment will come at this particular junction, which has this particular magnitude. So, it is always. Since it is a point load is there or we can say due to this moment, always the loads are not in a uniformly distributed kind of that. So, we have a triangular shape of that of bending moment, because of the linear relation M into A by L. So, this is starting from 0 and goes up to the maximum gives a triangular shape of a bending moment.

But, if you go to the left-hand portion, again as we discussed that, since a pin joint is there, the starting point. So, starting a point is always 0 in terms of bending moment. And now, this bending will… The middle portion if I am talking, it is always tend to move this beam in a lower direction. And to balance this sagging – always the other half, we have this portion – hogging portion is there. So, starting from 0, and we have MB into L. So, this kind of… Because at this particular point, we have a moment – additional moment is applied; so there is abrupt change is there from MA by l to MB by l, because the point of application of these distances are different at this particular point A; and at this particular point, B is there. So, we can say that, starting from this point to this point, the variation is from this MA by l to dropping is there – MB by A, MB by l; and it is going to the 0. So, this kind of bending moment will come in this kind of beam, where the loading conditions are of couple is there. So, now, as far as the couples are concerned, it does not… Only it is contributing much in bending moment, because whenever the bending moments are concerning, then always at what point you are applying the moment – additional moment is there. So, we can simply include those kind of things.

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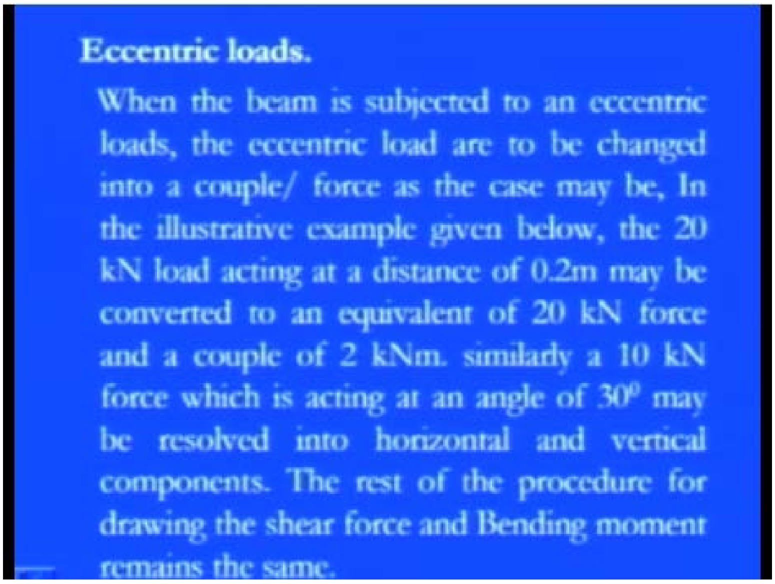

But, when eccentric loading is there; that means when a beam is subjected to a kind of not exactly at a middle point or some point, eccentricity is there. The eccentric loads are always changed into the couple or – oblique – you can say the force as the case may be in the example, which we are going to discuss; you will find that, whenever… If I am saying that, these points at the feasible things are there – means if we have a beam and these points are exactly on the middle portion, there is no eccentricity will come. But, if there is eccentricity is there, due to the eccentric load, because how much eccentricity is there and what is the magnitude is there; the total couple will come from the distance – eccentric distance into the magnitude of that load. So, this will give you a couple. And due to that couple, the similar kind of observations will come as we discussed in the previous case.

So, in one of the example, we can say that, the 20 kilo newton, which I am going to show you, a 20 kilo newton load is acting at a distance of let us say 0.2 millimeter. Maybe you can convert this thing in this 20 kilo newton and couple. So, this point load is there. So, there is an impact of this point load of 20 kilo newton plus – whatever these eccentricity is there, this will induce a special kind of couple. And the total magnitude of that couple is 20 into 0.2, that is, 2 kilo newton meter.

So, now, we can say that, it has a combination. So, if any eccentricity is there on this particular beam, you can simply take that particular portion, find out that how much distance of the eccentricity… – means due to eccentricity, how much distance is there. And then whatever the magnitude is there, this is additional couple is there apart from whatever the point loads are there. Or, we can say that, if we have a 10 kilo newton force, which is acting at an angle of 30 degree – means it is not exactly a middle portion just like the vertical forces are.

If it is acting at a 30 degree, maybe we need to resolve in two main forces: one in horizontal and one is in vertical way. And the rest of procedure for drawing shear force and bending moment will be same. The meaning is pretty simple for these kind of things that, whenever we are thinking that, if there is a load, which is eccentric, there is the two components, which has to be considered for analysis. One is a point load, which has a same magnitude, which is acting by point load. And other one is the load whatever magnitude is there of the concentrated into how much distance, which has displaced from the centre point; so eccentric distance. So, it will induce a couple. So, these two components has to be considered for analysis. While if you are saying that, the load is there, which is not vertical, which is at some angle is there; so you need to resolve these two forces in a cos theta, R cos theta and R sin theta let us say. And then you need to consider for our analysis that, if vertical forces are there, then how these vertical forces are acting; and if horizontal forces are there, then how these resistive forces are coming due to this angle load. So, these two special cases were there.

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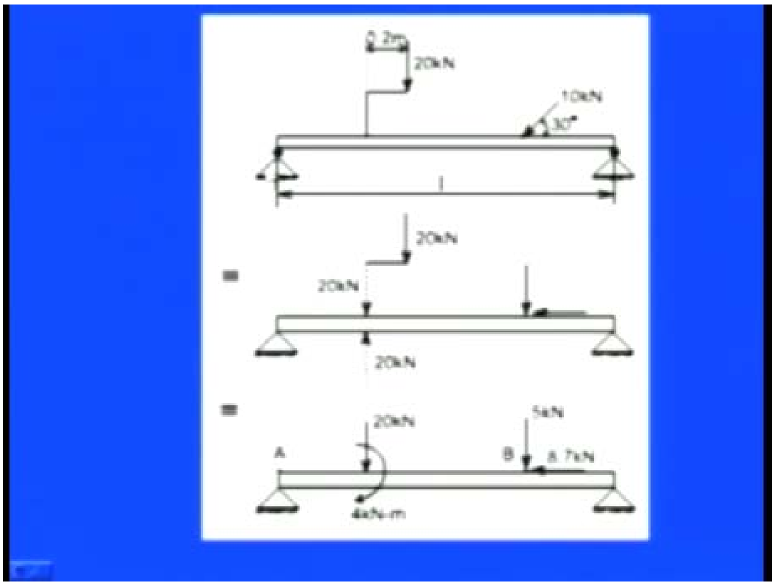

So, we can say that, as we discussed that, we have the simply supported beam. And instead of the point load, what we have? We have… This is a kind of nature, where we have a handle. So, if we are keeping a load on the extreme end of a handle, then what will happen? Since we are keeping at this particular point – this point; so there is a vertical load is there. So, we need to assume that, there is a vertical load, which has a magnitude of 20 kilo newton plus there is a couple. Or, we can say there is a moment, which will tend to move the beam into this direction, or we can say the clockwise direction, because we are applying this load in this direction.

So, the bending moment will be like that, or we can say the moment is there in this way and the load is there. So, if you take the resultant of this; if you are considering this case, then you will be ending up with two components as I told you. One, which is the magnitude of this load. So, it has a 20 kilo newton; and one because of this 20 kilo newton plus this handle distance. Because of that only, this bending moment is coming. So, the moment is of this into this. So, we have 4 kilo newton meter. So, these two components has to be considered for analysis; otherwise, this whatever the analysis is coming due to only this load will be of a failure.

And then the other case, which we discussed that, if we have an angle of let us… If we… The load, which is of any magnitude is applying at an angle; then you need to resolve that cos theta, which is in this direction and sin theta, which is of a vertical direction. So, once you resolve those things, you have the both components. At this particular point, you have the tan… This is sin theta; this is cos theta; and you resolve; then you will have 5 and 5.7 at this particular point. So, while analysis, we cannot ignore any of the force components, because the total resolution of these forces is always giving in two component and then you need to consider for analysis; that, if we are talking about a vertical part, then these vertical forces are… If we are talking about the horizontal part, then these horizontal forces are coming into the picture for our analysis part. So, this was a new case, where these two special things were there: one is the eccentricity; and one is the resolution of this force.

Then we have a different kind of situation, where both like point load as well as the unit load; or, if we have the uniformly distributed load and their two types of uniformly distributed loads are there; that means if we have a combination part; if the loading changes or we can say there is an abrupt change of loading is there; then what will happen? How the beam will react on that part? And what will be the net change is there in the shear force as well as the bending moment diagram we are going to discuss.

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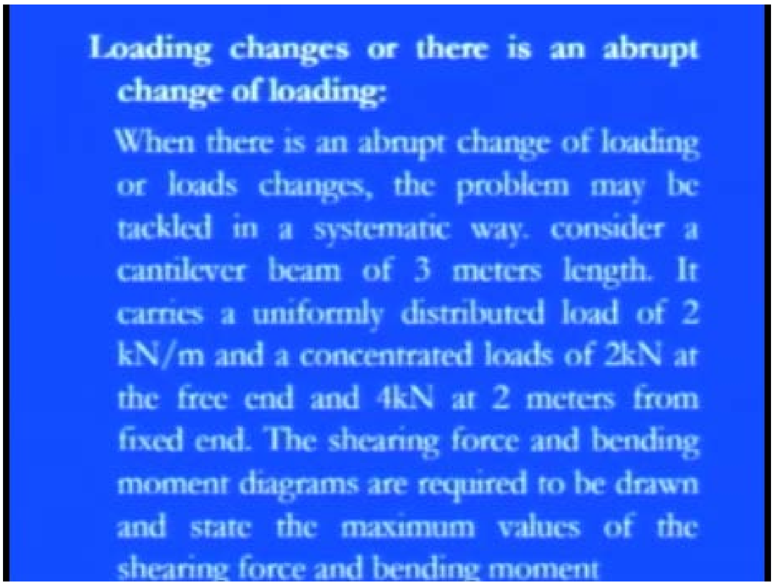

So, when there is a abrupt change of loading or load changes, the problem may be tackled in a symmetric way. Just total… means the same system is there; there is no change in that way whatever the symmetricity, which we have assumed. Again the same systematic pattern is there in that way also. So, if we consider a cantilever, now, just take an example of 3 meter let us say the length. And it carries a uniformly distributed load of 2 kilo newton per meter; that means we have a cantilever beam; one end is fixed; one end is free. And on that, over an entire length of this beam period, we have this – UDL is there, which has a magnitude of 2 kilo newton per meter. And also, apart from this, we have a concentrated load of a magnitude of 2 kilo newton at the free end and 4 kilo newton at 2 meters from a fixed end; that means we have… Just try to visualize this situation; we have a cantilever beam; this is rigid part; and this is the free part. Or, the entire length of this, we have like the UDL, which has a magnitude of 2 kilo newton per meter.

And then what we have; we have a concentrated load – 2 kilo newton at free end. So, there is a 2 kilo newton per meter is a UDL at this point. And additionally, there is a 2 kilo newton of this; plus we have the 4 kilo newton, which is 2 meter apart from this fix end. So, from this… Or, we can say 1 meter from this free end, we have one more additional load. So, the magnitude of that load is 4 kilo newton. The shearing force and bending moment diagram are required to be drawn. And the state of maximum value of the shearing force and bending moments are always required for that. We need to check it out that, actually, how these… Because of these three different load conditions; that means one is UDL and two are concentrated load at different positions, what the interaction is there of these forces of these – the loads. And then due to this interaction, actually, how they will impact on the shear force as well as the bending moment.

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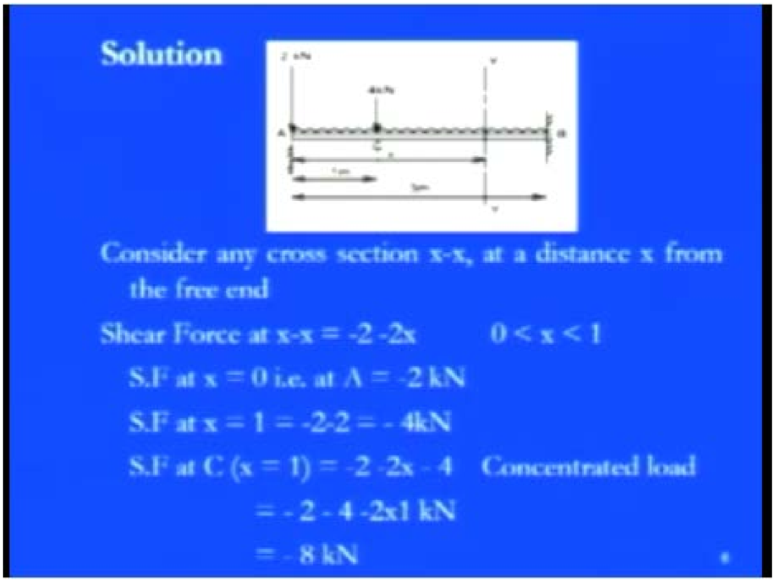

So, here it is we have discussed that, the same bending… This is… The beam is there at this… The cantilever is there; this is the UDL of 2 kilo newton per meter. And then we have at this particular extreme corner, we have a 2 kilo newton. And 2 meter from this distance or 1 meter from this distance, we have a 4 kilo newton. But, after visualizing these things, again we need to think that, actually how we can start. So, as we told, you need to cut the section. So, we cut the section at this end. This has a distance of x at this Y-Y distance.

So, what we need to do here; once we know that, this distance is that; that means this is a total 3 meter; this is total x; this is 1 meter. We will start that. That if we have a cross section X-X, which has a distance of x from free end, then what will be the shear force? Because we are starting from the shear force only. So, we have a shear force at the X-X distance; any distance x is minus 2, because it is coming in the minus direction; this force is there. So, minus 2 minus 2 of x for distance, which is less than l, but, greater than 0; that means starting from the free end and this, we have a distance at some places, but less than this distance l or we can say the 3 meter.

So, now, at this particular shear force, variation is what? It is starting, because this is the main contribution at the free end only. We are starting from the free end. So, be careful at that. Only we need to assume this one. So, minus 2 because of this end; minus 2 into x, because we are just talking about the x distance. So, the total load from this UDL is coming as 2 x. So, the interaction of these two forces: point load as well as the UDL will be ending up at minus 2 into minus… – minus 2 minus 2x. So, now, since x is a variation only; so we are starting from x equals to 0. So, when we are talking about x equals to 0, there is both – 2 minus 2. So, what we have at the point A, where 0 is there; the shear forces only minus 2 kilo newton, because 2x has gone. So, only the contribution is there. Due to this force loading – point load and UDL, is coming only due to this point load and it has a magnitude of 2 kilo newton. Hence, the sign convention says that, it is ending up in this particular direction. So, we have minus 2 kilo newton at point A – free end.

But, if we are going up to x equals to l; that means, if we are going for x equals to 1 distance, where the additional load is there; so only we are ending up this x is at equals to 1. So, we have minus 2 into 2 into 1. So, we have minus 4 kilo newton. So, starting from point A to go to the point C – or, C means where the 4 kilo Newton additional load is there, will be ending up in a shear force at minus 2 kilo newton to minus 4 Newton.

Then, shear force at point C; that means when x equals to 1 is there, same concentrated load minus 4 is there. So, we can simply induce this additional one, because this minus 4 is coming due to this 2 kilo newton and 2 kilo newton per meter. But, now, at the C point when we are saying that, there is an additional contribution is there from this 0.4 kilo newton. So, you need to add that force. And we will be ending up at minus 2 minus 4 and minus 2 into 1. So, we have the total load is minus 8 kilo newton. And that is the concentrated part is there.

So, one has to be very clear that, we need to consider the inducing effect of the previous load plus there is abrupt change is there; then how this abrupt change will come into the picture when we are adding those loads, because as we discussed in the previous cases also that, in a shear force, only what we are doing? We are simply doing algebraic sum of these loaded. So, here what we have; we have a combination here that, we have a UDL plus we have these two point loads. So, if we are talking about a point A, then only there is a load contribution is there from point as well as the UDL. But, at point A only as we found that, since x equals to 0. So, the main contribution or the total contribution is coming due to this point load. But, if we move to point C, then there is a contribution from minus – the 2 kilo newton load at extreme end, 2 kilo newton per meter load also up to the distance 1 and additional 4 kilo newton is there. And due to that, there is an abrupt change will come. So, at point C, we can say that, the total resultant force will be of minus 8 kilo newton.

So, now, if we are going beyond that point… Now, we have discussed up to 1 meter only. So, if we are going up to let us say Y-Y distance, then what will happen? What we have? We have at this point, the 2 kilo newton load. And at this point, we have… The total resultant is minus 8 kilo newton. Now, beyond that, here we have the total resultant is coming due to 2 and 4 kilo newton plus due to this 2 kilo newton meter. And it will be ending up and there is no additional component is there of a force in this entire length of that – that means in this 2 meter of this particular beam. So, we will be ending up that actually. Whatever the combination is coming due to these… or we can say whatever the resultant force is coming due to these three forces, they will be contributing equally up to this point. And there is no abrupt change will be there in that part.

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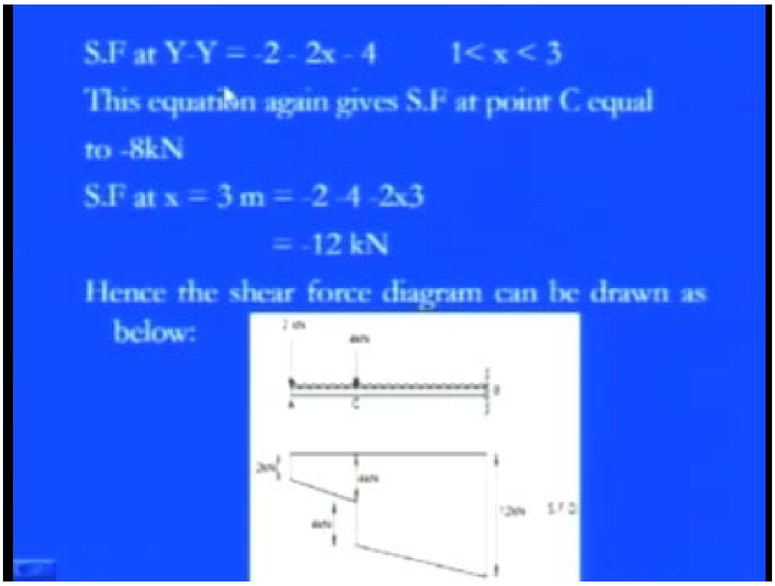

So, shear force at Y-Y section if you are talking; the same was there – minus 2, because of the 2 kilo newton as I told you; minus 2 into x distance because of the UDL, because 2 kilo newton meter is there. So, the x distance – minus 2 into the total x distance; and minus 4 because of the additional 4 kilo newton load was there in the downward direction. So, this is valid up to like x for greater than 1, because up to 1, we have discussed and less than 3. So, in between this region when there is no additional part is there, this is a valid equation.

And now if you are keeping that, at x equals to 1, the same C point is there; and we discussed that, it is at minus 8 kilo newton. And if you are talking about it at equals to 3; that means the extreme junction corner we have – minus 2 into minus 4 into minus 2 into 3. So, we have the total – minus 12 kilo newton; that means the additional minus 4 kilo Newton is coming because of the UDL part, because what we have? We have the 2 meter distance is left and 2 kilo newton per meter is the intensity is there of the UDL. So, the entire length is 2 meter into 2 is there and it is going downward direction. So, minus 4 additionally component is there beyond point C. That is why we have the total shear force at the extreme corner is minus 12.

So, now, we can simply draw this shear force diagram for this kind of loading of a cantilever beam that, starting from minus 2. So, this is minus 2. Then, at this point, we have minus 8. So, right from 0 to minus 2 at the beginning; then minus 2 to minus 8. And then we have the total change is there of this minus 8 is there because of the 4 and 4. So, this kind of the linear component will come. So, this 4 and this 4 makes the total 8. And this 8; and this is the total 12 newton is there in the minus term. So, this is the negative component all along and we have a kind of shear force diagram for this. And now, if we focus on the bending moment, because here as far as the bending moment is concerned, we need to write that, actually how the interaction will be coming because of the additional component of these loads are. So, for that, we need to write the bending moment equation at again by considering the X-X section.

So, here if we just again cut the X-X section of this kind of loading, for just starting from the free end, where point loading is there and the UDL is there; then what we have? We have the moment due to the point load is minus 2 into x distance. So, minus 2 into x. Then, because of the UDL. So, UDL – the total force component is minus 2 into x. And where it is applying? Always applying at the centre distance – x by 2. So, the total visualization of the component is minus 2 into x into x by 2. So, we have the total equation is minus 2x minus 2 x square by 2, or we can say the x square.

So, now, if you are keeping the values of x for different reasons, we have at x equals to 0, total bending moment is 0, because it depends on absolutely the x distance. And if you are saying that; if you are going up to the 1 – x equals to 1 meter; we have the total bending moment is minus 3 kilo newton meter; so that means starting from 0 because of the free end going up to minus 3 kilo newton meter, because it will just going in a shape of a hogging. So, obviously, it is in the negative direction is there. And then now, go beyond this 1 meter; that means at C point, what will be there. So, again cut the section of Y-Y and see the feasibility. So, for that, what we are doing here?

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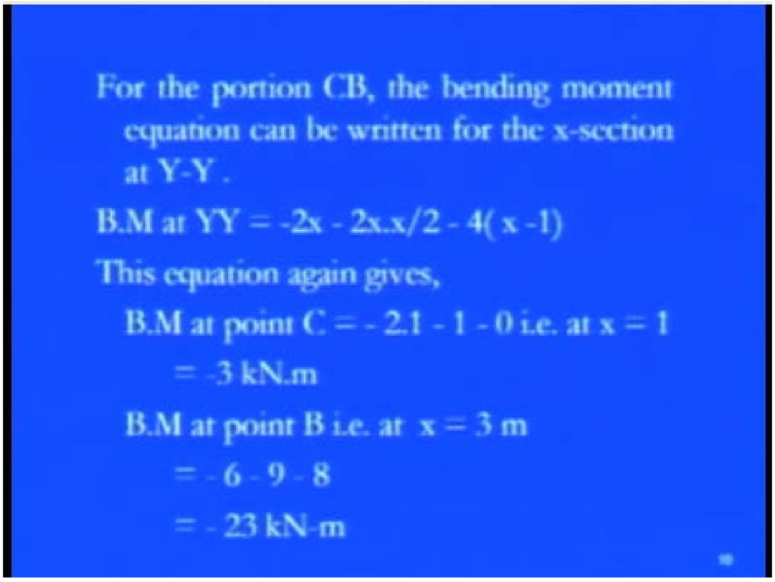

For up to distance of Y-Y, since the distance is x; so the contributions are coming from this 2 kilo newton, which is going in downward direction. Contribution is coming from 4 kilo newton; always going downward. And contribution is coming from 2 kilo newton per meter because of the UDL. So, we have a bending moment at Y-Y distance is minus 2 into x, because of the point load; minus 2x into x by 2 because of the UDL; and minus 4 into x minus 1 because it is starting from x equals to 1. So, x minus 1 for the entire length of 3 meter.

So, now, at point C, where x is equals to 1, you simply put the x equals to 1; then we have minus 2 into 1 minus 1. And since x minus 1 is 0; so there is no contribution is coming in terms of the moment, because the point load is… There is no eccentricity is there; point load is exactly acting on point C. So, no bending is coming due to this 4 kilo newton load at point C. So, only there are two contributions are there. So, we have x equals to… This bending moment is equals to minus 3 kilo newton meter. But, if you go beyond that point; means if we are going up to x equals to 3, then we have the contributions equally from 2 kilo newton, 4 kilo newton and the UDL, which is 2 kilo newton per meter. So, if you keep the value of x equals to 3 in this equation, then we have minus 6 minus 2 into 3. So, minus 6 minus x square is there. So, 9 square and minus this 4 times 3 minus 1; that means, the 2 into 4. So, 8 is there. So, total bending moment, which is at x equals to 3 is minus 23 kilo newton meter.

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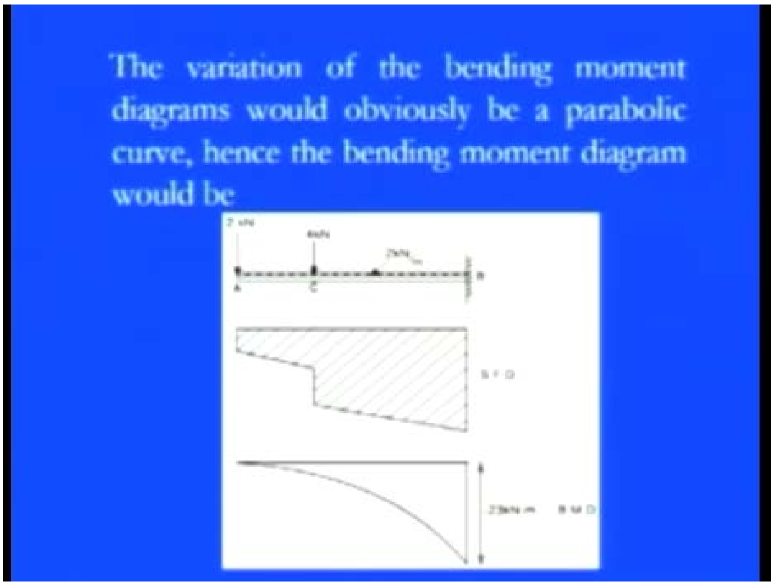

So, we found the two different values of the bending moment at the different junctions. So, at the middle portion, we have at point C, we have the bending moment, that is, minus 8 kilo newton meter because starting point is 0 as we observed. And at the extreme corner, we have the minus 23 kilo newton meter. And since there is a contribution in this bending moment due to the UDL also; so we have as we discussed that, always there is a non-linear relationship; that means we have a parabolic sections. So, if we draw these things by considering those values plus the consideration of these UDL in that, this bending moment, we have the total visualization is starting from 0 now coming at this. So, it is at this point, we have the minus 8 kilo newton meter.

And then we have at the extreme end, the total bending moment is there, that is, minus 23 kilo newton meter. So, this is the bending moment diagram; this is shear force diagram. So, if this kind of the interaction is there, we found that, wherever the change of this additional load is there or that means whenever there is an abrupt changes are there in these kind of loads – additional load. Then always we found that, there is a straight contribution is there, because of the algebraic sum of the load in the shear force diagram plus there is the… Since it is a UDL part was there; so we have the kind of smooth change is there in this kind of the bending moment, because at this particular point, straight reaction is there of these two loads. But, these because of the 2 kilo newton meter up to the 3 meter of length, always going up to the maximum value. And we have in this particular case, minus 23 kilo newton meter because of these additional components – 2 kilo newton meter – 2 kilo newton, 4 kilo newton and 2 kilo newton per meter of this kind of loading.

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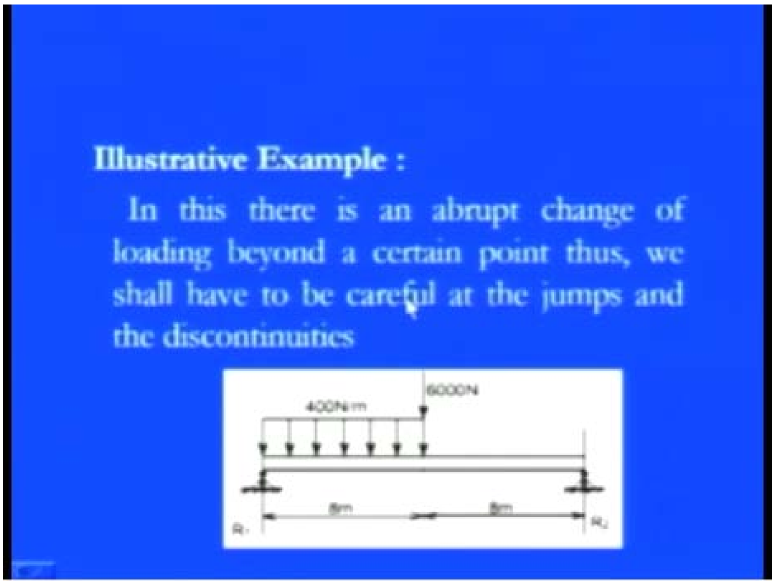

So, now, this was a typical case. We would like to discuss again one more case in which different sense of loading is there; that means again it is the combination of load. But, in that, this uniform distributed load is just distributed for a span of let us say some x meter, not for entire length of beam; plus there is addition of a point load is there. Then, what will happen? So, in this case, there is an abrupt change is there of a loading beyond a certain point. Thus, we will have to see carefully that, how these jump phenomena or we can say the discontinuities will come in the shear force as well as the bending moment diagram.

So, here the case is like that; we have a simply supported beam of total length is 16 meter. And then half of the length, we have a kind of loading of uniformly distributed load, which has a magnitude of 400 newton per meter. And at the extreme corner; that means at the middle portion of the beam, there is the additional point load or concentrated load is there of the magnitude of 6 kilo newton. And then we would like to just check that, what the kind of interaction is there in between this UDL, because at the extreme corner of the UDL, we have a point load. So, how this point load as well as the UDL will contribute in terms of shear force and the bending moment; our interest is that. So, for that, first again we would like to see that, how the reactions will come. So, R 1 and R 2 – if I am saying that, these two reactions are there; so what are the reactions through which we can say that, under the application of these 2 UDL and the point load, our beam is entirely in the equilibrium portion.

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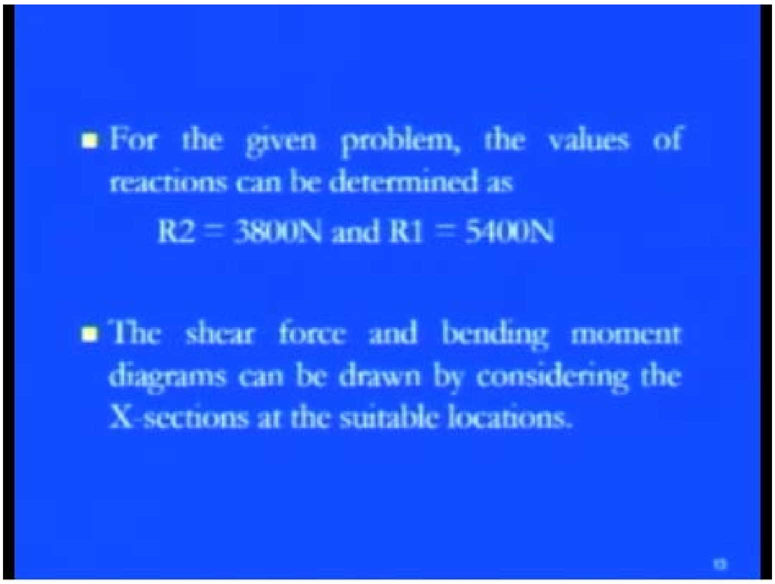

So, first, we would like to see that, what the values of these two are. So, first calculating those values by equating the forces plus the moment, we have R 2 is 3800 newton a R 1 is 5400 newton. So, once you have the values of R 1, R 2, you can easily find it out those things by getting two equation: one by force balance equation and one by moment balance equation, because the load is the UDL as well as the point loads are there; equally they will contribute in both of the term: force balance as well as the moment balance. So, by equating these two equations, you have the two unknowns. So, you can easily get those values, because the force and moment is known to you as per the distance and the applied force.

So, once you have this R 1, R 2, then you can easily get those values shear force and bending moment diagram, because of those whatever the reaction forces, which are acting towards the vertical upward direction. And the applied force, which is there due to UDL and the point load, they are acting on the downward direction. So, now, you need to be careful to choose that, how the shear force conventions are there and the bending moment conventions are.

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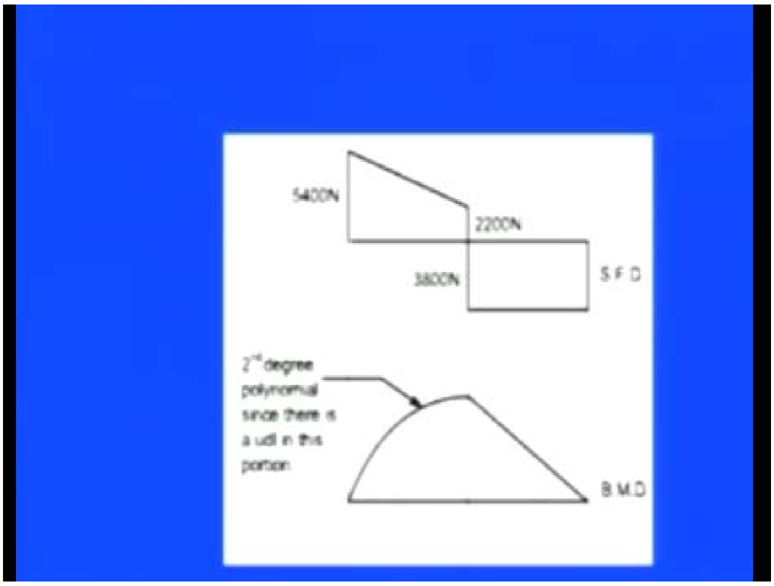

So, now, as we have R 1, which is 5400 – extremely at this particular point, where only UDL is there; there is no contribution of the point load. Only what we have; we have a UDL at this particular point – the starting point of this UDL. So, only the contribution will be… because this is force, is coming in the vertical direction; this force is coming in this downward direction. So, we have a positive part. And since only R 1 is contributing at the extreme end; so it is equals to F equals to w or we can say F equals to R 1, that is, 5400 newton. So, starting form that, then now, go up to 8 meter. So, up to 8 meter, we need to check it out that, what will be the contribution of the shear force if first of all the UDL is there.

So, if I am taking x; what is the component we have? This let us say w kilo newton per meter is there. So, in this case, we have the UDL. UDL plus we have the additional point force, that is, the 6 kilo newton. So, because of that, there is abrupt change is there at the extreme corner and since it is a point load is there at the abrupt change. So, because of this variation is coming, because of the UDL; UDL is only applying at this section. So, starting from 5400 newton to 2200 newton is coming due to this UDL. So, now, you can simply generate the equation by taking X-X section staring from x equals to 0 to x equals to 8 meter. So, in this section, it will be the contribution is there and then it will be the contribution is there from the UDL.

And then there is an abrupt change is there, because of the point load, which has 9 or we can say 6 kilo newton. So, here just 2200 to going up to below 3800. So, total is we can say 600 newton is there. So, it is coming due to the additional part. So, this slant is there because of UDL and the abrupt change is there because of the point load. And then there is no force contribution is there at the extreme end corner what we have; we have the R 2, which is equals to 3800. So, we have a perfect – we can say the rectangular is there right from this section. Right from you can say this section to this section, there is no changing of load or there is no additional form of load is there. So, we cannot say that, we cannot get anything an additional component. So, there is straight variation of these shear forces are. And this is F equals to minus reaction R 1 or we can say minus this 3800 of the total newton forces. So, this is all about our shear force diagram.

But, what about this bending moment? So, again, since what we have; we have the force contribution is there at the extreme end of the simply supported beam of this, is due to this UDL plus we have a point load, which is at the center point. So, again at these two junctions, the A and B, where R 1, R 2 reactions are there; they are always contributing in the bending moment is 0, because these are the junction point. And then now, we would like to see that, what the variations are there. UDL is therefore, one-half portion of this beam. So, it will contribute in terms of bending moment. So, since the UDL is there; so we have the second degree polynomial is there; that means we have the square term – parabolic term is there. So, we have this variation is there up to this point. And then there is a point load is there acting exactly at the mid portion of this beam.

So, now, there is no UDL contribution is there. So, we have a slant is there in the bending moment. So, you can say that, just you can visualize straightaway that, now, this is the contribution of UDL; this is the contribution of point load; so how they will play; what will be there since the point load is there. So, we have a linear relationship. If UDL is there, we have at non-linear relationship; parabolic term will come. So, you can visualize. And then by the numerical values, you can calculate what the total maximum bending moment is there or what the variation is there. So, we can get this kind of this bending moment diagram starting from this to the parabolic term. And then there is a slanting height is there. So, this is the bending moment diagram; this is the shear force diagram. And one can easily get those diagrams once they have a feeling of the interaction of the forces as well as the individual force application on this particular beam.

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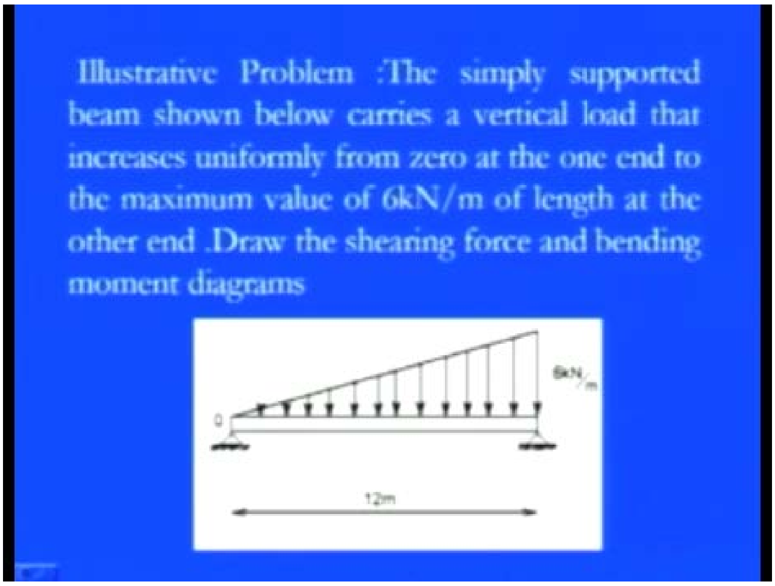

Now, we just move to the new case in which we have a simply supported beam that carries a vertical load, which uniformly increases. Till now, what we discussed that, UDL is there; but, it has a constant intensity at all the points. Let us say if 2 kilo newton per meter is there as we discussed. So, 2 kilo newton is acting at individual like the distances, not the variation. But, here now, we have a case in which the vertical load is there and it is uniformly increasing from 0 at one end to maximum at extreme point, that is, at the 12 meter, we can say – as we can see in this diagram. And that it has a value of 6 kilo newton per meter. So, now, we would like to find out that, how the shearing will occur and how the bending moment will react due to this particular kind of loading.

So, now, we have a simply supported beam, triangular loading is there. So, due to this triangle loading, what will happen; the reaction will be coming; the reaction will be acted. As per the visualization of this particular diagram, you can say that, at this particular junction, we have a maximum value of this reaction; at this particular junction, we have a minimum value, which we need to balance this particular beam under the application of this triangular loading. So, now, just keep in mind that, we have a 6 kilo newton per meter of the intensity and we have the total 12 meter of the beam length.

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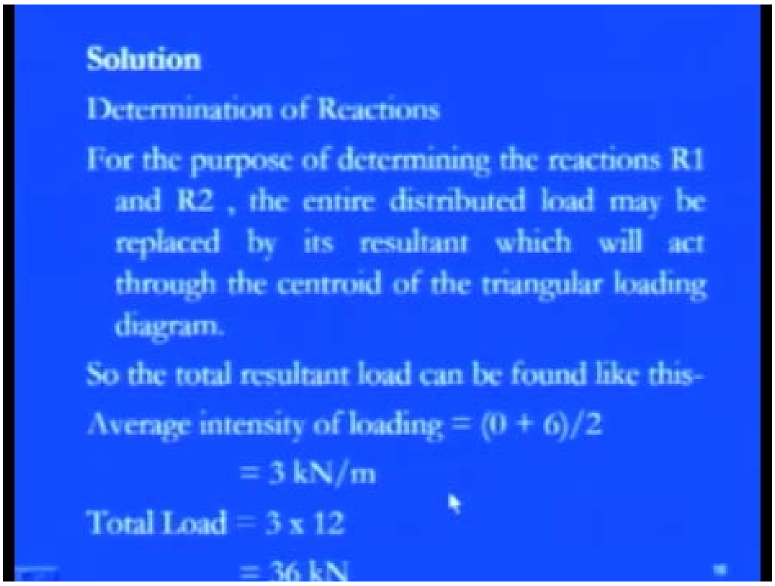

So, by considering those, first of all, same thing that we would like to find out the reaction. And we assume that, R 1 and R 2 – the 2 reactions are there at these two points. And then since it is a triangular loading, we need to find it out that, what will be the resultant is there of this triangular loading, which will be acting at a centroid of a triangular part, because it is a triangle. And always center of mass is at some centroid point is there. So, first of all, we need to locate that point and then we would like to find out the reactions of those things. So, first of all, the average intensity of this load is 0 at 1.6 at another point. So, 0 plus 6 by 2. So, we have 3 kilo newton per meter.

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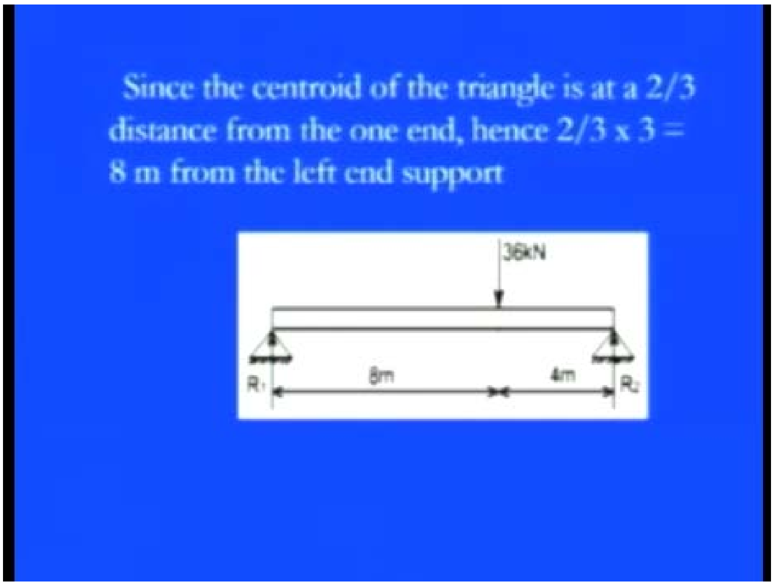

Now, the total load if you want to find it out, the average intensity is 3 kilo newton per meter. The total beam length is 12 meter. So, the total load is 3 into 12. So, we have a 36 kilo newton. So, now, we have a total load. If you just go for the average loading, then we have 36 kilo newton. And now, first of all, again since it is a triangular form is there0, always centroid is acting at two-third of the distance from one end. So, if you are taking two-third distance of one end; so we will find that, actually from one end, we have a 3 distance. So, we have this total distance is this one. So, 2 by 3 into you can say just put the total 12 meter. So, we have the total distance – 2 into 4, that is, the 8 meter from one end support.

So, if I am asking that, we have total 12 meter, one end is this. So, from here to here, the total 8 meter distance or from here to here we can say the 4 distance. And the total magnitude is 38 kilo newton – 36 kilo newton. So, now, you have a perfect visualization of this kind of 4. Again I am repeating those things that, if you have a triangular loading, first, since it is a triangular form; so at one point, it is 0; at one point, it is maximum. Go for average one. Once you have the average intensity of loading, multiply with the total load, where it is acting. So, now, you have a feeling of total load.

Now, since it is the triangular form is there – perfect triangle is there; so we know that, the centroid is always be there at two-third from one end or one-third from another end. So, you can calculate that, where the location of the centroid is. And once you have the location of the centroid is, you can simply have a feeling that, this is the magnitude and this is the location. So, now, in this case also, we have a reaction R 1 and R 2 at these two points. We have total magnitude 36 kilo Newton, which is acting at this particular point – 8 meter from this distance; 4 meter from this distance.

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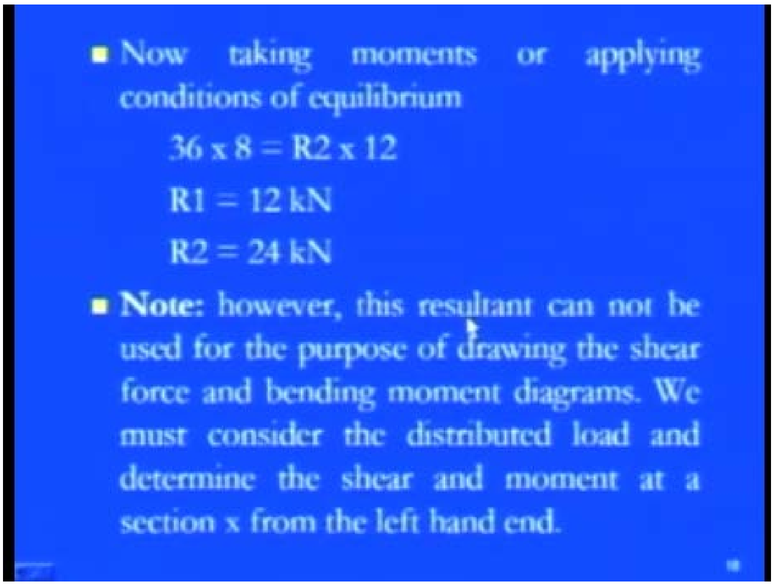

So, now, with that, we can easily calculate that remaining reactions. So, 36 into 8 from one end is equals to R 2 into 12 by taking moment at this particular junction R 1. So, we have this R 1, which is 12 kilo newton; and R 2, which is 34 kilo newton. There is one note. However, the resultant cannot be used for the purpose of drawing shear force and bending moment diagram.

We must consider the distribution of load and determine the shear force and the moment at a section X-X from the left end because… Why this thing is there? There is a variation and there is we are assuming that, though there is the variation is uniform, but in actual way, always it is whatever the displacements are there of the micro structure within that particular… Or, we can say the shear distribution is there; it is not exactly that in an average way. So, we need to check it out that, where the minimum loading is there or where the maximum loading is there, how this deformation or how the shear forces are playing their key role and how the bending moment will come. And then only, we can go for the actual – this design of the beam. So, for that; for the sake of the safety, always we would like to consider… Just for the reactions are okay for average, but, in real moment, we need to see that, how the variations are there in that way.

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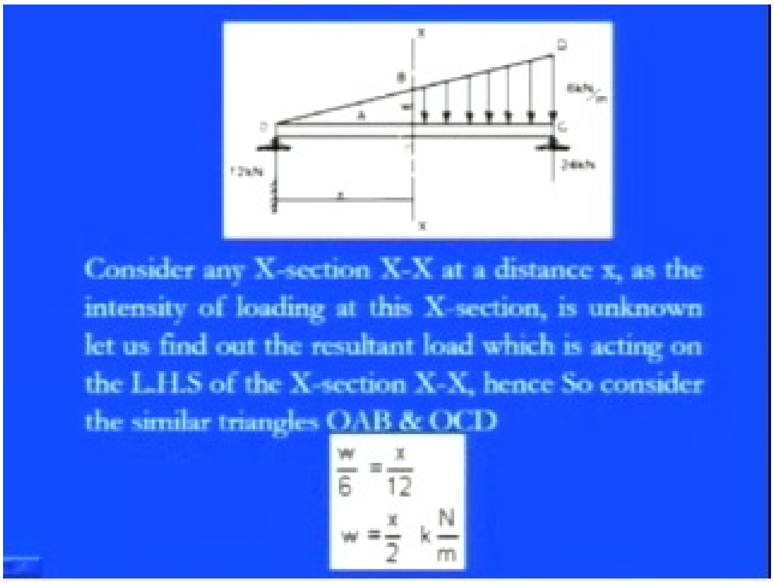

So, now, again we are going with the same processor that, cut the X-X section at one point. So, this is the X-X section. Starting from this point, these are R 1, R 2, which is 12 and 24 kilo newton is there. And this is the total intensity – 6 kilo newton per meter is there. So, consider the section X at particular distance x is there at X-X. The intensity of loading, which we would like to find it out with 6 kilo newton into x is there. And then what we need to do here; since there is a variation; so we cannot take directly as 6. So, what we can do; we need to find it out that, at particular distance x, what will be the intensity? Because it is uniformly varying. So, if it is a UDL, then you can say intensity is constant, but it is uniformly varying. So, first the main intention is to find it out those things. So, it can be easily find it out by taking the two symmetric triangles.

So, here we have a triangle from A, B and this portion; and we have a triangle of ADC. So, by taking, what we have the same angle is there; this height and this height as at this angle and this angle is same; this angle and this angle is same. So, it is we can say that, there is a symmetrical two triangles are there. So, by the symmetric, we can say that, w by 6; that means whatever the intensity of w is there; which we want to find it out at this particular junction or we can say at B, which is w. So, w by 6 is equals to x – the total distance by 12 – this is total. So, we have this w, which is at x distance is equals to x by 2 into the total part. So, we can say since it is x and it is going that; so it is x by 2 kilo newton per meter. So, now, we have at x distance, we have the intensity of load is x by 2.

And from that we can go ahead that, now, at distance of this, x is there; then what is the feasible loading as well as the kind of these intensities are there.

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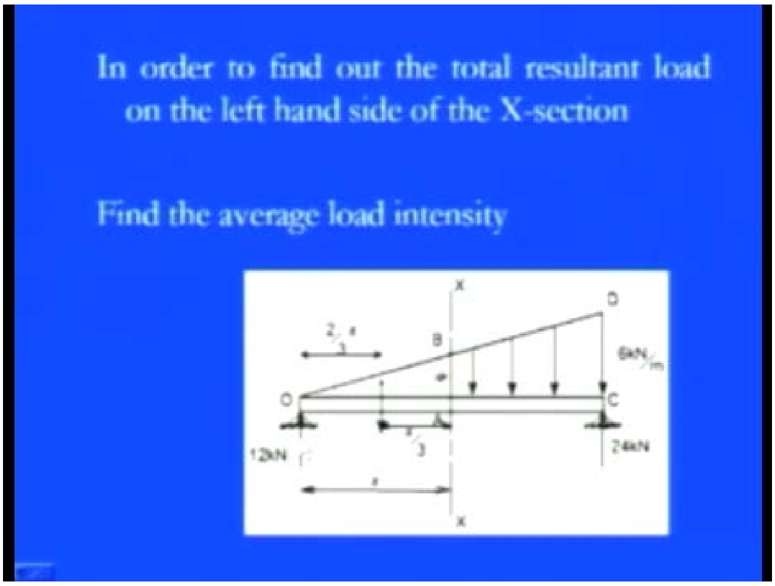

So, in this figure, you can easily visualize that, kind of part that, at this point, since it is w; w is equals to x by 2 is loading and it is locating at this particular point. So, two-third of x is there or we can say one-third of x is there. So, we can find it out that, how shearing will be occur at this due to this particular diagram. And then we will extend beyond this particular reason.

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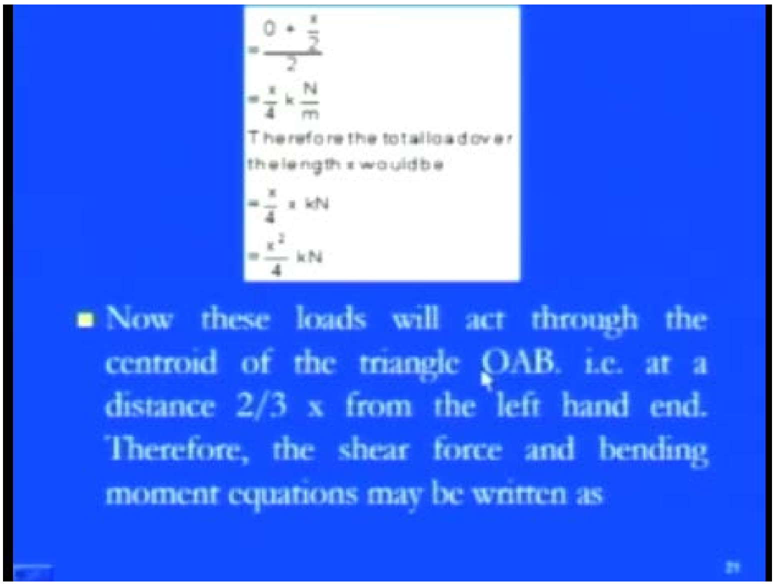

So, by taking that point, what we have? The average load is nothing but equals to 0 at one point – x by 2; which is at the distance of x is there. So, total is x by 4 kilo newton, that is, the average load. So, now, once you have the average load, you can easily find it out that, what will be the shearing part is there in that. So, if we want to find it out those things, then the total length is nothing but equals to the length or which this uniformly varying load is there. So, we have x by 4 into x or we can say x square by 4 kilo newton. It is acting… The total load is there acting at two-third distance from left-hand corner.

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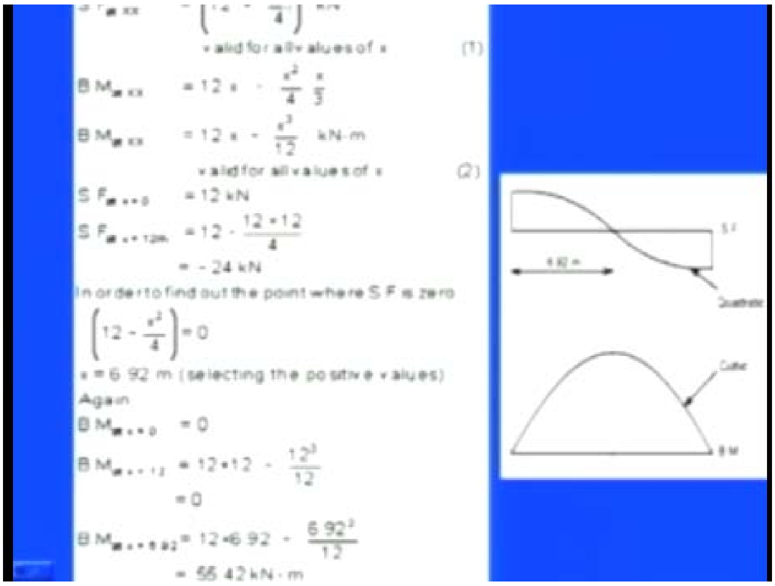

So, now, starting from those regions, we have in the final way that, the shear forces are 12 minus x square by 4 kilo newton. So, now, you can vary those things with the value of x, you can get. And bending moment is there – into 12 into x minus x square by 4 of a of x by third, because it is acting at one-third of this distance. So, now, you have the total 12x minus x cube by 12. So, now, we have a cubic term. So, square term is there in a shear force; cubic term is there in a bending moment. So, by keeping those values, you can get the shear force as well as the bending moment diagram. So, you have a parabolic term right from this way; you have a cubic term in this way. So, we can draw the bending moment as well as the shear force diagram of this nature, which has the values – bending moment at 0-0 at these two points; and the maximum is there at the 55.42 kilo newton meter.

Similarly, shear force is there – 12 kilo newton at x equals to 0; that means starting point; and minus 24 is there at this particular shear force. And both are in the nature of the quadratic term because of the square term. So, here in this section, we discussed about the interaction of the forces that, how these… If we have the couple is there or with this UDL point loads are there; and if the eccentricity is there; or, if the angle is acting at an angle – these forces acting at an angle; then how these resultant forces are coming out from these and what will be the direct impact is there on the shear force and as well as what will be the direct impact is there on bending moment.

So, now, in the next lecture, again we would like to see the different kind of loading that, if we have in that; if the additional torque is there; that means we have this UDL plus additional torque is there; then how they will react on a beam. And how we can say that, the maximum bending will be occur at a particular this-this locations. So, our main aim is to calculate that, what are the locations, where we have the maximum shear force or minimum shear force or whatever the location, where we have a maximum bending moment, so that whenever we design a beam, the design considerations are there based on whatever the analysis is.

So, thank you.