Strength of Materials
Prof. Dr. Suraj Prakash Harsha
Mechanical and Industrial Engineering Department
Indian Institute of Technology, Roorkee

Lecture - 24

Hi, this is Dr. S.P. Harsha from Mechanical and Industrial Department, IIT Roorkee, I am going to deliver lecture 24 on the course of the Strength of Materials and this course is developed under the National Program on Technology Enhanced Learning. Prior to start this lecture, because you see in this lecture we are going to discuss about, how to draw the shear force and bending moment diagram.

We just want to discuss about the previous lecture to refresh your ideas, that you know in the in the previous lecture we discussed that, if we have a beam and if you know like beam is under various kind of loadings. Then how you know like we can relate the shear force as well as the bending moment, because you see you know like, we found that if a beam is there irrespective of what kind of loadings are there irrespective of whatever the supports are there.

Then how these bending moment variation is there or how the shear force is there and that also you would you know like we discussed about that what exact correlations are there. So, in that the previous part of the previous lecture, we discussed about that, if we have beam and if it is loaded by point loading or if it is the cantilever or a simply supported beam is there.

Then how these force balance is there and because of the force balance you see we simply you know like the cut the section in between and you know like, we separate it out these we mean to two different sections. And these two sections you see we analyze that, what the resultant force is there and if the resultant force is acting irrespective of what is a downward or upward.

Then, what is a relation is there in between this portion and you know like the other portion, then we found that there is you know like the resultant of these applied forces means you know like, when we applied the force from upper side. And, there is a reaction forces are there from the down ward side, resultant of these things comes out with the shear force. So, we decided that actually when the shear force is there then how we can you know like confine into a sign convention.

So, we found that you know like if the shear force on the left potion of the beam is acted like; that means, on the tough side like this and bottom side like that, then you see like a we need to take the positive shear force and on the right side. If the shear forces are like that you see, if it is going on top side and if it is on the bottom side, then it is of the negative shear force.

So, you see just keep this thing in your mind that actually on the left side, if your left portion is going upward and the right portion is going downward, you need to take the positive shear force. And, on the right hand side if your right portion is going upward and left portion is going downward, then we need to take the negative shear force.

Similarly you see, we discussed about the sign convention of the bending mound because you see you know like the resultant of the applied forces also is going towards the bending moment. Because, whenever the shears force is there the bending is also started to occur and again you see you know like a kind of virtual bending moment is there with in that particular structure.

So, if you are saying that if this beam is under you know like the balance condition or the equilibrium condition, then we need to analyze that how you know like these variation is there and what is the resultant is there on that. So, for that what we need to do we need to first of all find it out, what the sign convention is there, which is correlating to the shear forces. Because, you see both are interacting to each other and both are simultaneously applied to the concerned beam.

So, you see actually, if you have we know like the beam and again like, if you cut the section by dotted line, if you remember that part, then we found that, if due to the you know like these applied forces on the left portion. If the resultant of you know like, these force, if resultant of these forces going towards upward direction; that means, you see if we have a tendency of beam due to these applied forces is going upward direction ; that means, you seen the clockwise direction.

And, you see if you go to you know like the other part of that that was the right part, The resultant of those forces tending bend go towards the upward direction; that means, this direction is there the bending that is known as a sagging portion. If this is sagging is there, we are always taking the positive. And, you know like the bending moment and if it is a hogging; that means, you say if it is going downward from both left as well as the right hand portion; that means, you see the counter clockwise and the clockwise direction.

If it is like that, then it is known as the hogging, you know like the portion or we always take the negative bending moment. So, you see these are the two main sign conventions are there for considering shear forces as well as the bending moment. Then, you see we discussed that actually, if we have a simply supported beam or if you have you know like cantilever or if we have you see you know like on that particular, these both of the portions.

If we have a point load or UDL, if UDL is there; that means, uniformly distributed load is there, then since the uniformly distributed load is always acted just Newton per millimeter or kilo Newton per millimeters. So, what we need to do here we just first of all would like to see that what is the centre of mass is so first to locate that part and multiply W into d x, whatever the d x length is there. Then, you see you have a concentrated load, then the total account, the concentrated load will be coming into the movement.

So, we see we discuss that actually what exactly the relations are there in between the shear force bending moment and the applied load. Then, we found that the shear force is nothing but equals to the partial derivative or exact derivative of d M divided by d x; that means, you see whatever the resultant shear force will come, where ever the change of bending moment is there with the domain.

So, since we are taking our beam in the x location or we can say the longitudinal axis, so our variation in a domain in x direction only. So, we can say that the shear force having a relation with the bending moment in that form shear force F is equal to d M by d x. And, simultaneously we can also correlate the shear force as with this point of loading, so we found that the loading point W, which you see kilo Newton per millimeter is there or we can say kilo Newton meter is there.

We know, we can simply set up the relationship in between that, the W is equals to d f by d x or we can say that W is equals to the second derivative of d 2 M by d x square, the meaning is pretty simple. Now, if we have a beam which is under loading of universally uniformly distributed load the relation is pretty simple, that the load which is applied it cause the shear force in a domain of the x in a first derivative form, that is d f by d x or we can say that it cause the bending moment in the second derivative that is d 2 by d x square, if we have you know like the variation in the x axis.

So, you see the relation is very, very important, now if I am saying that you see with that relation what we have now, two more relations were there, which is very important, which we are going to discussing in our lecture, that first F is equals to d m by d x. So, if I am saying that we have the point where there is no shear force, because you see we are saying that this beam is under balanced condition; that means, if the variation of the shear force is there.

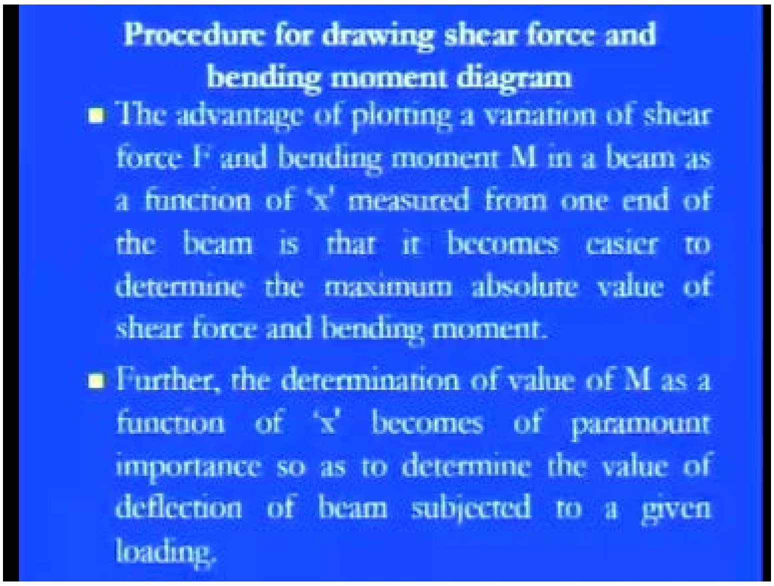
That, there is a certain point, where there is no shearing force; that means, you see d m by d x is equals to 0, that means where you see the shear force is 0, we have a constant bending moment, because you see d M by d x 0; that means, the movement is constant. So, keep this thing in our mind that, if you want to calculate you see any portion; that means, you see in the beam, that where is the maximum bending moment, where is the minimum bending moment, where is the maximum shear force, where is the minimum shear force.

We can easily correlate, you see with the using of these relations and that you see you know like in the coming lecture we are going to discuss all these issues there. So, first we would like to draw the bending moment and the shear force diagram, just to have a feeling, that you see where a beam is there. You know like irrespective, whether cantilever beam or simply supported beam and you see either the points loads are there or universally this hum uniformly distributed load is there.

Under, these applications of load what kind of you know like the shear forces are coming, what is the nature of those things are there, what is the magnitude is there. And simultaneously, what you see you know like the bending moment variation is there and what the magnitudes are there. So, first of all would like to have this kind of feeling for that the fore, most application is that, we need to draw the shear force as well as the bending moment diagram.

And through, that you see we can easily know that actually how these variation is there and we can find out the nature of the this shear force as well as the bending moment diagram, under that application of this force. So, here in this lecture 24, you see we would like to discuss first, how you know like we can draw the shear force and bending moment diagram.

(Refer Slide Time: 08:51)



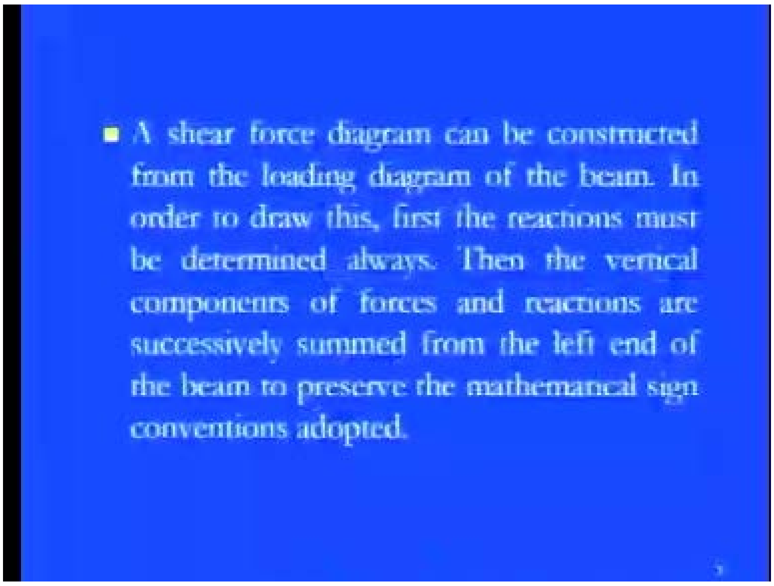
So, here you see on your screen we find that we have a procedure for drawing shear force and bending moment diagram to have the feeling. So, the advantage of the plotting variation of shear force F, which we are using F generally and bending moment to M in a beam as a function of x, because we found that this F is nothing but equals to d m by d x or W is nothing but equals to d f by d x or d 2 M by d x square, that mean, the variation and shear force as well as variation of this bending moment is absolutely varying with respect to the x.

So, they are the function of x, because they are varying them not having the constant value the x. So, if I am saying that, if I just want to plot the shear force F or bending moment M in a beam as a function of x, which is measure from one hand of a beam is that. It becomes easier to determine maximum absolute value of shear force as well as the bending moment. Because, you see you know like if that is what I told that actually we just want to have a feeling, that how these shear force variation is there and how this bending moment variation is there.

So, you know like we can easily find out the actually, if this kind of loads are there, where is the bending the maximum bending moment is there, where is the you know like maximum shear forces are there. So, you see only you know like have a feeling, if you know we can draw the shear force as well as the bending moment diagram, for that determination of value of M as a function of x becomes a paramount importance.

So, as to determine the value of deflection of a beam subjected to a given loading, which is very, very important, because if somebody wants to design the beam, you should know that actually, where is the maximum deflection, where is the minimum deflection. So, that you know like, we can simply put the factor of safety against that kind of loading, so that, whatever the displacement or whatever the deflection is there, it can be minimized or else we can go for the material property.

(Refer Slide Time: 10:40)



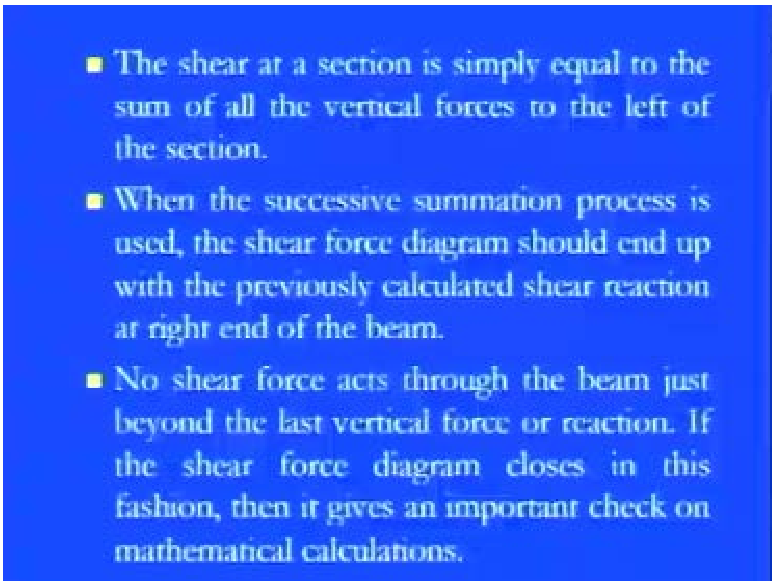
Now, you see comes to the shear force diagram first, a shear force diagram can be constructed from the loading diagram of a beam, you see here because, how these loading are there based on the loadings. We can easily find out that what the algebraic sums are there of a applied force; that means, because we know that, whatever the forces are coming, they are coming in terms of point load irrespective or with the UDL.

So, how you know the resultant of these magnitudes of these forces is there, we just want to find it out by using this algebraic sum. So, for that what we need, we just first the reaction must be determine, always because you see, we have simply supported beam or we have a cantilever in which you know like, this one rigid end is there. Whenever, we applied the load, always there is a reaction is coming from the support end or where ever the supports are there either in respective of pin joint also.

So first of all, when we know the amount, whatever the amount, which we are you know like putting on the beam, first we would like to see, if the beam is in the statically, you know like in determinant form, then what the reaction forces are. So first of all, we would like to see, that what reactions are there and how these reactions are there and how these reactions are varying.

Then, you see the vertical components of the forces and reactions are successfully summed from the left hand of a beam to preserve the mathematical sign conventions as adopted. Because, we know that since we are cutting the section of a beam in right as well in left as well as the right end. So, we would like to see that, how these shear force variation is there and once we know whether it is a positive sign and negative sign. Then, we can easily sum up those forces with the corresponding sign convention and then, we can draw these shear force diagram with the extended sign conventions. So, you see here after getting the real feeling of the algebraic sum of the forces, we can easily put those shear forces on a beam.

(Refer Slide Time: 12:33)



The shear at a section is simply equal to the sum of all vertical forces to the left of the section that is, what we discussed. When, the successive summation process is used, the shear force diagram should end up with the previously calculated shear reaction at right end of the beam; that means once you started here force calculation from the left end. Whatever, you can even start from the right end, but it is pretty easy for us to analyze a beam, under the action of forces.

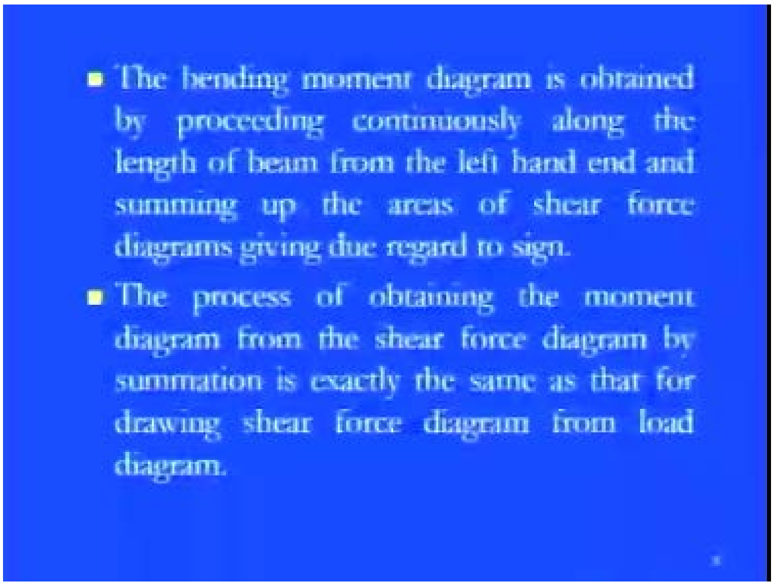
If, you starting from the left, so that you see since it is a function of x, so we can easily find it out, the resultant of these forces. If, it is started from a extreme end or if the middle force is there or it is pretty close to this, the left end or even the reactions, whatever the reaction which are coming towards the vertical directions. We can easily find it out that actually how these forces like these the algebraic sums are there.

So, better to start from the left end and then find it out that what exactly this particular shear force variation is there toward the right hand direction. So, you see we can easily calculate the shearing reactions at the right end of the beam also where have the supports are there. No shear force act through the beam, just beyond the last vertical force or reactions that is a very, very important thing.

Because, when we have a support and whatever the force application is there, if you see like this beam is there, here under end you see this reaction is there and what are the vertical forces are there. If there collinear or coinciding, then there is no resultant is there, there is no shearing is there, if there is no eccentricity and if it is going beyond the support reaction or if there is no vertical forces are there at the particular end.

Then, we cannot go to then we cannot consider the shear force beyond this region, if the shear force diagram, close in this fashion. Then, it gives importance, just important task actually on a mathematical calculation that. If, you go beyond those things or we can say if the extension is there, then how this variation is there, because in that you will find the bending moment in comparison with the shear forces.

(Refer Slide Time: 14:39)



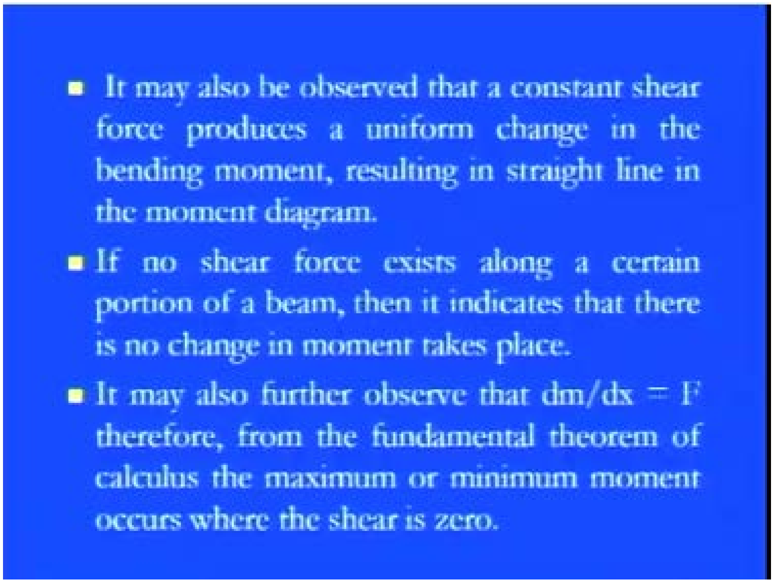
So, bending moment now is real good importance of that, so for drawing the bending moment diagram is always obtained by a processing continuously along the length of beam. Because, M is a function of x from left hand end and summing up all these areas of the shear force diagram, giving due the regard of to the sign. That means, we know that the relation between the shear force and bending moment is what is F equals to d n by d x or we can say M is nothing but equals to integration of F into d x.

So, whatever the area which is covered by the shear force diagram, will give you the bending moment, whatever the area is there, because M is the integration of F into d x. So, here you see from that the formula, we can easily calculate the movement, if you know the area under which this shear force diagram is coming and only the problem is that actually what the sign conventions are there. So, whatever the concern sign conventions, we can put simply on the bending moment also.

So, the process of obtaining the movement diagram from the shear force diagram by summing up is exactly the same as that for drawing shear force diagram from the load diagram. Because, so we discuss the W is equals to d f by d x, so F is nothing but equals to integration of W into d x. So, you see whatever the load diagram is coming, if you simply draw the load diagram and whatever the area which is under this load diagram will give you the shear force.

So, these are all analogous is there only the quantitative as well as the qualitative changes are there, just like you see if we want to find it out the shear force. Then, what you need to do you need to draw the load diagram, just check it out the area under that particular diagram will give the shear force. Similarly, you see if you want to find it out this bending moment, what you need to simply draw the shear force, take the area under that particular curve and find out the bending moment. So, these but the important thing is that actually, what the direction of these shear, the forces are or what the direction of shear forces are there and that corresponding the shear force diagram and the bending moment diagram can be easily drawn.

(Refer Slide Time: 16:50)



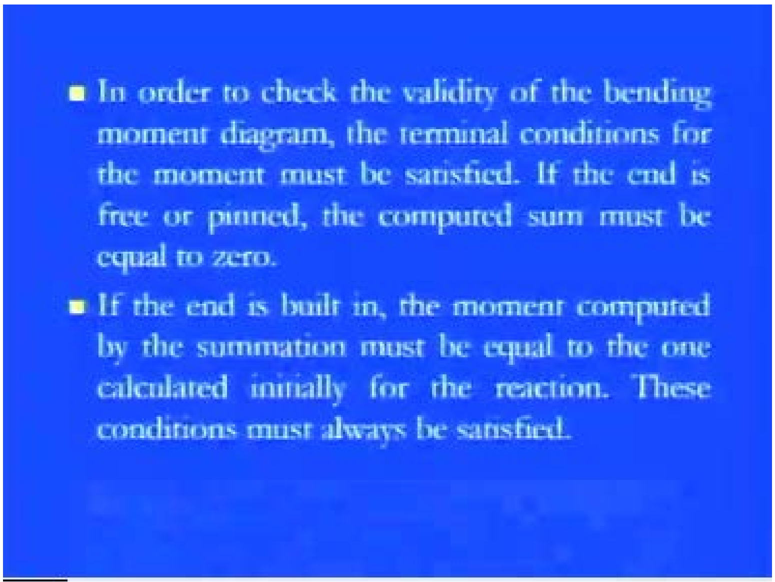
So, you see here it may be also observed that at a constant shear force produce a uniform change in a bending moment resulting in a straight line, in a bending moment diagram. Because, you see if I am saying that the bending moment is constant; that means, d m by d x is also constant, so obviously see, if am saying that there is a constant change is there in the bending moment with respect to x, so there is straight line is coming in the moment diagram.

If, we have a constant shear force, if there is no shear force is exists; that means, F is 0, a certain force on of a beam, that it indicates that, there is no change in the movement is there. Simply, you will find the d m by d x is 0; that means, you see there is no shear force is there, so it has least contribution to induce the bending inside the being portion. So, it may also further observe that the d m by d x which is F as I told you from the fundamental theorem of the calculus.

We can easily get the value of maximum or minimum movement, where the shear force is 0, because you see generally, if you want to find, let us say, if M is a function of x and if you want to find it out its maximum or minimum value. Then, what we are doing here, we are simply deriving the first derivative of that function putting equals to 0 point, the location of that, where it is maximum or minimum. And, then you see simply put those values in the main function and get the maximum or minimum value of the the function is.

So, here you see we are dealing with these kind of lesson with the bending moment d m by d x 0. So, we can get the location where the maximum or minimum bending moment is there and once you put that value in the main function, you can get the real values. That actually where at what location, you can get the maximum or minimum bending moment and what is the value of bending moment is concerned.

(Refer Slide Time: 18:39)



So, in order to check the validity of the bending moment diagram, main thing is that, the terminal conditions for the movement must be satisfied. Because, if it is not satisfied, then there is eccentricity is there in the beam or we can say that, whatever the bending moment, which is coming due to the application of load, will tend the beam in particular rotation, beam is not in the equilibrium form or there is a rotation is there in a beam.

So, we cannot say that actually, whatever the beam or the movement which is considerable under the action of these forces are real satisfied or it is good; that means, there is a faulty part is there in the consideration of the bending moment. And, if the end is force or a pinned means the reaction forces the component total, whatever the components are there.

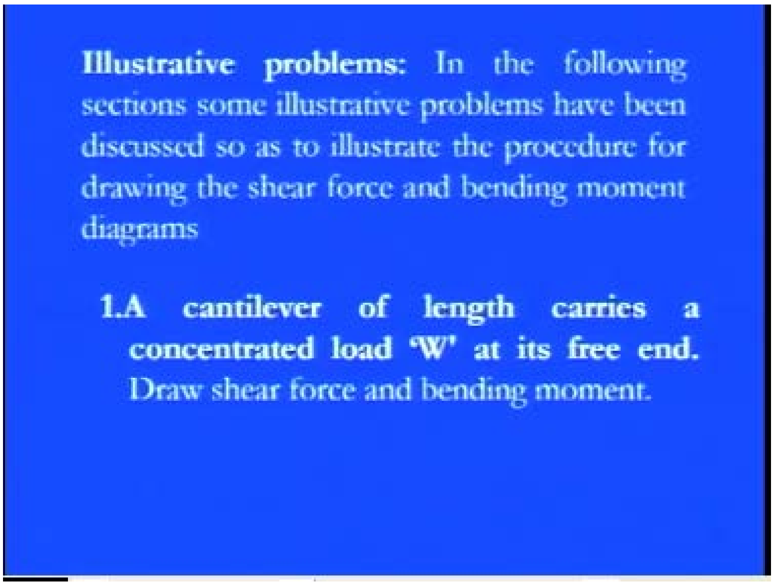
Under these force, these application of these forces computed some must be equal to 0; that means, whatever the forces are coming in as I told in the individual section. **T**hey, must be well balanced under the application of these forces and we can say that, this pinned force or whatever this line force is they are simply inducing in bending moment in the equating of a 0 part.

If the end is built in whatever this end, they are in within built in portion is there, the bending computed by the summation must be equal to the one calculated, initially for the reaction, because there are in built system. So, when we are talking about the movement, which is coming due the eccentricity, they are always be calculated by the initial, whatever the initial reactions are there from the system concerned, when you apply the load and these conditions must always be satisfied.

If, we are saying that the bending moment is there or shear force are there in a beam and they are the application of these load conditions. We need to satisfied these condition and then, only we can say that, whatever the condition which we observed or whatever the sign convention which we took to analyze these the shear force as well as the bending moment. There are really, they are correct and whatever the values are coming they are perfectly in a right direction.

Now, you see whatever we discussed the in these to calculate the shear force and bending moment and also, we discussed about that actually, how we can what which conditions have to be satisfied to check it out. Whether, we are drawing the shear force diagram or bending moment diagram, perfectly or not, what will be the condition, when we have the shear force is 0 or we have the shear force is you know like the constant, then how these bending moment will vary. So, all the discussion, we would like to see in our real, these numerical problems, so first of all we are taking some of the problems to check it out those phenomena’s.

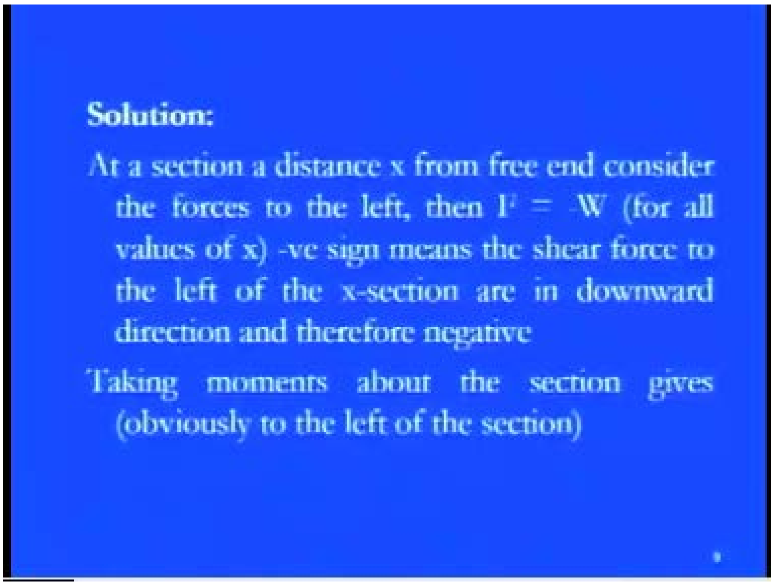
(Refer Slide Time: 21:08)



So, in the following sections some is illustrative problems have been discussed as to illustrate the procedure for drawing shear force and bending moment diagram. And, only we are just changing the domain to have a feeling that actually, if the different forces are there and different domains are there. They are then, how we can show the shear force as well as this bending moment diagram for those things.

So, first of all cantilever, we have cantilever; that means, when we are saying that a cantilever; that means, we have a beam one end is rigidly fixed up and one end is free. So, if you want to apply the load, only we have a free end, where one free end, where we can applied load irrespective, load bending moment, whatever like that, if you have starting from a cantilever of a length carries, the concentrated load W; that means, only we have a point load at a free end. So, it is pretty simple condition. this end you know this is fix and the load is here at the extreme corner. So, this load will tend to move this beam toward this direction this rigid and tend to put anti movement to balance this condition. So, due to that, we have shear force, we have bending moment, how to draw those things, now the process is like that.

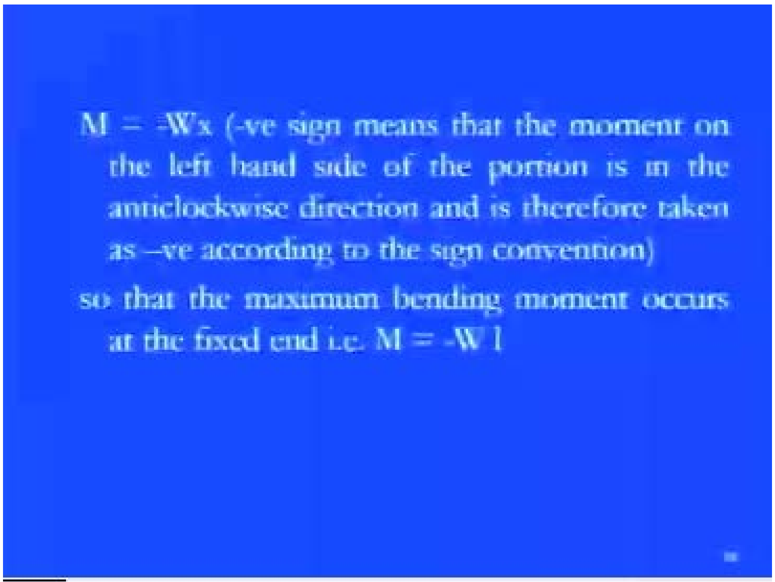
(Refer Slide Time: 22:25)



First of all at a section, just take a section at distance x from free and consider the forces to the left, so always go from that portion as we discuss starting from left end portion. Draw, you know like just that this distance x, that we have d x section from this left hand section x. And, then you see we know that since the forces are going towards this direction; that means, what we have the shear force are like that, so F equals to minus W for all values of that segment x.

Negative sign means, the shear force to the left of this cross section are in downward direction. And therefore, it is negative as I told you like a if it is in the this way or if it is in the this way and then the how sign conventions are. So, here you see you know like the on the left hand side, how what we have the shear forces towards the downward direction and you see we have a negative F is there. And, now taking the movement about this cross section, which we have discussed what we have now this value F is nothing but equals to minus W and corresponding the bending moment will come.

(Refer Slide Time: 23:28)



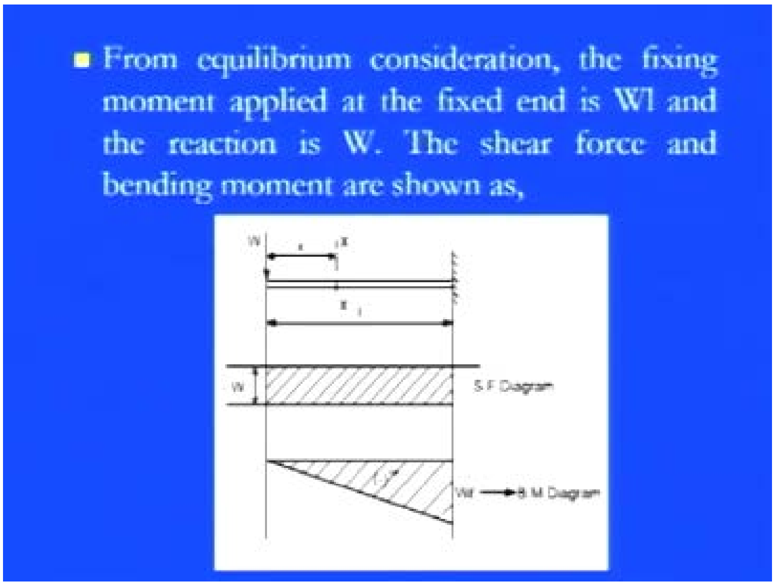
So, bending moment is nothing equals to minus W into x minus sign again, because you see it is just going to move this particular bend in this hogging portion. So; that means, we have negative part in that, so negative sign means the movement on a left hand side of the portion is in anticlockwise direction. And therefore, you see you know like taken a negative according to the sign convention or you can straight away go that, if it is in this hogging negative sagging positive.

So, this is pretty common you know like based on your I Q, you can simply concerned those sign conventions for this shear force plus minus or this sagging, hogging for bending moment plus minus sign. So, that the bending moment, now occurs at fix end that is M equals to minus W into L, because you see you know like we are taking this L is the length. So; obviously, you know like if you are going for entire portion of beam, the total length of L is because again, this is a very common phenomena.

If, you want to analyze anything as we discussed we better start from small cross section take that at x axis, we have this particular cross action and then the take the movement are the forces. And, then sum up all those forces for a small sections, you can easily find it out the resultant shear force as well as resultant bending moment. So, now you see here as I told, we have this prismatic bar; that means, whatever the beam is there, that is uniform cross section.

This point load is there, there is no loading variation is there, only we are putting the load at the extreme corner of this end. We have the resist support, so we can simply consider with the symmetric portion or symmetric part of this particular beam, we can say that M which is equals to minus W x, which also equals to minus W l. So, this now you have the bending moment minus W into l force, which is minus W. So, corresponding you see with those values, you can simply calculate, this total shear force as well as the bending moment and you can draw. Because, now you have a clear feeling with the sign convention, that actually how the variation the shear force as well as the bending moment is.

(Refer Slide Time: 25:30)



So, from the equilibrium consideration the fixed movement applied to this fixed end, because you see we have cantilever. So, we have a fixed end, which is equals to whatever the bending moment is there, W into l and reaction which is due to the shear force is W. So, come to that point, what we have this cantilever, so the cantilever is this one, this is the rigid end, this is you know like the free end and as I told you know like we have a point load at the extreme corner of the free end.

So, this is extreme corner of the free end, the load W is there, which is the concentrated load and now you see it simply cut the portion at x distance from the left end. So, now we cut the section x 2, x 1 and we have you see this distance is x, so you see the total length, if I am saying that this x. So, the total length is l, so you see for this particular portion.

Now, we have seen that the shear force minus W, so since it is minus W and it is consistent, there is no force is there in entire length of the beam except these portions. So, this force must be the; that means, this force must be same; that means, there is no contribution is there from any of the force component. So, if I am saying that, this is minus W, so it has to be constant and this is you know starting from this origin, where this is you see the W, the F equals to minus W is there.

From that end to that end, this here force will be occurring on the beam and it has a constant value minus W and it is summing up to the extreme end of the beam. Up to the right hand, I should say, so this is my shear force diagram, so it is pretty rectangular form nothing is there and the rectangular form of this particular, this height is minus W. Because of the applied load and it is consistent up to that portion, so we have shear force diagram. So, once we have shear a force diagram, then it is pretty easy for us to calculate the bending moment also.

So, the shear force is nothing but it is F equals to minus W irrespective, whether starting from the cross section x or from the entire beam length. So, now If, we see on the same beam and this you know like due to this applied load, it we have bending moments. It will vary from the origin of this load and it will go up to the maximum, where the fixed corner is so because, this M is nothing but equals to minus W into x.

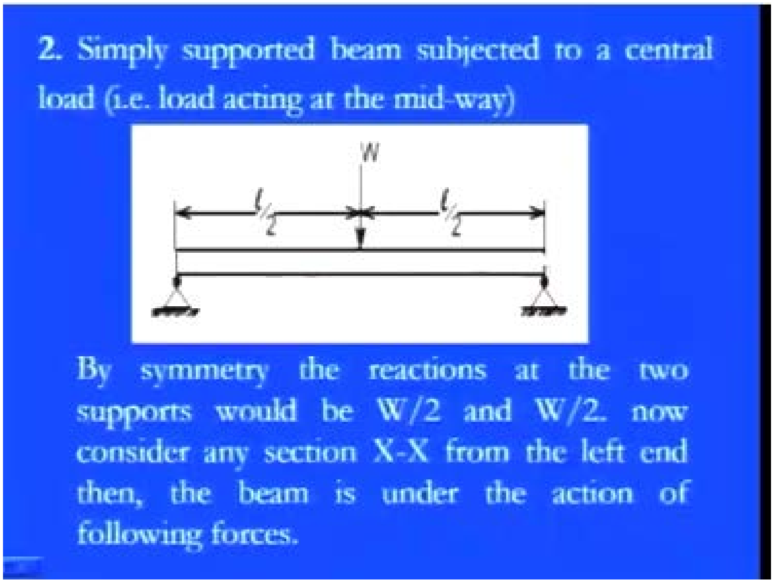
So, if you keep the value of x equals to 0, probably we have M equals to 0, so at this particular point, where this origin is there, this bending moment is 0. Now, if you keep this x's values 1, 2, 3, if you keep on increasing those things the bending moment will increase, it will increase up to the extreme corners. So, if we keeping x equals to l, what we have M equals to W l, since there in the hogging way, so it is in the negative way.

So, if we see on this particular diagram, will found that, it is variation is in this, in triangular way starting from 0 to maximum of this corner and the maximum value is minus W into l. So, will find that this variation is pretty triangular way or there is a linear variation is there, as you keep on moving in the variation of x. So, you see here, it is pretty simple from this cantilever beam.

If, we have W which is a single loading condition, if we have a single loading condition, then the contribution in the shear force as well as bending moment is from only that load. So, this is the applied load, the shearing load is coming from at the reaction part, so from these two part, what we have simple shear force diagram rectangular part and the triangular part bending moment diagram is and the maximum value of the shear force is minus W.

The maximum value of the bending moment is minus W into l, the minimum value of the bending moment is 0. So, like that you see we can easily calculate all those things for a cantilever. Now, the next case, if we are thinking about that if we have a simply supported beam, which is again subjected to a point load or we can say the concentrated load and the location of this concentrated load is exactly at a midpoint of the beam. So, now we see what we have this beam is supported as these two extreme corners and the point load is there, the central position just see on this particular diagram.

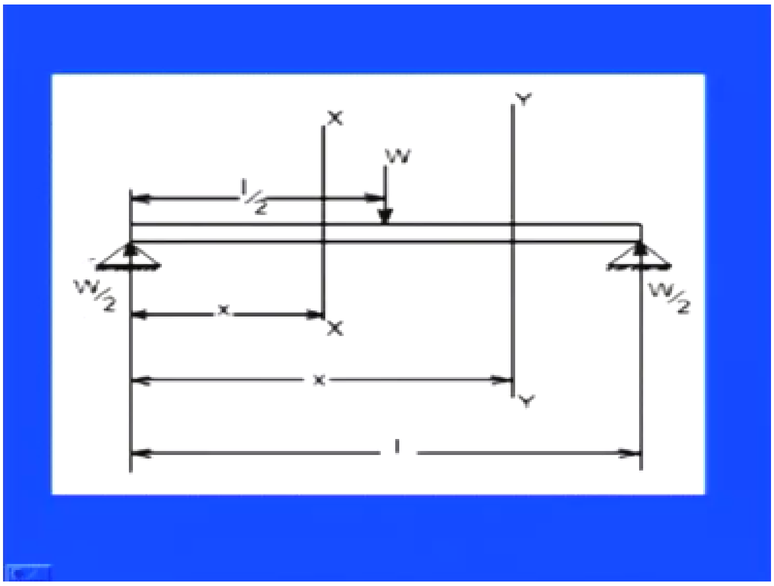
(Refer Slide Time: 29:40)



This is a simple, the uniform cross sectional beam, these two supports are there and when we apply the point load at the mid section, you see l by 2 in l by 2; that means, you see we have the total cross section at the total length of beam is l. So, by the symmetricity is there, because the point load is at the centre point, the reactions are coming at extreme corner the length is l by 2, l by 2.

So, from the symmetricity, we have a similar reactions on the vertical top, this upward direction and then the magnitude of this reaction forces are also same, equal and the same direction that is W by 2, W by two. Now, if we consider any cross section at any of from left end the portion, then the beam is under the action of these forces, we have symmetric reactions are there from both of the end. And then, corresponding we can find it out what will be the shear force and the bending moment is.

(Refer Slide Time: 30:38)

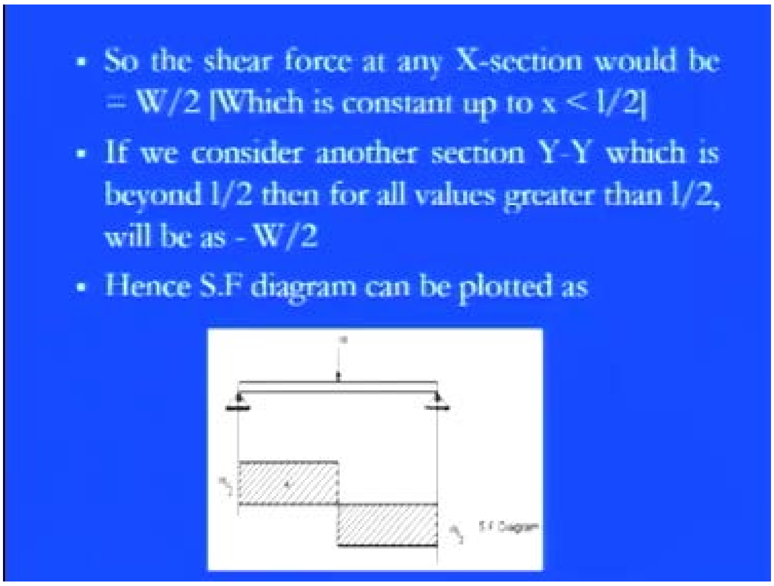


So, see now considering the same part, you see we have this, l length same cross sectional beam, these reactions which have the same magnitude W by 2, W by 2 and acting towards the upward direction. And, you see this point load is there, which is just acting downwards and because of that we have the shearing action in this portion as well as in this portion also. So, in that if you go for the left hand portion you know like what we are doing here we are first considering the X x portion.

So, with this X x portion at the distance x, if you look at this point, then you will find that you have the force which is going upward direction at the extreme corner and this is positive, this is going towards the downward direction. So, if we are checking this shear force in this direction, we have a positive shear force, similarly if you go on the other round like that you see this section.

Then, you will find that this force is W is going downward direction and the reaction W by 2 is going on the upward direction. So, this one is there; that means, we have negative shear force. So, by considering this positive and the negative, since they are equal and opposite and you know like in the direction; that means, they are canceling each other and we can say that the beam is under static equilibrium condition, under the application of the force.

(Refer Slide Time: 31:45)



So, by considering that concept, what we can do here we can simply find it out, that what is the magnitude of the shear forces at the cross action, that would be always W by 2 which is constant up to X is less than l by 2. Because, after X equals to l by 2, we have additional W, so you see it will add up in that particular part, so now we just want to see that, what will be the shear force diagram.

Then, the shear force diagram is automatically, since it is starting from this corner which is a positive corner. Because, as I told you this one is a shear force diagram the portion which is positive, so if you look at this point, then you will find that this is a beam at which load conditions are and this is the positive W by 2 up to this end. And then, you see since it is positive W by 2 now minus W is coming.

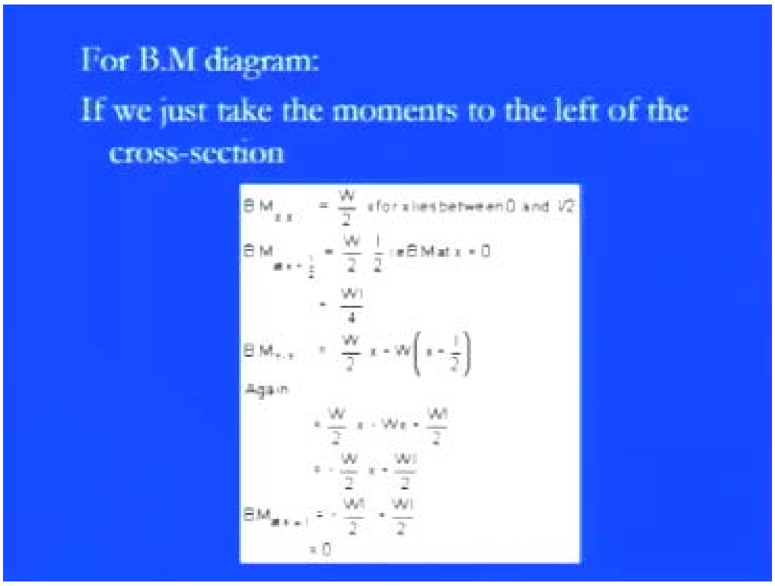
So, minus W will cancel out this positive W by 2 and additionally this minus W by 2 will also become and this minus, because it is going downward from this end minus and this is a positive one. So, this like W and W by 2 will come, so the resultant will come in the minus W by 2 constant up to a starting from this point to this point, so this is minus W by 2.

So, we have two rectangles one is positive of the left end portion one is negative at the right end portion. So, we can easily find it out that, actually what the shear direction are and that how the shear forces resultants are coming and how we can draw this shear forces. If the point load is there at the extreme corner of a simply supported beam, now we would like to see that, how what the variations are there of a bending moment, if this kind of shear force diagram is.

So, the shear force, we found that the variation of shear force is exactly along with the load application on the beam, So, the corresponding changes are there in this bending moment also, that because, we told you know like discuss that, this either shear force as well as the bending moment, they are the function of the X. So, we would like to see that this bending moment variation is so for that we need to calculate the bending moment different sections.

Because, what we have simply supported beam, do these reactions and the midpoint, we have the W, so for that again, we cut X x section. So, first of all we would like to check that, what is the variation is there in the bending moment at the Xx cross section and for that starting from the reaction on the left end corner W by 2. So, if we start from that point on, if we just see go towards that side up to the middle portion.

(Refer Slide Time: 34:07)



Then, we found that this bending moment at. X x cross x is nothing but equals to W by 2 into x. Because, it is just lies in between 0 to l by 2 and for that, we can simply keep that if we start from a x equals to 0 and if you are going up x equals to l. Then, we have two different values, so bending moment at x equals to 0 definitely it will be 0, because it is W by 2 x and it at l by 2, where you see the point load is there, we have bending moment at x equals to l by 2 which is W l by 4.

So, you see here what we have at starting point; that means, the reactive point, since it is starting at the bending moment. So, we have a 0 value, if you move towards the middle direction, where you see the point load is there, we have the maximum portion, that means, that is W l by 2 and then you see even if you go beyond X x point, that means Y y point.

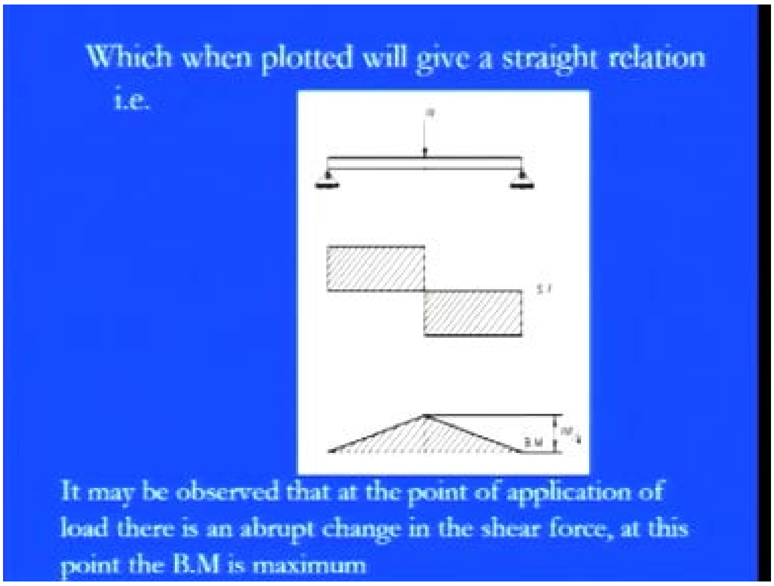
Then, we have different cross section; that means, then we have different you know like all together variation of the bending moment and that is equals to W by 2 into x minus W into x minus l by 2. Because, that portion of this is x minus l by 2, that distance, so now we would like to take this interactive effect of both the point. So, one we have W by 2 into x and one we have W into x minus l by 2.

So, you see what the interaction is there, so minus sign will you know like gives you the clear interaction and by keeping by doing this manipulation, what we have W x by 2 minus W into x plus W l by 2 or these portion. If you calculate then, you have minus W by 2 into x plus W l by 2, so now what we have the variation with the x.

So, now if we are changing the value of x with irrespective to of 0 2 irrespective to l by 2 l, then what we have, when we are keeping x equals to l by 2, then we have the same value as W l by 4. And, when we are keeping x equals to l, then we have the similar value minus W l by 2 plus W l by 2 or it is equals to 0; that means, you see the concluding part is that we have a simply supported beam. Two reactions are there, at the two corners, simply this concentrated load is there at the middle point.

So, the inductive effect as well as shear force we saw, but here the inductive effect is that, we have the 0 bending moment at the extreme two corners, where the pin joints are there. And, we have you know like the maximum bending moment that the magnitude is W l by 4 at the middle portion of that where the point loads acting.

(Refer Slide Time: 36:36)



So, you see here, if you want to draw that part, then what we have the shear force and now this is bending moment, these two reactive point 0 0. We have the maximum, it is varying like that in a this triangular one or we can see you know like these one slight variation is there. So, upward and downward at these two points 0 this is maximum W l by 4 and this is you see the overall feeling of the bending moment.

So, you see here the meaning is pretty simple, if we want to design a beam which is simply supported and the point load is there. We can easily find it out that, the bending moment gives you a maximum value, where there is a change in the shear force from positive to negative or negative to positive whatever like that. So, you see here this we can we have a clear feeling of this, from this diagram this is shear force from positive to negative.

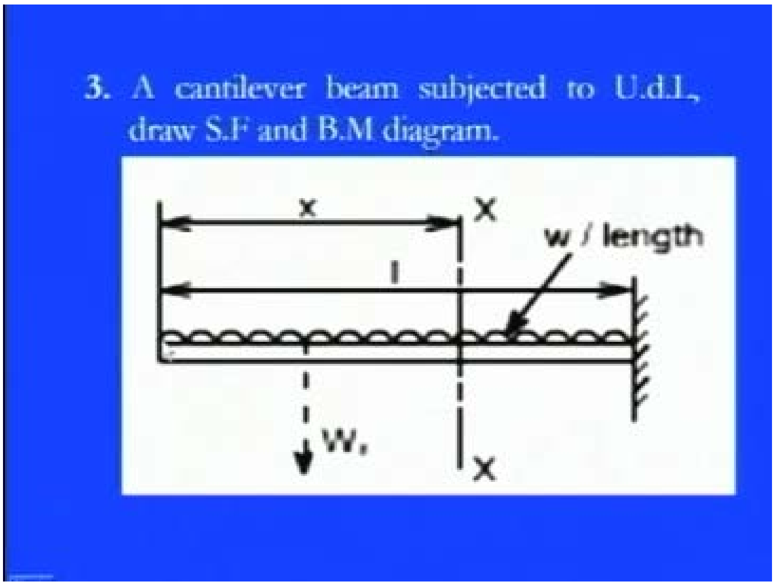
So, this one is the point, where the variation F is there, which is equals to d m by d x, it gives this, so it has clear feeling change of this, will give maximum bending value. So, this W l by 4 is the maximum value and the W positive is the maximum shear force W, negative is the minimum shear force and this is the shear force diagram. So, it may be observe at the point of application of this.

There is an abrupt change is there in a shear force, a straight change is there at this particular point the bending moment is maximum and that gives F equals to d m by d x. So, you see whatever we were discussed in the mathematical way, it was clearly visualizing from these diagrams, that if we are saying W which is nothing but equals to d f by d x or if you saying d 2 M by d x square can be easily find it out. Actually, there is a variation straight way, if the load application is there, so abrupt changes are there in the shear force.

Similarly, you see if you saying that the F is equals to d m by d x, wherever abrupt change is there, we have maximum bending moment. So, all these you see you know like the factors they are very much correlating and we can also have a clear feeling, if you draw the shear force as well as the bending moment diagram for any kind of loading for either simply supported or this the cantilever is.

So, now the third case is that that we have a same cantilever beam which the extreme end is rigid one end is free. Instead of point load, now we have a uniformly distributed load which have intensity of loading is W; that means, kilo Newton per millimeter; that means, W per unit length. So, and we would like to find it out that, what will be the shear force as well as the bending moment diagram is...

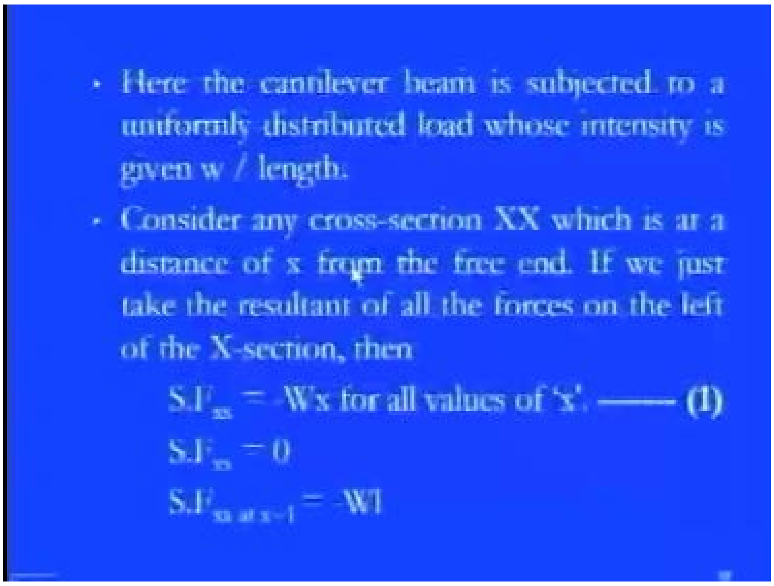
(Refer Slide Time: 39:06)



So, starting from that, it is the free corner, this is the fixed corner and this is the UDL which has a intensity W per unit length. So, from that now, what our main concern is that what will be the shear force is there due this application of load, because when the load application is there. Then, the bending will tend to move towards the downward direction, but there is some shearing force will occur, at the junction where the where the rigidness there.

So, from these the junctional point how the reaction forces are coming to balance those forces which are coming due to this load intensity and how we can make the this beam in a equilibrium manner. So, for that what we are considering again the same similar processes which we discussed in the previous section, cut the beam in the X x section at the X distance for that, what you need to find it out. Then, what will be the intensities there, so that intensity is W x and the tendency of this W x, the intensity of load is towards the downward direction.

(Refer Slide Time: 40:03)



So, now for that, so what we need to do, we need to find it out that, how the shear force is varying, so here you see you know like that load is W per unit length. So, the total load is W, if I am considering the X segment, so the total concentrated load is W into X, so now W into X is there. Now, this is acting on the downward direction and the reactions at the junction as I told you, where the junction is there the reactions are coming towards the upward direction. So, what we have the shear forces in this direction.

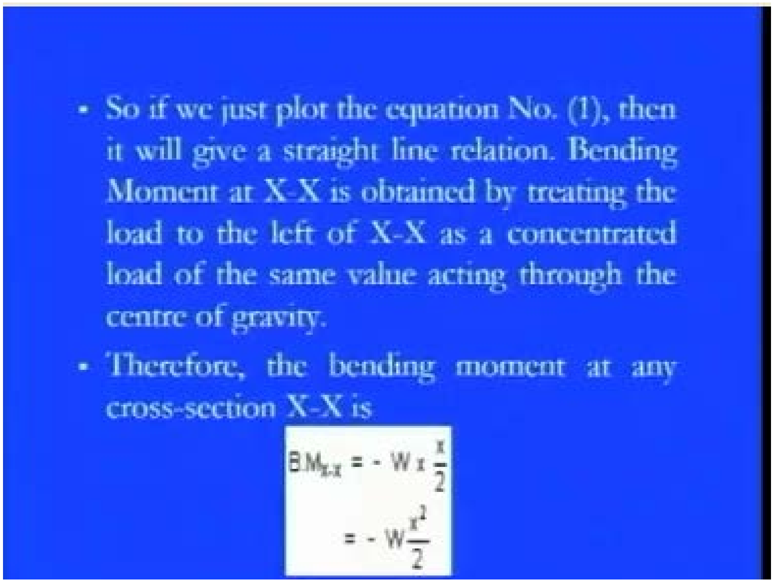
So, if have this kind of shearing forces is as I told you the sign convention is you have to take the negative shear forces. So, for that up to the X x cross section of the beam, the shear force at this particular X x, where started line was there is minus W into X, because W is nothing but what we have this total, this small W into X is there. So, W is this one the intensity of load into X, so this is your shear force which is acting at the X x section and it minus sign as I told you because of this.

So, this is there for equation one and if I am saying that if starting from X equals to 0 there is no shear force. So, X when if I am saying that, if shear force at X x section at the starting point, where x is equals to 0, you always shear force is 0 is there and it has a maximum value where the reactions are coming on the top up way. So, shear force is x equals to l you know like it is equals to minus W into l.

So, you see here it is a clear variation of the shear force from starting point to end point, because in the previous section we found that, the shear force was very consistent. If starting point it has a maximum value middle point, it is the constant or if any addition is there is a algebraic sum is there, but here this is a entire length of beam or entire expand of the beam is loaded by this W.

So; obviously, we can easily calculate, because of the intensity of this load, what the variation is there, so the variation is at starting 0 to maximum minus W into l. So, you see here, again if you want to plot that, this the shear force at X x section is minus W into x, were we can say 0 or minus W l, it will clearly give you the straight line relation and the bending moment.

(Refer Slide Time: 42:23)

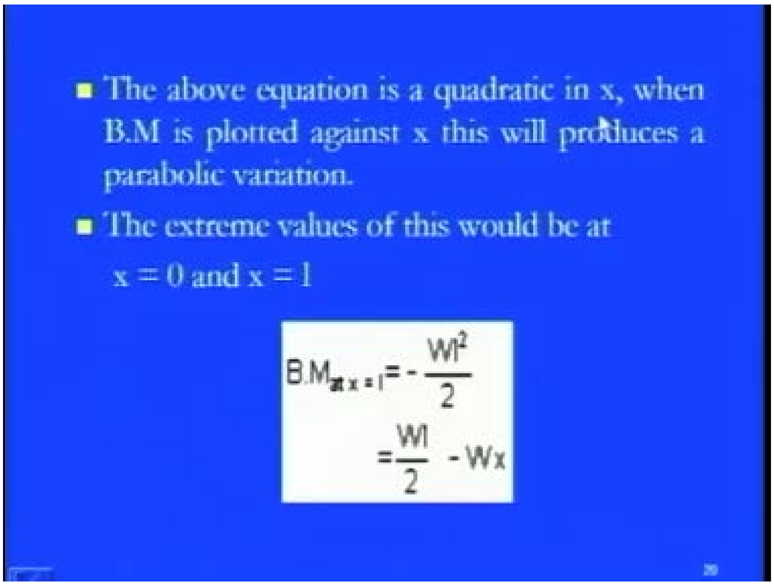


At the XX section obtain by simply treating the load to the left, always you see because the it is this bending is coming from the left which is the free end and it will tend to move towards the downward direction. So, in the left end of this XX section will you know like concentrated load of the same value acting through the centre of the gravity, because again, I told you that we need to find out that, what will be the CG there, where these force intensity will come.

And therefore, the bending moment can be easily calculated by at X x section is minus W into X, that is the load into X x by 2, so this is the location. Where this because, it is a symmetricity is there, so it will always act at the half of the portion. So, X by 2 is the location, where W x loading is acting, so due to that you see the bending is coming. So, bending moment at XX section is nothing but equals to minus W into X into X by 2.

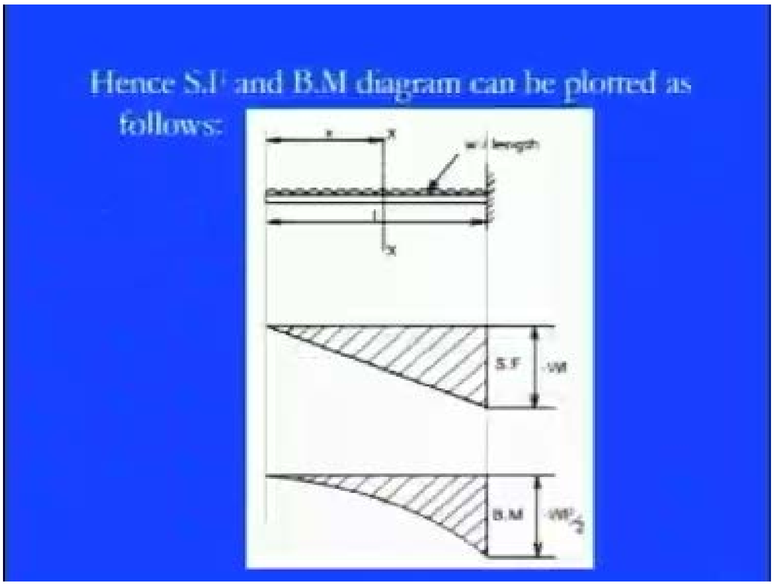
So, we have bending moment which is you know like it is a pretty is difficult to you know like the kind of behavior is coming it is minus W X square by 2. So, now it is a square term is there, so again you see you will have a different kind of relations, it is not the straight relations, now it is sort of the different relations as.

(Refer Slide Time: 43:34)



In terms of say, I told you and bending moment is plotted against x, this will produce a parabolic relation, so as I told you it is not straight relation, it is now a parabolic relation. So, x square equals to 4 a y, so staring from x equals to 0 to x equals to l, if you are keeping x equals to 0; obviously, there is no bending moment at that particular point. If you go up to the l, we have you see minus W l square by 2 or we can say you know like x at other portion, we have W by W l by 2 minus W into x.

(Refer Slide Time: 44:03)

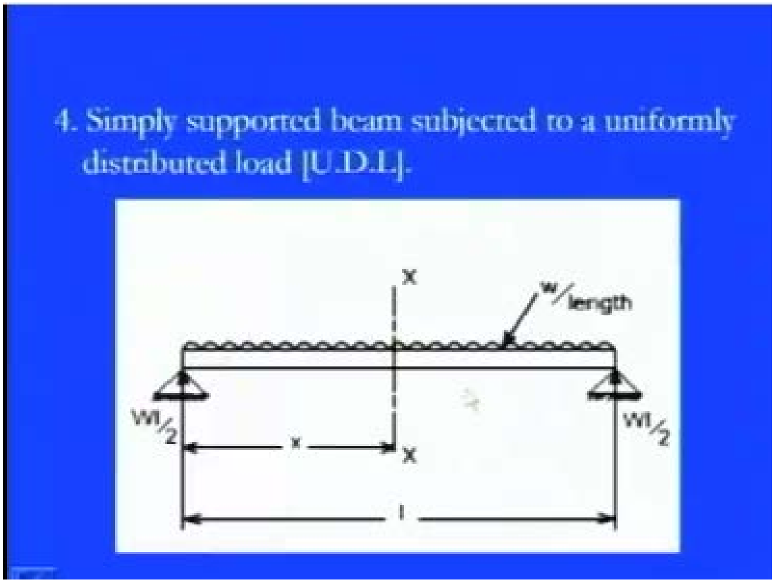


So, from that you see now, we can easily plot that the bending moment as well as the shear forces and for that the same cantilever beam supported by these, the rigid support and this is free end, you see the UDL is there the uniformly distributed load which has the intensity W per unit length, take the cross section x and as I told you the shear force was minus at the one corner, it was 0 and at extreme corner it was nothing but the minus W into l.

So, starting from 0 to minus W equals to minus W into l, will be the give the maximum shear force, we can simply plot a straight this line, which will give you the real feeling of the shear force, when it is UDL is there. So, since you see the reactions are coming, this is the total intensity is there, so obviously, at these particular point, this point look at this particular feature at this point. We have the maximum shear force, so we have to be carefully design, these things and this magnitude of the shear force is minus W into l.

And then, what we have now a parabolic relation is there in between the variation x square with the load condition. So, you see we can easily find it out that, at these the extreme corner the extreme end corner and free end we have the 0 bending moment and at extreme end corner we have minus W l square by 2. So, since it is a square term is there, W x square by 2 was there, at x equals to l W l square by 2. So, at this corner we can easily plot those things in a parabolic way, so we have you know like the bending moment diagram in a parabolic or in if bending moment is there in the shear force, it is the triangular way.

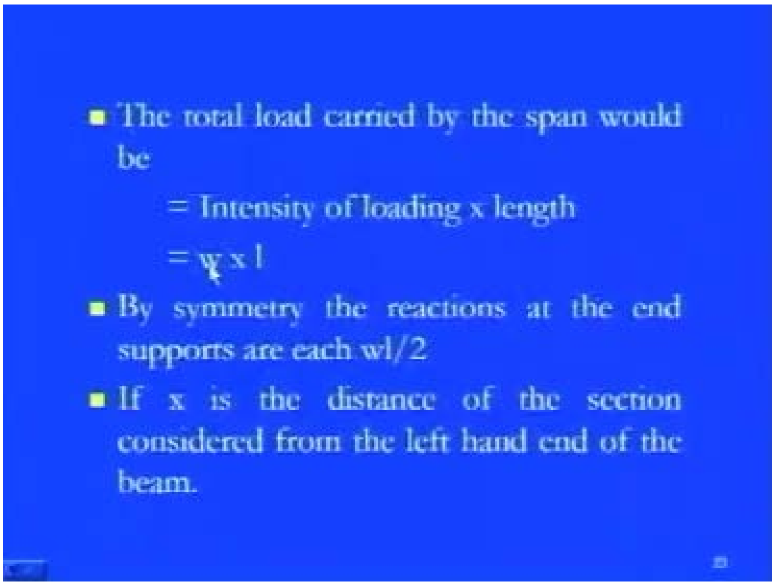
(Refer Slide Time: 45:40)



So, that was condition when the cantilever was there and now, if apply the similar uniformly distributed load on the simply supported beam, the similar kind of the behavior is there. If these two extreme corners point load is this a concentrated load, which can be easily get it, if you know the distance this l and W is here. The total W into l is there, which load is applied and due to that the shear force as well as the bending moment will come the reactions are there which is due to the symmetricity.

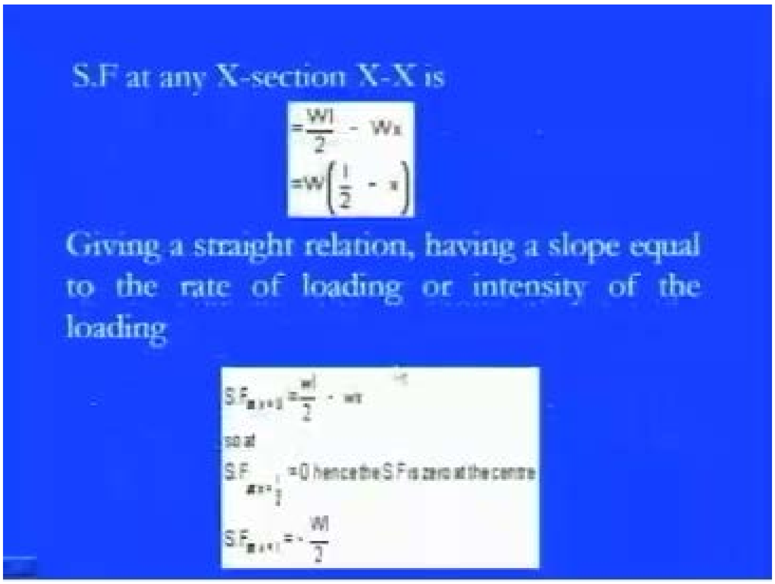
Always, will be equal to W l by 2, at this point W l by 2 this point and both will be acting at the top up direction. So, now if you want to compute the shear force, what is that this W l by 2 is going upward direction, the load which is W into l will be going in the downward direction l by 2 relation l by 2, point. So, this and now the shearing reactions, so what we have a positive sign on the left end portion.

(Refer Slide Time: 46:36)



So, now here the same which I discussed the total load will be carried by W into l and by the symmetry of the sections, we have w l by 2, w l by 2.

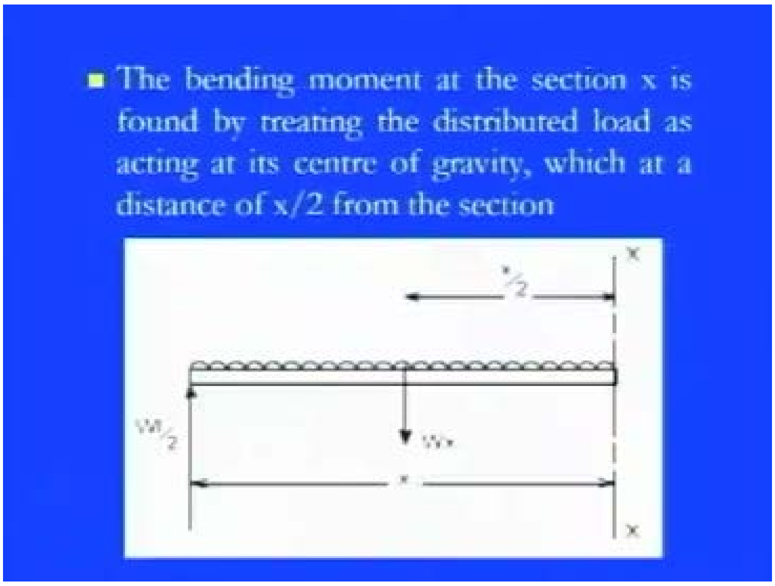
(Refer Slide Time: 46:45)



The reaction forces are and then at XX cross section the shear force is W l by 2, because of the reactions the this particular reaction which is going upward direction and you see the middle portion W into x is there which is the total intensity of the load into x. So, this will go in the downward direction, so we have a positive side here W l by 2 minus W x are we can say, if you take the W common then we have l by 2 minus x.

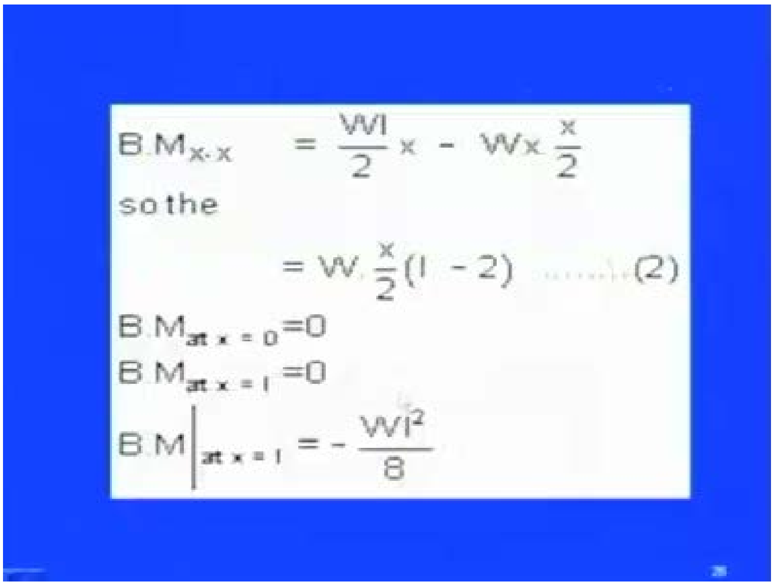
So, now by starting that what we have it is a straight line relation square cubic term is there and having a slope which is equal to the rate of loading or the intensity of this loading is this is this is W. So, at x equals to at x equals to 0, what we have we have a W l by 2, so what we what is that now at the beginning, we have a shear force it is not the 0. We have a shear force which is equals 0 part and then, if you go towards on both of the side, then what we have minus W l by 2.

(Refer Slide Time: 47:39)



So, concerning with that at x just W l by 2, W l by 2 and x by 2 is there, which is W x is the central position, what we can simply get.

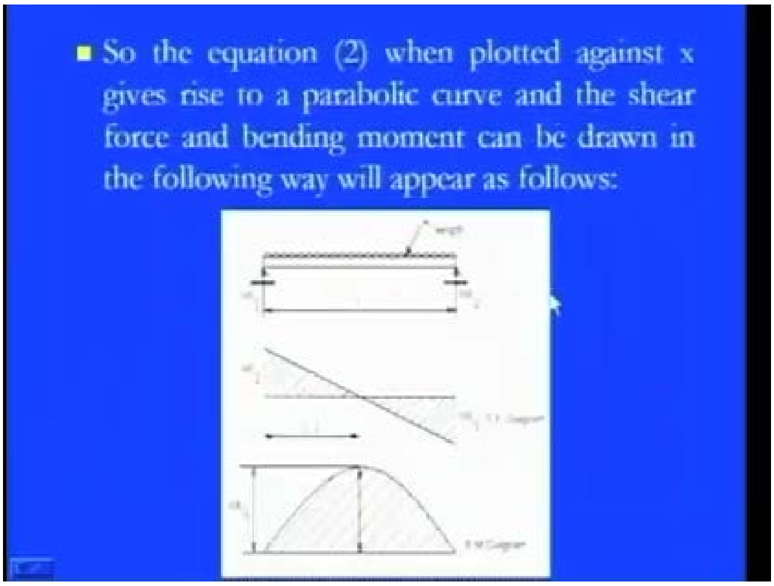
(Refer Slide Time: 47:50)



We can get those bending moment also from those equations, because whatever the load is coming, it is coming from the centre and the reactions are going on the upward direction. So, we have bending moment at X x section, which is cutting at the middle portion, W l by 2 that is the total reaction into x, the reaction is coming from the left hand side, this one into x minus W x which is the load for the X x cross section, the intensity, W into x the total load into x by 2.

So, by taking some of W end x by 2 is common, what we have we have l minus 2, so if we are starting from 0 and going to the l, so if I am starting at x equals to 0, this will gone, if I am doing at x equals to l, so obviously, you see l by 2 and this will 0. So, I mean to say that if simply supported beam is there and the two reaction points are there, the bending moment will be 0 and the bending moment will be maximum, where you see x equals to l by 2.

(Refer Slide Time: 48:50)



So, if you keep those things, then you have minus W l square by 8, so by summing up of all the relations, we have pretty simple diagram first of all the shear force diagram, these two reaction force W l by 2, W l by 2. So, we have the positive W l by 2 in this way, because it is going in this direction and then it is in this direction this one, so it is a negative direction and variation is this, so plus W l by 2 to minus W l by 2.

This shear force diagram will be like that and bending moment, because you see it is simple kind of these parabolic relations was coming. So, we have the W l square by 8 as I just told you the middle portion but at this two extreme corner it was 0, so starting from the 0 going up to the maximum, where you see the change of shear force is. So, it is pretty again, it is verifying you the theory, that F equals to dm by d x, where ever change the abrupt changes are there in the shear forces from positive to negative.

The bending moment will be the maximum and that a value the value is W l square by 8 and it is like that, so in this chapter, we discussed the different four different cases. The first case was very simple that, if we have a simply supported beam, if we have a cantilever and all point loads is there at the extreme end corner, there is no load contribution is there. So, the shear force and since it is going the down ward direction in this particular way, we have negative.

So, a simple rectangular was there and the same bending moment was pretty, the straight line triangular ways, there are both in the negative. And then, we discuss the second case, if we have a simply supported beam and the point load is there, the central position, then what kind of variations are there in the bending moment as well as the shear forces. Then, we found that since in both of the cases it is pretty, straight relations are there in the shear force.

As well as the bending moment, as the W is equals to d f by d x are d 2 m by d x square, but as we go in the UDL part; that means, uniformly distributed load and then since shear force. It is a force part W into x is W is a intensity of the load, which is W per unit length, so whatever the total load is coming that is W into x W into l. So, in the shear force it has a clear contribution, that it is starting from you know like 0 to it will above for the maximum at the extreme corner.

If, the cantilever beam is there or if you have these last case, where the simply supported beam was there, then it has a clear variation from W l by 2 positive to minus W l by 2 negative shear force, but if you are talking about the bending moment. Then, it has a parabolic relation; that means, now there is not a straight line relation is there, if we have a UDL, we have a parabolic part irrespective of l square by 2 or if the cantilever beam is there or we have W l square by 8, if the simply supported beam is there.

So, we must know that, if the kind of variations are there, if whatever the kind of variations are there, it is due to the application of the load, that how the loads are applying on this particular beam. And, what the corresponding interactions are there in between reaction as well as the application of load.

So, that we discuss, but you see still, we simply discuss that only one load is acting; that means, there no interaction is there of the different kind of loading. We discuss starting from the point load only, that point loads are there on a cantilever or simply supported beam. And then, in the next part, we discuss, if a UDL is there on a cantilever or simply supported beam.

Now, in the next lecture we are going to discuss about that, if we have both two types of loadings are there together; that means, if we have a beam which is the l length is there in the one portion of the beam is subjected by a point load. One portion of a beam is subjected by a UDL, than how the interaction is there and that two, this is only the loading.

We also want to discuss that, actually if there is a movement application is there; that means, if any twisting moment or any couple is acting on a beam, along with the variety of the forces. Then, how these forces and the movements are interacting to each other and what will be the resultant shear force as well as the bending moment is. And those things, you just want to see on the shear force as well as the bending moment diagram with kind of variations. So, that is in the next lecture, we are going to discuss all these things.

Thank you.