Strength of Materials

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Lecture - 23

Hi, this is Dr. S. P. Harsha from mechanical and industrial engineering department IIT Roorkee. I am going to deliver my lecture twenty third on the course of strength of materials, and this course is developed under National Program on Technological and Enhanced Learning. Prior to start this lecture I would like to refresh your previous concept because you see, we are going to use those concept in this particular lecture.

You know, like in, in last lecture we have discussed about, that if we have a object or if we are having kind of any bar, this prismatic bar or kind of that and if you are saying, that there, there is a change in the dimensional of, you know, like if particular in the longitudinal as well as in, you know, like this lateral directions, then we can simply termed as this particular substance as a beam. Like you see, you know, like we said actually, if whatever this, either we can say the thickness or the diameter in the lateral direction is very, very thin as compared to, are very, very small as compared to length of the beam or the longitudinal dimension, then we are, you know, like defined that part as a beam.

So, you see, here you know, like we defined the beam in many ways and then you see, we categorized the beam by, you know, like the various, like the categories are there. In that first we defined, that actually if we have a beam, the simple beam, then we can say, the beam is either of the straight beam or then the beam is of a curved beam. And if you want to analyze those beams under the application of any force, then you see, we found, that both the straight beam as well as the curved beam has a different theory altogether. Because you see, in the curved beam we have a curvature theory and then you see, the corresponding, the bending stresses or the shearing stresses will be coming. So, that part, you see, we are going to discuss later on.

Then, we discussed, that actually, if you know, like we can classify the two main types of beam, first based on what kind of supports are. So, first we found, that actually if one end is rigid and other end is free, that means, you see, you know, like the cantilever or we can say the beam, which is supported by one end rigidly, we termed as the cantilever.

And you see, because of the rigid support they would not allow this kind of beam, either in terms of the rotation or the translation. So, this, you know, like beam is of having a different kind of nature. And then you see, would like to see, that if the kind of forces are applying, then actually what will happen.

And the second kind of beam, again based on what supporters are there. So, we can say, that actually if the simply supported beam is there, that means, you see, if the two pin joints are there at the extreme end of that kind of beam, then always the reaction forces are there and the reaction force are in both, mutually perpendicular direction in x as well as the y direction.

And then you see, we also found, that actually if this pin joint, or we can say, the pinned, you know, like this whatever the pins are there and if they have rollers, that means, you see, we are allowing this beam to move in x direction. This also, this is also kind of cantilever beam with roller supporter in the pin joint. So, you see, here in, that we found, that only we have a reaction force, which is vertically upward direction. So, this, these two types of beams, we can say, simply based on, that how these supporters are.

Then, we also discussed, that actually we can also classify this beam based on what kind of loadings are there. So, we, we found, that actually if the loading is, just you see, in the entire length of beam only at some portion or we can say, that actually at some of the effective area just where the loads are there and they are the, either the hanger side or we can say, they are either, you know, like just actually concentrated parts are there. These kind of loads are known as, either the concentrated load or we can say, since the effective area under these loads are very tiny as compared to this, the total length of the beam, so we can say these are the point loads.

So, generally you see, if you are classifying these kind of concentrated load or you know, like where the effective area is very, very small, we are always using the point load and they, we are always showing these kind of point loads by an arrow. That actually, you know, like because they are concentrated. So, you see, they are coming in terms of 10 Newton, 20 Newton, 100 Newton, like that. And they are always applying either vertically downward or vertically upward because you see, actually, if as, as we discussed about the definition of cantilever we found, that whatever the load application is there, they are always perpendicular to the axis of the beam.

So, you see here, either it will come on vertically upward direction or vertically downward direction because you see, we have a longitudinal axis as the horizontal. So, this was, you see, the one kind of loading.

Then, we found another kind of loading, that in which the load is distributed over short span of beam or we can say, the entire span of the beam. So, if it is distributed in a short span of beam, then we found, that it is nothing but equals to this non-uniform distributed load. That means, you see, you know, like only one part is affected over which these loadings are there. This is not the point load because you see, they are already spread it out over a certain part of the beam. So, we can say this is the distributed load but non-uniform.

But if, if I am saying, that this whole, this whatever the load conditions are there and if it is spreaded all across the beam, then it was this uniform distributed load or generally, we are termed as the UDL. So, uniform distributed load is always, you know, presented over the entire length of the beam. And whatever the kind of supports are there, irrespective of whether it is a cantilever beam or simply supported beam, these kind of analysis is always important.

So, you see, here if we are talking about the non-uniform distributed load always or UDL uniform distributed load always, we are not saying, that it is 10 kilo Newton or you know, like the 100 kilo Newton or 10 Newton, like that. It is always be termed as 10 Newton per millimeter. That means, you see the effectiveness or we can say, that the magnitude of these kind of loads are having a constant part like 10 Newton. But since they are, you know, like spreading in a particular millimeter or centimeter or even, you know, like sometimes we can say if it is this beam is of this kind, you see, you know, like where the bridges are, there we can say in terms of the kilometer also.

So, if we are taking in that term, then we found, that what is the rate of loading is. So, always, you see, if we are talking about the non-uniform distributed load or uniform distributed load, we always termed by Newton per millimeter or kilo Newton per millimeter or whatever like that. So, always you see, we can simply distribute these kind of beams in these two categories.

And the third category was there in which always we found, that actually, when we have a dam where this water distribution is not uniform at particular end, we have the, you know, like that maximum load condition, at the other end we have a very minute or we can say the negligible kind of load conditions are there. So, we can term it as a triangular loading.

So, in that you see, we would like to find it out, that if you want to find it out the, what is the impact is there. So, in that we would like to see, that what exactly the total impact is there. That means, where is the center of mass of this kind of loading is, and then once you locate the center of mass for this kind of non-uniform distributed or we can, just you see, you know, like the triangular loading is there. Once you locate that point, then you can easily find it out, that actually what is overall impact is there because at one point of, you know, like at one corner of this beam we do not have any loading, but at another corner of the beam we have the full loading. So, you see, we will discuss in, you know, like in this lecture, that actually what the exact impacts are.

In the last section I told you, that you know, like whenever the kind of loading is there on an a beam irrespective of whether it is simply supported beam or a cantilever beam, always our main intention is, that how these, because you know, like when the loading is there, then the bendings are there. So, how these bending is forming in that and you see, we would like to show on a diagram. So, these diagrams are known as the bending moment diagram because you see, it, you know, like this different, different loadings are there, the different, different section and the reactions are there from the joint or supporters.

So, how they are playing, you know, like on the entire length of beam, our main, you know, like interest is just to see what is the bending moments are there, and we would like to show these kind of bending moments on a bending moment diagram.

Second, you see, whenever the bending moments are there or we can say, the bending is taking place on a beam altogether, you see, at a particular section or in the entire, you know, like the sections of the beam, we just want to see, that how this shearing is taking place. Because you see, one load is acting on top of that towards the vertical direction the reaction is going from bottom to upward direction, so always the shearing is there.

Then we would like to see, that what exactly the shear force diagram is and then what the, what is the relation in between these bending moment as well as the shear forces diagram. So, these all, you see, you know, like the kind of technical terms, which we used here, we would like to see what the impact is there, as well as, as far as this cantilever or simply supported beam is concerned. So, here you see, in this lecture we would be starting with this bending moment and shear force diagram and then we would like to draw for different, different kind of load conditions as well as different, different kinds of beam also.

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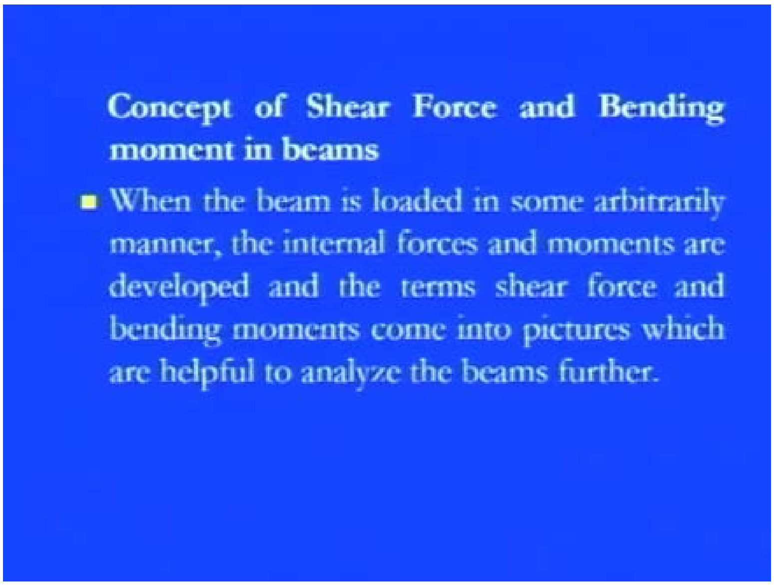

So, here you see the concept of the shear force and the bending moment in the beams. First of all, when the, when the beam is loaded in some arbitrarily manner, whatever you see, you know, it is loaded in, in terms of a point load or you know, like the UDL, uniform, uniformly distributed loaded or non-uniformly distributed load, whatever the kind of load is there, arbitrary manner you see.

The internal forces and the moments are developed basically, definitely, you see. Whenever we apply something, the internal resistances are coming out from, you know, like the beams or we can say, that the internal intensities are there. And due to the internal intensity of these forces, the shearings are there and these kinds of shearing are always bending as well as the shearing, shearing stresses.

So, this internal moment, the internal forces and the moments are developed and these, you know, like these terms, shear forces and the bending moments come in to the picture, which are you know, like helpful to analyze the beams further. So, you see, here these two are the, you know, like the dominating parameters. Though you see, there are other impacts, but these two are the key features to analyze any kind of beam.

So, if you would like to design a beam, then we have to see, that actually how these, you know, like this bending moment as well as the shear forces are generating, generating in the beam and how we can analyze and in which section is, you know, like ((Refer Time: 10:50)) by these kind of, either the bending moment as well as the shear forces.

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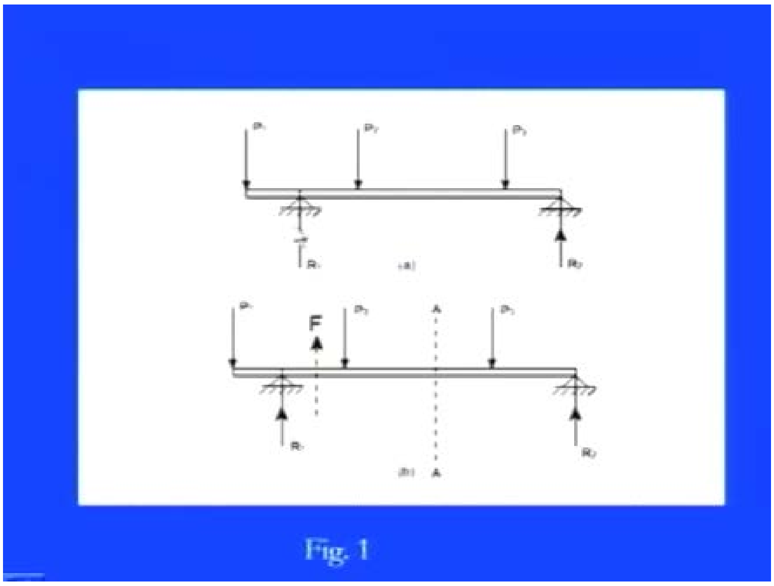

Fig. 1

So, now our main thing is just to consider the beam, you know, like as shown in this figure, that we have a beam and in this beam we have the two supporters. So, here you see, this beam, which is having the length l is supported at these two corners. One is the overhang part is there, so this end is free and it is supported by that. So, this is the overhang portion and this is exactly supported at the end.

So, if we see this figure we will find, that there are two reactions, are the two reactions forces are coming from these two joints in the vertical upward direction, as I told you earlier. And then we have the three different forces, P 1, P 2 and P 3, and they are just you know, like acting on the downward direction.

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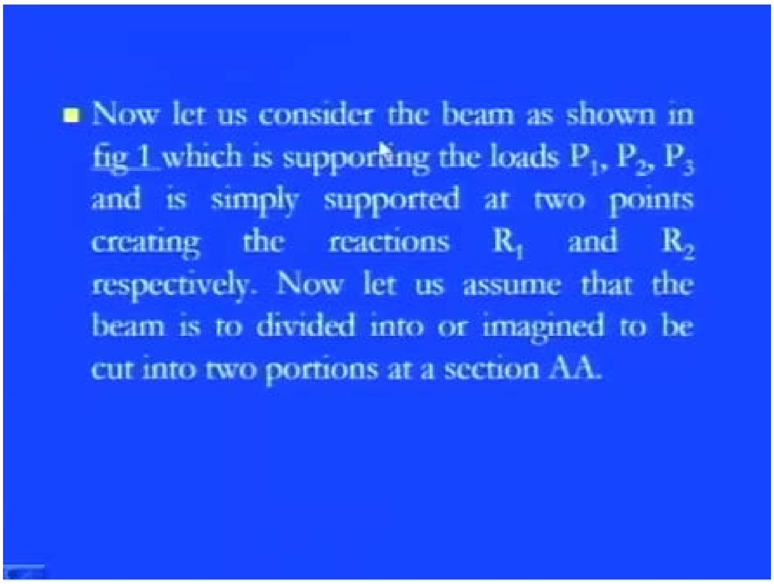

So, this beam is, you know, like supporting by the loads P 1, P 2 and P 3 and simply supported beams are there because you see, the two different joints are there. And these two contacting points where the supporters are there, we have the two reactions, as I shown in the previous diagram, R 1 and R 2. Now, you see, you know, like we just assume, that the beam is to, you know, like divided in to just an imaginary way by AA section, as you see this one.

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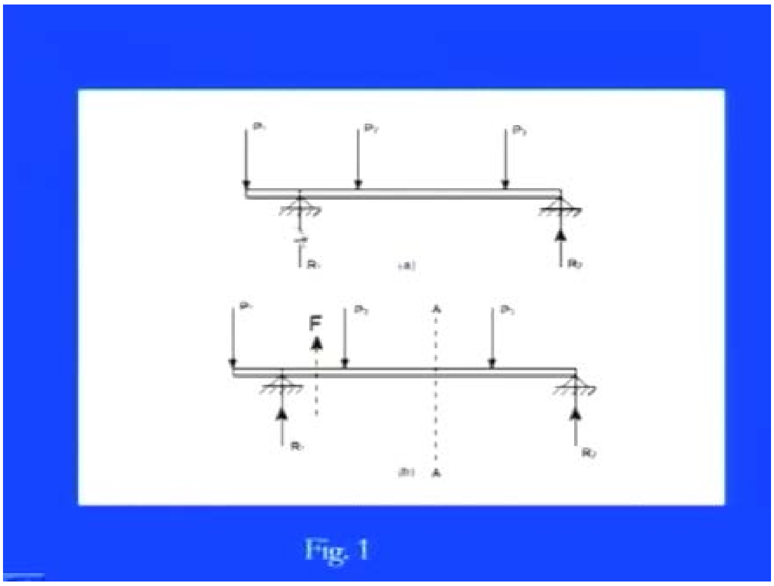

We just divide this into 2 2 2 category. You would like to see, that how these force distribution is there and under these forces, you see, the applied load reactions and whatever like, that how the stresses are being generated or we can say, induces in this particular beam and how we can do the analysis. So, for that we just want to make it easy, you know, like easy analysis.

So, what we did here? We simply cut this action at AA. So, if you see this beam carefully, then you will find, that at the left end corner of the beam what we have? We have, of the two different forces, which are coming from upward direction, these are P 1 and P 2 and there is one force, reaction force, which is going up in upward direction. So, if you just see the feasibility of these forces, then you will find, that I am just talking about the left hand side only. So, you will find, that we have a reaction force on top up direction we have a load on bottom direction. So, there is a shearing is there.

So, definitely if you, if you see on this particular, this portion, just view on that particular part. If you see this portion or this portion we have the shearing action and due to that this beam will go in this kind of bending moment. So, always you see, we have a bending as well as shearing together because of these load condition, either the left side. And if you just see on the right side also, you will find, that this reaction will just, you know, like move this, this beam in upward direction and this load will tend to move in the downward direction.

So, we have a shearing action and due to this shearing action the affected area of this one or we can say the affected length of this one and also, you see, due to that it will always try to bend in this direction. So, you see, we have the bending action as well as the shearing action together. So, we would like to analyze both, both of them, you see, the bending as well as the shear in a separate manner. And then we would like to combine both the thing as well for our analysis, that what exactly the interaction is there in between the bending moment as well as the shear stresses.

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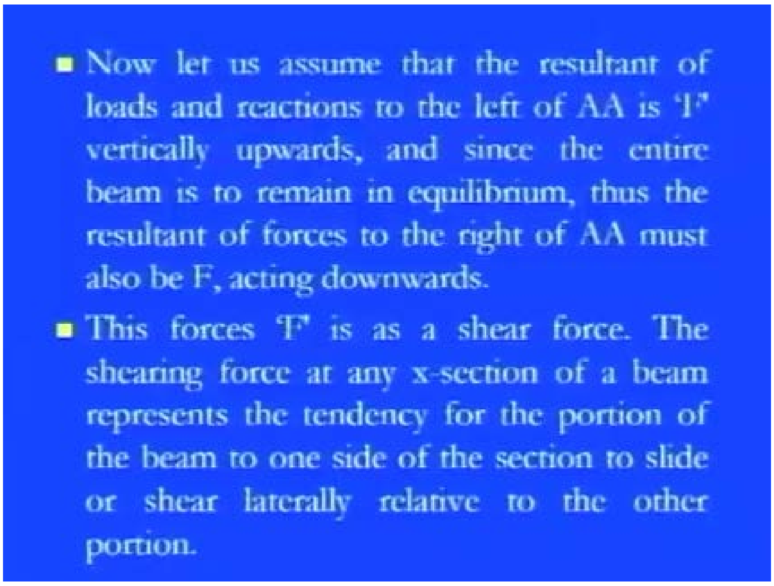

So, now you see, you know, like again we would like to see, that what exactly is happening. So, let us assume, that the resultant of the load as well as the reactions, the loads P 1, P 2, P 3, as well as the reaction R 1 and R 2, to the left in the corner of AA. As I told you, you see, there were in that the two forces were there, P 1 and P 2, and the reaction was there in R 1 is just F 1.

So, you see the resultant, as I show you, and in dotted one is just F 1 is there, the resultant force vertically upwards, which is acting. And since the entire beam is to remain in equilibrium ((Refer Time: 14:29)) because you see, as I told you in, for any kind of, you know, analysis in the strength of material subject, whatever the forces are there, even they are applying.

And if we are saying, that our object is just stationary, that means, whatever the forces, they are acting from outside and whatever the forces, which are coming from the inside, that means, the reactive forces, they are well balanced and we can say, that the system is in equilibrium. So, here also we are assuming the similar kind of situation that we are applying the forces form the top up direction, P 1, P 2 and P 3, we know like in the downward direction and the reactions are there from the downward direction towards the upward way. So, both are well balanced and the beam is in the equilibrium position in the left as well as the right corner of AA dash line.

So, if we go for that then we will find, that resultant of the forces of the right direction of AA must be also F, which will act in the downward direction. So, you see, here if we, you know, like just cut, as I shown you, that we are simply cutting the beam in AA section. So, whatever the forces, which are generating on left hand side, as you know, like just discuss there, actually the resultant is F 1, which is acting upward direction. Similarly, the complimentary part will come just to balance this beam, which is equal and opposite to this particular force.

So, we have the resultant part of F also, which is in the right, this right direction of AA section, which will be always equal in magnitude F and act in the downward direction. So, that is why, you see, we can say, that this beam under the application of these forces with the reaction forces are applied forces, it is well equilibrium manner. So, this force F, which we are saying, that the resultant force is nothing but the shearing force because you see, these are the resultant. These, these force is the resultant due to P 1 P 2 from upward direction and reaction force from the downward direction.

So, you see, here this, always shearing this beam we can say, that this is nothing but the shear force and the shearing force at any section, whatever we can say, the cross-section of beam represent the tendency of the portion of the particular beam on one side of the section to just, you know, like sliding part or we can say shear laterally relative to the another portion. That means, you see, you know, like it is just always going in lateral direction just to give an intact.

That is what you see generally if you just go to the previous sections, which we discussed about the shear part. Shearing is always coming not in the axial way. It is a plane stress, which is always acting right from, you know, like circumferential part. So, here also if you go for that, we have a beam and this shearing is always going in the lateral direction to one part, to another part and it has direct impact, impact if you take any cross-section

And we discussed many cross-sections, like we have either a rectangular cross section or a circular cross section or even we have a I section beam. In all of the sections, you see, you take any cross-section you will find, that whatever the shearing forces are coming, they have a direct impact and they are going in the lateral direction relative to one portion to the another portion of a beam.

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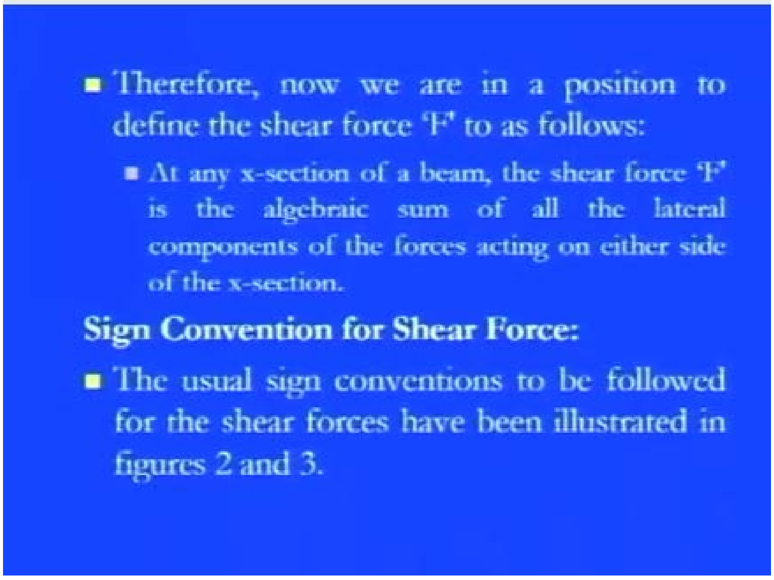

So, you see, now we are simply, you know, in the position to define the shear force and at any cross-section of a beam the shear force is the algebraic sum of all the lateral components of the forces acting on either side of the cross-section.

So, you see here, as I told you, that what we are doing here, we are simply taking, you know, like just in the previous section we are taking the two portion only. So, if, even if you divide this beam in many, many cross-section means, many, many, you know, like the segments. And if, if you are taking, you know, like this shearing force at one segment, by the application of force simply sum up those, linearly sum up those forces and you will get the overall impact of this shearing forces on the beam.

So, you see here, you know, we are saying, that this is the algebraic sum, that means, it is a linear sum is there of these forces because they are applying all across of the beam. And you will find, that since we have same cross-section of a beam, so they have, you know, like they are always setting up in this beam in a very uniform way.

Now, you see, our main interest is, that since the shearing forces are always there due to the application of any kind of force on a beam, so our intension is, that how to analyze that. So, for that, you see, what we have done? We simply set up that certain sign conventions are there for this kind of shearing forces, whether you see, it is going in the, resultant is upward or resultant is downward. We just want to give such certain, you know, like the notation, so that if you want to compute. Because what we are doing here?

We are simply taking the algebraic sum. So, you see, whatever the directions are there, we just want to compute all those in the linear way.

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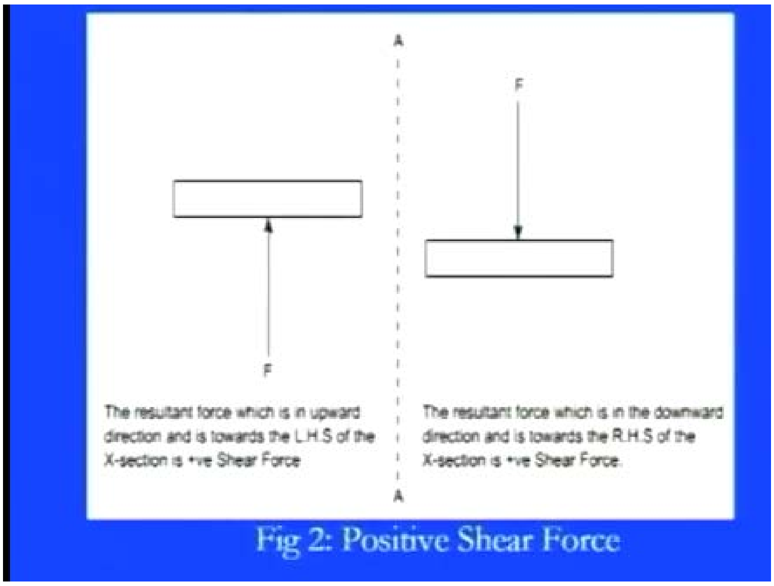

Fig 2: Positive Shear Force

So, our first focus is, that what is the sign conventions are there for these kind of shearing forces? So, you see, the usual sign conventions, we simply categorize into two main figures. So, figure two is simply showing, that if we have the resultant force on left hand direction in upward direction. That means, you see, the resultant force, which is upward direction and towards, you see, the left hand side of any cross-section is a positive shear force.

So, if you see shearing force, the resultant shearing force on left side of any beam, the cross-section, you see, if I am saying, that this is the section and the resultant is going in upward direction. And on the left hand side, you see, the resultant force is going in the, you know, like the downward direction. Always we are going for the positive shear force. Just keep this thing in your mind, that whatever the, you know, like these point applications are there and reactions is going in top ward direction and the force applied is in the downward direction, this side. Whatever the shearing is coming in between this section is the positive shear, positive shearing force.

So, you see here, you know, like we have just now denoted, that one sign, that if this kind of force applications are there because you see, you know, like the basic definition of the force is what is the point of application of these forces. So, how they are, you know, like attracting or we can say, how they are, you know, like joining this particular point, this is the main criteria here. So, here if it is going in this way, always it is a positive and vice versa. And you see, if it is going, that means, you see if on the left hand portion if this resultant force is going downward direction and on the right hand portion, if the resultant force is going in the upward direction, then we have a negative shear force.

So, you see here, if you want to draw any shear force diagram or if you want to compute the algebraic sum of these shearing forces at different, different segments, if you see the resultant force in which this, on the left hand side, you see it is going to upward direction and the right hand side it is in the going downward direction, just put the positive shear force plus F.

And you see another section of that. If the resultant force is coming in the downward direction on the left side and on the right side if it is going upward direction, then you just take the minus F or we can say the minus V or the negative shear force. So, this is the sign convention for the shear forces, well, irrespective of even of, for an entire length of the, you know, like this beam cantilever or simply supported beam.

If it is the under the application of various forces from the downward as well as the upward, we just want to see, that what is the feasible situation is there. That means, what is the resultant is coming out of these forces and due to that we can simply denote, that whether it is the plus F or minus F or generally, we have denoting this shearing forces by V also sometimes.

So, we can say, that whether it is a plus V, minus V or what. So, corresponding changes will come in the, you know, like in these summation and net resultant will come on, you know, like this particular, this cantilever, cantilever, simply supported beam. And we can say, that the total impact of the shearing forces are like that. So, here you see, now, now we just analyze those for shearing forces.

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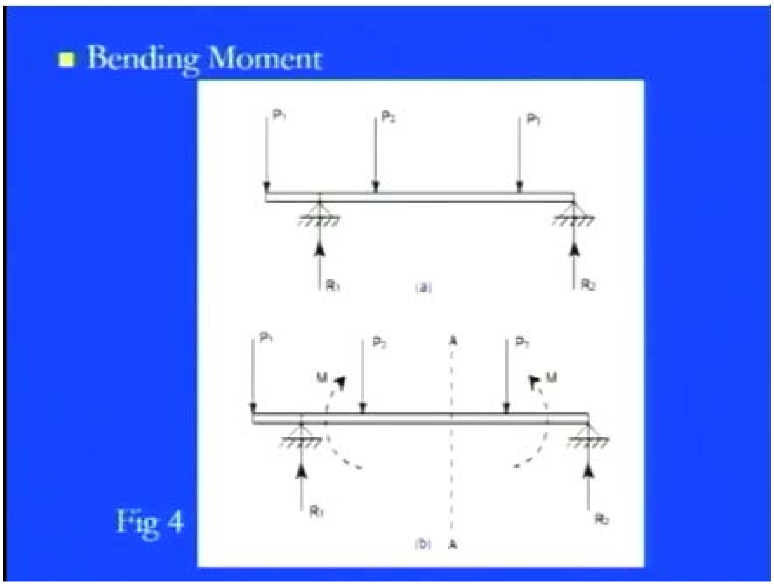

But you see, as I told you, you know, like due to this application of forces we have the bending also. So, now, we would like to see, that how this bending moment will occur in the, in this particular beam. So, now, again we would like to go for the previous case. So, you can see on your screen that we have a figure, same you know, like the cantilever beam with these two supports. And we have the same these three loads are there. So, you see here what we have? We have these three supports, P 1, P 2 and P 3 and we have reaction forces, R 1 and R 2.

So, due to that now, again what we want to do here? We just want to see, that what is the bending moment because you have already analyzed the shearing forces. Now, our intention is to see, that what is the nature of the bending moment is. So, for that again, what we did here simply, you know, like cut the section here by AA. So, we have this section AA. We have a left portion, we have a right portion from this AA. So, now, if you go for the left portion, then you again you see, we have two main, this points loads are there on top of that. So, these two point P 1 and P 2s are there, which are acting to the downward direction and we have a resultant, which is upward direction.

So, if you see the real feasible situation here what the resultant is? The resultant means, you see, in terms of bending, these two forces will tend to move this, this beam in the downward direction. So, you see, the resultant will come towards, you know, they are pushing in to the downward direction. But in between these two, means, you know, like the location of these point loads are different. They are not exactly on the same position.

And in between these two forces what we have? We have a point load, which is acting in the upward direction. So, you see, if you go from the now look at, from the, you know, this AA section. So, if you look from this position you will find, that this, this beam overall resultant of this, that this beam, beam will tend to move in this direction.

So, you see, here the dotted line, which clearly show you, shows you, that this is the, the, tending position of this bending, most of this bending moment is. So, we have a bending moment M and the direction is showing in this way. Similarly, if you go for the right, right direction.

Then, again you will find, that we have a reaction from bottom, you know, like the bottom part at this position. And we have, you know, like this P 1 is there on this, towards the downward direction. So, if you go for the overall impact of the, then you will find, that we have a bending moment, which is going in upward direction and the tendency of this kind of thing is this, which is, which can be easily shown by this dotted line, and we have a M magnitude of these things. So, you see, here we have a bending moment, which is going in this direction. So, this direction is there. So, means, that actually this overall impact of these forces on a beam will give you the overall, you know, like this bending moment in this particular cross-section. So, now you see, we would like to analyze that part.

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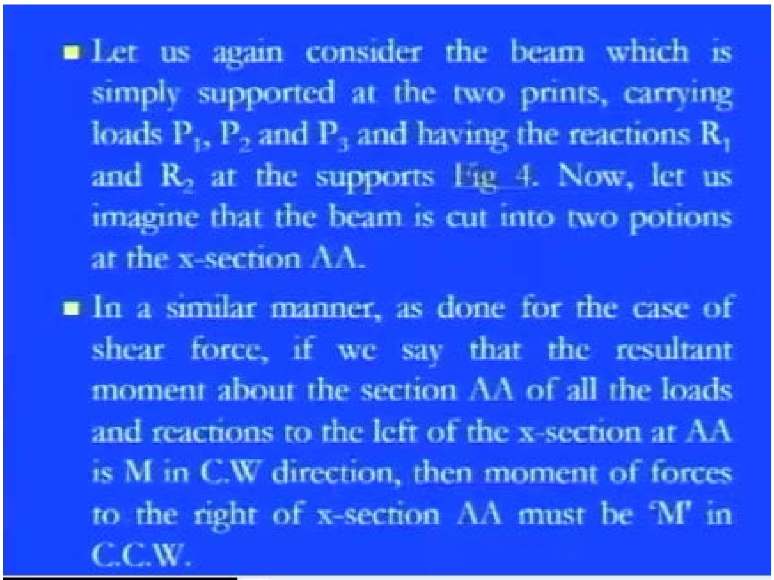

So, again you see, all those things, it is pretty clear, that actually we have, you know, like the P 1, P 2, P 3 forces are there and these reactions are there in this, this particular figure, you know, like all those things. And now, we just want to again cut the section, which we did it. And in a similar manner, again what we did here we just want to see that you know, like what is the overall impacts are there of these forces in the left hand side and what are the overall impacts are there of this forces on the right one, right direction.

And now, you see, here we found, that the reactions, you know, like in this sections, you know, in the particular left, left sections, that overall impact is coming in the clockwise direction, you see, this part. So, you see, you know, like whatever the tendency of the beam is there, it will tend to move in a clockwise direction. So, similar way, you see, we just want to, you know, like make this particular beam in an equilibrium manner.

So, what will happen? You see, the complimentary, this kind of bending moment will be formed in another section to make the beam equilibrium. So, automatically, you see, you know, like on the other side the bending moment will be generated because of the application of force. And this, this will held to be, you know, like have the same magnitude, but in the counter clockwise direction.

So, you see, that is what this is termed, you know, like bending moments are there and we can say, that now we would like to see, that whether if you see the force under this force application, whether this bending moment is going in this, this direction or in this direction. That means, you see, in the upward direction or in the downward direction. So, accordingly, you know like our main focus will be there, that how, you know, like the bending will, this beam will be reacted on this kind of bending moment.

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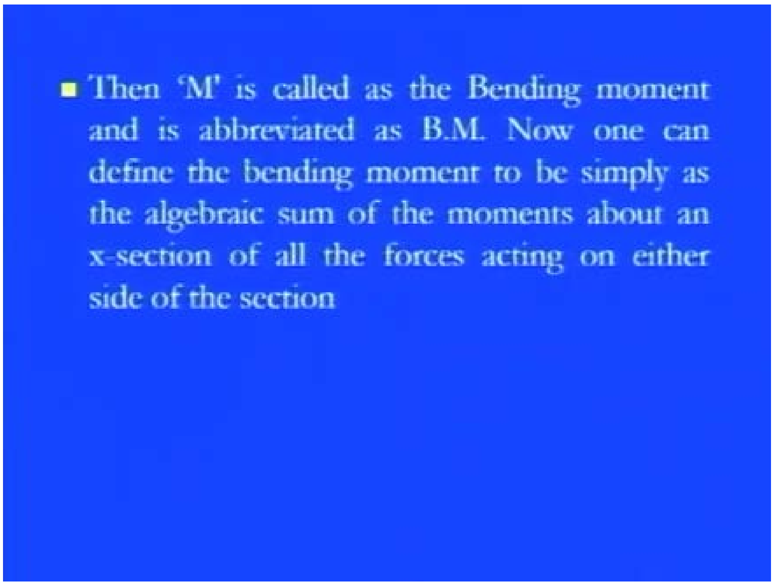

Generally, you see, we are termed as this bending moment as a M or generally, we are writing B dot M dot, is a usual name of this bending moment.

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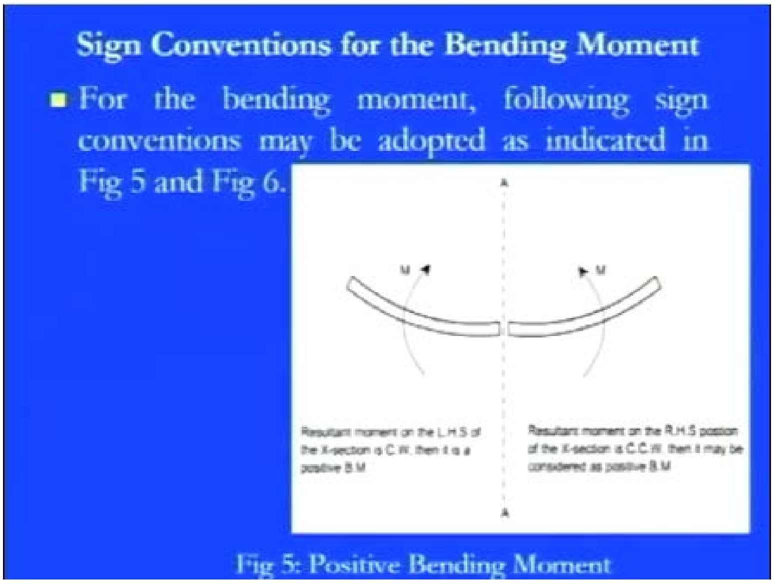

So, now you see here, as usual, which we defined the sign convention for shearing forces, here we would like to define the sign convention for the bending moment.

So, for that you see, the bending moment, again as I told you in the previous section, that what we have? We have on the left hand side of the AA section, you can see on this particular diagram on the left hand side.

Now, if these, whatever the resultant of the forces are coming, if they just tend to move this beam in the clockwise direction on the right hand side, they are just tend to move this beam in the counter clockwise direction. We are saying that this is the positive bending moment. So, means, you see, if we have the resultant bending moment under the application of all these forces in the upward, upward direction, that means, you see, both are going in a upward direction. It is the positive bending moment.

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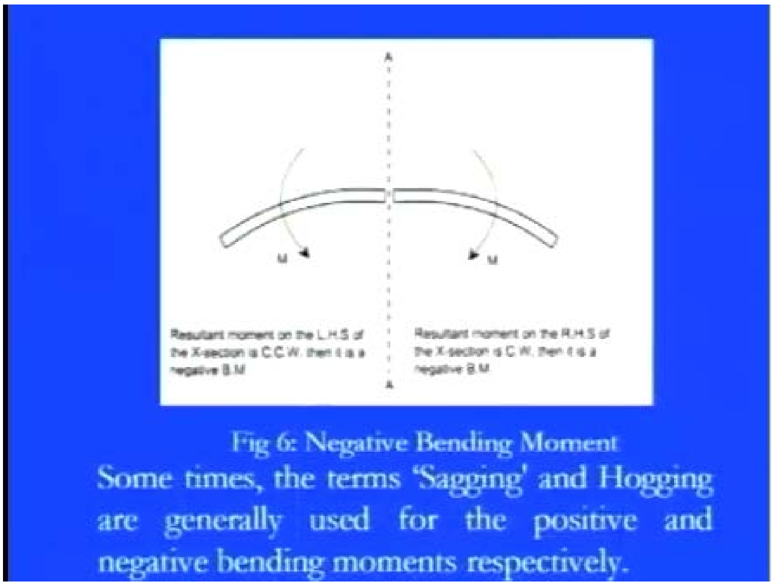

And due to the resultant of all these forces on a beam, if they just tend to move in anti, with this, you see in the left hand side if they are just moving in a counter clockwise direction and the right hand side, if it is moving in clockwise direction. To balance those things if we have the tendency of the total beam in this direction, we can say, that this is the negative bending moment.

So, you see, sometimes the term, you know, like this sagging and hogging are generally used for positive and negative bending moment. So, you see, the sagging termed is, sagging means, you see, it is as per the shape itself. The sagging means, the positive bending moment and hogging means, you see, in this particular shape. So, generally we are termed as the negative bending moment.

So, you see, here now we have the sign convention. For shear force we have the sign convention for bending moment as well as the shear force sign convention is conformed. We are the, as far as the shear force sign convention is concerned we are saying, that this and this is the positive direction; this and this is the negative direction for shear force, as far as the, as well as this bending moment is concerned.

The sign convention, if it is in the, you know, like the sagging mode, that means, you see in this mode we are saying, that the positive bending moment. If it is going in this direction, we are saying, means the hogging direction, we are saying, that it is a negative bending moment. So, just keep these, you know, like the direction in your mind and then you see, corresponding numerical problems can be easily solved or we can say, that we are taking these science for our, you know, like just algebraic sum of these, either the shear force as well as the bending moment. Now, you see, here we would like to see, that actually after, you know, like putting those sign conventions now how they will act. That means, what is the bending moment as well as the shear force diagram is…

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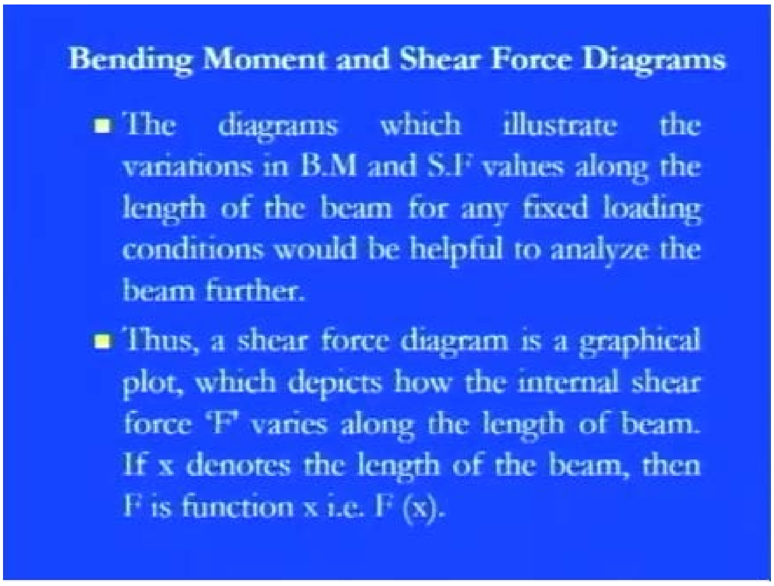

And then you see, you know, like here the diagram, which illustrate the variation of the bending moment. And the shear force, shear force values along the length of the total beam for any fixed loading condition would be helpful to analyze the beam further. That means, you see, now we would like to see, that under these application of the forces how, you know, like shear forces are varying. It is, you know, will see for particular section you, you must find, that it is positive after, you know, the addition to that, that portion, you will find sometimes the negative.

So, how they are, how the variation is there of the shear forces or how, what, what kind of variations are there of the bending moment and what the resultant is there. And then you see, after getting the resultant what is the interaction is there in between those, you know, like the shear force and the bending moment. This is the matter of concern and we would like to see the total impact of those things.

Thus, the shear force diagram is a graphical plot or a graphical representation, which depicts how the internal shear force, the F, you see, which, which we have shown, that the resultant shear force on the left as well as the right direction varies along the length of the beam. And if you see, that you know, like in the next figure, you see, if x denotes the length of a beam, then F is a function of length.

That means, you see, you know, like always if you are saying, that if the concerned portion or the effective portion under which these forces are acting, if it is x, then how they are acting. So, you see here, in, in the next diagram I am going to show you, that actually what the shear force is there, but the shear force is a function of the x and we will see the effect of.

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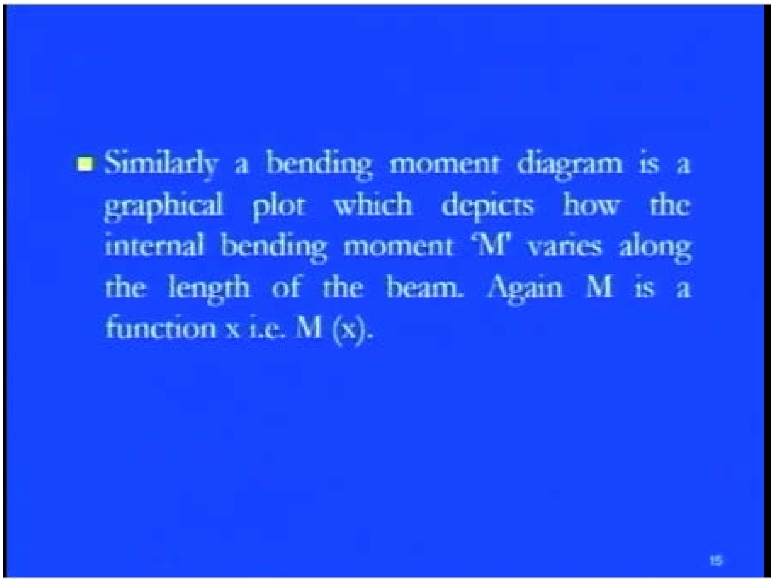

Similarly, a bending moment diagram is also a graphical plot or we can say, graphical representation, which depicts how the internal bending moment M or B dot. M varies along the length of beam. And again, you see, we are saying, that since you know, like we are taking the individual, individual sections of this beam. So, M is also a function of x.

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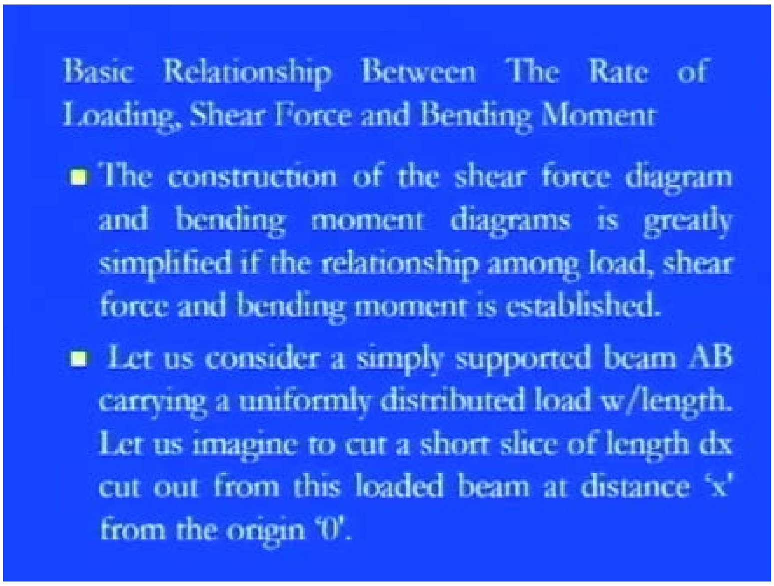

So, now, here you see, you know, like in these kind of actions, which we found, that whether the shearing force or the bending moment, since they are the function of M. So, how they will vary, now it is our matter of concern. So, we would like to first set up the basic relationship between the rate of loading, shear force and bending moment because whatever the rate of loading or the kind of loads are there, the point load, even you know, like concentrated load. Due to that, you know, like the shear forces are generated, generated in those cantilever beam or a simply supported beam and due to that the bending moments are there.

So, now our main intention is, that what the relations are there in between the shear force, bending moment and these kind of loadings are. So, for that the construction of the shear force diagram and bending moment diagram is really, you know, like the simplified way, just if the relationship among the load shear force and the bending. This bending moment is stabilized.

So, once you have, you know, like as we have discussed. We have either a cantilever beam or we have either a simply supported beam. And on that also if we have a point loads, only just as I shown you in the previous figure, that only the point loads are there, P 1, P 2, P 3 under which and then you see, the reaction forces are there. There also, in terms of the point loads, which are, you know, like acting on the vertical direction.

So, under these load conditions what is the real impacts are there and what the relations are there? The mathematical relations in between, you know, like the force, applied force and whatever the shear, resultant shear forces are coming for individual sections. And then you see, what is, what are the bending moments in those particular sections.

So, now we would like to make, you know, like we would like to set up the relations for these kind of thing, but it depends that what kind of loadings are. For point loadings are there or we have this UDL, the uniformly distributed load or non-uniform distributed load. And other thing, you see, whether we are taking a cantilever beam or whether we are taking the simply supported beam because you see, the nature of the reactions are coming from the kind of supports.

So, you see, whatever the forces are coming from the external side and whatever the forces, which are generally generating inside, would like to set up the relations. So, basically what we are doing here? We are simply, you know, like trying to set up the relations with the using of what the physics of the beam is.

So, let us consider a simply supported beam. So, now, you see, we are taking a simple case of a beam, which is a simply supported beam. The total length of the beam is, you know, like L. I am saying that and we are simply notating A and B, carrying a uniform distributed load. So, here, you see, we are not taking, you know, like the point load, as we discussed in the previous case, we have a UDL, so uniformly distributed load. And always, you see, we are showing the UDL by whatever the w per unit length. So, kilo Newton per meter or whatever you see, per unit length.

And let us design to cut short, just slice, you see what we are doing here? We just want to focus, that what is the overall impact is there. So, for that just cut a slight or cut the short portion of length, dx, just cut out from that loaded part at a distance x where you see, you know, like we have the origin.

So, what we did here? We are starting from a point, which we are saying, that the origin is there and from x distance we simply cut the portion and we just want to analyze under these application. That means, you see, under this rate of loading what kind of the shear stresses and what the kind of bending moment is.

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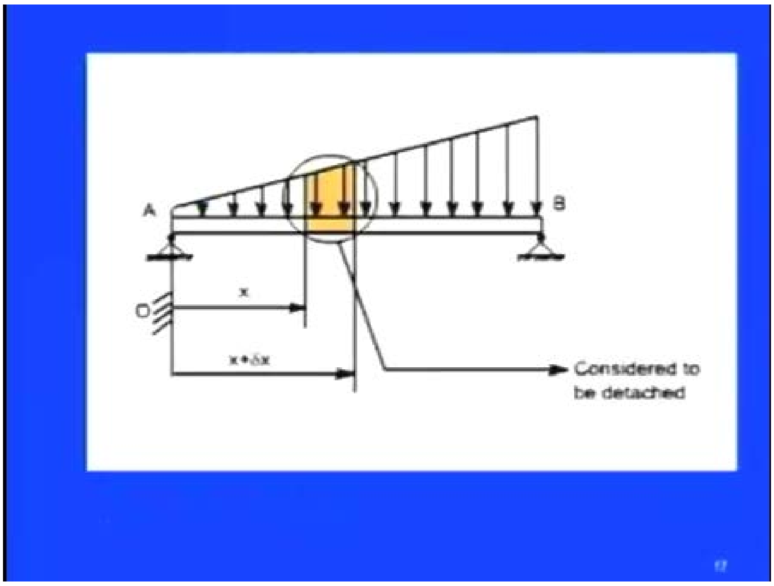

So, now here is this diagram is. So, what we have? We have the entire length of beam, this, and of this entire length of beam, this rate of loading is that. That means, you see, we have a UDL, uniformly distributed load. And this load, you see, we, here we have the maximum intensity of this loading and here we have the minimum intensity of loading. And you see, you know, like this load is just varying all across the entirely, entirely, you know, like length of beam. So, we have, this is the uniform distributed load, UDL is there.

And then at two points what we have? We have the simply supported beam. So, the reactions are coming from these two beams in the upward direction. So, you see here, these two upward directions are there and now as per, we discussed what we are going to do here. We are simply taking, you know, like the small section of that just to see, that how these shearing forces, as well as the bending actions are taking place on this particular beam.

So, for that our, just for study, you see, we are simply cutting this portion. So, this circle shows you our main interest or we would like to focus on this particular portion, which has a distance x from the origin. So, this is our origin, you see, and this is the x. So, this length of the beam is x and you see, we are taking the portion of dx. So, this length, the extreme end of this is x plus dx. So, total you know like the length in terms of the horizontal way is just dx. So, now, we would like to see that under the dx section how these forces or we can say, under these, due to these, this uniform distributed loading, how these shearing action and the bending will taken place.

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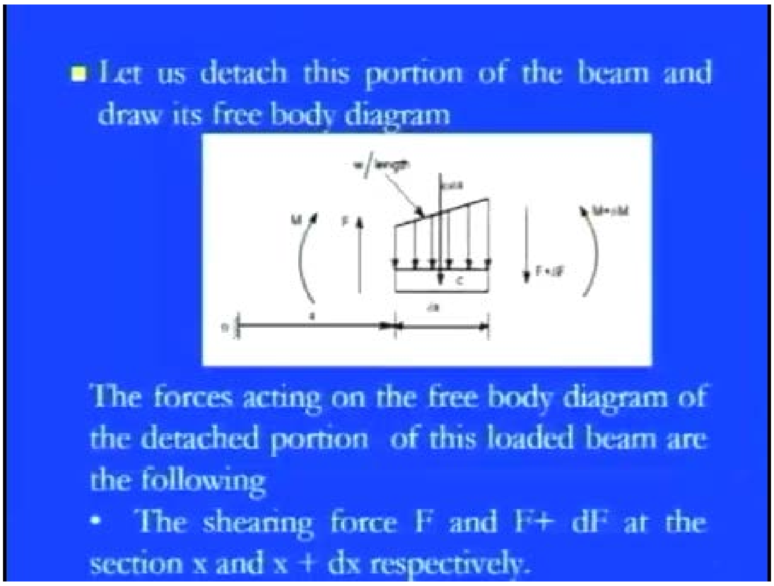

So, you see here, as we discussed the, let us, you know, like the detach, the portion of the beam as well as, you know, like just draw its free body diagram. So, just concern that particular force, we have a small portion. In the small portion, as I told you, what we have? We have a simple, you know, like the length, which is dx. So, once you take this dx and from the origin how much distance is there?

This is the x, so the total distance is x, x plus dx and then you see, you know, like. So, the center point at which the overall, as I told you in the previous, previous section, that if we have a UDL, we would like to see, that what, where is the center of mass is acted. That means, what is the overall impact of this loading is. So, the overall impact of this loading is coming from this direction, which is going in the downward part.

So, at this particular point C we can say, that this loading is coming towards the downward direction. So, now our main intention is, beam is coming in terms of the rate of loading, so that what is, that of loading by this distance delta x, you will have the load. So, w into dx will be the applied load, which is acted on this point.

Again, focus this, the location. You can easily find it out, that this is the C location where this point of, you know, like the center of mass is acted. And at the center of mass, you see, always the load is coming towards the downward direction because of the nature and the magnitude is w into dx. So, since this dx is there and this w is varying the rate of loading. So, w into dx will give you the magnitude of load.

Now, so now, you see, what we have? We have the reaction forces, which are coming from downward direction. We have the, this w into dx, this is the magnitude of the load, which is going towards the downward direction. So, now if we are cutting the portion on the left hand side as well as on the right hand side, we have two different, you know, like the shearing forces as well as we have two different bending moments also.

So, now you see, just go to the left end portion, what we have? We have the shearing, this reaction force, which is going on upward direction. The w into delta x is the total load, which is coming in the downward direction. So, you see, this is the, this notation is there.

So, as you go with the convention of this notation you will find, that what we have? We have a positive shearing force or we can say the resultant force. You can see the diagram, this F is going in upward direction and if you go on the other side portion. So, F plus dF is going in this direction. So, total impact, that means, you see it is going in this direction. The total impact of the shearing force is in this direction. We have a positive shear force.

Now, you see, we would like to see, that what the, this bending moment will behave? So, if you see the resultant of the bending moment, which is, you see, you know, like on the left end force, the bending, this whatever, you see load is coming here and the reaction forces are going in this direction. So, they, they have a tendency to bend this particular, you know, like the beam in this clockwise direction.

So, obviously, you see, on the other direction it will be counter clockwise direction. And after a dx situation, what is the magnitude? The magnitude is M plus dM. So, you see, here this M plus dM will move in this way. M will move in this way, clockwise and counter clockwise. So, we have a sagging situation. So, again you see, what under these loading condition, w into dx and under these, you see, the reaction forces. We have a positive shear and the same time we have a positive bending moment.

So, you see, here again the forces acting on a free body diagram, just you see, detach the portion, is loaded beam, are just following the shearing forces on the left hand side is F. The shearing forces on right hand direction is F plus dF. And you see, since you know, like the cross-section of the, this the cross sectional length is x and dx is there.

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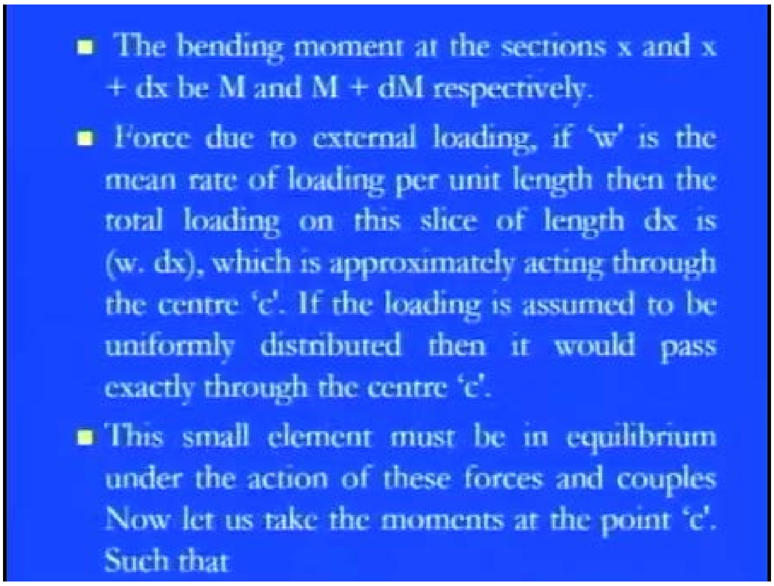

So, corresponding changes are there. So, the bending moment section at x as well as x plus dx action, that is, this left as well as the this, this right hand direction is M. And M plus this, this dM, just you know, like the small section is there. So, forces, due to the external loading, this w F, you see, is the mean rate or we can say, the rate of loading is there.

And if w x is the total load is there, so they are acting at center C, as I shown you in the previous diagram. And if this loading is assumed to be, obviously, we are assuming that it is a, this UDL, that means, uniformly distributed load, then it has an overall impact on a point C, which is nothing but equals to w into dx, as I told you.

And the small element must be, must be in the equilibrium position because whatever the forces, they are coming on the left as well as on the right direction, or whatever the bending moments are coming in the left as well as in the right direction. They are simply counter balancing into each other. So, definitely we can say, that this section is in balance condition.

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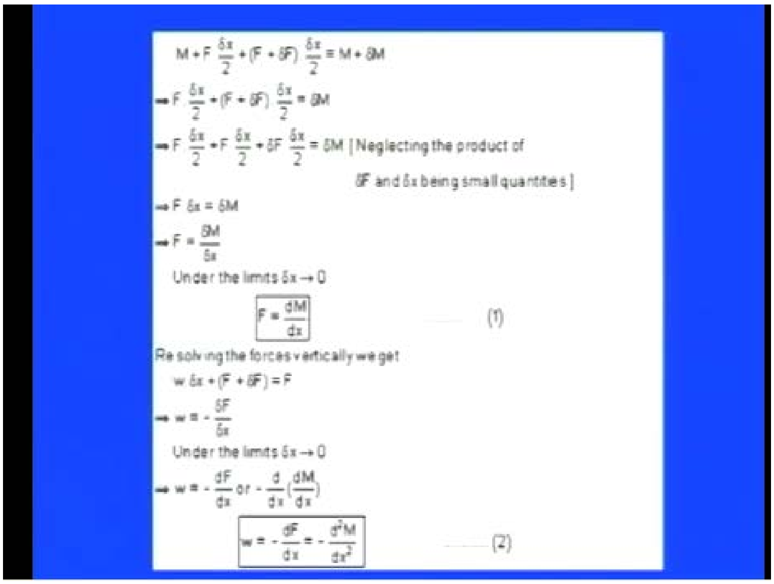

And after applying, but again keep this thing in your mind, that whatever the forces, they are coming, the resultant force is on x direction. The resultant forces F plus dF in the x plus dx direction; they are in this section. So, we have a positive shear force. And as well as you see, you know, like as well this bending moment, it is this M is going in a clockwise direction, M plus dM is going in anticlockwise direction. So, this is a sagging condition and what we have? We have a positive bending moment.

So, with the consideration of this plus and minus sign, now we would like to analyze those things. So, come to the, come to this feature, that what we have at this extreme end? We have M plus, this M is the bending moment, which is you know, like tending to this move, this particular beam into this section. So, M plus this moment is coming due to the shear force, that is the F into, you know, like the center point is there, the C point to this is dx by 2.

So, F into dx by 2 is the moment due to the shear force plus, you see, on the other side you see this bending moment is there, which is coming from the other side. So, plus F plus dF into del x by 2 from the C to this portion, the delta x by 2 is there and on this action, you see, this F plus dF is going in this direction. So, you see it will be in the minus way. So, this plus F plus dF into dx by 2 is equals to M plus dM, which is in the counter clockwise direction.

So, meaning is pretty simple, that you see, here if we have this, an element and if all these, you know, like the moment as well as the shear forces are acting, first of all what we need to say? We, we need to you know, like assume, that under the application of these all forces, our total element is in equilibrium position as once it is in equilibrium position, whatever the forces, which are applying on those things, they are supposed to be balanced irrespective whether the forces or the moment.

So, here you see, we are just going with those conditions and we assume, that since it is a balanced part, so what we have? We have a balanced moment condition. So, either the moments are coming due to the real bending moment or the moments are coming due to the shear forces. They must be well balanced and they must be interact like that. So, this equation satisfied that condition.

And now, you see, again by cancellation these M M and by, you know, like manipulating those things what we have? We have F into dx by 2 plus F plus… So, we can see, that the moment is simply cancelled out by both of the way. So, we have this equation F into delta x by 2

So, on the left hand side whatever the moment is coming due to the shear force, it is there, plus the right hand side moment due to the shear force, F plus dF into delta x by 2 equals to delta M because M has cancelled out. And now we know, that whatever the, this thickness, which we are considering here in terms of delta x or you know, like the delta F, they, they have a very, you know, like less impact on these moment equations.

So, what we need to do here? Whatever the multiplication is coming because of this delta x and delta F, even their impact is very, very less. So, we can simply neglect those terms. So, you see here, what we have after doing this particular, say, that neglecting the product of a small f and the small x because of the small contribution or we can say, towards the total impact or we can say, the total towards the total bending moment or the moment portion also.

What we have after concluding that part? We have F into delta x by 2 plus F into, you know, like this delta x by 2 because of these, these forces, F into delta x by 2 plus delta F into delta x by 2 is cancelled out, as I told you, equals to delta M. So, you see, if I cancel out this, this portion, then we have F into, you know, delta x by 2 plus delta x by 2. That means, F into delta x, which is equals to delta M, which is on the right hand side portion.

So, you see, if I am concluding that part for small section where the shearing forces as well as the bending moment is applied, we have the final conclusion as far as this shear force and bending moment is concerned. F is equals to dM by dx. Or if I simply apply, this dx tends to 0. I have the shear force, which is equals to the rate of change of bending moment in the x domain. That means, you see, what is, that wherever the change of moment is there it will simply apply the shear force on the section.

So, if you just remind those things from this equation. If you, you know, like just from this equation one if you just remember, that we have a cantilever beam and on this particular beam we have the forces, the force application is there from the top up side and the force is coming from the bottom side.

So, meaning is, that wherever the change of the moment is there with respect to x, it always gives you the shearing action and whatever the, due to shearing action the force is coming out, that is F. So, we can say, that F is nothing but equal to dM by dx and it is a very good relation in between the bending moment as well as the shear force.

Because you see it is giving you clear feeling, that wherever the bending moment is there and wherever the change of bending moment is there with respect to this domain x, always the shearing forces appeared in that particular part and we need to calculate the shear force by simply changing dM by dx.

So, you see, now again, just these kind of analysis came when we simply balanced the moment equation. But now, you see, if we balance the vertical forces because you see, if you just remember the figure, then you will find, that we have the vertical forces all along acting on this bending, this cantilever or the simply supported beam.

So, now if we would like to see, that since you know, like the load is coming from the top up side, which is at point C, is w into dx. This is the total load, which is coming from top up side and the shearing force, the forces, which is, you see, are there from both of the side, one side, which is F, one side, which is supporting this part is F plus dF.

So, you see, by balancing those condition what we have? We have one side, which is going upward direction, that is the shear force F and on other side we have, you know, like the shearing forces F plus dF, which is coming towards downward direction. And also, you see, the one force, which is coming due to the applied load, that is, W into dx. So, if you balance those things, w plus dx, w into dx plus F plus dF will be equated by this force F. So, F is equals to, if you see this particular condition, F is equals to w into dx plus plus F plus dF. Or you see, as I told you, you know, like whatever the conditions are there, if we balanced those things, F F will cancel out. So, we have w equals to minus dlF by dx.

And you see, whatever the, you know, like the small segment, which we are considering here, dx is, if it is tending to 0, then you see, we have, you know, like simply differentiate in a real manner. So, we have w equals to, dF by, minus dF by dx or we can say, that it is minus d by dx of dM by because F, you see, from this first equation we can simply replace this. So, we have d 2 M by dx square.

So, the meaning is pretty simple, that whatever the load application is there, the load is coming on point C, you know, like. So, whatever the load application is there, the magnitude of this load can be easily calculated by the first derivative of the shear force with the x domain.

So, whatever the change of force is there in the x domain will clearly give you that what the load application is, means, what is the basic cause of this shear, shear forcing is. So, here the change of force with respect to dx will be equal to F. The negative direction itself, you see, it has a different meaning that what the direction according to which direction you are choosing, this minus sign will come. Or else, we can say, the second derivative of the moment with respect to dx will give you the load.

So, you see, here if we want to correlate, so that is what you see our intention was, you see, that we just to want to correlate the applied load with the shear force as well as bending moment. So, the basic relation is w, which is the applied load, is nothing but equals to minus dF by dx or it is equals to minus d 2 M by dx square, which is a very important relation. Because you see, from this relation, whatever the kind of the beam is there, irrespective of simply supported beam or by cantilever beam, all with the boundary conditions are changing or whatever the kind of loading is there.

Once you take the segment or once you know, that this, this, this is nature of the load, which is due to this actually, the, this shearing or the bending moment is coming or this is the nature, you see, of the load, it is varying like that. Once you know these things you can simply put in terms of, you know, like the force or in terms of moment and you can get the load or you can say, that actually what the intensity of the load is there at the different, different points, like at x equals to 0 or x equals to 1, 2, 3, whatever like that, you can easily get those values. If you put those values in this dF by dx or this d 2 M by dx square and if you can simply put those boundary conditions as well.

So, now you see we have the two basic conditions, two basic formulas, I should say. Formula one, which gives you the relation in between the shear force and the bending moment and formula two, will give you the relation in between, not only in between the shear force and bending moment, but also with the applied load w equals to the dF by dx or we can say, the d 2 M by d x square. And minus sign is always, you see, gives the direction of these forces.

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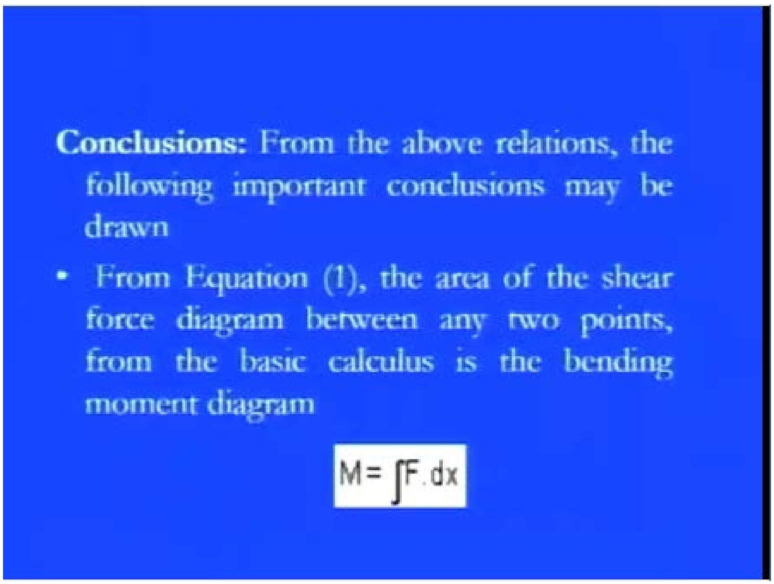

So, now we would like to conclude that part. From the above equations, whatever you see, the two equations, the following important conclusions we can simply drawn. First, the equation was, which, which was correlating the shear force and the bending moment. So, from the equation 1, the area of the shear force diagram between two different points from, you know, like the basic calculation, we can simply calculate the bending moment from, that is, bending moment is nothing but equals to integration of this F into dx.

That means, you see, what is the intensity of the force is into, if you multiply with the total, you know, like where this force is applied, means, what is the effected. This is, the segment is there, you can simply get the bending moment. That means, you see, you know, like it is pretty simple, that if you know the force and if you know, that what is the point of application of this force is there, you have a bending moment. So, always keep this thing in your mind, that actually if a beam is under the various application of load, always bending moment is there and it can be calculated by simply integrating the applied shear force, or what is the, this resultant shear force into the distance, whatever the distance is there.

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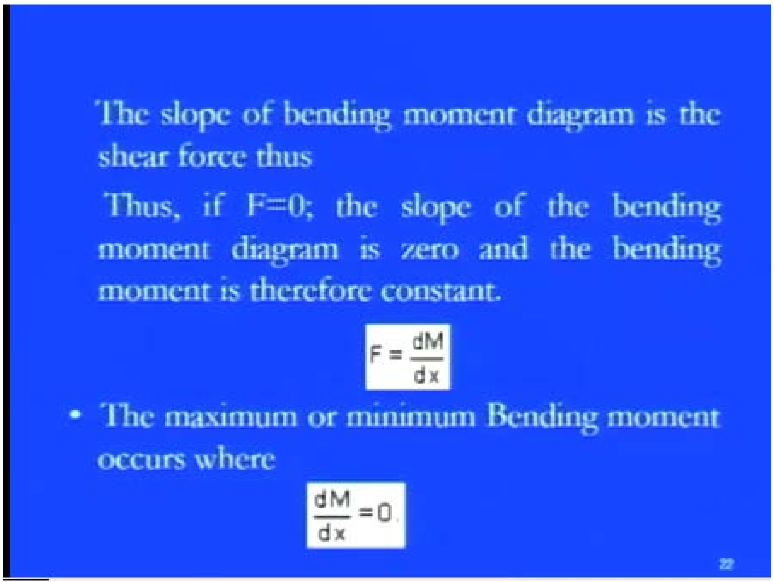

The slope of bending moment is nothing but the shear force; that is the key part. If you, if you know the slope of the shear force, that means, if you have the value of M and if you just take the slope of that, you have the value of the shear force. Means, you see, if we have F equals to 0 that means, the slope of bending moment diagram is 0. The bending moment is nothing but the constant.

Or we can say, you know, like from the first equation, F equal to dM by dx. If I know, that the slope is 0, that means, dF, dM by dx is 0. We have the 0 shear force. Means, you see, if there is no slope, that means, if you see, you know, like the projection of these particular bending moment on these cantilever beam is 0, then you see, there is no shearing is occur in between and due to these particular forces they are simply counter balancing. That means, there is no eccentricity in between these applied and the reactive forces and the maximum or minimum bending moment occurs always when dM by dx is equals to 0.

So, that is what you see, you know, like we can simply get. Just remember these conditions because whenever we apply, you know, like these condition on either shear force or bending moment diagram, these are real, you know, like applications for calculating these parameters.

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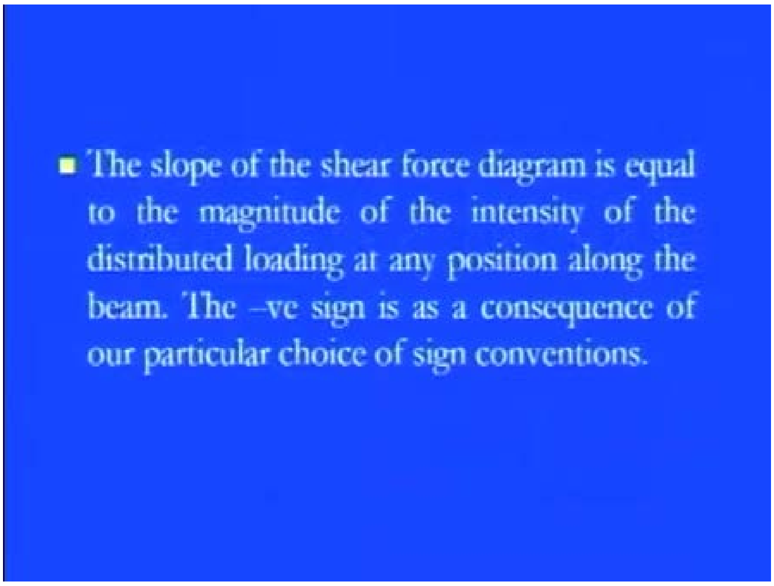

And then you see, you know, like the last part the slope of a shear force diagram is equal to the magnitude of the intensity of distributed loading. Just keep this thing in mind. And whatever the slope is there, you see, of the shear force diagram, because the slope of the bending moment will give you the shear force, but the slope of a shear force will give you the magnitude of the intensity of the distributed loaded at any position of the beam.

And whatever the sign is coming in the negative sign is simply a consequence of our particular choice of the sign convention, as I told you. Like, if you want to calculate w, which is nothing but equals to minus dF by dx or w, which is minus d 2 M by dx square. Minus is always there, that how you are considering the sign convention or how you are considering the load application on that particular beam.

But the important thing is, that actually what is the slope relations are there in between the intensity of the load with the shear force or between the shear force. And these slope of the bending moment is always gives you a clear cut value and it, it gives you the real feeling of bending moment as well as the shear force diagram, the shear force.

So, you see here, you know, like in this chapter we discussed about, that if you know, like a beam is under the application of the loading always, you see we found, that the shear force is there and also we found, that actually the bending moments are there.

And you know, like here, we know in the last section we found, that there is a straight relation in between the load application, shear force and the bending moment and also, we found, that we can simply get the sign convention by, you know, like just watching, that how these forces are acting on a beam.

If you see, you know, like the two forces are acting on a beam and they have, you see, on the left end direction if they have, you know, like the resultant, which is going on this upward direction. And in the lower direction if you see my sign, this hand notations, this is the positive shear force and this is the negative shear force. So, just keep in, this in mind, this is the negative shear force, if these two forces are just going in this way, the negative shear you need to consider. If they are going in this way, that is the positive shear force.

And then as well as the same sign convention, which we discussed, that if the bending moment is, you know, like sagging part is there. If the bending moment is going in this way, means, if the beam is going on the bending side in this way, sagging portion, we have a positive bending moment. And if it is going in this downward direction, then you know, like towards, that is clockwise, as this clockwise and anticlockwise direction, you know, like we are saying, that this is the negative bending moment.

So, this part we discussed and in the next, you know, like lecture we would like to discuss, that if we have, you know, like the real numerical values, that if we have a beam, which is under, you know, like the point load or if we have this, this uniformly distributed load or if we have a simply supported beam or a cantilever beam, then what exactly these theories are applicable. How we can calculate, you know, like this shear force and the bending moment and also after calculating these values at different, different points we would like to draw the shear force diagram and bending moment diagram to have a clear feeling, that what the variation is there of the shear force and the bending moment or the entire length of a beam if this beam is under the application of variety of the loads. So, you see, here you know, like in the next lecture we would like to discuss this kind of issues.

Thank you.