Strength of Materials
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Lecture – 21

Hi, this is Dr. S. P. Harsha from Mechanical and Industrial Department, IIT Roorkee. I am going to deliver my lecture 21 on the course of the Strength of Materials, and this course is developed under the National Programme on Technology Enhanced Learning (NPTEL).

Prior to start this lecture, I just want to refresh the prior discussion as we discussed in the previous lectures that, you know, when we have a circular shaft or any circular bar is there, and whenever, you see any twisting moment is applied on that, or any torque is applied on that, then, you know like, the shear deformations are there and this circular bar is under the pure state of shear stress. And due to the shear stress, you know like, it is always there starting from the center position and it is going maximum on the circumferences. So, shear stresses are always maximum on the top of the surfaces of a circular bar - that we proved.

And then, you see, you know like, we found that whenever the shear stresses are there, since these load application or we can say the torque application or any twisting moment application is just within the elastic deformation - this assumption we made; this assumption we made basically, and we also, we made that… we put the assumption that whatever material which are using that is purely a homogenous material.

So, under those conditions, we simply apply the Hooke's law, and we found that whatever the shear stress is coming due to that the shear strains are there, and with those things, you know like, there is this angle of twist is there, and we would like to relate all those coefficients together with the using of the generalized Hooke's law. So, with that we formed the basic equation for the torsional part - if you remember the T by J equal to tau by R is equal to G theta by L.

So, that, you see, you know like, we found that simply whenever we are discussing about the tau, it is absolutely related with the shear modulus of this inertia. And then, also we discussed if any shear stresses are increasing with the radius, obviously, it is going, starting from the center and going towards outward direction with the maximum value. Or also, we found that actually whatever the shear modulus of rigidity is there always associated with the how much angle of twist is there. And if we want to, you know like, measure the whatever the angle twist or we can say the deformation or distortion due to the shear stresses, we can simply measure with the using of the shear strain.

So, with that, you see, you know like, we after that we, you know like, discussed that whenever this kind of shear deformation is there, then how we can, you know like, correlate with the using of shear modulus of rigidity. So, you see, we found that if we have a continuous bar, then this equation is very, very well valid. But if we have, you know like, this solid circular bar or hollow shaft, then what exactly the comparison is; that means, you see, if you are using the same shaft, just one, with the same diameter and the length is same, but one is the solid shaft and one is this hollow shaft - or we can say the circular shaft in which there are two diameters - then we compared both the things with the same diameter and with the same material.

And we found that only the difference what the two diameter, and we found that there is, you see, you know like, the shear stress is maximum, you know like, the 6.6 percentage shear stresses are maximum in, you know like, hollow shaft as compared to the solid shaft.

And also we, you know like, discussed that if we cut the solid shaft exactly from the middle point, and if we remove that portion to make the hollow shaft, then, you see, you know like, the weight comparison there is a total 25 percent weight reduction is there, but the shear stresses are more - 6.6 percent more. So, meaning is pretty simple that actually if you would like, you know like to design a circular shaft based on our weight criteria, then always we would like to go irrespective of whatever the shear stresses are then we would like to choose the hollow shaft.

But if our design criteria is absolutely based on that shear stress is there, and, you see, whatever, you know like, the applications are there, and due to these applications, if shear stresses are dominating in the nature and we want to design the our circular shaft against those shear stresses, then probably we would like to, you know like, choose the solid shaft irrespective whatever the weight criteria is there.

So, that means, you see, you know like, we can compare those parts and then we discussed that actually if we have not the uniform bar - that means, not the uniform prismatic bar, but if we have, you know like, the stepped bar, means, you see, two or three different diameters are there, then how we can, you know like, find it out these, you know like, the torque as well as these, you know like, if the two different torques are there or three different torques are there altogether, then how we can find out the deformation or we can say the angle of twist and then corresponding, you say, since we are measuring on the basis of the shear strain, so how we can find it out the total shear strain and based on that actually how we can, you know like, set up the relationship.

So, in that we found that instead of because it is a different, different areas are there, effective areas are there, and you see, since we have the different effective areas, so obviously, you see, the J value will be different based on that T value is different. So, probably, you know like, we would like to, you know like, put the summation criteria that for small, small segment take one part. So, second part, third part, and sum up all those parts to get the final value.

And then, you see, in the final version, we discussed in the previous lecture that actually instead of the stepped bar, if we have a continuous bar, but the shape of that bar is in a tapered section, that means, there is a continuous bar. So, instead of, you know like, taking the different, different segment, what are we doing here? We are simply taking a small section at some point. We can say generally we used x distance from the left hand side of the depth of the dx. So, you see, we would like to check that actually what exactly the deformation is there of that particular part, by taking T by J equal to G theta by l and then put, you see, just we will calculate the theta x for that particular segment.

And then, instead of putting the summation action, what we are going to do? We are simply, you know like, using these integration sign and this, since it is the… whatever, you see, the cross sectional area is there, it is, you know like, uniform all across this particular bar. So, we simply put the zero to this - whatever the length of this area is there, the area is there - and simply we put the integration sign within the dx formation.

So, pretty simple, that actually if we have the stepped bar, then you better use the, you know like, the sigma, the sigma notation for individual sections and if we have the this tapered section bar in a uniform cross sectional way, then probably you are going to use the integration sign.

So, you know like, all those discussions were made on that, if you see, we have a bar irrespective of the prismatic or non prismatic bar and it is under the effect of twisting moment or the torque, then, you see, you know like, the T by J equal to tau by R equal to G theta by L. So that we are using, and if you want to calculate the maximum shear for either the solid shaft or circular shaft, then, you see, tau maximum we calculated at 16 times this torque, which is applied, divided by this d cube.

So, you see, you know like, the shear stress is maximum with the torque applied as … usually you see if more, more torque is there, more shear stresses are there, and second, you see, it is a reciprocal relation with the diameter. So, this tau maximum is inversely is proportional to 1 by d cube. So, you see, you know like, we set up those relations and we found that actually how the shear stresses are, you know like, distributing if we have this solid circular bar or this hollow shaft. So, this kind of discussion we made in the previous lecture.

Now, you see, you know like, we would like to continue this concept and we would like to apply this concept on the spring; spring as an element. So, now, you see, here, if we have a closed coil helical spring, means there are various forms of the springs, which we are going to discuss in our lecture. So, first of all, you see, if we are talking just general spring, which is a closed coil, you see, because, you see all the coils are closed; means, closed right from beginning to end, and if it is subjected to an axial load, then what will happen?

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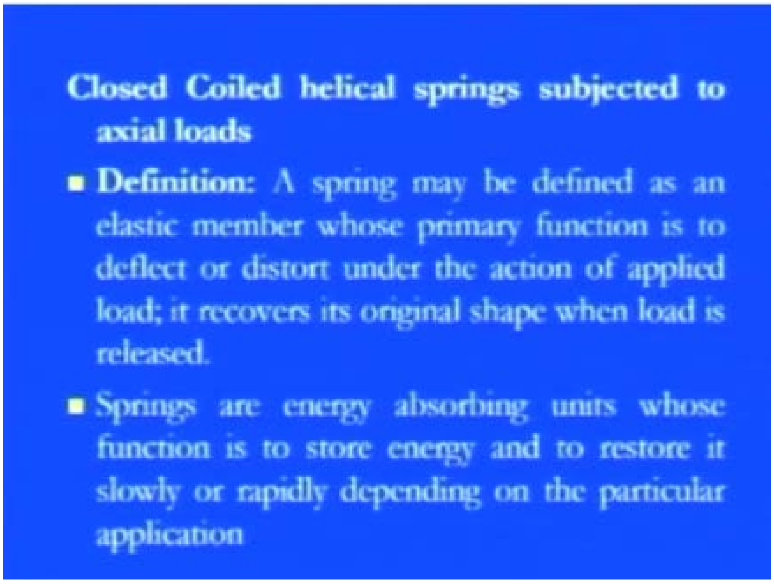

So, you see, first we would like to define a spring, that a spring may be defined as an elastic member whose primary function is to deflect or distort under the action of applied load, and it recovers its original shape when load is released. That means, you see, you know like, whatever the spring is there, the thin spring, thick spring, or whatever the diameter is there, always we assume that whatever the deformation is there under the application of any load on the spring, it is just the elastic deformations. So, once you release the load, the spring comes to its original shape; there is no permanent set of deformation will be there within the spring format. So, this is the basic definition of a spring.

And the basic function of the spring is such as the spring are always energy absorbing units whose function is to store the energy. So, you see, when you apply any load is there, so it is simply stressed. So, whatever the energies are there it is simply converting into the kinetic one. So, once you release the load again, it will simply squeeze into original form. So, whatever the energy is there, it is simply converted into the potential one. So, you see, it is simply absorbing the energy when you apply the load. So, you know like, it is a kind of storage; it always… spring is device in which you can store the energy.

So, to restore it slowly or we can say rapidly depending on the particular application. So, what kind of applications are there accordingly the spring acts. So, the meaning is pretty simple, that actually spring is always acting in an elastic deformation and the basic purpose of the spring is to restore the energy. So, you see here, if you want to use the energy in any of the function based on what its application is there, the spring is acting correspondingly. So, now, you see, you know like, as I told you that we would like to define that what types of spring are there.

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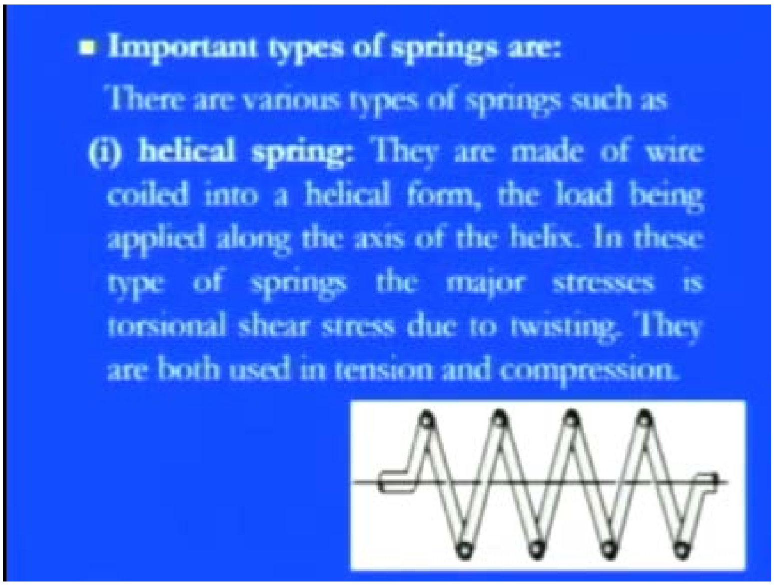

So, there are, you see, various types of springs such as, first we are saying as usual that helical springs are there; that means, the helical groups are there in form of the coils of the spring. So, they are made of the wire coiled into a helical form. So, you see, helix angles are there at a particular thing, you see, just if you cut the section from the center sign, then we will find that there is a helix angle at a particular, you know like, these axis form. And the load being applied along the axis of a helix. So, you see, whatever the load application is there, it is always just, you know like, applied along the particular axis of the helix.

So that, you see, whatever the elongation or the compression is there, it is along with that particular axis of the helix only. So, in these type of a springs, the major stresses is the torsional stresses due to the twisting. Because, you see, whatever the kind of, you know like, the moment or we can say the couple is there or any kind of twisting is there due to that the first stresses which are forming in these kind of springs are only this shearing stresses or we can say the torsional shear stresses are there. And they are both, you see, you know like, used in either in the tension side or in a compression side. Because, you see, you know like, they are formed along with the helix part just, you see, along if you go with these axis, so whatever the compression, you see compression is there or whatever the extensions are there always due to these kind of actions, the kind of stresses which are occurring in these coils - you see, you can see these spring format - the kind of stresses are there in these, you know like, the elements of the spring always it is torsional shear stresses.

And then, you see, you know like, it all that what the sustainability is there of these kind of spring simply based on how these joints are functioning. So, you see, when you are stretching these, you know like, joints are simply coming together, they are simply closing together, towards the helix angle they are moving. And then you see, they are simply restoring the energy in this form or vice versa is there. When you are in compression then, you see, it is simply, you know like, consisting the energy and they are just going apart from, you know like, these angle of the… this axis of the helix. So, this is, you see, the basic form of a spring which is known as the helical spring. And the second form of the spring is the spiral spring. As per its name, you see, it has a spiral coils in form of, you know like the spring, and again you see, it is functioning exactly as the normal spring.

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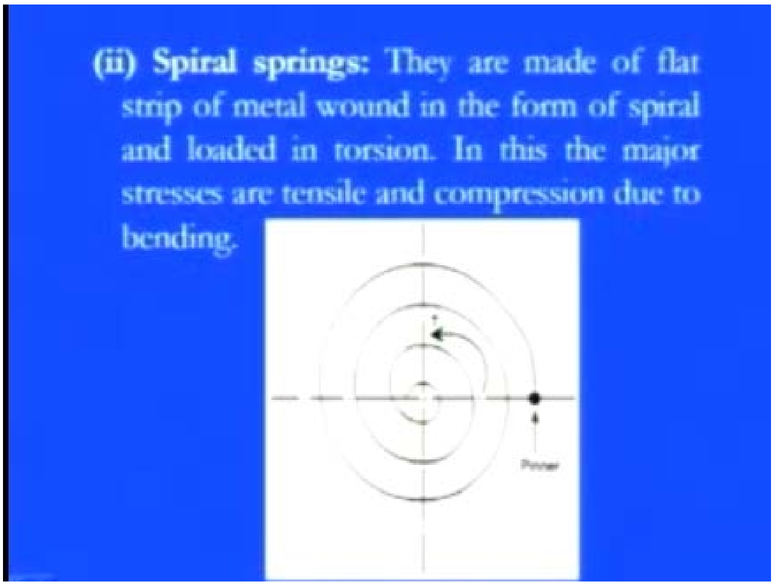

So, they are made of , you know like, the flat strip of a metal wounded in form of the spiral and, you know like, it is loaded in the torsion only. So, in this, you see, the major stresses are tensile in compression due to the bending. So, you see, if you see, this particular figure, you will find that what we have, we have a simple, you know like, the strip of metal, which is wounded in form of a spiral one. So, that is why, you see, it is known as the spiral spring. So, you see, whenever a torque is applied, you know like, the kind of squeezing is going on if we are turning in a clockwise direction or if we are turning in a anticlockwise direction, you know like, these are just coming outer wards.

So, you see, irrespective whether it is in the tensile form or it is in the compressive form, but always the bending action is happening on these kind of springs. So, you see, if we have a… means if you apply the load, you see, it will simply, you know like, the load is transferred from this way and it is always, you see, transferring like these things. So, you see, it will just try to expand. So you see, we have a tensile form or you see, if you simply stress this kind of thing here if you stretch in this way, so you see, these kind of load transfer is there along this particular line, and all those, you see, these segments, they are simply affected by this kind of load application and they are trying to squeeze, so the compression is there.

Meaning is pretty simply, that it is always being loaded by torsion, but the bending action is forming due to the bending, you see, we have either the tension or the compression forms of the stresses. So, you see, here we have, you see, you know like, all the kind of springs, but they are under the action of the torsional way only.

And then, you see, if you go for the next spring which is very common, you know like, application in all kinds of vehicle is the leaf spring. So, you see, if you see any kind of vehicle, you will find that we have a leaf springs, just on the bottom of these vehicles and they are, you know, perfect absorber of the energy. So, whatever the shocks are coming because, you see, you know like, they are simply absorbing the energies whatever the shocks are of the impulse are coming from the road or of the jerks are coming from the road towards the, you know, vertical direction on the vehicle, they are simply absorbing those shocks and whatever, you see, the material or even the human beings are sitting, they are feeling more comfortable because of the shock absorber. So, we can say that these leaf springs are a perfect absorber as far as the vehicle is concerned, vehicle design is concerned.

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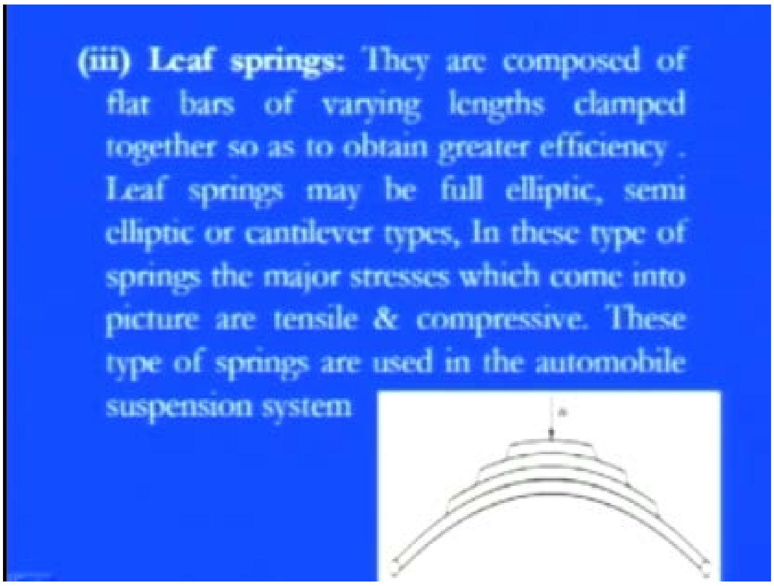

So, they are composed of… if you are talking about the leaf spring, then they are composed of flat bars. So, you can see here, what we have. We have bars here. So, they are, you know like, the flat bars are there in… we are simply, you know like, composing those things only the main key feature is that they are having varying lengths So, you see, at the bottom you will find that the, you know like, this bar is of the maximum length, then we are simply reducing the length and, you see, they are simply just attached on each other. So, that whatever the jerks are coming they have simply compression in this proper way.

So, the meaning is pretty simple that leaf springs are always composed of the flat bars of varying lengths as shown in this diagram, clamped together so as to obtain a greater efficiency. So, you see, you know like, if the compact design is there, whatever the jerks are coming, they can simply be distributed all amongst these, varying length cross sectional, you know like, the composite bar and it can be easily absorb those kind of shocks. Leaf spring may also be, you know like, fully elliptical, semielliptical, or cantilever type.

So, you see, we have three basic types of the leaf springs are there based on what kind of applications are there, a corresponding, you know like, the leaf springs are usable. So, generally, you see, we are using the fully elliptical for, you know like, the bigger vehicles. But if we have, you know like, the small vehicles then probably you can use the semielliptical sometimes or, you see, for different application the cantilever types of leaf springs are also usable. In these types of springs, the major stresses, which come into the picture, are the tensile and compressive. So, you see, as usual as you have seen in the previous helical springs or, the you know like, the spiral springs, again in this kind of springs also - in the leaf springs - we saw that the major contribution of this stresses are due to the tensile and the compressive. And these type of springs are used in the automobile suspension system. So, they have are perfectly good for automobile, this automobile industries or we can say the vehicle system where we can simply put this leaf springs to get the perfect absorber.

So, you can see in this particular diagram we have, you know like, the varying length of the cross sectional bar and whatever the load applications are there it is simply, you know like, distributed amongst that. So, whatever the stresses are being formed we can say that actually, we can perfectly design the safe spring based on what the number of coils of springs are there; means how many number of, you know like, the bars are there. And how these, you know like, the compactly designed irrespective to each other like this to this, this to this, and this to this.

So, this is, you see, you know like, the third form of springs. So, now, you see, we discussed about the three basic forms and all three basic forms irrespective of if you are talking about, you know like, the helical spring, or if you are talking about these leaf spring, or if you are talking about this spiral spring, they have their own application. Though you see, the stress formation on these kind of stresses they are very common, but the applications are different.

So, you see, you know like, if you are, if you are simply watching that, you know like, the axial loads are predominating. So, definitely, you see, we are going for the helical spring. So, that whatever the stress formations are there, we can clearly design those springs and we can see that actually how these stresses are being forming and what kind of deformations are there. But if you are, if you are talking about the impact or the jerks or this kind of impulsive forces, then probably we are going for this helical springs, this leaf springs as we discussed in the previous section.

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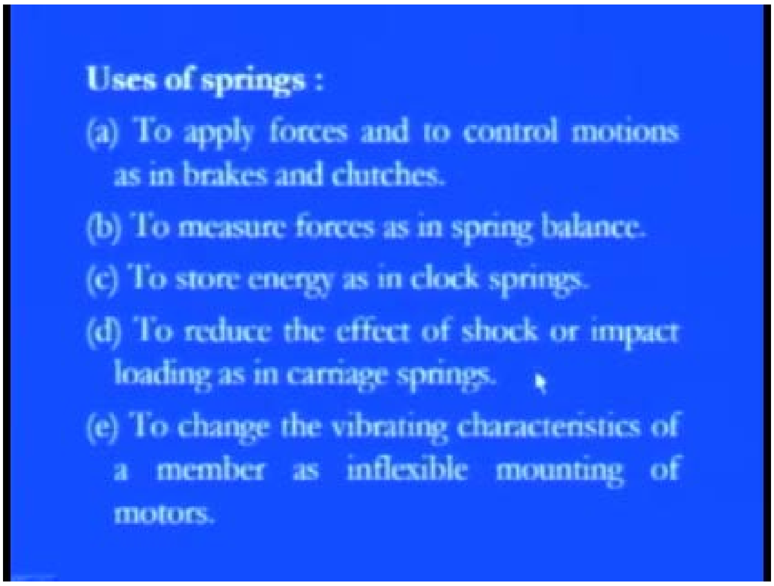

So, now, you see, we would like to see that what exactly the uses of the springs are. So, first, you see, to apply the force, to apply the forces or to control the motion as in brakes and clutches. So, you see, if in any of the automobile the two main, you know like, the devices are there through which, you see, you know like, either we can apply the kind of this braking forces or we can say to just shifting the power transmission either the brakes or the clutches they are absolutely based on these springs.

So, you will find that if you want to transfer, you know like, the power from one gear to another it is simply, you know like, the spring device is there which simply, you know like, freeze; we simply, you know like, gives you the free kind of motion in between these parallel shafts of the gear. Or we can say whenever we are talking about the brake when you push the brake, you see, these springs are there which control the motion of your foot. So that it is not that actually when you apply the brake immediately it simply goes in impulsive way and suddenly it stops, no. It is going in a very smooth way, because when you apply your force, it simply gains the energy, and this energy is simply releasing, just releasing in form of the transformation.

So, this is the basic use of these springs, that actually, they simply apply the force or we can say rather if they are simply transmitting the force and they also control the motion in form of either brakes or we can say in form of the clutches.

Second use is to measure forces as in the spring balance. So, you see, this is the perfect use. Wherever we are, you know like, balancing those things always if you see, any kind of, you know like, the balancing machine it is simply based on the spring. Or if you want to even measure the force always spring is there, because F equal to k x n. Always, you see, that actually what the stiffness of the spring is there and how, you see, you know like, these deformation is going on under the application of this force. Once you know the deformation, once you know the stiffness, you can find out the force. Or you can see, you see, whatever the weight measuring machines are there, always they are simply based on the spring balance.

So, if you want to measure the force, so if you want to measure the weight of any person or we can say any object, then these, you know like, whatever the machines are there they are absolutely based on these springs. Or we can say if you want to measure the force spring balance is the basic concept to measure those kind of forces. So, this is the second use of the spring.

Third is to store the energy as in clock spring So, you see, here you know in now-a-days, you see, all these clocks are basically based on the batteries, but if you go for any older clock, simply you see, the coiled spring, the spiral spring was there, those were, you know like, the flat foils are there and they are simply wounded in the coil form, and we are keeping those things. So, once you squeeze that, you know like, by the key, they store the energy and, you see, corresponding the release of their energy according to the minute or we can say the hour, or in particular, you know like, these the simultaneous relation is there in between the hour and the minute, those clock wires.

So, meaning is pretty simple that if we have the dial of this clock, and if we have, you know like, the two indicators for - irrespective of whether the minute or hour - simply they are getting the energy from those springs only. And, you know like, there is a perfect use of these application of or we can say these springs are there that actually they are simply restoring the energy and they are releasing as per their use.

And the fourth use is to reduce, you know like, the effect of the shock or impact loading as in the carriage springs. So, you see, you know like, the perfect use as we, you know like, discussed in the previous section, but if we have, you know like, the leaf springs then whatever the shocks are coming or whatever the impacts are coming from any of the input excitation, not from this roads, but from many of the input excitation, it can simply absorb those shocks and, you know like, whatever the load transmission is there to the upper section is very smooth; it is linearly varying. So, that is why, you see, we are using in all those vehicles, in the automobile particular, these kind of leaf springs to absorb the shocks or impact.

And the last use is, the main use is here, to change the vibration characteristic of a member as inflexible as this mounting of a motor. That is a perfect use, in main, in these industry particular, that if you want to, you know like, there are various vibration characteristics are there. And if we would know the basics of vibration then, you see, I just want to refresh those vibration concept that whenever we are talking about the vibration, the main phenomena is coming that actually, you know like, that how the displacements are there. The displacement means actually how this, you know like, they are varying from their own position.

So, if you are talking about the vibration any single degree of freedom or two degree of freedom, then the main equation is coming that mass into acceleration plus stiffness into displacement plus damping into velocity is equal to whatever the external excitation is there. So, here, you see, if you are talking about the spring, then how much deformation is there in any of the particular section like k k into x, k into x is there. So, whatever, you see, you know like, the stiffer part is there and how the displacement is coming in that part they are simply measuring.

So, you see, if you want to change the vibration characteristics, always we are putting the springs as an absorbed part. So, that, you see, you know like, whatever the vibrations are coming it can be easily absorbed or we can say we can simply restore the energy. So, we can simply change the vibration characteristic of a member as in a inflexible mounting of particular motors, in terms of the electrical motors or we can say, you know like, the induction motors are there through which the power generation is. So, these are the basic uses that is why, you see, the study of the springs are very, very important.

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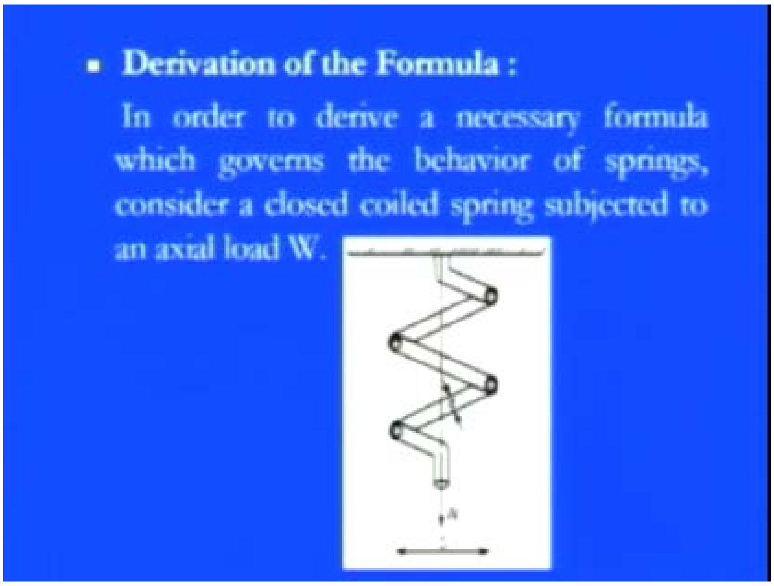

Now, you see, we would like to derive the formula for a spring. In order to derive the necessary formula, again, we would like to first see that what the exact mechanism is there of a spring and if under the load application how they are deformed. So, you see, here we are, you know like, if, you see, this particular figure they will find that we have a spring which is simply, you know like, hanged on a particular this wall. So, this is my datum on which this spring is hanged. And if you apply the load in towards the lower direction - this w - then there is a kind of expansion is there. So, now, you see, you know like, we would like to see the behavior of the spring just considering a closed coiled spring which is, you know like, subjected under the radial load W.

So, as I told you, you know like, whatever the deformation or the extension is there it is just along the axis of the helix. So, you see, here we have the diameter and whatever that, you know, twisting form is coming, it will form or the shear stresses are coming, they are coming in form of these coils. So, you see, we would like to see that what will happen exactly. So, we have, you know like, first of all we would like to define the terminology - basic terminology - or, you see, basic parameters of this particular spring. So, we have used like this capital D is there, small d is there, w is there. So, we would like to first define those things.

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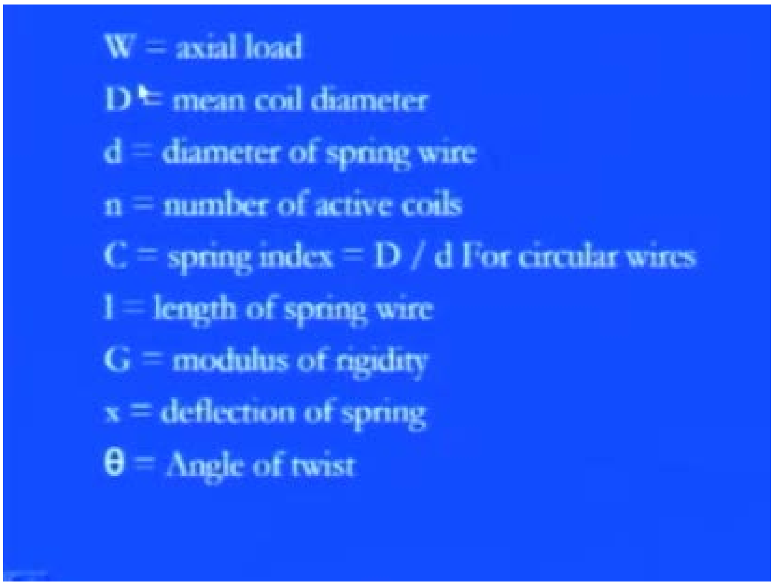

So, here it is, you see, as I told you w is nothing but the axial load which is in terms of the Newton always, because we are applying at the extreme corner - the bottom corner - of the spring. So, how much, you see, you know like, these radial load is there in terms of Newton and due to that actually how much deformation is there in that particular spring, that is the matter of concern.

Second as I, you know like, show you on the bottom that we have a capital D. Capital D is nothing but the mean coiled diameter from one junction to another junction. So, you see, you know like, we are simply putting the cross section and we are measuring that what exactly the capital D is, because you see, whatever the twisting is coming, it will come along this particular D or we can say the mean coiled diameter.

Then we found that we have the small d is there in those particular junctions. So, that small d is nothing but the diameter of the spring wire, because you see, all those helix are coming with those junctions of the spring wire. So, we would like our interest, because our interest is there to see that what the diameter is, because you know like, the shear stresses are coming all along with this small diameter.

Then, you see, we have the n, which is the number of active coils; that how many coils are there; means actually how many, you know like, these two junctions are there. So, our main interest is the number of, how many number of coils are there. So, if you want to design any spring, the n is also an… it is also taking an important… this is playing an important role to design those things.

Then we have one constant that is the spring index, which is nothing but equal to D by d for a circular wire. So, if we have a circular wire, then we can simply calculate the spring index, which is nothing but equal to mean coil diameter divided by this diameter of the spring wire.

And then, you see, we have, you know like, if we stress those, you know like the, all those wires you have the total length of the spring wire which is nothing but the small l. And since, you see, whatever the deformation is there on the spring, under the action of these W, whatever you see, you know like, the shear stresses are coming or shear strain is there as I told, you know like, we are only interested to go up to this elastic deformation or we can say the generalized Hooke's; Hooke's law is there. So, for that we have this shear modulus of rigidity is there. So, as usual, you see, we are showing by G.

And then x, you see, because of the load application there is a deflection. So, how much deflection is there we are measuring with the x. So, x is the deflection of a spring.

And theta is the angle of twist that actually how the twisting is there if the shear stresses are acting on that particular way. So, now, you see, by defining all those terms, now when the spring is being subjected to an axial load as I shown you in the previous diagram, the wire of the spring, you know, gets to be twisted like a shaft.

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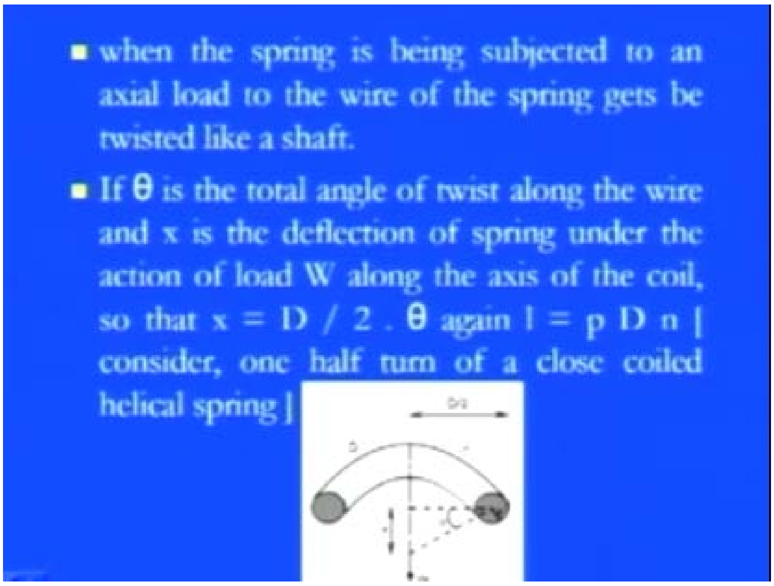

So, as you can see this particular diagram, we will find that whatever the wires of the springs are there, which has a diameter of, you know like, the small d in between and the total capital D diameter is there. So, it is simply twisted, exactly like the shaft. So, you can see in this particular diagram, that we have a twisted shaft, and the angle of twist which we are measuring is the theta. And, you see, whatever the deflection is coming in that spring is simply x. So, this is the load application and this is the deflection is there and if you want to measure the deflection, what we have? We have the angle of twist.

So, you see, if I am saying that this particular diameter… because the total diameter is D which is the, you know like, the mean diameter is there of the coil. So, probably we can take this D by 2 here. So, now, you see, if the theta is the total angle of twist along with the wire, and x is the deflection of the spring under the, you know like, action of this radial load w along the axis of the coil. So, we can say that whatever the x is coming this by simply, you know like, this radius, the radius equal to r divided by, you know like, that how much deflection is there. So, by that formula we can simply calculate that x is nothing but equal to this total D by 2 into theta.

So, this D by 2 into theta will give you the total deflection or we can say that if we are simply stretching that, you know like, this spring up to the total length, and total length is nothing but equal to how many number of coils are there into, you know like, the p into D. After considering those things, you see, you know like, for one-half turn of the close-coiled spring we can say, that the theta which is the angular deflection, is nothing but equal to 2 x by D.

So, you see, here just remove this formula that whenever, you know like, under this action of this radial load towards the downward direction if any angle twist is there and the deflection x is there, then the angle of twist of that particular spring part is nothing but equal to 2 times of the deflection divided by the mean coil diameter. So, we going to use this particular formula in the next slide.

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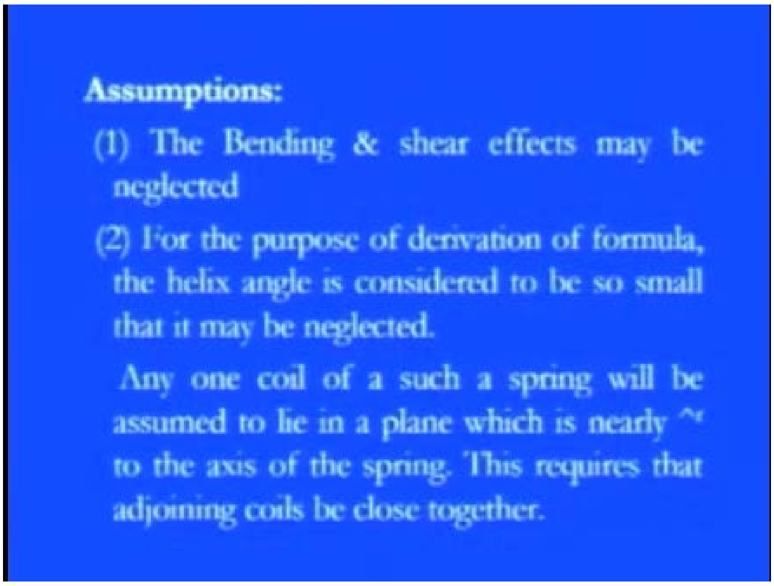

So, now, you see, before starting the analysis, you would like to put certain assumptions here. So, the first assumption is that we are simply neglecting this bending and the shear effect in that; because right now, you see, our main concern that whenever the load application is there, due to that load application how the tension and the compression is taking place in that. And, you see, due to that, how shearing parts are coming along the helical part. So you see, our main concentration is just in the shear stresses of those kind of springs. So, you see, we are simply neglecting the bending as well as the shearing effect in those things; though, you see, they are acting here, but to make a, make our, you know like, easy analysis we are ignoring that part.

Second assumption is for the purpose of the derivation of the formula the helix angle alpha - generally we are using the helix angle alpha - is considered to be so small that it can be neglected, because you see, since, you know like, the shearing part is coming along the axis of helix. So, definitely there is a deviation is there in the helix angle. So, we would like to measure that part, but it is so small that as compared to the other this angle of twist, so we can simply neglect that part.

So, you see, any one coil of such a spring will be assumed to lie in a plane, which is nearly perpendicular to the axis of a spring. And, you see, this requires, you know like, the adjoining coils of a… just whatever the adjoining coils are there, this simply requires to coming these coils to close together. So, we would like to see that actually what exactly the impacts are, impact is there under the action of these radial load, and how this shearing stresses are being formed in that kind of closed coil.

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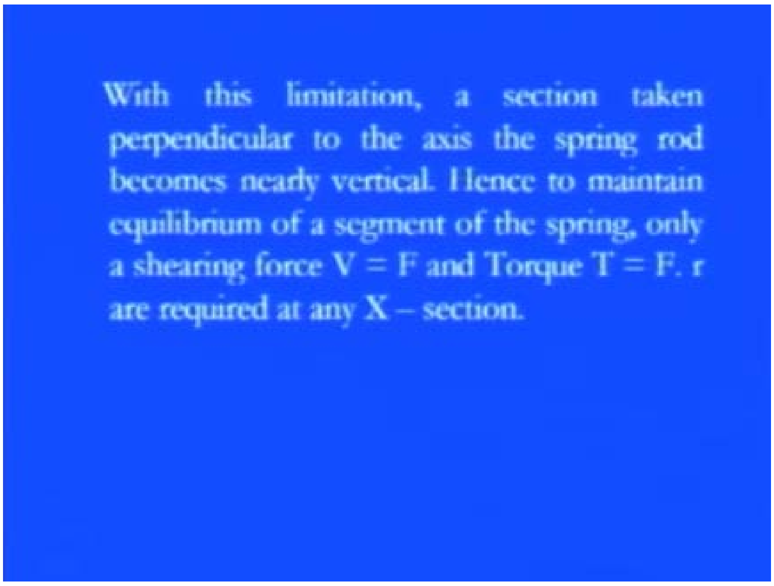

So, we have seen the limitation. So, within this limitation a section has to be taken perpendicular to the axis of the spring rod, which becomes nearly vertical. So, what has happened, you see, we just want to maintain the equilibrium of a segment of the spring, because if they are coming too close together, you see, always, you see, we just tried to maintain that actually it should be there under whatever the shearing action are there, shearing action is there, it has to be there within the equilibrium position of that particular segment of the spring.

So, only a shearing force V equal to F we can say, V the shear force and so whatever the force is applied, we are taking as a shearing force V. And the torque which is coming due to the shearing forces F into r are just required at the X section, you see, we simply curve the X section.

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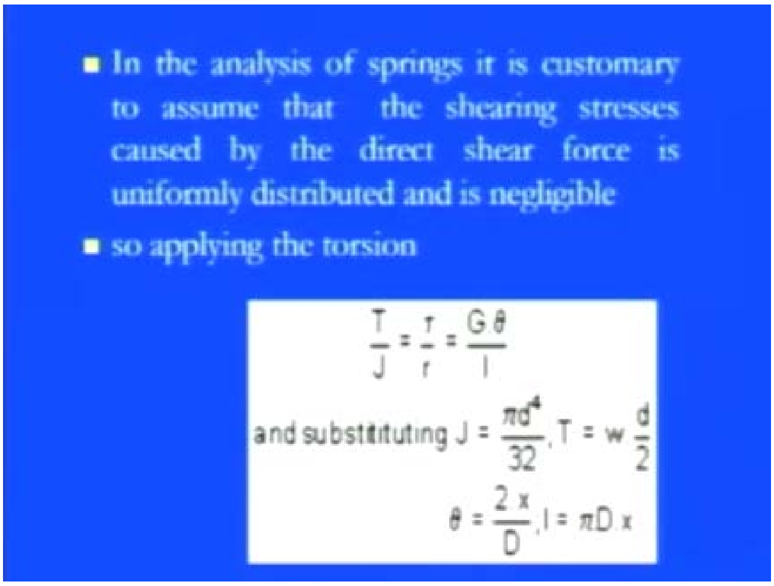

So, once, you know like, you did it that the shearing action, we can say that whatever the complementary shearing stresses are coming, they can simply maintain the equilibrium of that particular segment of the spring. So, in the analysis of the spring, it is customary to assume that the shearing stresses caused by the direct shear force is uniformly distributed and it is negligible, because, you see, if it is not uniformly distributed then we have a, you know like, the stress concentration for that and due to the stress concentration always there is a uniform shear deformations are there or distortions are there. If you want to measure the shear strain probably, you see, this is the weaker section is there where these cracks or the spares are there or it if it is not uniformly distributed.

So, you see, we need to be very careful that actually whether the shearing stresses which are inducing due to the application of these kind of forces, it has to be uniformly distributed. So, you see, once we know that actually, the shearing stresses are there and under the action of this torque, we know that the standard equation is nothing but equal to T by J, which is equal to tau by r is equal to G theta by l.

So, now we would like to substitute whatever the different components of the, you know like, these particular equations are and if you want to compute those things for a spring element, then we can simply compute as the J, which is the section modulus of, you know like, these inertia is equal to pi d 4 by 32 where the d is, you know like, the spring wire diameter is there, because the shearing is coming all across the axis of the helix. So, this diameter is the effective under the action of this torque.

So, this is, you see, the G is there, the J is there, pi d 4 by 32, and the torque which is coming is nothing but equal to when you apply the load and it is just at the d by 2. So, w into d by 2 will give you the torque. And as I told, you see, under the action of these, you know like, the load we have the deflection, and the deflection is coming at the angle of - this twisting angle - theta. So, we made already this relation that x is equal to d by 2 into theta or we can say the angle of twist theta is nothing but equal to 2 x by D.

So, it will be, you know like, this theta is equal to 2 x by T and, l which is, you see, you know like, the total length is there which is pi into D into x is there. pi D is the total, you know like, the area into, you know like, once you multiply with this, you know like the x then probably you will be ending at the total length of these things. So, by keeping those, we can simply calculate that what is the spring deflection.

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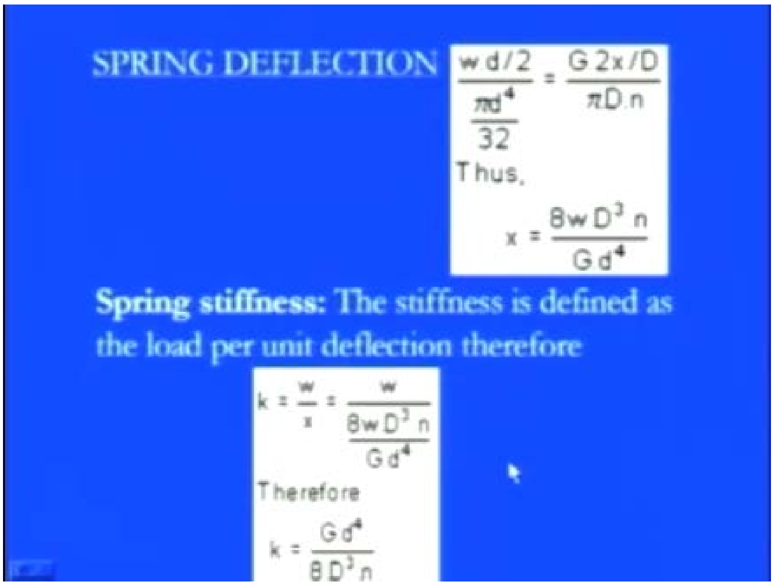

So, now, you see, just we are keeping those values there in the first equation and we would like, we found that w d by 2 which is, you see, you know like, the torque divided by the J, this is pi D 4 by 32 is equal to G into theta which is, you see, you know like, we already calculate the 2 x divided by D divided by pi D into n, that is the l.

So, G theta by l is this value and T by J is this value. So, from that we can simply calculate that how much is the spring deflection. So, this x value will be equal to, you know like, when we calculate these 2 by 2 will cut out. So, these use the 4. So, we have the 8 into w into, you see, the capital D cube, which is you see like, these forms will come into, you know like, the D into the number of coils divided by G into d 4.

So, what we have, if you see this deflection formula it is absolutely depends on that how much load we are applying. So, it is directly proportional to the load applied and then it is the non-linear term, you know like, non-linear variation is there with the diameter - mean diameter of that. So, x is proportional to… w x is proportional to D cube and x is also linearly proportional to the number coils. So, if you have more number of coils and probably, you see, we have, you know like, the extension is pretty more, but it has in it… just inversely proportional to small d. So, if we have more thicker wire, probably you know, it will provide more stiffness and we have the less deflection is there.

So, that is what, you see, x is proportional to 1 over D 4 and, you see, G is the shear modulus of rigidity which is the material property. So, the corresponding deflection is there, if we are you know like, having more this rigid material or we can say the stiffer material, we have definitely the less deflection is; so they have this reciprocal relationship.So, you see, here if we want calculate the spring deflection and this if this the spring is under the application of the radial load and due to this radial load if we have, you know like, the shearing action and due to the shearing action, if we have this angle of twist theta is there. So, we can calculate the deflection, x is equal to 8 times radial load w into capital D, which is the mean diameter cubic, into number of coils and divided by G into D 4 where D is the spring wire diameter So, now, you see, once you have the deflection, then probably you can calculate the spring stiffness, because the spring stiffness is nothing but equal to load per unit deflection.

So, you see, you know like, we are applying the w load and we have the x deflection. So, we can calculate the stiffness k is nothing but equal to w by x where w is the radial load divided by you can put this particular x which is nothing but equal to 8 WD cube n divided by G d 4. Therefore, what we have? We have the spring stiffness K is equal to G d 2 - the small d 2 the power 4 - that is nothing but the spring wire diameter 2 power 4 divided by 8 D cube n.

Please remember this formula because it is very, very important formula for any kind of numerical problems because, you see, when we are solving, you know like, the deflection or the stiffness, you know like, or when we are designing the spring these formulas are very, very important. That under if you apply, let us say 10 kilo Newton load or any kind of load you see, then how must deflection is there and what is the total stiffness is coming after this load.

So, these two formulas are important, once the deflection, which is nothing but equal to 8 w capital D cube into n divided by G d 4 or we can say the k which is the spring stiffness, which is nothing but equal to G into small d to the power 4 divided by 8 capital D cube into n, where n is the number of coils of the spring.

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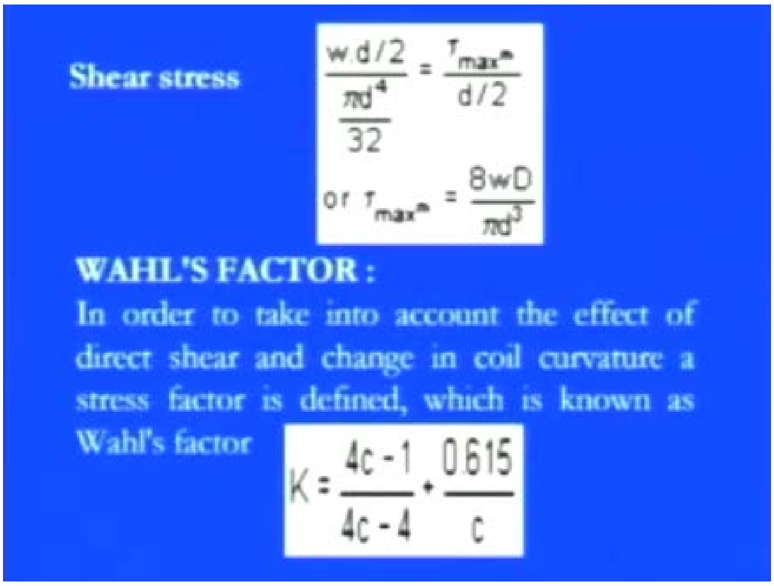

So, you see, these two formula after that, you see, you know like, once you have the deflection, once you have the spring stiffness, then you can easily calculate with the using of the main this torsional equation that what is the shearing stresses are there. So, you know like, we would like to calculate the shearing stresses and shearing stresses are always, you know like, maximum when this radius is exactly at the circumference.

So, when we have the d by 2 as a distance, then we have the maximum shearing stresses. So, again, you see, we would like to put this particular formula where T by J equal to tau by r, where r is nothing but equal to d by 2 - small d by 2. So, you see, here after keeping that formula w into d by 2 divided by, that is the T, divided by J, that is pi d 4 divided by

32 equal to tau maximum divided by d by 2. So, after, you see, manipulating what we have, we have the shear stress maximum is equal to 8 times w - w the radial load - into the mean diameter divided by pi d cube.

So, you see, the shearing stresses are directly proportional to what the diameter is there; more diameter, more shearing stresses are there because more of twisting is there. But, you see, if we have the spring diameter is more then we have, you know like, the less shearing stresses because the shearing is there along the axis of the helix. So, if more diameter is there, it provides more resistances. So, you see, we have, you know like, the whatever the stress formations are there, they are quite less as compared to the mean diameter.

So, this is, you see, you know like, the spring, you know like, the three main characteristics are there. One, we calculated that what the deflections are there under the load of - under the axial load; we have the stiffness, the spring stiffness under the application of the load; and we have the shear stresses under the application of the radian load. So, these three formulas are really, you know, important for any kind of calculation or if you want to design those things, we would be very careful to choose the appropriate value just for, you know like, the safe design of the spring.

Then in that, you see, there is a Wahl's factor is there and Wahl's factor is nothing but, you see, if you want to, you know like, consider the effect of direct stress. And we just want to see the change in coil curvature, the stress factor is coming and this stress factor is, you know like, known as the Wahl's factor.

So, you see, here the main thing is that actually we are going to consider the effect of the straight direct stresses along the axis of helix. So, what will happen, you see, if you are considering that, you know like, the direct stress, then straight away there is a change in the coil curvature, because, you see, right now we are taking, you know like, the straight part, but when there is shearing part is there, a kind of curvature form is there in this particular spring wire.

So, now, you see, if you want to compute that part, always you see, there is a factor; that factor it is known as the Wahl's factor, which can be easily computed on the basis of 4 C, and C, you see, we already, you know like, defined that it is the spring index, which is nothing but equal to the ratio of capital D by small d. That means it is the ratio of two diameters - the mean diameter of the spring divided by the spring wire diameter.

So, you see, here once you know the value of C, C is nothing but the dimensional parameter. So, 4 times of C minus 1 divided by 4 times of C minus 4 plus 0.615 divided by C. So, you see, based on those diametrical values that how the curvature will take place, and if you want to avoid the curvature, we have to design or we can say we need to take the value of K appropriately so that, this curvature - coil curvature - will not be formed. And the straight, you know like, this straight formation of this spring wire will be coming into the picture under the action of this radial load.

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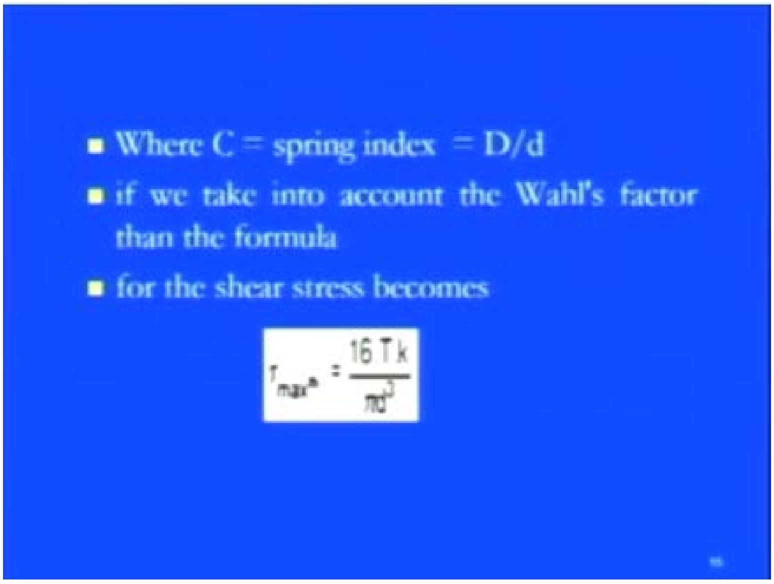

So, now, you see, here as I told you the C is the spring index D by d and if we take into account the Wahl's factor in that formula, only what we have? We have the shear stresses as 16 times of T into k divided by pi d cube. So, you see, here now straight away the shear stresses are the proportional to the applied torque and proportional to this - the Wahl's factor.

So, you see, whatever the Wahl's factors are there, according if we have more curvature part, probably you see, we have more shearing part, the shearing stresses are there all along this - all along the section. So, you see, when we want to design any spring and if we have these kind of Wahl's factors and the shear stresses are there, then we can set up the relation based on these things. So, you see, the tau maximum that is the shear stresses are nothing but equal to 16 times of T into k divided by pi d cube.

Now, you see, you have the shear stress, you have all those parameters, and you see, we are assuming that whatever the deformation is coming under the action of this radial load is the elastic deformation. So, you see, we can simply… and the basic purpose of this spring is to store the energy. So, whatever the storing energy is there, this is known as the strain energy.

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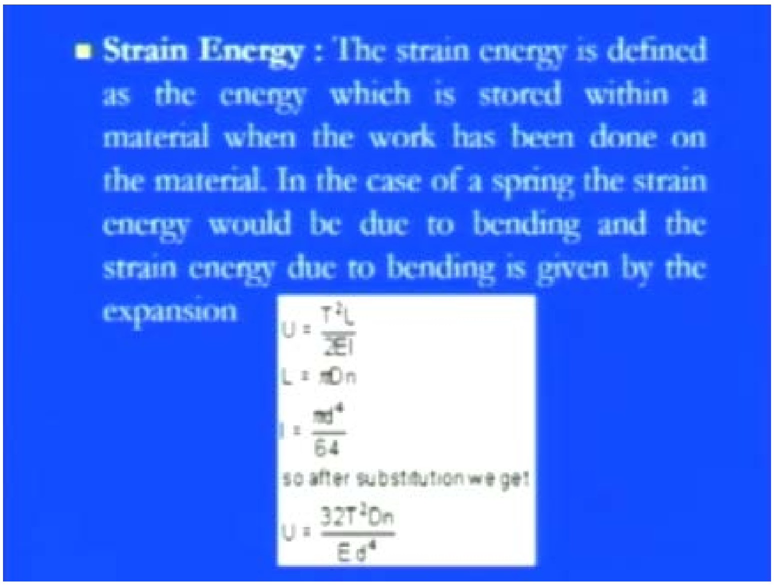

So, you see, the strain energy, we can simply define as the energy which is stored within the material or the spring material I should say, when the work has been done on the material. So, in, you know like, since we are using the spring, so the strain energy would be due to the bending, and the strain energy due to bending is given here in this equation that U - which is the strain energy – it is equal to, you know like, it is coming due to the applied torque, so T square L divided by 2 times E I.

And again, you see here, please keep this thing in mind that if you apply the load, whatever the deformation is coming, it is only the elastic deformation and this strain energy is defined for that region only. So, the strain energy is the function of T square that is the applied torque into the length of that particular spring divided by 2 E I.

And, you see, you know like, since if we are saying that the strain length of the L and stress length of the spring is the L, which is nothing but equal to pi d, which is the area into the number of coils. So, n is there or we can say the small l is nothing but equal to that the spring wire, you know like, this length. So, we have pi d 4 divided by 64. So, you see, after keeping those values, we have the strain energy is equal to 32 times T square D into n divided by E d 4.

So, you see, here again what we are defining, what we are trying to define here, the strain energy is nothing but the function of that what the torque is there due to which this bending action is forming, and then what the mean diameter is there. So, it is directly proportional to that n, how many number of coils is there, because more number of coils more, you know like, the strain, we can say, absorbing capacity or strain this energy saving capacity. So, when we are saying that we can simply, you know like, store the more and more energy, if more number of coils are there, then this spring is very good or we can say it can easily compensate whatever the shocks are coming. So, whatever the shocks, we can say that actually the shocks are coming, we are simply designing based on the capital D divided by the small d; that means, the spring wire diameter.

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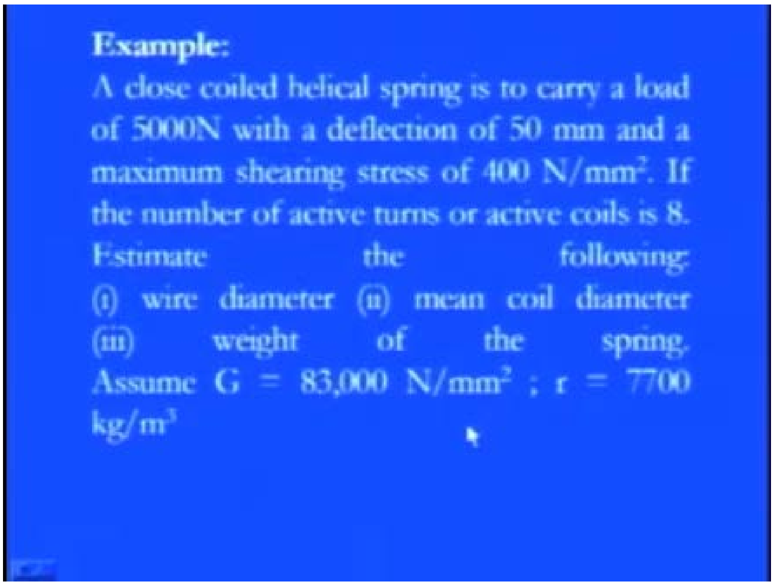

So, now we would like to, you know like, whatever the formula, which we derived here, we would like to put those formula in the real example. So, we have an example as if we have a closed - coil closed coil helical spring – and it is simply carrying a load of 5000

Newton or we can say 5 kilo Newton with the deflection of 50 millimeter. And the maximum shearing stress under that particular 5 kilo Newton load is 400 Newton per millimeter square. And if the active turns of or we can say the total number of coils are 8, then we would like to find out that what is the spring wire diameter - the small d; the mean coil diameter - capital D; the weight of the spring and, you see, with that we are assuming that the shear modulus of rigidity - the G - is nothing but equal to 83,000 Newton per meter square; generally it is always given in terms of the kilo Newton or the mega Newton particular, but it is in this way, and we have the radius r, this r is nothing but equal to 7700 kilo gram per meter cube.

So, you see, with the consideration of all those parameters we would like to find out these three parameters. So, here it is, you see, if you want to calculate the wire diameter, again we are going with the same equation T by J equal to this tau, tau by r. So, by keeping those values T w d by 2 divided by pi d 4 divided by 32 equal to tau maximum divided by d by 2, after keeping those values, we can simply calculate the capital D which is our main concern. So, capital D is equal to 400, which we are applying here, divided by d by 2 into pi d 4 divided by 32 into 2 by w.

Or we can say this D, this outer, because, you see, we want to calculate both the diameter. So, the outer, the mean diameter - capital D- is nothing but equal to after keeping those values what we have, we have 0. 0314 d cube. Or, you see, you know like, after we just want to keep those things. So, that once we have the one more equation, we can simply generalize those things or we can say once we know the value of either this D or this d we can simply find it out the vice versa part. So, here, you see, the same thing if you know the deflection formula, you can simply keep that formula and x is nothing but equal to 8 w D cube n and divided by G d 4.

So, now, you see, you know the maximum value, the applied load you know, you know like, and you know the how many number of coils are there, you know the G value, you know the… you know like, these kind of values. So, we can simply put that how much deflection is there. So, deflection was 50 millimeter and the value of w was there and all those particular parameters are there. So, we can simply calculate that actually what is the value of the small d. So, small d is nothing but equal to it comes around 13.32 millimeter. So, once you know the small d keep that value and get the value of capital D.

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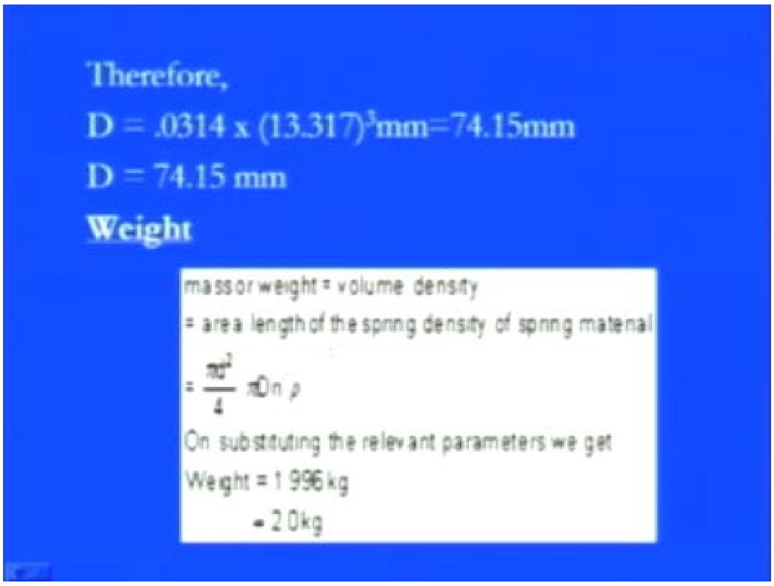

So, you see, we have the spring wire diameter, you have the mean diameter of that. So, you see, capital D is coming as 74.15 millimeter and if you want to calculate the weight of that, weight is nothing but equal to, you know like, the mass or we can say the weight is volume into density or density is nothing but equal to mass per unit volume.

So, now, you see, you know the volume; volume is nothing but the area into length of the spring, this spring which is pi d n into, you see, the density of the spring material which is already given as 7700. So, you see, by keeping all those values we can simply find out the weight which is nothing but equal to 1.996 kilogram or we can say the spring weight is 2 kilogram. So, the meaning is pretty simple; if you know these, you know like, the different values, and if you know the formula, you can simply put those values in the formula, and you can get the desired output. So, but the key feature is that, whatever the formula which we derive, it is simply, you know like, valid just for the elastic deformation.

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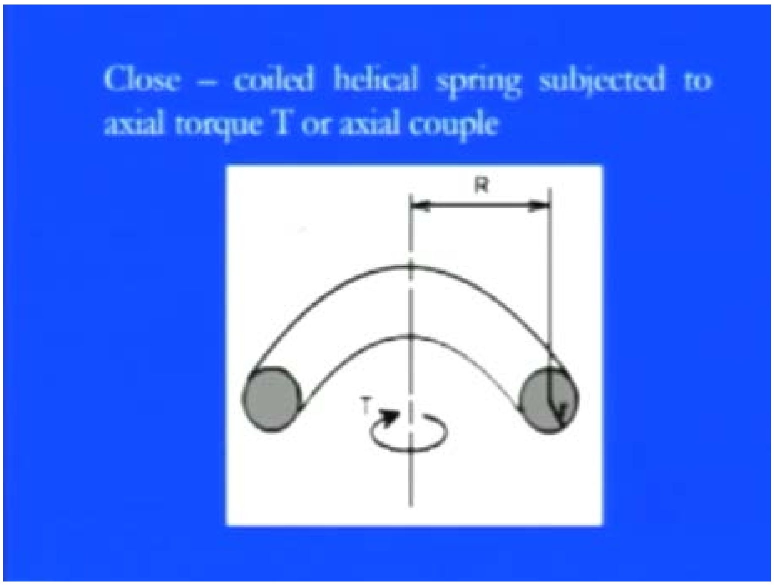

So, here, you see, now we would like to see the another case, that if we have the closed coil helical spring, which is subjected to the axial torque or we can say the axial couple instead of just, you know like, giving the radial load, we have the axial couple which is just try to twist that part. So, now when the effective area is like that, you see, this is simply put in the curved form and what we have, we have, you see, you know like, the shearing action is there all along this particular action. So, under the action of this T, we would like to see that actually how they are, you know like, significantly affect the axial couple.

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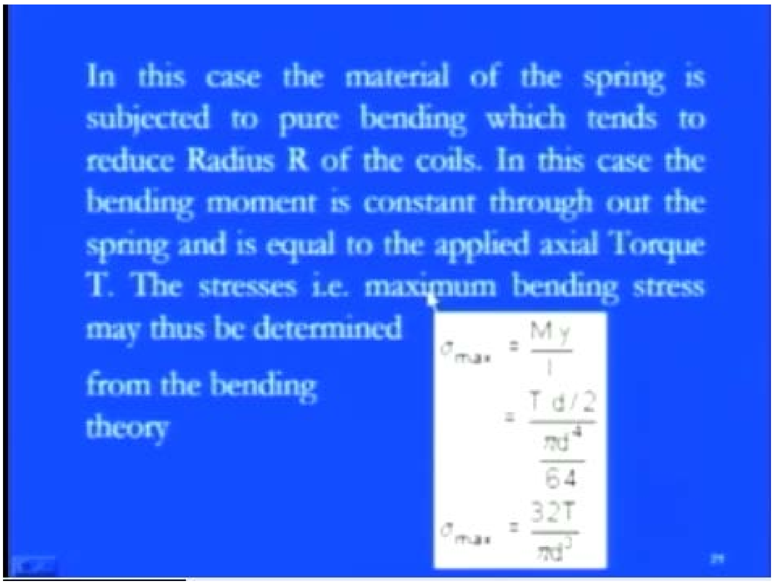

So, you see, in this case the material of the spring is subjected to the pure bending, which tends to reduce the radius R of the coil. So, you see, it is just tending to come in the curvature part, and in this case, the bending moment is simply constant throughout the spring, and you see, we would like to even put this particular assumption here, which is equal to the applied torque, you know like, what the applied torque is there and due to this actually how this bending will take place.

So, if we want to see the stress, which is nothing but the bending - the maximum bending - stress is always coming as the main bending formula, the bending moment because whenever the bending action is there, the bending moment into the total y. Whatever the, you know like, the distance is there from the neutral axis divided by the I or I is nothing but equal to pi d 4 by 64 that is the inertial factor.

So, you see, we have the movement T into d y is nothing but the dy 2 divided by pi d 4 by 60; the 64 we can calculate the maximum shearing bending stresses is equal to 32 d by pi d cube. So, always keep this thing in your mind. So, whenever the spring element is there, and if it is under the action of this twisting torque, the maximum bending stresses are forming in this spring coil and this is equal to 32 by pi d cube.

So, you see, here, you know like, this was the basic application that if we have radial load or if we have the twisting movement then what the changes are there in form of the stresses.

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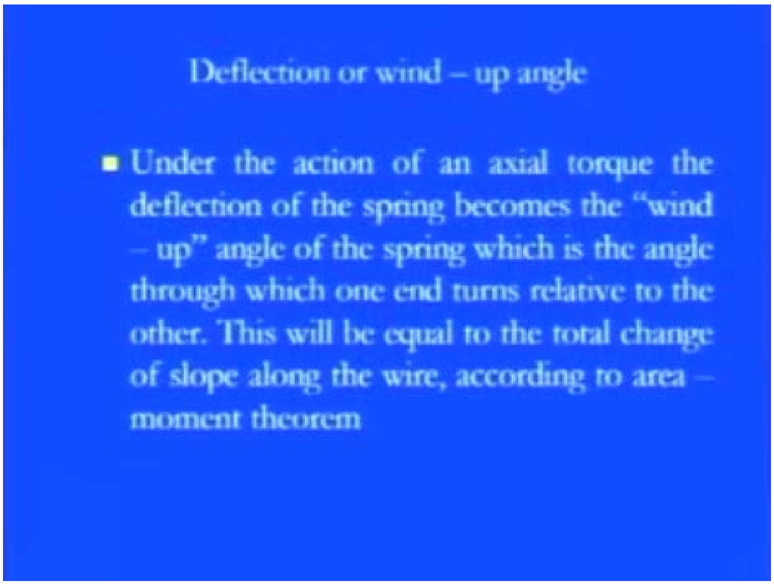

Now, you see, our last part of this chapter is if the deflection is there or wind up angle is there, then how, you know like, they are simply forming under the action of these forces. So, under the action of any axial torque. the deflection of spring becomes wind up angle or we can say, you know like, it is just they are just trying to close, just trying to close as as much as it can, and you see, it just it is forming, that actually it is just winding up. So, we are saying that it is a wind up angle. Generally we are saying that is a angle of twist, but here it is nothing but the wind up angle of the spring which is the angle throughout, you know like, one end turns relative to the another. That means one end is coming close to another end and this will be equal to the total change of slope along the wire according to area moment diagram.

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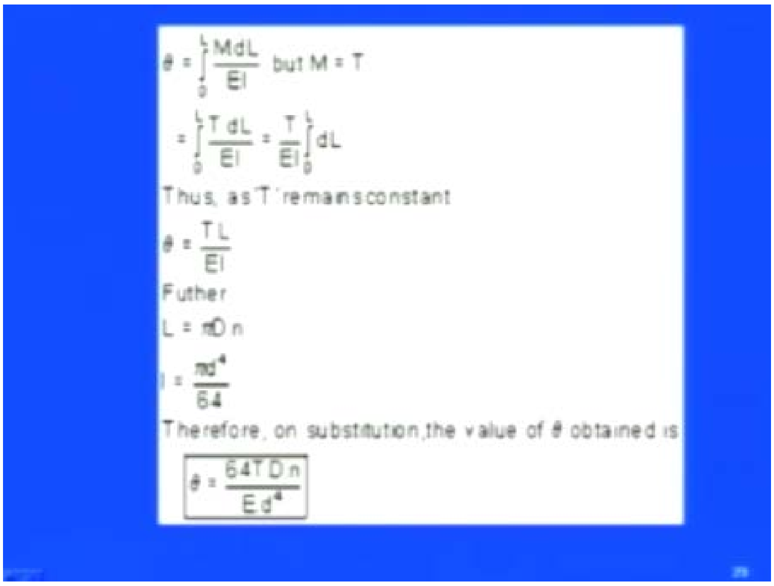

So, you see, if you want to calculate that, then it is nothing but equal to theta is equal to 0 to L and the total moment applied, or we can say the torque applied into the d L that effective, you know like, this whatever the segment of this particular length of this spring into E into I or we can say, you know like, the applied torque is there. So, T into d L into E I T and E I are nothing but the constant parameters. So, we can simply find it out the total length L.

So, the total theta or we can say the wind up angle is nothing but equal to T L by E I or since L you know that it is a unstress length of the spring. So, it is pi d n or I we know that, you know like, the section a modulus is there. So, it is pi d 4 divided by or we can say the moment of inertia I is there pi d 4 by 64. So, we can calculate the wind up angle for this kind of spring is nothing but equal to 64 d, d into n divided by E d 4.

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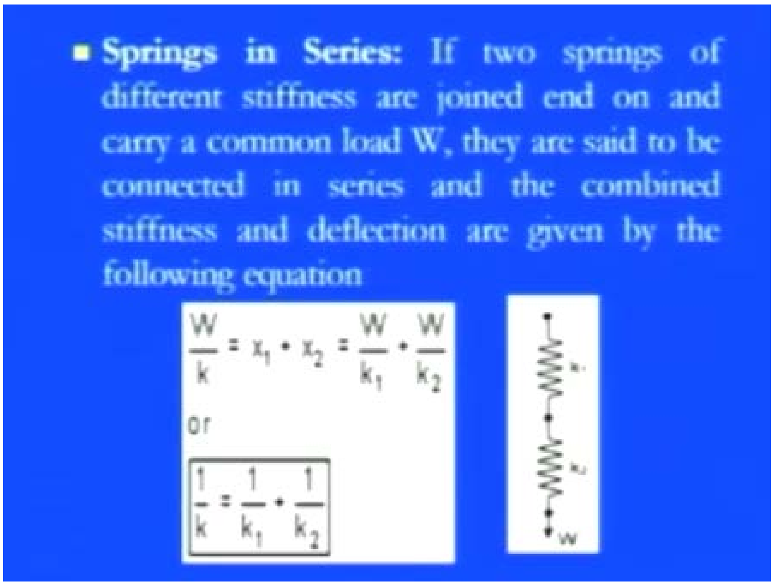

So, you see, here we see that how these springs, you know like, these are taking place whenever, you see, you know like, under the action of any twisting moment or under the action of the radial load like that, and as usually see if these springs are in the series form; that means, you see, you know like, if these springs are well connected to the series, we can simply form those that what the stiffness is there, the stiffness is 1 by k equal to 1 by k 1 plus 1 by k 2.

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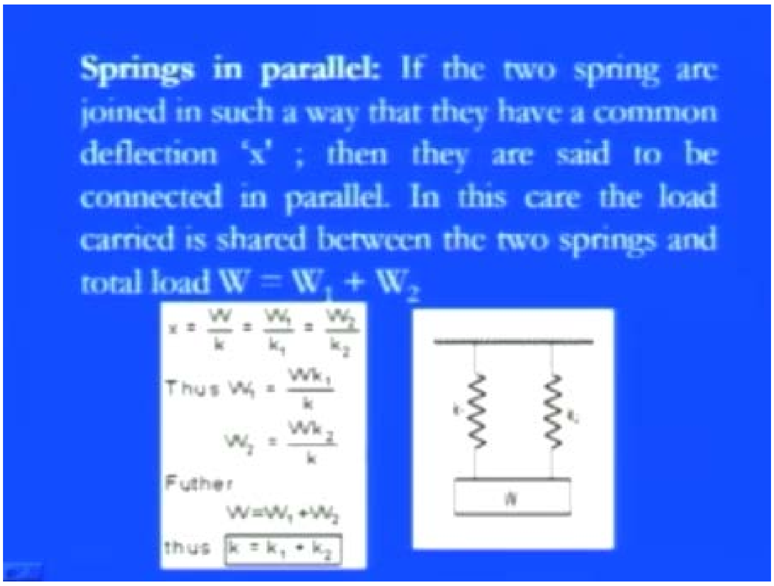

And if this springs are in the parallel form and the radial load is acting, then stiffness is nothing but equal to the linear relations k 1 plus k 2.

So, in this chapter, you see, we discussed all those terms which are related to the spring that, you see, if the axial load is there or twisting moment is there, then what the angle of twist is there or wind up angle is there? Or if the springs are there in the series form or the parallel form, then how the stiffness is taking taken care of? If it is in the shear, then it is k is nothing but equal to k 1 k 2 divided by k 1 plus k 2; if the springs are there in the parallel form then k is nothing but equal to k 1 plus k 2.

So, in the next lecture, you see, we are going to see that if we have a uniform bar and if it is under, you know like, under the various loads, then how the bending will form? If it is point load is there or if it is the uniform distributed loads are there. So, now our focus is on the specially beam or we can say the rod.

Thank you.