Strength of Materials

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Lecture – 20

Hi. This is Dr. S. P. Harsha from Mechanical and Industrial Engineering Department, IIT Roorkee. I am going to deliver my lecture 20 on the course of Strength of Materials. And this course is developed under the National Programme on Technology Enhanced Learning (NPTEL). Prior to start this lecture, I just want to refresh you like that, what we have done or what we have discussed in the previous lecture.

As you know that, actually, if we have rotating shaft and, you know like, under this particular shaft rotating, if any kind of twisting moment is applied or any torque is there at the extreme corner or there is a couple is there at the extreme corner, then what will happen. Then we found that wherever you have seen like this kind of torque is there, then this shaft is purely under the state of first shear stress and whenever you see, you know like, we are just keeping our torque under the elastic deformation only.

Then we can simply setup the relationship between the shear stress which is there, and due the shear stress whatever the shearing strain is coming out, whatever the distortion is there, the angular twisting is there, you know like, we can simply setup the relationship.

So, the shear stress is there, shear strain is there, then you see, you know like, the modulus of a… shear modulus of rigidity is there. And then, when we are talking about that, the applied torque is there and due that the shearing stresses are there, the shearing stresses are there, and angular twist is there, and the shear modulus of rigidity is there. We were just trying to, you know like, setup the relationship, and you know, like we observed that there is a straight relationship is there in between all these technical terms as T by J. T is the applied torque and J is the this modulus of, this section modulus of inertia or we can say whatever, you know like, the polar moment of inertia is there for the area.

So, as you see, you know like, for the inertia if you are talking about this, so T by J is equal to tau dash by r or tau by r, whatever. You see, if I am saying that the shear stress is there at tau for any radius r, so T by r or tau by r, tau by capital R or tau dash by r is equals to G theta by l.

So, this relation we setup, but to setup those relation, you see, we assume many things like, you know, like that we have whatever the material of this simple circular shaft is there is a homogeneous material, in which all the elastic properties are uniform in any of the direction is there. Then second, also our main assumption, was there that whatever this torque or the load application is there through which this stresses are forming, the shear stresses forming, this is under only the elastic deformation.

So, we can simply define all those elastic constants like the shear modulus of rigidity, like the stress or strain, whatever you see or we can say, you see the Poisson ratio or whatever the last you know. Or we can say Hooke's law is valid within that and also apart from these two, do these two begins since we assume that cross sectional area of the circular shaft is uniform throughout the length of this particular circular bar; with that also we assume that the uniform cross section means actually it does not have, you know like, the different, different segments.

So, if we see, if you want to compute the shear stresses for a small section, we can simply put, you know like, the integration to capture the whole, either the deformation or, you know like, the torque or whatever like that. So, these kinds of assumptions, which we made to, you know like, analyze this T by J is equals to tau dash by r is equals to G theta by l. So, that is that what you seen, you know like, we discussed.

And also, we found that actually if we are talking about the shear stresses or shear strain, the shear strain is always can be easily captured by knowing the total distortion, that actually how much total distortion is there. Or once you know the total distortion the vice versa is there the shearing stress can be, the shearing strain can be easily computed. So, that, that is what we discussed the, you know, like in the… and then also we calculated that if you know the applied torque and if you know, you see, due to this, you know like, what are torque is there or if you know that what is the power capacity of the shaft is there, that, you know like, what is the power developing because always the rotating shaft, the basic feature of the rotating shaft is power transmission. So, once you know that how much power you are transmitting and you see what is the speed of the shaft is there, then you can also calculate the applied torque. So, means how much torque is there by P equals to two pi N T by 60 into 1000 as in kilowatt.

So, this kind of discussion, you know, like, which made in the previous section, so, again you see we are going to start in this our lecture 20 with the small portion of that, that actually, what exactly, you know, like the things are there, how we setup the relations, and then, you see, we will move that actually once, you know like, we are setting up those relations, then how we can, you know like, solve that numerically, that what the numerical real applications are there of this kind of formula, under these particular assumptions.

So, here you see, you know like, again the simple torsion equation, which is the basic torsion equation, is as we discussed, that T by J which is equals to tau by r, which is equals to G theta by l, or we can say that whatever the shearing stresses are coming, the responsible form for, you know like, inducing the shear stresses are G theta r by L.

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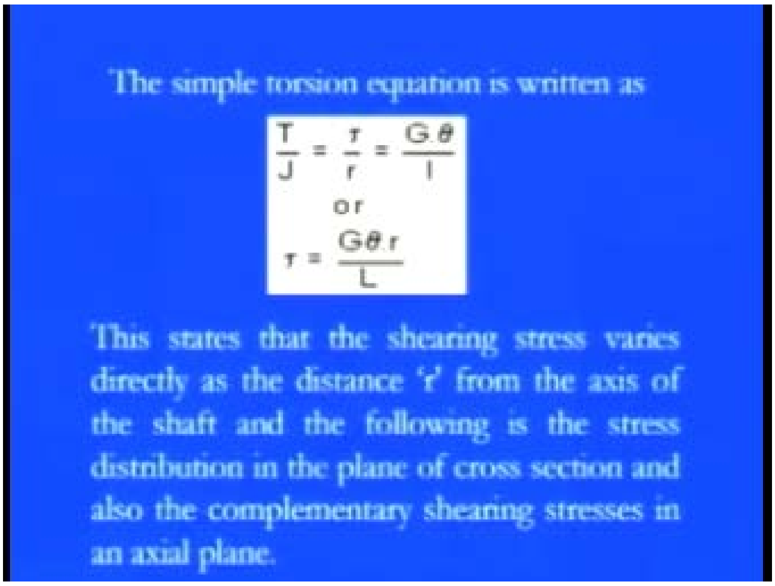

That means what is the…what properties are there, through which the shear modulus of rigidity is coming? How much angular twist is there under the application of this tau or the torque? And what is the radius is there? Because as you move, you know like, right from axis to the outer circumferences the shear stresses are increasing. And also we know that actually the shear stresses are always maximum at the outer periphery of that. So, you see how this r is increasing and divided by what is the total length is there.

This states, that is what you see, you know like, the sequence states that shearing stresses varies directly as the distance r. That is what you see, our matter of concern is from the axis of the shaft and the following stresses, you know like, the stress distribution in the plane. Now, you see we are going to discuss about that how this distribution of the shearing stresses are there in the plane of the cross section, and also, you see, how the complementary shearing stresses are, you know like, featuring against, you see, the applied shearing stresses r.

So, now you see, we would, you know like, see in the different section in the next slide that how these shear stresses are forming. And then, you see we will find that if you see this particular figure, then you will find that we have a cross section of the circular shaft and if you cut the portion, then you will find that this is origin through which the shear stresses are beginto form, and then as you move towards the extreme corner, and this is you see, you know like, the maximum tau is there.

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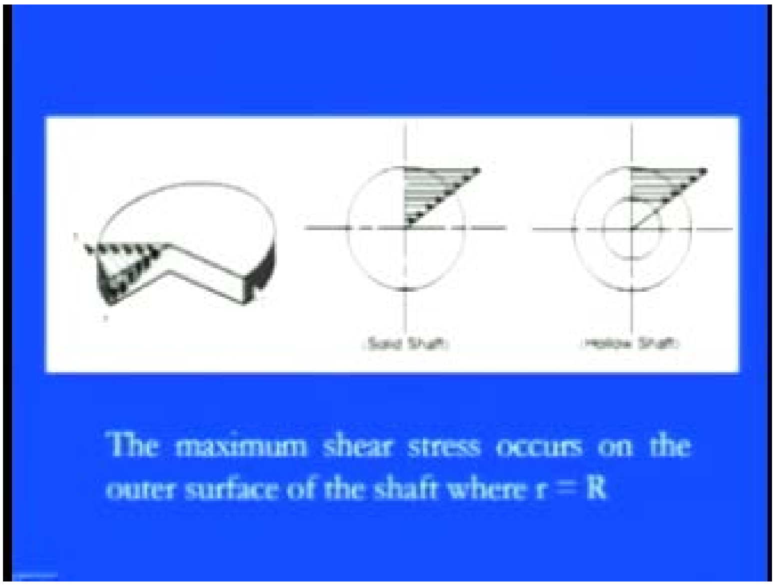

So, you will found that this is, you know, like my center of axis and either you are going in this direction, this you see here this is small, you know like, the stresses have begun to form, onset of the shearing stresses are there, and as you move further towards the outer circumferences or outer periphery of this particular shaft, you will find that the maximum shearing stresses are there. So, this as I told you tau is proportional to r. So, here as r is zero here, this see, there is no shear stress, and r when r is R - the capital - the maximum part you have tau is the maximum shear stress irrespective of this direction or irrespective of the downward direction.

So, here this, this, you know like, variation or this variation will clearly signifies that tau is directly, tau is directly proportional to the radius specified. So, for this region or if you simply say on the plane region, for a solid shaft, then you will find that we have, you know, like this is a kind of solid shaft and this is the ray origin of the solid shaft, and as you move further right from, you see, zero to the extreme corner, you will find that the origin, that is the no shear stress, but as you move towards the outer circumference or outer periphery of the shaft you will have the maximum shear stresses; so, this is the maximum shear stress.

Or even if you see that… actually if you have solid shaft, then we have, you see, this region where this solid, you see, hollow is there, and this is my inner diameter, this is my outer diameter of a solid shaft. So for that, you can simply signify that the stresses have beginto form at this particular region, because this is the hollow region is there. So, in that particular, in the micro structure all the layers of this particular solid region or intermediate region, they are simply inducing under the action of shearing action.

So, starting from this region, even at this particular point, we have some shear stresses and if you go beyond, then you can simply find that at the outer periphery of the shaft you have the maximum shear stresses. So, all and all, the main thing is that the shear stresses have beginto form from the origin of the shaft and it is always maximum at the outer periphery of the shaft. So, whenever the torque application is there, on any circular shaft, if you are simply cutting from the neutral axis, you will find that the shear stresses are not there on those neutral axis.

But when you start from, you know like, the neutral axis or the central axis, two outer periphery, you will find that the, all those, you know, distortions are coming right from that portion and it is that maximum distortion is there at the outer periphery, because the shearing action is maximum at the outer periphery. So, you see, you know like, as we discussed in the previous section also that, this tau dash by r is nothing but equals to G theta by l. So, tau is proportional to r or we can say that, you see like, the theta is there. So, angle of twist is also maximum in the corresponding section. As you see the r is increasing. So, this is, you know like, the coiled the signified terms are there, as you can clearly visualize in these kind of figures. So, the maximum shearing stresses occurs on the outer surfaces of the shaft where r is equals to R - capital R.

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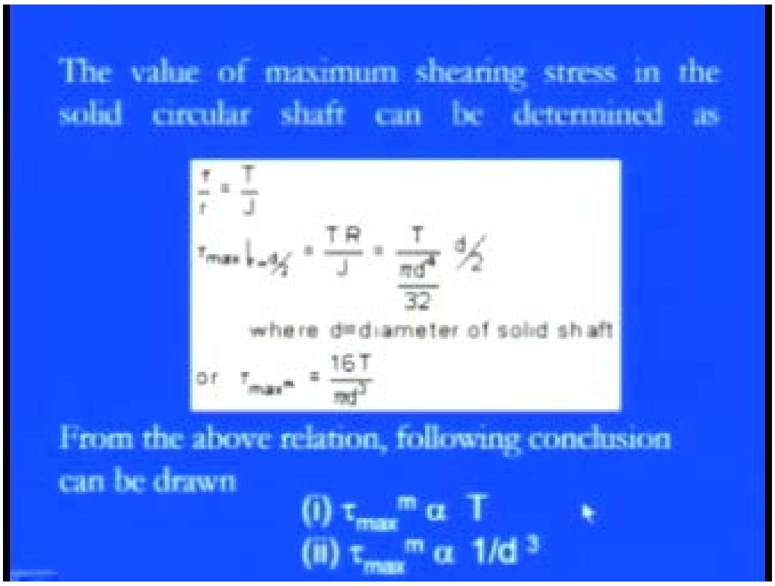

So, now you see, the value of… now we would like to see that what exactly is the value of the maximum shearing stresses are forming when we are saying that irrespective whether it is a solid shaft or whether it is a hollow shaft.

So, by visualizing that previous figure, we found that we have a maximum shearing stresses at the outer periphery where the maximum radiuses is there. So, again by considering the same formula tau, tau by r is equals to T by J, now our main interest is where the maximum shearing stress is. So, now you see tau maximum, which we are taking, as, you know like, the r is equals to d by 2, the maximum radius which is equals to T into R. So, R is now we are taking as a capital radius. So, this T into R divided by J or we can say T, J is nothing but equals to pi d 4 divided by 32; this is the polar moment of inertia.

So, if you are keeping those values and if you convert this R into d by 2, we are, you know like, having a kind of formula as T into d by 2 divided by pi d 4 by 32, where d is the diameter of the circular shaft. So, we can get the tau maximum in terms of this diameter, and this, whatever the torque applied is equals to 16 T divided by pi d cube.

So, please remember this thing that we have, whenever a torque applied is there on a circular shaft and due to the torque applied, you see, we have this shearing stresses are forming and they are forming right from this central axis to the maximum of the radius, the total circumference - outer circumference. That means, you see, it is passing from zero r to the maximum R - capital R.

So, you see, here if we are talking about that term, then we have the tau maximum, that it is the maximum shearing stresses, they are nothing but equals to 16 T divided by pi d cube, which is a very good formula for any kind of numerical problems. So, from these relations, we can simply conclude that the maximum shearing stresses are the function of applied torque, because, you see, tau is proportional to T; so, obviously, you know, like whenever more torque is applied, more couple is there, more twisting, you know like, moment is there.

The corresponding shear stresses are always maximum, because you see, you know like, the more and more distortions are there or there shear stresses is nothing, but you see, these are internal intensity of resistive forces across the plane. So, whenever the more torque is there, more internal resistances are coming out from the circular shaft and corresponding you see more stresses are there. So, obviously, this is quite signifies that tau maximum is always proportional to T.

The second relation is there tau maximum is proportional to 1 by d cube. So, as you move further, as, you know like, it is a inverse. So, as we have more and more diameter it is a cubic term, and you see, corresponding the shearing stresses are there. So shear stresses are more for ,you know like, the more torque, but it is always less for more diameter. So, you see, if we have the small shaft more torque is… if more torque is there more shear stresses are there. So, this can be easily concluded if the using of this formula. So the tau maximum formula is 16 T divided by pi d cube.

So, now you see, once you have the torque, whatever the torque is applied on a circular shaft, you can simply calculate that what will be the power transmission as we have discussed in the previous case, that whatever the given power transmission is there and if you know the speed of shaft, that is N rpm, N is, you know like, the revolution per minute, then probably, you know like, you can simply get that how much torque is there or once you know the torque you can simply get the required power.

So, power is nothing but equals to T into this omega; omega is nothing but equals to the angular rotation. So, it is always equals to… since it is angular rotation is there, so how much total periphery is coming? So, it is 2 pi into this N. N is the total rotation - that how much it is rotating per minute. So, 2 pi N by 60 will give you the total angular rotation of the shaft and once you multiply by the torque applied you will have the total power.

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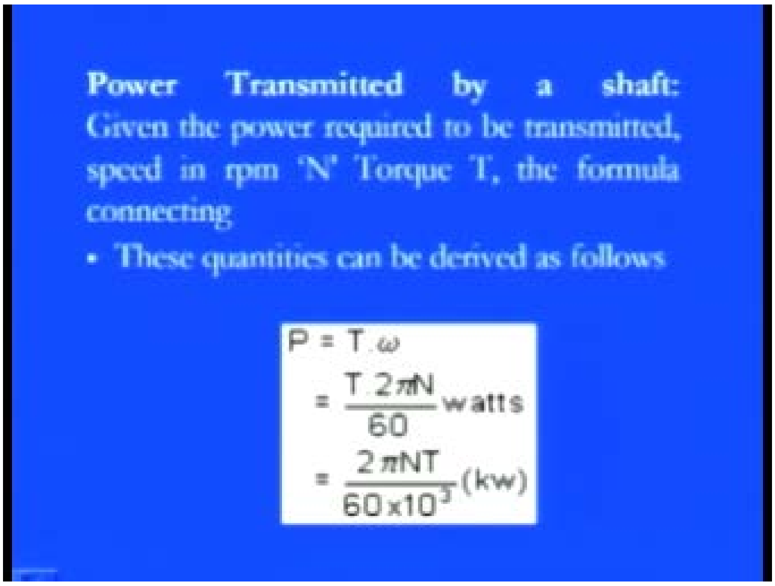

So, P is nothing but equals to T into 2 pi N by 60 or in terms of watts or you see if you want to calculate in terms of kilowatt then we have 2 pi N T divided by 60 into 1000. So, you see, you know like, always we are calculating for power that, this much power can be easily transmitted from the shaft. So, probably, you see, once you know the power, once you know the total rpm, then only you can calculate that.

If you want to transfer this much power, this much torque can be easily generated or if you know the applied torque, if you know this N rpm, you can simply calculate the power generation or power transmission. So, this is, you see one perfect application of this shaft is there or circular shaft whenever the torque application is there and whenever the rotation is there. So, now you see, you know like, once you have this kind of relationship you can again, you know like, get this particular one term as which we discuss the torsional stiffness. Because, you see, whenever we are talking about a kind of a, you know like, this stresses, strains, and all those kind of things, one must have to know that actually how much stiffness is there; that means, how much deformation or the distortion can be taking place under the application of the applied torque. So, once you know that this, this is hard or this is more, you know like, stiff is there against the applied torque, then probably you know one can, one can easily define or design the applied torque, so that, you know like, the less deformation or less distortion is there and corresponding less, you know like, this stresses can be formed.

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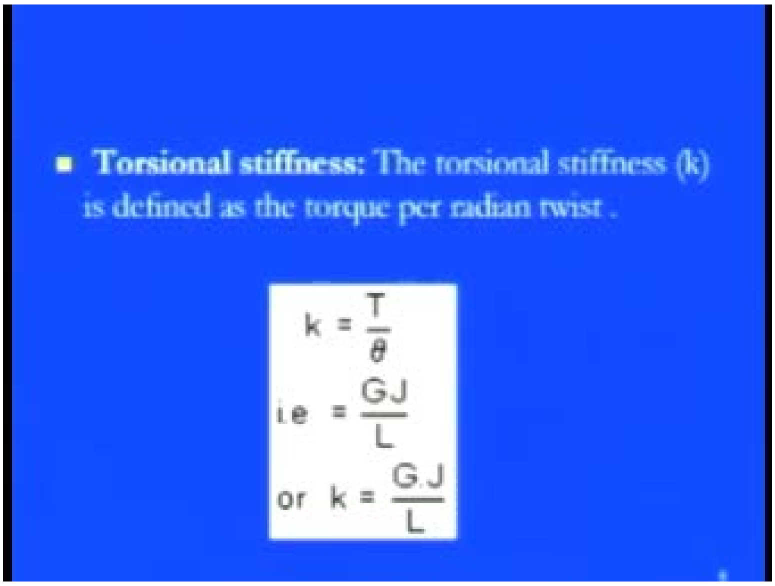

So, again you see this term - the torsional stiffness - is nothing but equals to, it is the ratio of the torque per unit what the twisting is there. So, k is nothing but equals to T, which is applied torque divided by what is the angle of twist. Or we can say that it is nothing but equals to G J by L or we can say k is equals to G J by L. So, k is not only the depending parameter on the torque as well as this twisting part, the twisting, this angle, but also it also depends on that what the material property is there. That means which material you are using? The material is more tougher, the material is more stiffer against the load. So, corresponding value as G is coming the shear modulus of rigidity.

And then, what is the area? Means what is the section modulus of inertia is? So, you see, section modulus of inertia is absolutely dependent on what the area is pi d 4 by 32. So, again, you see, if you have taken a small diameter, definitely, the corresponding stiffness is there. So, it depends on G J and divided by the total lengths; if more length is there, definitely, you know like, the less is stiffer properties are there, because you see, you know like, it is distributed all along the circular shaft of the length. So, obviously, you know like, we have the lesser, this deformation is there in that particular shaft.

So, meaning is pretty simple. You see, if you want to calculate the power transmission or if you want to calculate the torsional stiffness of a circular shaft that can be easily calculated if you know the applied torque. And if you know the rpm, then this power is there, and if you know the angle of twist it is the torsional stiffness is there based on the applied torque. So, from the ductile material source, since we are talking about this stiffness, that means the deformation, it is always, you see, we are going with the which kind of material is there. As I told you, the G is there, so it is absolutely clear this property of material.

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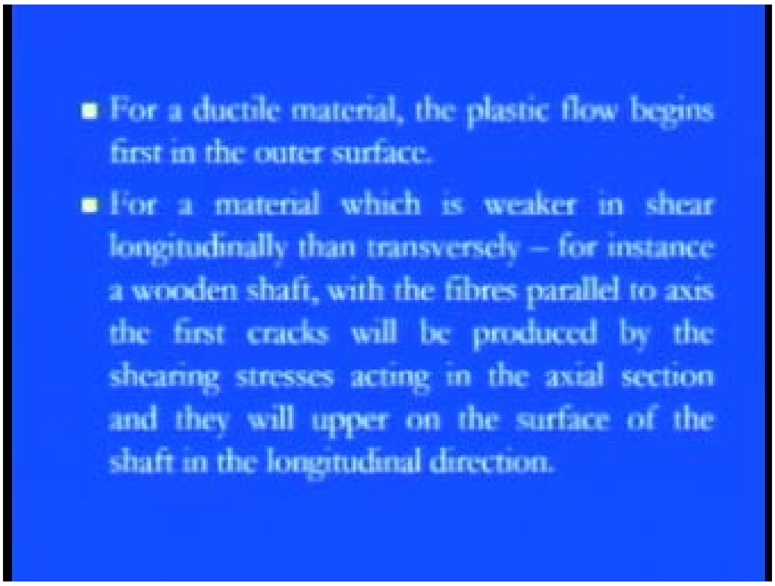

So, for a ductile material, the plastic flow begins first in the outer surface, because you see, as we know that when you apply this, when you apply the torque, we have the maximum shearing stresses are there at the outer periphery. So, whenever you see there is a deformation is there in the layers, the outer periphery, firstly, you know like, gives you the kind of plastic flow. So, plastic flow the onset of the plastic flow is starting from this outer surfaces only under the action of this applied torque.

So, for a material, which is weaker in shear - please focus on this point - if we are taking a material, which is weaker in a shear longitudinally than transversely, that means, you see it is weaker in shearing part as compared to this transverse; that means, the torsional part. For instances, if we have a wooden shaft with the fibres parallel to the axis, means we have a wooden shaft and all those fibres in the wooden shaft, they are just parallel to the axis towards the axis.

So, the first cracks will be produced by the shearing stresses acting in a parallel axis; that means, you see, you know like, the first, the onset of the shearing stresses are always or we can say the plastic flow begins from the outer surfaces, and then, they will simply, you know like, all those, you know like, the shearing stresses, they will come on the upper of the surfaces of the shaft in a longitudinal direction. So, you see, first they will start from the outer surfaces and then they will distribute accordingly. So, if this kind of material is there, but if we have in case of material which is weaker in the tension than in shear.

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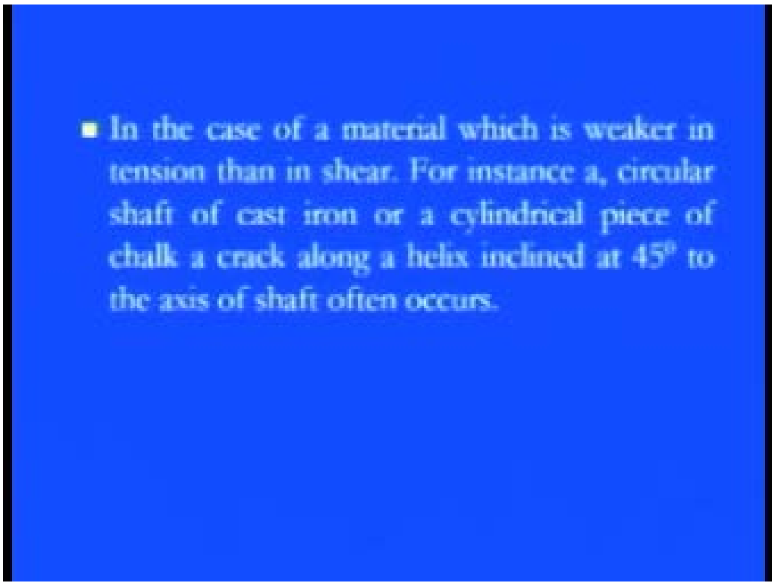

So, now the things are different. In the previous case, we had a material, wooden shaft is there, which is weaker in the shearing action as compared to the tensile action, because, you see, always you will find that the wooden parts are always… they can bear up to maximum tensile; they can be long aided. But whenever you see the torsion or the twisting is there, they are simply, you know like, the layers are coming out from this surfaces and the rupture is starting or we can say the plastic flow is starting from that.

But if we have a material which is even weaker in a tension side as compared to the shearing, for instance, if you are taking a circular shaft of cast iron - cast iron is a ductile material. So, in a ductile material always we found that they are, you know, the whatever the layers are in the… whatever the layers are there, they are well absorbed in this or they are well very much harder. So, they are they can be easily absorbed under the compressive action.

But if you are applying the tensile loading, then these layers are easily taking out from this particular origins. So, if we have a circular shaft of the material of the brittle - like the cast iron or a cylindrical piece of a chalk - like whatever the chalk is there which is absolutely a brittle material; no this ductile it is there, that means percentage elongation is very small, a crack along a helix inclined at 45 degree will be starting to the axis of the shaft. That means, you see, what we have, whenever we are simply applying this kind of twisting, for this kind of material of the cast iron bar or we can say a cylindrical piece of cast iron, of this chalk particular, if these, you know, like this kind of material is there and if you apply this loading, then always the crack starts in a helical way.

So, you see the arc forming on the outer surfaces, and they are, you know, helical with the inclination of 45 degree to the axis of shaft. So, this is a perfect, you know like, this kind of two special cases are there, and if you want to, you know, like analyze those kind of shearing actions under the application of, you know, like the torque, always we would prefer that actually, that the material should be ductile and it should, it can, you know, like resist the maximum shearing stresses, so that if we want to, you know, like rotate or if you want to simply transmit the power, that kind of material is always applicable if the, you know, like resist a maximum shearing stresses. So, that is what, you know like, if you want to explain that kind of phenomena, then the explanation is pretty simple, that this all things are, you know like, there.

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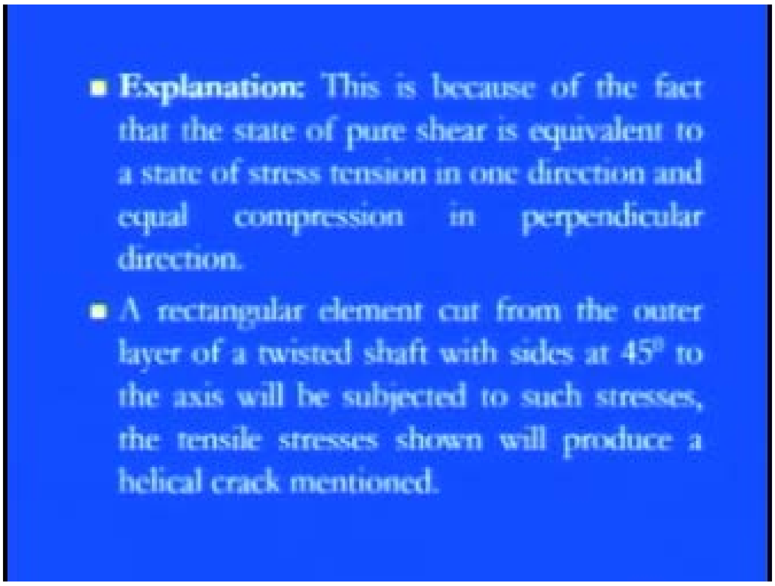

This is because of the fact that the state of pure shear stress is equivalent to the state of stress in tension in one direction and equal. Similarly, you see, equal in the compress, in the perpendicular directions. So, that is what you see, you see when we are saying that there is a tension is there in a particular one direction, obviously, mutually perpendicular direction we have a contraction or we have a compression.

So, a rectangular element cut from a outer layer of a twisting shaft with the sides of 45 degree to the axis, will be subjected to a such a stresses and the tensile stresses can be easily, you know like, produces with the helical this crack monitoring. So, if you, if you are talking about that we have a chalk of this particular material, any material of the chalk, or we can say if we have a cast iron this circular shaft, and if you are applying the twisting moment, and which, you know like, always, since they are, you see very, they have the less tendency towards the tension, but they have a very good things as far as the shear stresses are coming.

So, you see, whatever the deformations are there, as we discussed that actually starting from zero at the central axis and they have the maximum, you know like, the shearing stresses at the outer periphery, when you see, when you are simply cutting the angle at 45 degree.

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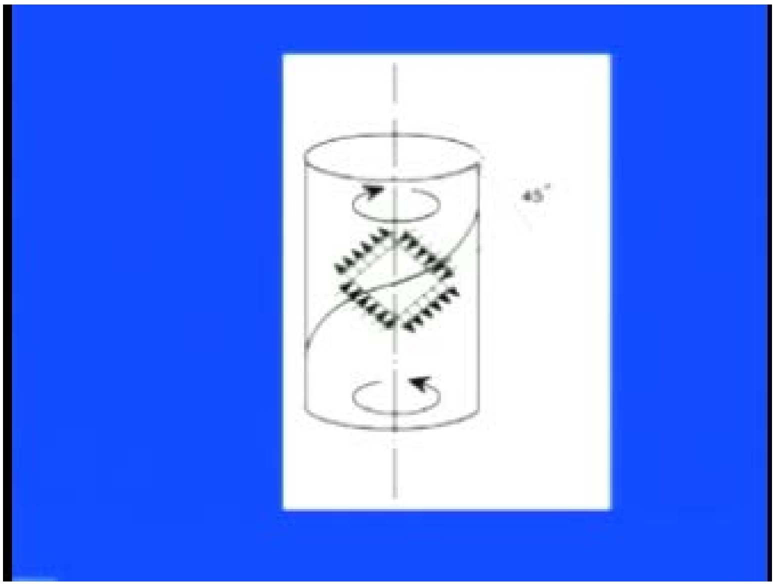

So in this figure you see we can clearly show that, we have a circular shaft here and, you know like, starting from this region what we have, you have pretty similar, you know the symmetric regions are there, but as you move further in this 45 degree angle, you know like, we are having this 45 degree. So as the helix angle is there, you see this is the helical shape is there. So, as you apply the torque there, stresses are forming and they are forming at the maximum at the 45 degree, and they are in the shape of the helical, and you can see here this stresses, so they are in the tensile way. At the one end, you see it is simply tensile, but in the other region we have the compressive in the nature. And these are, you know like, the applied torque is there, this is the resistive torques are there, and they are simply forming in the helical shape at maximum at the 45 degree. So, this is the 45 degree which can clearly shown in this particular diagram.

So, you see, you know like, as we have a different kinds of shape and if you are talking about, you know like, the material which is good in tension and less in shear stresses, those you see, you know like, this wooden plates are there with which they can clearly exhibit this kind of relation, but if we have, you know like, if they are good in tension.

If they are not good in tension, but they are good in shearing, then you see these kinds of, you know like, the examples are perfect for just to show the shearing stresses are occurring as well as the tensions are.

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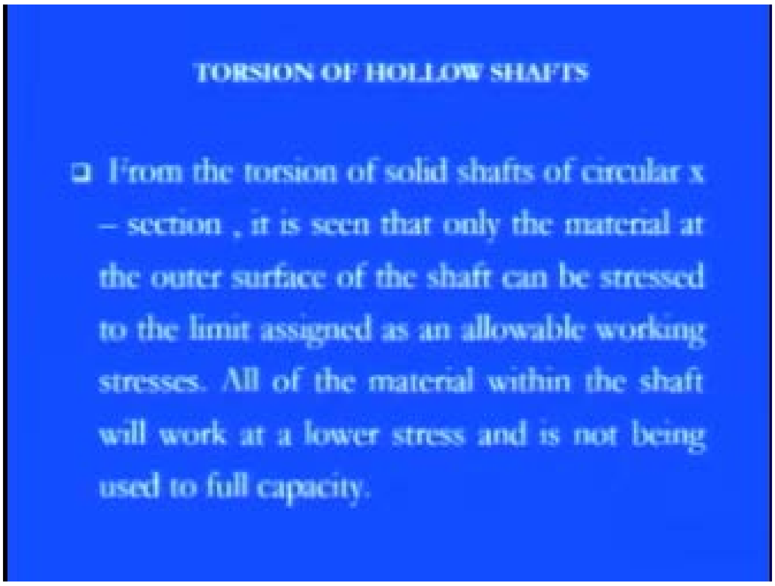

So, now you see in the previous section we discussed about that kind of phenomena, but now you see, if we have, if we want to transmit the power, and if we have a hollow shaft, so what will be, you know like, the torsions are there within those hollowed shaft? We would like to setup the relationship for that.

So, from the torsion of the solid shaft of a circular cross-section it is seen that only the material at the outer surfaces of a shaft can be stressed to the limited assigned as allowable working stresses. That means, you see, know like as we discussed that if we have a circular shaft and if we want to see that how the shearing stresses distribution are there, always we simply cut the section and found that in … we showed in this particular transparency also that as we are moving from, you know like, zero central axis to outer periphery starting from zero, and you see, you know like, this kind of layers are there; that means, you see it is uniformly varying all across the outer surfaces.

So, as far as the solid shaft is concerned what we have, we have you know like, the uniform material is there, it is homogeneous is there. So, all those displacement under the action of twisting, you see, this displacement is uniform. So, the corresponding shearing stresses are also starting from zero to maximum is there at the outer periphery.

So, you see it is always to be defined that whatever the stresses are there, if the stresses to be, to be you know, defined in the limited way as in allowable working stresses, and all the materials within the shaft will work at lower stress, and is not being used to full capacity, because you see, you know like, we simply apply and we apply the torque within the elastic deformation. So, we cannot go beyond certain limit.

So, that is what you see now, what we are doing here? Here simply we are assuming that since it is a solid shaft and it is twisting, and though you see it has a uniform distortion all along the material, but we cannot go beyond a certain lead. So that actually, the permanent set or whatever the cracks can be started to form, and then, when the cracks are there or when there is a permanent distortion is there, the stress concentrations are there, and whatever the theory which we have applied, it is at valid.

So, that is what you see here also what we are doing here? All the materials within the shaft it has to work with the lower stress limit, so that once you remove the load, it comes to its original way, and there is no stress formations are there within the element whenever the unloading is there, and it is not being used for the full capacity.

So, now you see, thus in these cases where the weight reduction is an important, because weight is also a key feature in that it is advantageous to use a hollow shaft. So, hollow shaft has its own unique feature and in discussing the torsion of the hollow shaft, the same assumptions will be made as in case of the solid shafts. So, you see here also as far as the hollow shaft is concerned, we are again putting the same assumptions, and just I want to, you know like, repeat those assumptions so that actually it is just refreshing you.

That, you know like, for solid shaft, we simply assume, the biggest assumption that actually our material is homogeneous and also isotropic. So that whatever the elastic properties are of the material, it is uniform and it is uniform in all the direction; it is showing, it is exhibiting all these unique properties in all three directions. So, one is that.

And other assumption was that actually it is simply following the Hooke’s law; that means, whatever the distortion or the deformation is there under the action of shearing stresses, it is elastic region. So once you remove this torque or once, you see, remove the load applied, the body comes to its original shape without any kind of permanent set of deformation. So, this was the second.

And third was, you see, whatever we are taking the cross section of the circular shaft, it is, it has to be uniform. So, in this also for this hollow shaft is concerned, we have to be very very careful that actually whatever we are taking the two different diameters, that both diameters are to be supposed to be having a uniform cross-section. There is no distortion is there that at the initial step we have bigger diameter or whatever the difference is there in the diameter, at one particular point of time it is more, at one other point of time it is less. So, this is not supposed to be there; otherwise, you see, whatever the theory which we are going to develop it is invalid.

And then you see, you know like, the another assumption was there that whatever the material is to be there, it is, there is no cracks or spares or nothing has to be there. That means, there is no stress concentrations are there on the material. The materials well, you know like, surface finishing is there. So, whenever we just want to apply the torque, you see, whenever if because if any kind of distortion is there initially or any cracks are there, and if you apply, you know like, the torque, then it is simply showing the weaker section of the material. So, it has to be uniformly polished or we can say the surface finishing or whatever you see the honing, lopping all those process are to be applied just to have a perfect surface. So, that if you apply the load, these material show a uniform distortion or we can say that whatever the displacements are there of these microstructure, the material from one point to another point, it is simply exhibiting the similar kind of nature under the application of the load. So, these all assumptions are very much valid for the hollow shaft also. So, the general torsion equations as we apply, this in case of torsion also solids are will be holding a good amount of the similar kind of nature.

Like you see, now like this torque is nothing but equals to 16. As we discussed that 16, you know like, T divided by pi d cube. So, this was there the tau maximum for this solid shaft or we can say, you know like, the T by, this T by J which is equals to tau dash by r which is equals to, you know, G theta by l. This is also, you know like, valid for the hollow shaft also as well for the solid shaft. So, now you see, you know like, you would like to again discuss all these kind of issues for hollow shaft, as we discussed for the solid shaft.

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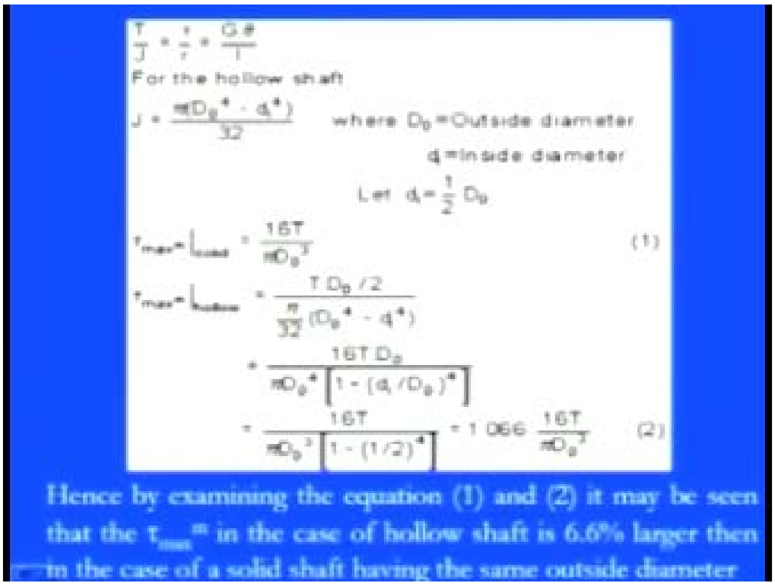

So, here, you know like, we have T by J as we discussed which is equals to Tau by r which is equals to G theta by l. So, for solid shaft what we had? We had, you know like, J G which is which was the nothing but this polar moment of inertia, which was nothing but equals to pi d 4 by 32, because d was the solid shaft diameter.

But here, you see, what we have. We have the hollow shaft. In the hollow shaft, we have two different diameters. So, you see here the difference comes first here at the section modulus of this inertia. So, here it is, J is nothing but equals to pi, D outer periphery diameters. So, outer, D outer to power 4 minus d inner to the power 4 divided by 32.

So, D, D 0 is nothing but the outside diameter. d i is nothing but, d i or we can say d 1 whatever, you see, is nothing but equals to inside diameter, and d i, you see, if we are saying that, actually if we are simply sectioning these hollow shaft exactly at the middle of the portion, that means if I have a relationship in between d i and D o as 1 by 2; that means, d i is equals to D 0 by 2 and if I am keeping that in value of J, then what I am having? I am simply having the maximum shearing stresses. And because you see, you know like, as we setup the relationships in between the shear stress and the radius that tau maximum is there always when it is proportional to R. So, you know like, we have shown actually starting from zero, origin, if I going up to the maximum radius, that means, you see, outer periphery the shear stresses are maximum. So, by taking that condition what I am having? I will be having the maximum shear stresses, if I am considering d i is equals to D 0 by 2. So, by keeping this relation here, what I am having? I am having maximum stresses. As far as a solid is concerned, we already derived that it is nothing but equals to 16 T divided by pi Do cube. So, this is for solid shaft.

Now you see what I am having here? I am having here this hollow shaft, this tau maximum for hollow is nothing but equals to T times D 0 by you see here, simply from this tau is G into theta into r by l. So, by keeping those things, or by, you know like, taking J, what we have? We have T into, you know like, this D 0 this, whatever the r is coming. So, dt into r. So, T into D 0 by 2 divided by this J; J is coming from this portion. So, pi by 32 D 0 to the power 4 minus di to the power 4.

So, by now, you know like, analyzing those things what I have? What I am having? I am having 16 T divided by, you know like, 16 T into this, you know like, this 32 will be cancel out from this 2. So, we have this 32 will be coming on the top half side. So, 16 T into D 0 divided by pi, if I am taking D 0 out. So, D 0 to power of 4 what I am having? I am having 1 minus. So, this will be 1 minus d i by D 0 to the power 4.

So, now you see what I am having, simply if I am keeping this value here, I will be having this 16 T divided by pi D 0 cube into 1 minus 1 by 2 to the power 4 or what I am having just if I am calculating those terms I will be having 1.066 into 16 T divided by pi D 0 cube. So, if you, now you see, you can simply visualize that if we have a solid shaft and if you want to calculate the maximum shear stress, the maximum shear stress is nothing but equals to 16 T divided by pi D cube 0 pi D 0 cube and if you are calculating the maximum shearing stresses for hollow shaft, then we have 1.066 into 16 T divided by pi D 0 cube.

So, now you see, if you are comparing these two equations what we have. So, we found that the maximum shear stresses which are forming in the hollow shaft is 6.6 percent larger than as compared to the solid shaft. If we are taking you see, you know like, the shearing stresses maximum at this di by D 0 is there till half. So, now, you see, now you would like to see that once you have the 6.6 percent is more shear stress is.

So, what about the weight consideration? So, how much weight reduction is there exactly if you are going for the same, you know like, the consideration of the hollow shaft. So, considering a solid and hollow shaft for the same length l and the same density rho. Only with the consideration of d i by D 0 is half, you can see that we have a solid shaft with outer diameter D 0.

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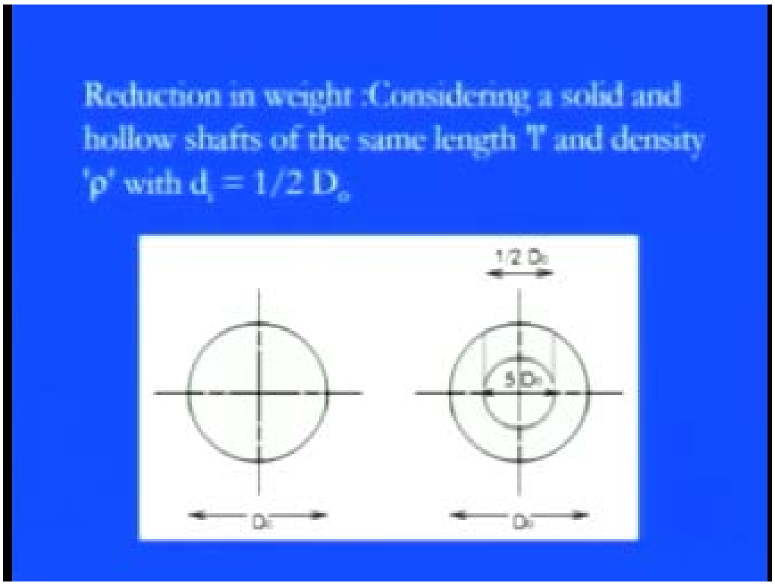

And we have the hollow shaft with this inner diameter is exactly 0.5 D 0 and outer diameter is exactly half of this one. So, means we have, you know like, this half of D 0 and outer diameter D 0, with those consideration now what will be the weight of the hollow shaft?

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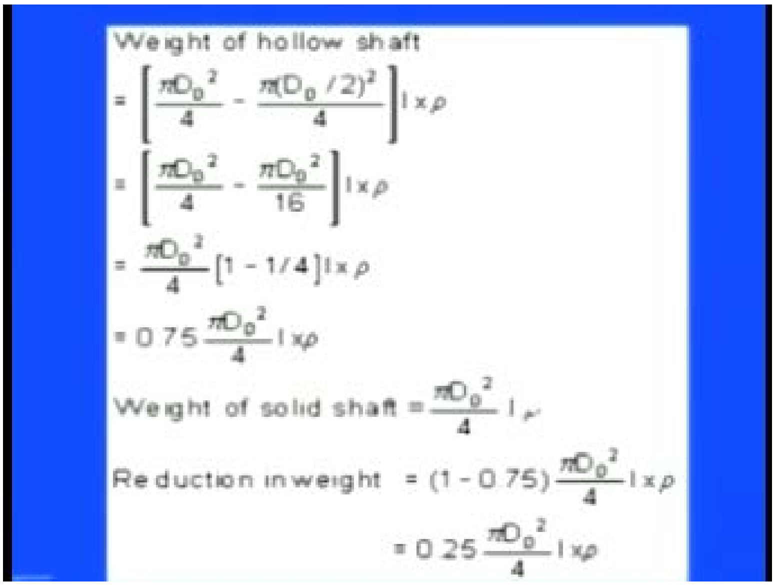

The weight of the hollow shaft is nothing but equals to pi D square 0 by 4 into l into rho. So, this is nothing but the outer surface of weight minus inner is pi d i. So, d i is nothing but D o by 2 whole square by 4 into l into rho. So, pi D square by 4 into l is the area into density will be the weight.

So, you see here with that consideration, if you are going for that, then you will be having, you know like, pi D 0 square by 4 minus pi D 0 square by 16 into a length of the shaft into the density will give you the effectiveness of the weight of hollow shaft. Or if you are doing, like you know, taking pi D 0 square pi D outer square divided by 4 take it out, then we will be having 1 minus 1 by 4, because 4 is coming out, so 1 is here and 16 is there. So, 1 by 4 will be there into l into rho.

So, if you know like, 1 minus 0.25 will be 0.75 pi D 0 square divided by 4 into l into rho. Meaning is pretty simple that actually we have the weight of hollow shaft, which is absolutely depending on that what will be the outer diameter into what is the length and what is the density? And since, you see, it depends on what kind of material which you are taking, the density will be coming as usual. So now, this is for the hollow shaft.

And if you are taking the solid shaft; so previous cases, if you are talking about the hollow shaft, we assume that the inner diameter is nothing but equals to outer diameter divided by 2, which is exactly we are taking in the middle portion of, we are taking out that portion exactly from the middle portion of the outer periphery. So, here if you are considering the weight of solid shaft it is nothing but equal to pi D 0, because we are considering that the whatever the diameter is there, that is the D 0 that pi D 0 square divided by 4 into l into rho.

So, this is the weight of solid shaft; this is the weight of the hollow shaft. So what is the reduction of the weight? That means, you see, if we are taking, you know like, the middle portion out that how much reduction in terms of the weight is. So, this is 1 minus, you know like, this minus this. So, this is exactly pi D 0 square by 4 l into rho minus 0.75 pi d 0 square by 4 into l rho. So, if you are simply minus it, then it is 1 minus 0.75 pi D 0 square by 4 into l rho. Or we have 0.25 pi D 0 square by 4 into l into rho. Meaning is pretty simple that, you know like, if you are considering about the weight, then the total 25 percent reduction of the weight is there.

If you are simply taking out, you know like, from a solid shaft the middle portion of the solid shaft is. So, total weight reduction is 25 percent, but, you know like, in terms of the shear stresses generating, because whenever the torque application is there, as I told you it is subjected to the pure state of the shear stress. So, the total shearing stresses are maximum 6.6 percentage larger in a hollow shaft as compared to solid shaft.

So, you can simply compare the situation that even you see wherever if you want to design any component, and the weight is the proper criteria, then we would like to put the hollow shaft as compared to solid shaft; but if we are talking about the shearing stresses and this the dominant parameter as compared to any weight, we say respect to weight, we are not bothering about a weight, then probably, you know like, we would go for a solid shaft region, because in solid shaft we have a less, you know, at the same torque application, we have less shear stresses while in comparison with the hollow shaft.

So, this is a quite comparison with the solid as well as the hollow shaft under the same application of torque and the same material as well as the same diameter is. So, this is a perfect comparison as compared to the solid as well as the hollow shaft. Now, we would like to apply those formulas in terms of the problem is specified.

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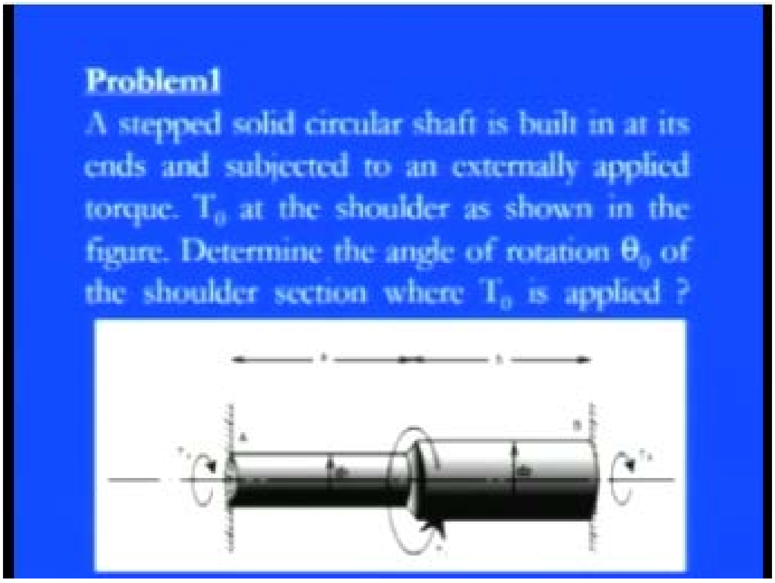

So, here you see, we have a simple problem which you can see on your screen the diagram, this diagram is nothing but, you see, we have a two different diameters are there. So, this shaft is known as the stepped solid circular shaft, which is built in its own, you know like, the ends are there; the both ends are rigidly fixed. So, this is a great condition that actually both ends, we were not allowing any end to freely rotate. As you discuss that actually one, in probably you know most of the cases, that one end is rigid, one end is free. So, you can simply, you know like, put the apply the torque and it is simply rotating, means the twisting is there, but here the constraint is pretty simple that both ends are extremely fixed.

And then, you see, if it is subjected to an external applied torque T 0 at the shoulder as shown in this particular figure. So, at this junction point now what we are doing here? We are applying a torque in counter clockwise direction as you can see here. So, now, there are two portions of that. This small portion, which has a diameter d 1 is total length of A is there, and the bigger portion which has a diameter d 2 has the total length as B.

So, you see now, when we are applying a torque at middle portion, we would like to determine that what is the total angle, angle of a rotation is there? The theta is zero, you know like, of the shoulders section where t 0 is applied. That means you see, where ever this particular shoulders action is there, the junction point is there, where we are applying a torque, we would like to see that what the theta 0; that means, what will be the angle of rotation is?

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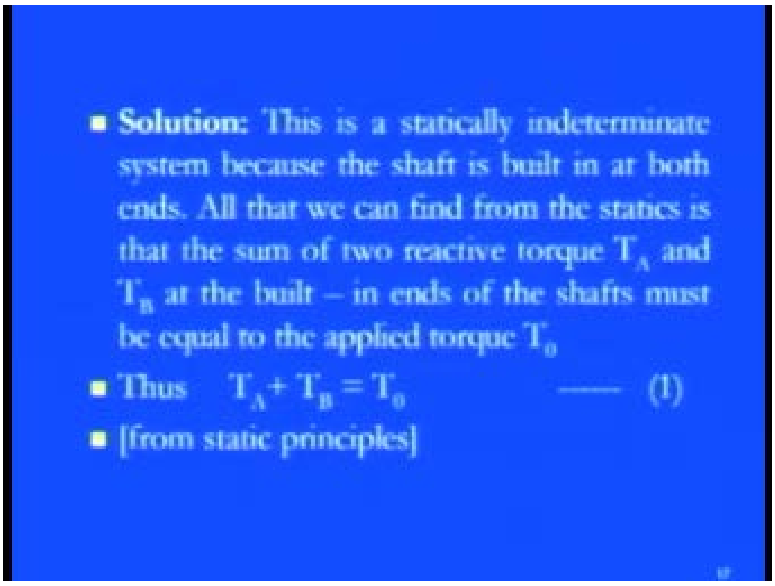

So, for that you see first we would like to discuss about what the steps to be followed. So, this action is statically indeterminate system because there are two systems are there, and both are rigidly fixed at the extreme two corners, and they are, you know like, joined with the junction, and at this particular junction we are applying a torque, and because the shaft is built in both ends, and both ends are fixed, and all that we can find from the statics is the sum of two reactive forces you see. Because as we apply the torque, there is a resistant torques are there at the two extreme corners though they are rigid, but they are tending to move in a opposite direction as compared to the T 0 applied.

So, you see, if you are simply applying… since and we are assuming that it is statically indeterminant system is there. So, these two reactive torques T A and T B at the two level string corners A and B are always built in these particular ends of the shaft. They must be equal to the applied torque; then only we can say that the system is in an equilibrium manner. So, by applying this particular statics what we can say that the T A plus T B, because both are applying in a same manner, but exactly opposite to the T 0. So, T A plus T B is equal to T 0, or we can say that T A plus T B means the total torque at the two reactive ends is minus T 0, the applied torque will be 0 or we can say that it is under the application of these torques, action and the reactions, the system is in well equilibrium manner.

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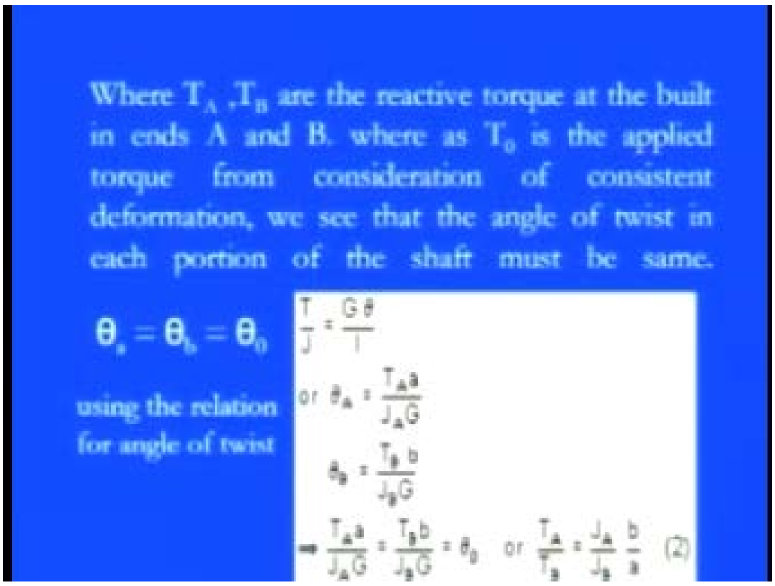

So, after you see, you know like, when T A and T B are reactive torques and they are you see, you know like, coming due to, you know like, the applied torque at these two extreme corners T B, at these two extreme corners A and B, and T 0 which we are applying at this contact point of these two shafts. So, after consideration of these, you know like, the torque what we are, you know like, we are thinking that this T 0 which is applied torque from the consideration of the consistent deformation.

We can simply see that it is, you know like, the statically indeterminate system is there. So, whatever the deformations are coming, at these, you know like, junctions as well as these two corners it has to be same; otherwise, you know like, this will not be in the, you know like, the system will not be in this particular format. So, if we are applying this condition that, you know like, the what are the twisting are there in the individual portion, they must be shown with the same value or we can say if theta A is equals to theta B is equal to theta 0, we can say that the system is well in the equilibrium manner or the statically indeterminate system is there.

So under the application of these torques. So, by considering these things what we can say that, we can say that, we can say simply that T by J which is equals to G theta by l as per the extended formula now we would like to apply this formula for different, different regions. So, first you are applying for this first action where the theta 1, where this d 1 is the diameter and A is the length, so for that, if I am saying that the angle of twist for this particular portion is theta A. So, theta A is equals to T A into A divided by J A G where T A is the applied reactive torque in the x action as we have already assumed into A is the total length divided by… because this l will be gone in that. So, T l by G J will be there as theta. So, T A into a, the l is the total length of a divided by J A, that is nothing but equals to pi d 1 to the power 4 divided by 32 into G is there.

So, G is a common because we are taking the same material for both of the portion of the shaft. So, G will be equal, and then, you see, similarly we can simply consider the angle of twist for the second region that is the b region, for which the total length is B and whatever this polar moment of inertia will be there as pi d 2 cube divided by 32. So, by keeping those things in consideration we have a theta b is equals to T B into B, which is the total length divided by J B into G.

Now, you see, we have already assumed that whatever the angle of deformation is coming in individual portion with the junction, it must be equal to retain the system is an statically indeterminate system. So, by keeping those conditions in our mind, what we can do? We can simply apply those values theta A is equals to theta B is equals to this theta 0 or we can say that TA into A divided by J A into G is equals to T B into B divided by J B into G, which is equals to theta 0 or we can say that T A by T B; that means, the ratio of the torque in these two portions because of the applied torque T 0 is equals to J A into… J A divided by A into B by J B.

So, you see here, these torque distribution in these two sections are absolutely depending on these dimensional parameter, that how much length is there and what is the diameter corresponding. Because, you see, if you are talking about J A by a, a is the length of this particular specified, you know like, the shaft, and J A is absolutely depending on this polar moment of inertia. So, what is the area is concerned? So, pi d 1 to the power of 4 divided by 32. So, you see here whatever the torque distribution is there, it has to be there on the basis of J A by a into b by J.

So, this will give you the torque distribution or we can say the ratio of the torque in these two portion of a shaft. So, you see here, we are simply assuming that modulus of rigidity G is to be same for both portion.

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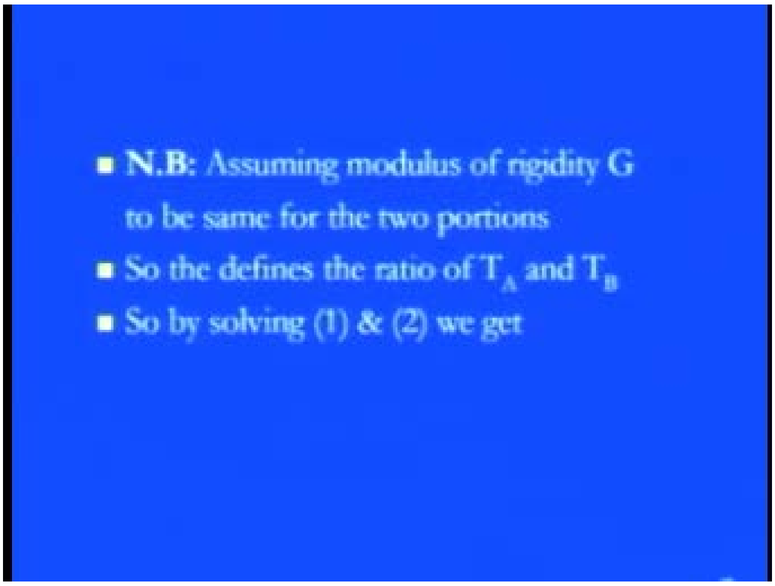

Because as usual, if you are considering the same material, it has to be same G, will be same. So, you see we need to define the ratio of T A by T B, once you know those things you see, we can simply solve those equations one and two, and then probably, we can get the theta 0. So, theta 0 is pretty easy to get, you know like, the value once you have all those parameter.

So, what we need to do here, you know like, these are the steps which you have to be taken care. You know like that what the parameters are to be given and what we need to, you know like, consider. So, after applying those conditions which we have simply discussed, they are the T A by T B is nothing but equals to, you know like, G A by J by A into B by J B. After putting those conditions, we have those values, and then if we are keeping in the first equation, you will be having theta 0, because theta 0 is nothing but equals to it is theta A equals to theta B. Or you can keep those values there in these respective formulas.

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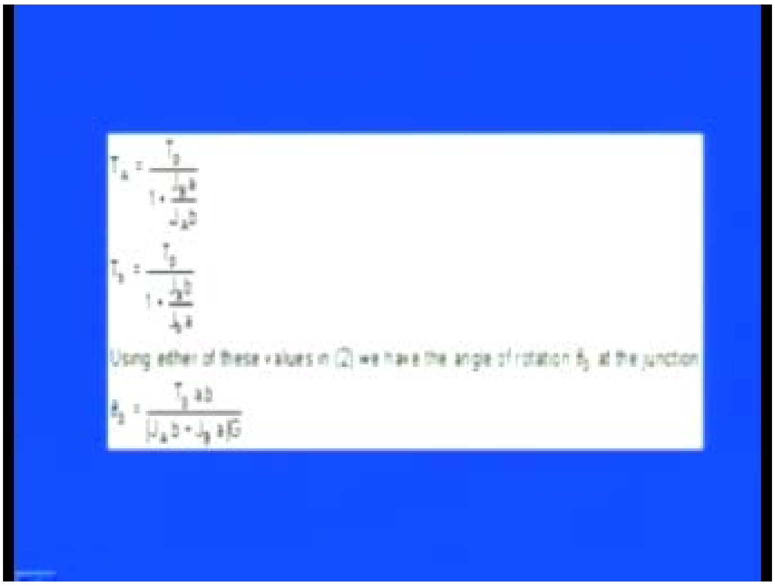

So, this is you see, you know like, these formulas again we are keeping those values in our terms. So, T A was nothing but equals to T 0 divided by 1 plus J B into a divided by J plus JA into b or we can say this T B is nothing but equal to T 0 divided by 1 plus J B J A into b divided by J B into a.

So, under these considerations, you know like, we have, you know like, these relations. So by keeping those relations, what we have we have the angle of rotation theta 0 at that particular junction where T 0 is applied. So, this is nothing but equals to theta 0 is nothing but equals to T 0 into a into b. So, a and b are nothing but the simultaneous portions of the length in these two corresponding sections divided by J A into b plus J B into a.

So, this is, you see, the total, what we can say, this shear, this moment of inertia is there plus whatever the section modulus of inertia into the corresponding length. So, this will give you the total effective areas under this load application. And you see, you know like, if you multiply with the G, then you have the total angular rotations. So, the angular rotation theta 0 at the junction will be computed by, you know like, both of the effective area section. So, what we have theta 0 is nothing but equals to T 0 into a into b divided by J A into b plus J B into a into shear modulus of rigidity.

Because we are assuming that G is constant for that material by keeping those values we can simply calculate theta 0 or one can be easily calculate T A and T B by keeping those values here. So, you see, you know like, these values can be easily computed by putting those value here J A J B, you know like, J A J B or this because you know d 1 d 2, so you can calculate J A J B. You know, A and B, so you can put those values here, and you can get this, because T 0 is applied torque. So you can get those values here easily T A and T B, and also the same time you can calculate theta b by keeping those values here.

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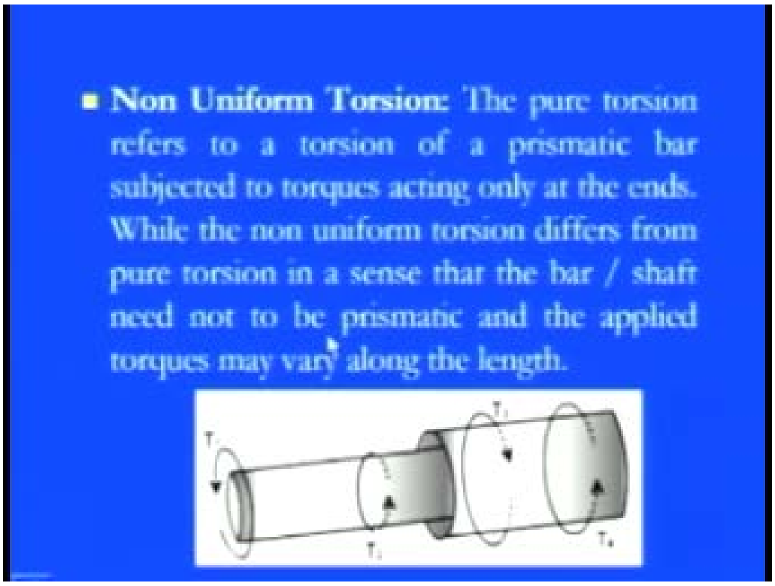

So, this was you see one like this one problem was there, only the key feature of the problem is we have a statically indeterminate system; that means, the system is rigidly fixed up at these two extreme corner. And we are applying a load at middle section and we would like to see that actually how this angular twisting is there, because you see, that is we are not allowing any shaft to freely rotate at its outer surface.

So, you see, we are putting the constraints here, at the extreme boundaries that whatever the angular rotations are there, it has to be confined under the action of this T 0. So, that is what you see, you know like, with that condition theta 0 is equals to theta is equals to theta B. And then you see, corresponding values of J, J B or A B or whatever you see, you can calculate this theta 0 at the junction point, that how much angular rotation is there if you apply the T 0.

So, now you see, in this particular portion, now we would like to see that if we have a non uniform torsion then what will happen. So, it means actually if the torsion is not uniformly distributed all across the section of this particular uniform bar, so then what will happen? So, you see here we are taking the pure torsion refers to the torsion of a prismatic bar, which we have discussed, subjected to the torques acting only at one ends, while the non uniform torques is such that it differs from the pure torsion, in the sense that the bar oblige the shaft or any, you know like, the section need not to be prismatic, and the applied torques may vary along the length; that means, you know like, it is not a prismatic bar and it is not acting by the same torque in an particular direction. Here you see we have a non uniform, you know like, the bar; that means, you see the normal prismatic bar is there and the applied torque which is, whatever the applied torques are there, they are always varying along that particular length.

So, now you see this particular figure, then you will probably, you know like, judge these kind of non uniform torsions are that we have at this particular extreme corner, we have the torque which is applying in the counter clockwise direction. And then you see, if you move further, then at a certain distance we have the different torque which is in the reverse order, and you see it is, you know like, it has a different magnitude as well it has a different direction.

And now you see, if we move further, then probably you will find the two different torques and the both are in the opposite nature, but and with the same, you know like, both are in the different direction, but with the different magnitude also. So, if we have a this kind of a specimen or a bar or a shaft, and these kinds of torques are there which are applying at a different segments and have different nature, then this kind of the total portion and the study is known as the non uniform torsions.

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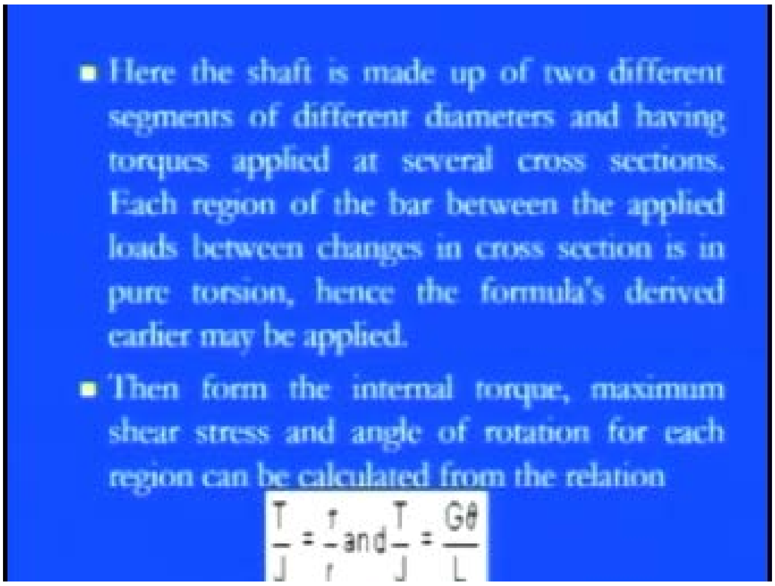

So, if you are considering those things, then you see what we need to do here, the shaft is make up of the two different segments as we discussed. The small this radius is there and bigger diameter is there, and both are having a different diameter as we shown, and having the torques applied at the several cross section such, you know like, these kind of shaft is always known as the non prismatic bars.

And each region of this non prismatic bar between the applied torques always changes in the cross section is in the pure either the torsion, hence you see we can say whatever the formula which we applied it can be easily applied here, because whatever the portions are there in the individuals sections again we are assuming that it is under, you know like, they are uniform in between these two portions. And we can say that whatever the torque applied is there, it is under the shear deformation and it is under the elastic deformation of the shear part. So, either the shear stress and shear strains are there; they are well proportional, they have the linear relationship. So, whatever the formula, which we have derived for the previous sections for a prismatic bar, it can also be, you know like, derived for the same this non prismatic bar. Thus from the, you know like, the internal torque and the maximum shear stresses and the angle of rotations for each region.

Now, you see. what we are doing. Here we are simply segregating these, you know like, the total shaft into the small, small regions where you see we are starting from one region and, you know like, we are ending where the next torque is there. And then, we are starting from that region and ending in the next torque. So, for individual torque application, and then we are trying to sum up, and also with the summation, we are also just see that actually what is the impact of these corners of the proceeding these torques. So, with the consideration of all the relative impact and the individual component we are simply measuring that what the angle of rotation is there with in those regions. And then, you see we can simply apply that these formula for individual regions like T by J is equals to tau by r or we can say that T by J is equals to G theta by L.

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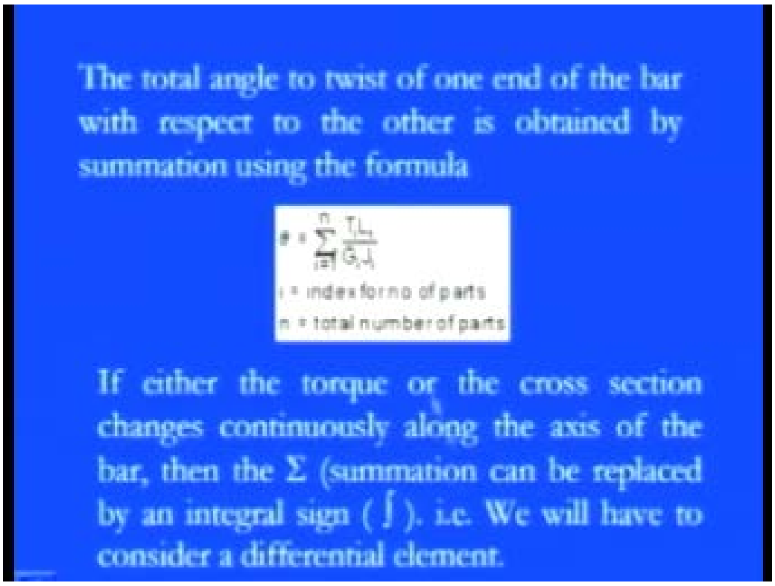

So, by considering all those things, so the total angle of twist of one end for one bar with respect to the another end is obtained by summation as I told you. So, you see, theta is nothing but equals to summation of… this is the summation term of i equals to 1 to n, where i is nothing but equals to for individual section that how many sections are there. So, it is T i L i by G i J i. So, you see, if I am talking about the first then you see the T 1 L 1 G 1 or J 1 is there; so, how much T 1 is applied, that is the very well known, because it is a applied torque if there. So, values very well known. L 1 is what is the effective length is there under the applied torque divided by the G which is the metal property.

Generally if we are taking G as a constant, then G will be constant for all those segments, and if you are even changing, if we have a composite kind of prismatic bar this non prismatic bar, then even for that actually we can apply this kind of formula and J 1 is, you know like, what will be the area of the modulus of this section is there. So, corresponding you see all we can simply compute those things, where i is the index for number of parts always that how many, you know like, whether we are considering two parts or three parts or four parts. So, will go for i is equal to 1 to 2 3 2 3 4 like that. And then, n is the total number of parts.

So, if either the torque or the cross section area are changing continuously along the axis of bar, then summation, you see, now here, we are, you know like, this summation which we are using is to be replaced by the integral sign, because, you see, if it is a uniformly well varying a bar then what we are doing here instead of taking the different, different segments it is linearly varying section. So, instead of taking summation bar, we can simply use the integration right from 0 to the extreme end. So, you see, we will be having, you know like, to consider the different elements in that part, and since you see as we have done in previous cases, take a small segment of that and then sum up that sections.

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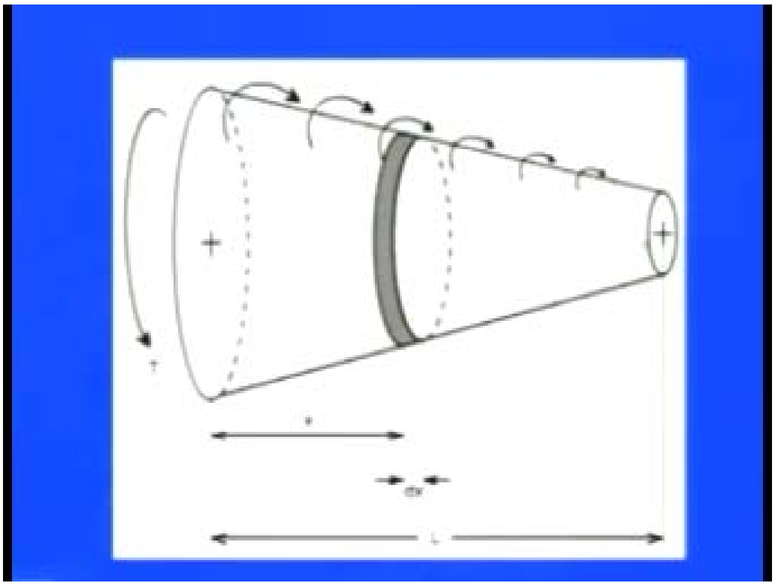

So, you see here if we have this kind of region where, the uniform cross sectional areas are there, it is a J, you know like, the small diameter and it is a bigger diameter, and you see this area is linearly varying from this end to that end, and it has a length of L.

What we need to do here, you know like, here this torque is there, you see, on the outer periphery we have the kind of torque applied. So, you need to take the small section, just cut the portion, take this small section, because it is in the x direction. So, the thickness of the section is d x and it has distance from the x, from this particular left end corner, which we have discussed in many cases. So, now you see, what we need to do here we need to simply, you know like, check the deformation for this region. Once you have the deformation theta for this region, because for this region you have this, you know like, whatever the torque applied T, you have you see what is the total length, whatever the length is there d x divided by what you have. You have J 1 and you see, you have the relative component.

So, once you have those J G l or whatever like that, you can simply compute the deformation for this region, then put the integration sign, because this is for dx region. Once you integrate for entire length, from you see, if I am saying that from 0 to L. So, by keeping 0 to L integration sign, you have the total deformation or the total this stresses along this particular uniformly varying cross sectional area.

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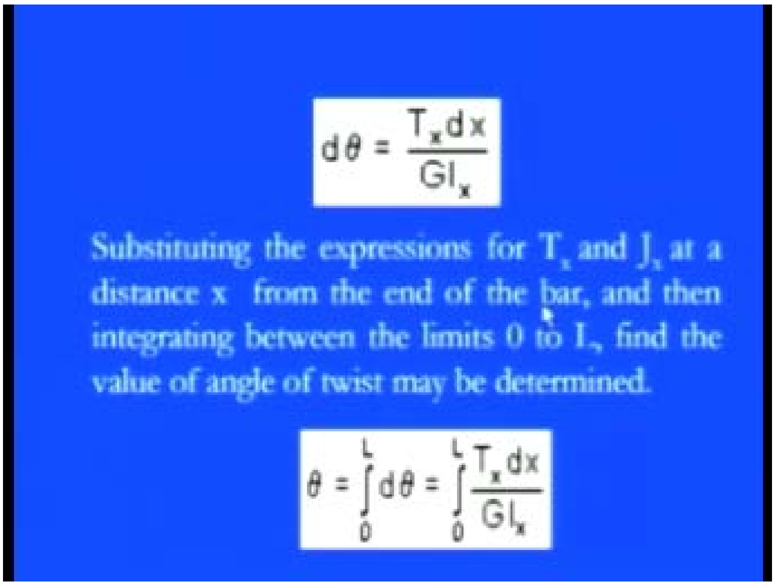

So, now you see what you have? You have the deformation for a small section as I told you d theta, for that small dx section, what you have you have the T x, which is the torque applied for this particular region into dx divided by G l x.

So, this is a specified region. So, for that you can simply compute the angular of rotation for the… or you can angle of twist for that, and now you see, substituting this expression for T x and, you know like, the J x for this at a particular distance x from the end of the bar. Then what you need to do? You need to simply integrate between 0 to L, as I told you, because it is varying from 0 where you see the bigger section is there, the bigger portion is there and it is going up to the L, where the smaller portion is there, linearly varying areas are there. So, what you need to do? You need to consider the total angle of twist for that, which is theta is equals to 0 to L d theta, and you see, simply if I am keeping d theta here, what I am having at the end? I am having 0 to L T x into dx by G into l x. That means pretty simple.

The total angle of twist is simply sum up of the individual segment deformation. So, you see, if I am saying that this is response for this much, you know like, the total twisting, so it has to be uniformly distributed. So, as I told you, see if I am talking about the uniformly distributed, you know like, the cross sectional bar or even, you see, if we have the non prismatic bar, simply we are applying the same theory as we applied for a prismatic uniformly cross sectional bar. So, with that consideration, you see, we can simply compute that what is the total angle of twist is there for this kind of uniformly tapered sectional bar is.

So, you see here in this particular section, we discussed many things about that what will be happen, you know like, what will be, you know like, happening when you apply a torque and we have a prismatic bar is there, then how we can relate; and if we have a solid shaft and if we have a hollow shaft, then what exactly the relations are there, in terms of the shearing part and we found that the shear stresses are always 6.6 percentage larger than a solid shaft if you are comparing on the same diametrical basis, the same material.

And also you see, in this chapter we discuss that if there are, you know like, the weights are the two different weights are there, then two different, you know like, the solid and hollow shafts are there, and if you are simply reducing the hollows at exactly at the middle portion, then the total weight reduction is of 25 percent from solid shaft. So, you see that if, if somebody wants to design, you know like, the shaft as on basis of this weight, then probably the one is going to go for, you know like, this hollow shaft and is somebody wants to design on the basis of their stress criteria, then solids shaft will be considerable on the basis of hollow shaft.

And then also, you see, if you have a stepped bar then how we can go for, you know like, considering with the theta 0 or theta A or theta B in the numerical problem we discussed.

And in the last portion, you see, if we have a non prismatic bar, then how we can go, how we can put the different, different segment, and how we can calculate theta for small, small segment, and then how we can sum up - this part we discussed.

And then if we have non prismatic, but if we have a prismatic bar, but in that linearly cross sectioning, you know like, the bar then, how we can go with the using of not summation, integration, this theta angle of twist, you know in the last, you know like, slide which we discussed. So, you see here, in this portion we discussed lot many things about the circular shaft irrespective of their cross sectional area or you see, we can say irrespective of their material properties also.

So, you see here this was, you know like, up to the solid shaft, now we are going to discuss in our next lecture, that what will happen if we apply a similar kind of torque to a spring, because, you see, once the torque is there on the spring, because you see once the torque is there on the spring then how, you know like, its material property or the number of coils or even you see the stiffness will vary, and how they are impacting on the deformation side or angle of twist.

Thank you.